MIT CSAIL

6.869: Advances in Computer Vision

MIT COMPUTER VISION

Lecture 5

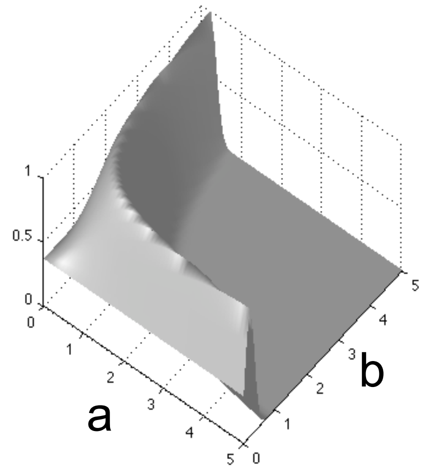
Statistical Image Models

Bayesian approach

Use P(a, b | y = 1) = k P(y=1|a, b) P(a, b)

Likelihood function

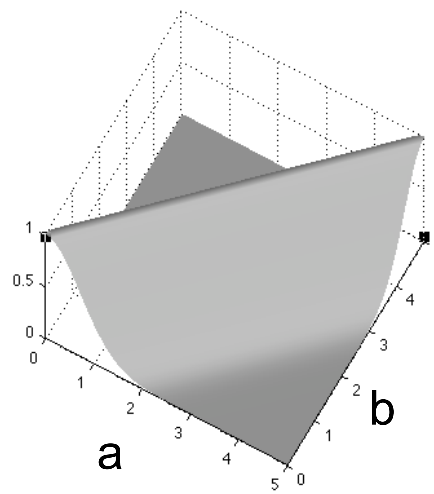
$$P(y=1 \mid a,b) = ke^{-\frac{(1-ab)^2}{2\sigma^2}}$$

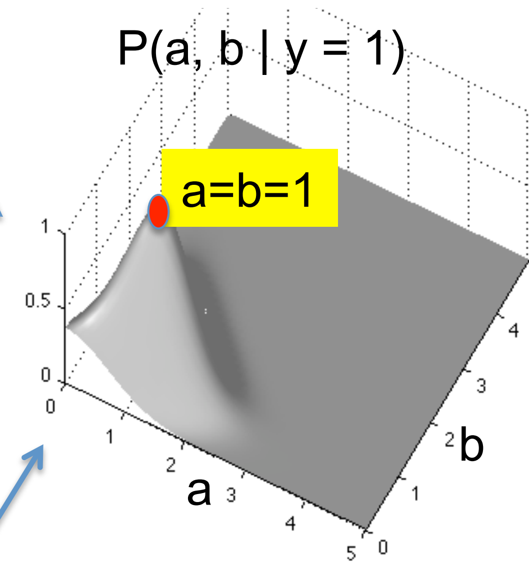


Prior probability

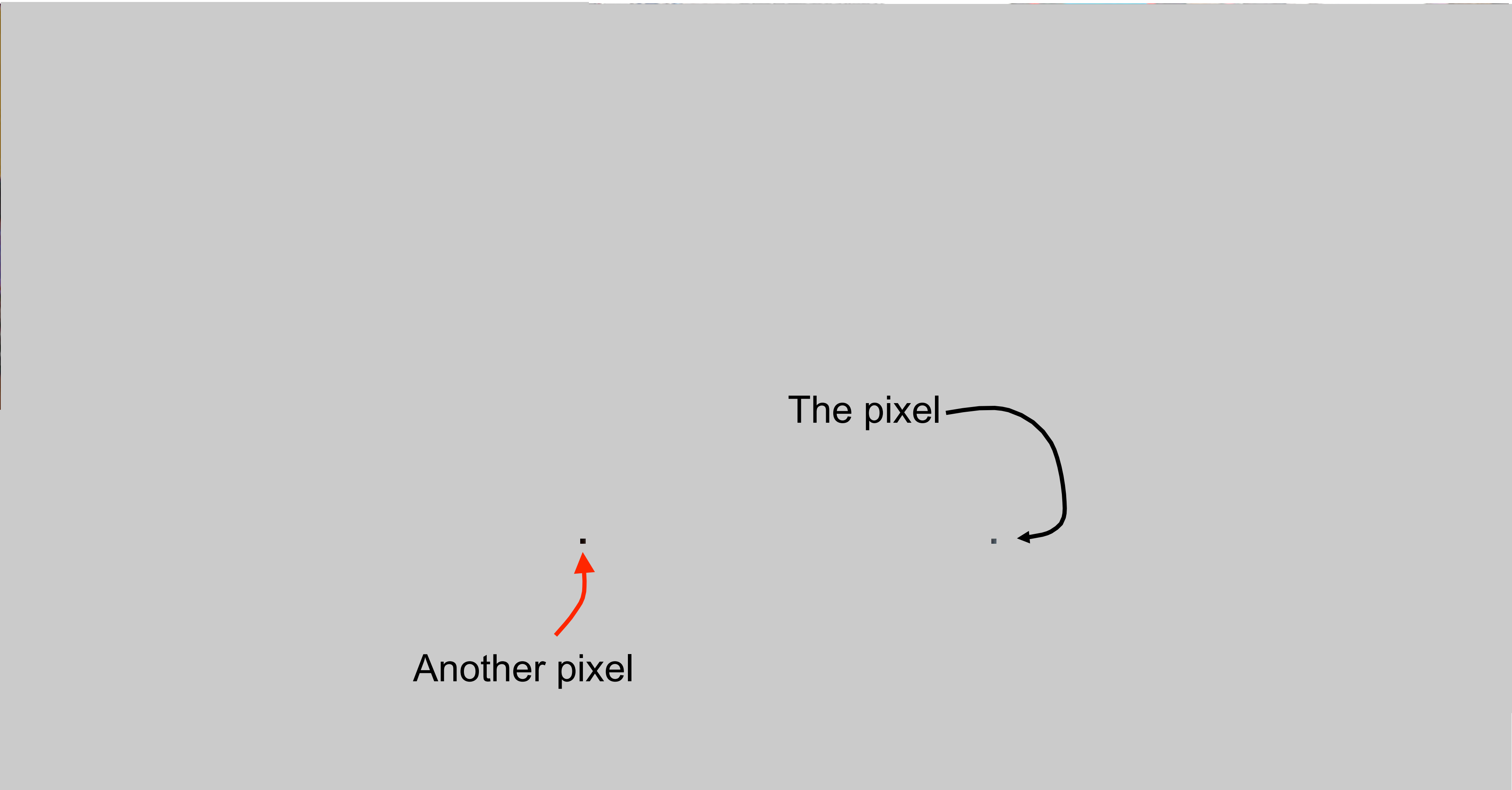
$$P(a,b) = ke^{-\frac{(a-b)^2}{2\sigma^2}}$$ If a>0, b>0

= 0 otherwise





Statistical modeling of images



$$C(\Delta x, \Delta y) = \rho\left[\mathbf{I}(x + \Delta x, y + \Delta y), \mathbf{I}(x, y)\right]$$

Gaussian model

We want a distribution that captures the correlation structure typical of natural images.

Let **C** be the covariance matrix of the imag

$$p(\mathbf{I}) = \exp\left(-\frac{1}{2}\mathbf{I}^T\mathbf{C}^{-1}\mathbf{I}\right) \qquad C = \begin{bmatrix} c_0 & c_1 & c_2 & & \cdots & c_{n-1} \\ c_{n-1} & c_0 & c_1 & c_2 & & \vdots \\ & c_{n-1} & c_0 & c_1 & \ddots & \\ \vdots & \ddots & \ddots & \ddots & & c_2 \\ & & & & & c_1 \\ c_1 & \cdots & & c_{n-1} & & c_0 \end{bmatrix}$$

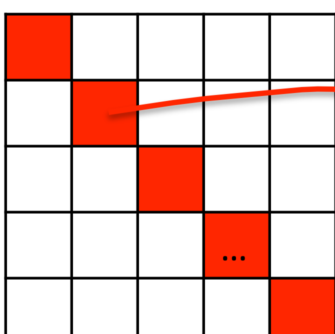
Stationarity assumption: Symmetrical circulant matrix

Diagonalization of circulant matrices: $C = EDE^T$

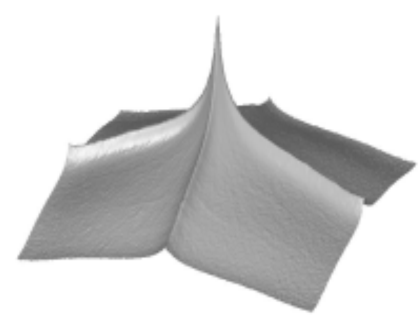
The eigenvectors are the Fourier basis

The eigenvalues are the squared magnitude of the Fourier coefficients

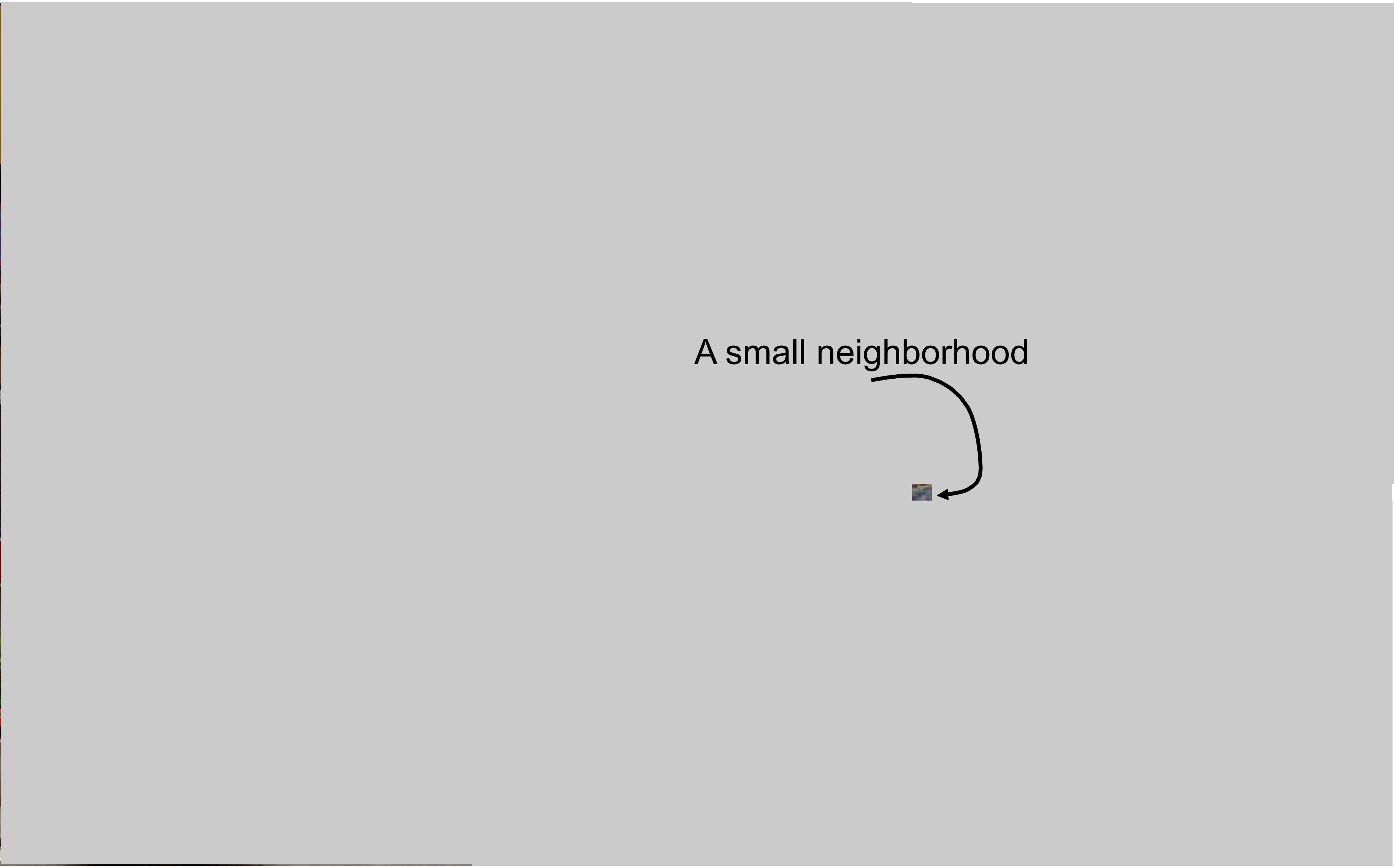
D=



$$|\hat{\mathbf{I}}(v)|^2 \simeq \frac{1}{|v|^{2\alpha}}$$



Statistical modeling of images



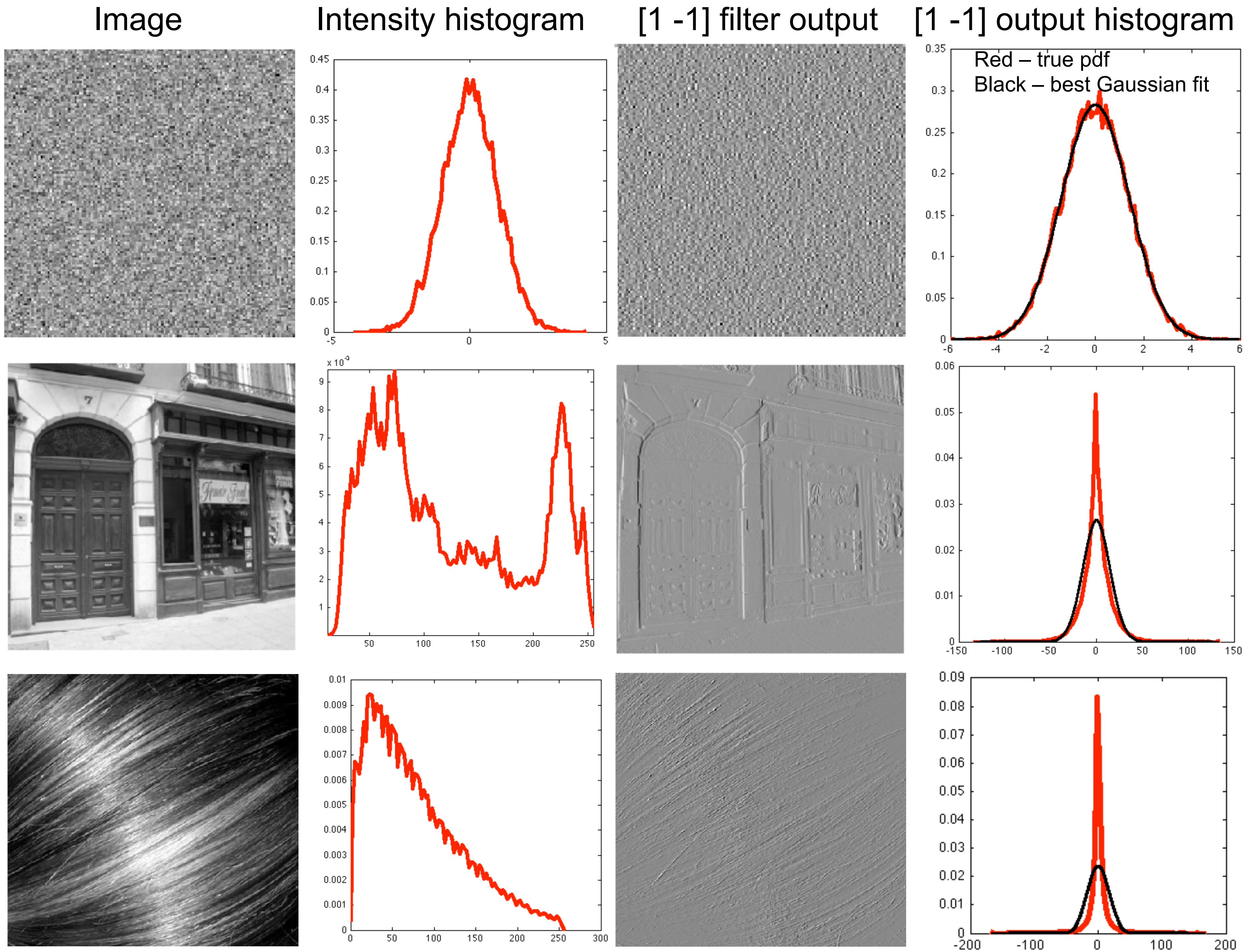
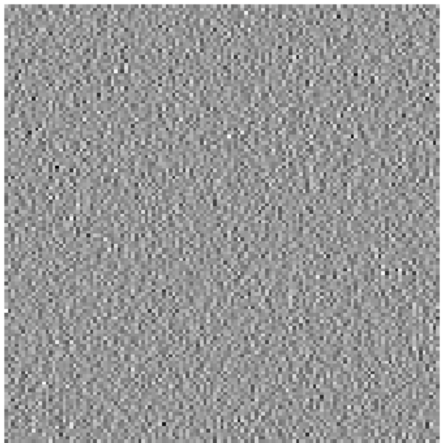
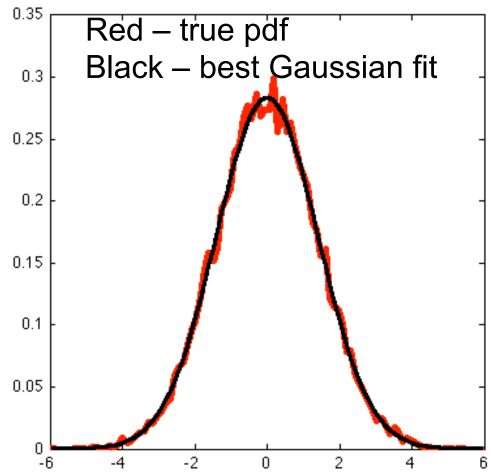


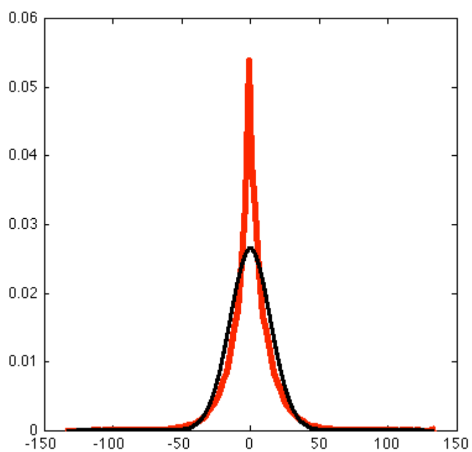
Image
Intensity histogram
[1 -1] filter output
[1 -1] output histogram
Red – true pdf
Black – best Gaussian fit

A model for the distribution of filter outputs





$$p(x) = \frac{\exp(-x^2/2\sigma^2)}{\sqrt{2\pi\sigma^2}}$$



$$p(x) = \frac{\exp(-|x/s|^r)}{2s/r\Gamma(1/r)}$$

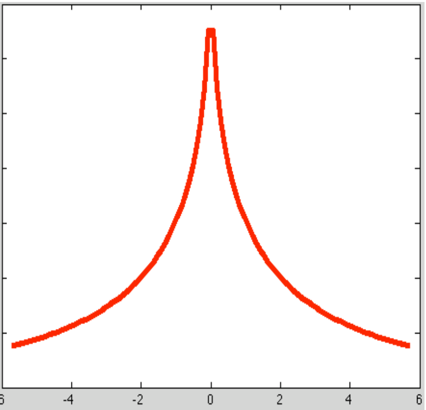
$r \sim 0.8 \ \ (< 2)$

Note: this is not a good model for ALL filter outputs

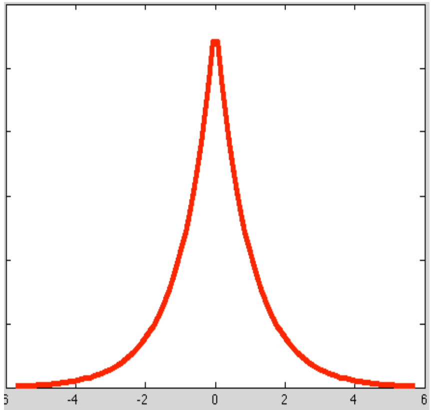
Generalized Gaussian

$$p(x) = \frac{\exp(-|x/s|^r)}{2s/r\Gamma(1/r)}$$

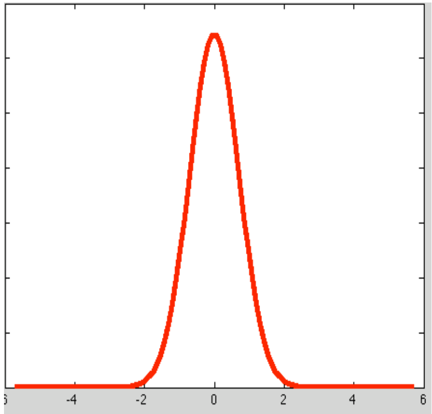
r = 0.5



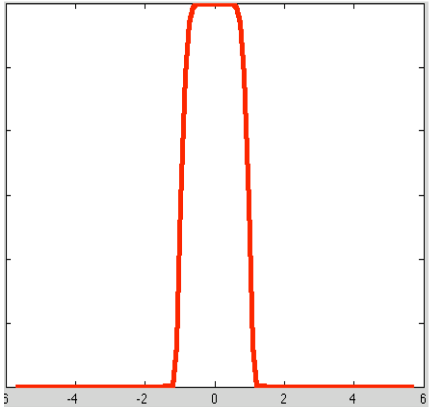
r = 1
Laplacian distribution



r = 2
Gaussian distribution



r = 10



Uniform distribution
r -> infinite

The wavelet marginal model

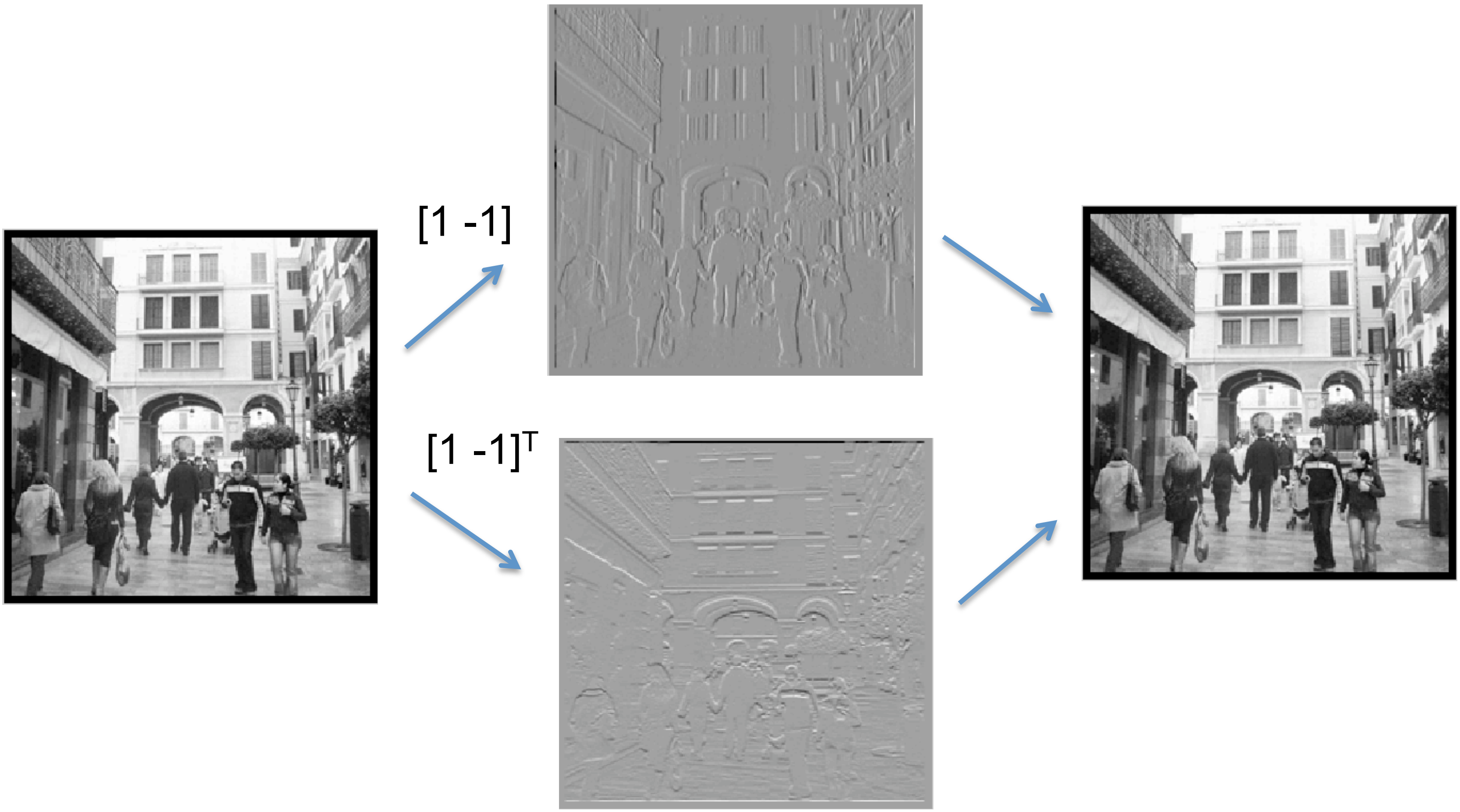
A small neighborhood

$$p(\mathbf{I}) = \prod_{k} \prod_{x,y} \mathrm{p}(\mathrm{h}_k(x,y))$$

All pixels and all outputs are independent

Filter outputs

The wavelet marginal model

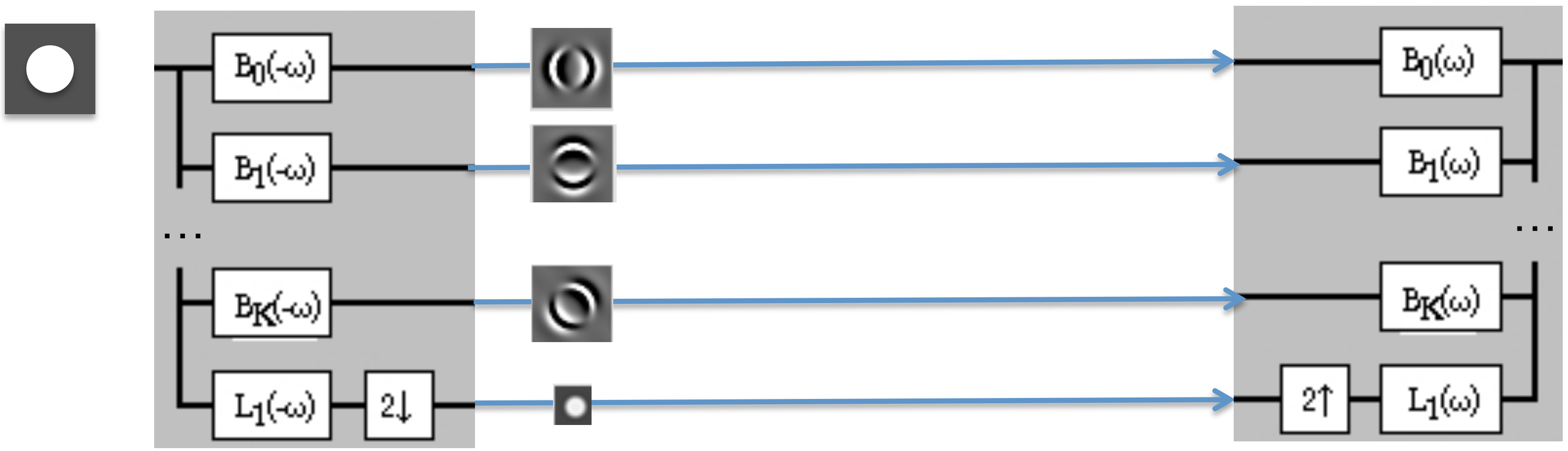


$$p(\mathbf{I}) = \prod_{k} \prod_{x,y} p(h_k(x,y))$$

Steerable Pyramid

We may combine Steerability with Pyramids to get a Steerable Laplacian Pyramid as shown below

Decomposition **Reconstruction**

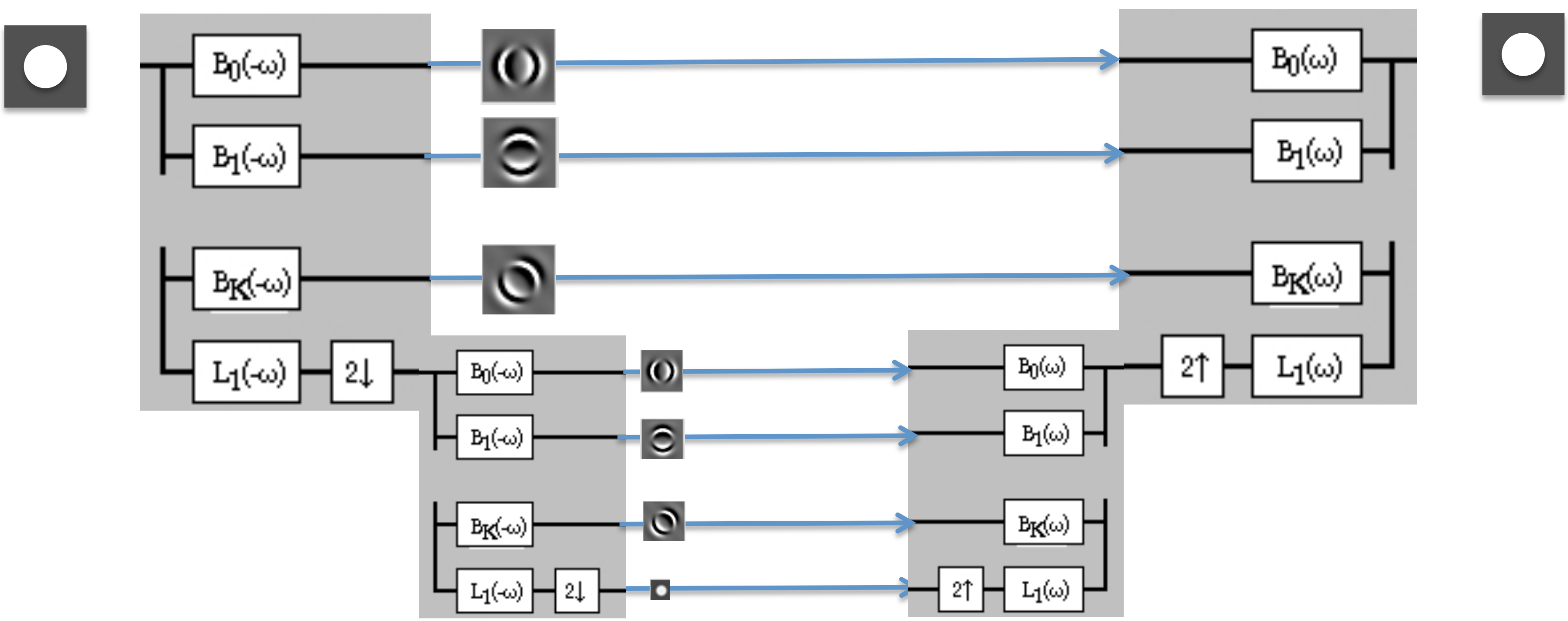


Images from: http://www.cis.upenn.edu/~eero/steerpyr.html

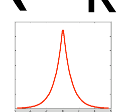
Steerable Pyramid

We may combine Steerability with Pyramids to get a Steerable Laplacian Pyramid as shown below

Decomposition **Reconstruction**



$$p(\mathbf{I}) = \prod_{k} \prod_{x,y} p(h_k(x,y))$$



Sampling images

Gaussian model

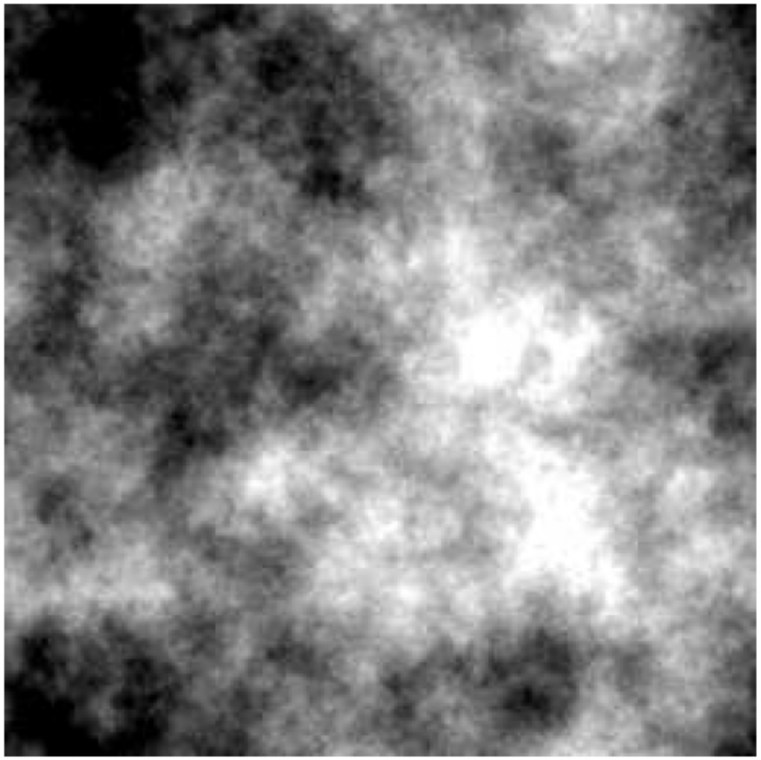


Fig. 3. Example image randomly drawn from the Gaussian spectral model, with $\gamma = 2.0$.

Wavelet marginal model

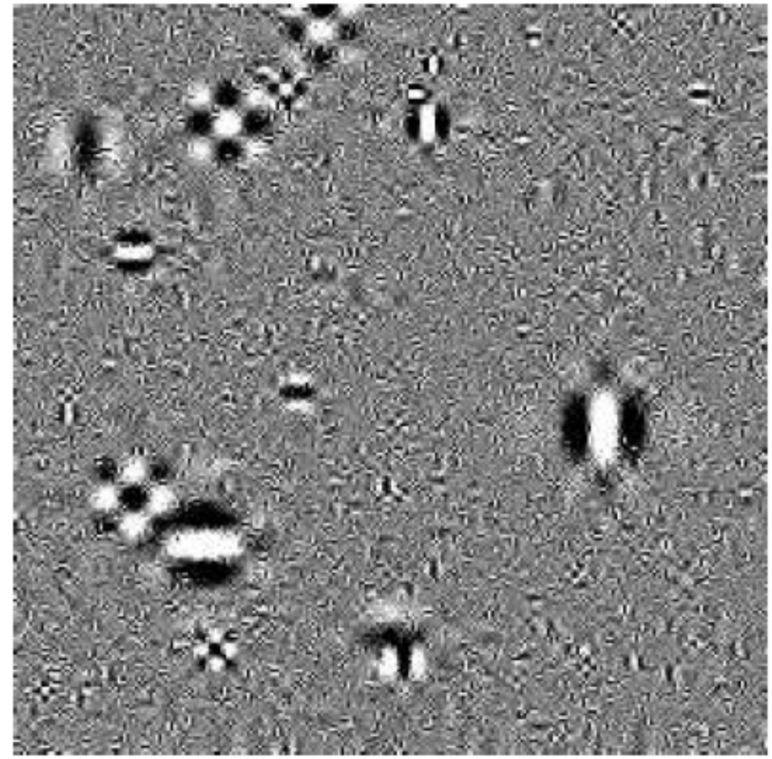


Fig. 6. A sample image drawn from the wavelet marginal model, with subband density parameters chosen to fit the image of Fig. 7.

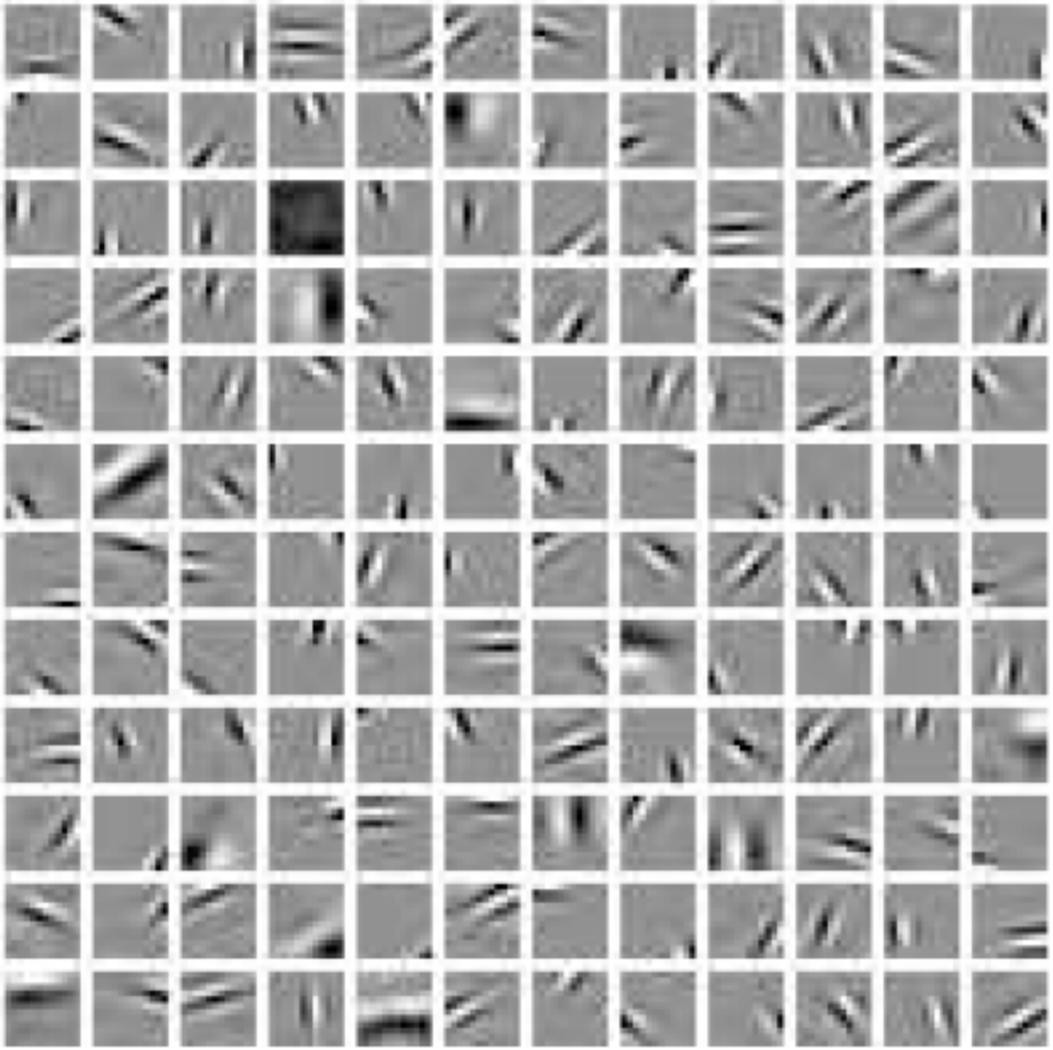
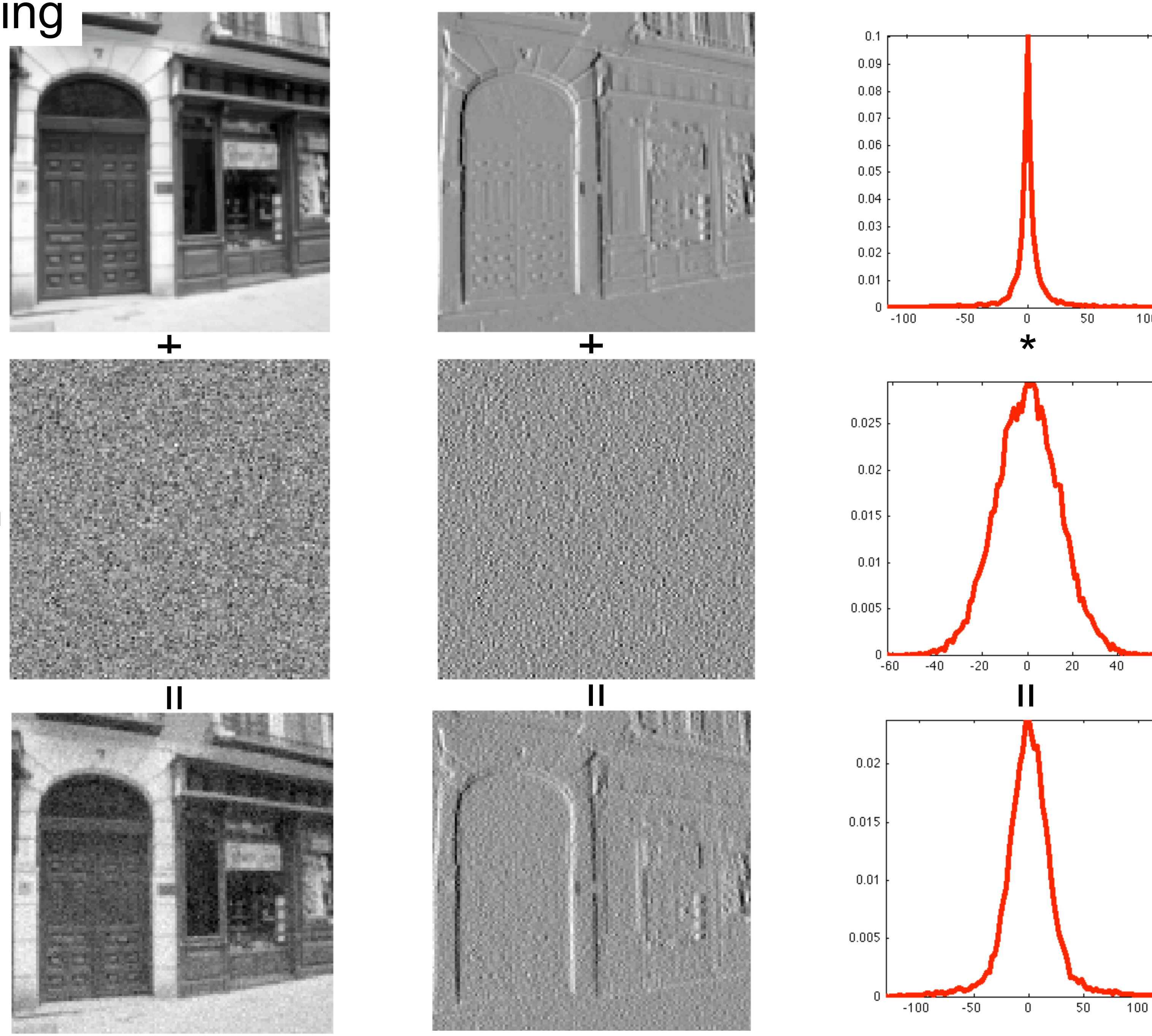


Fig. 5. Example basis functions derived by optimizing a marginal kurtosis criterion [see 35].

Denoising

White Gaussian noise

Noisy image

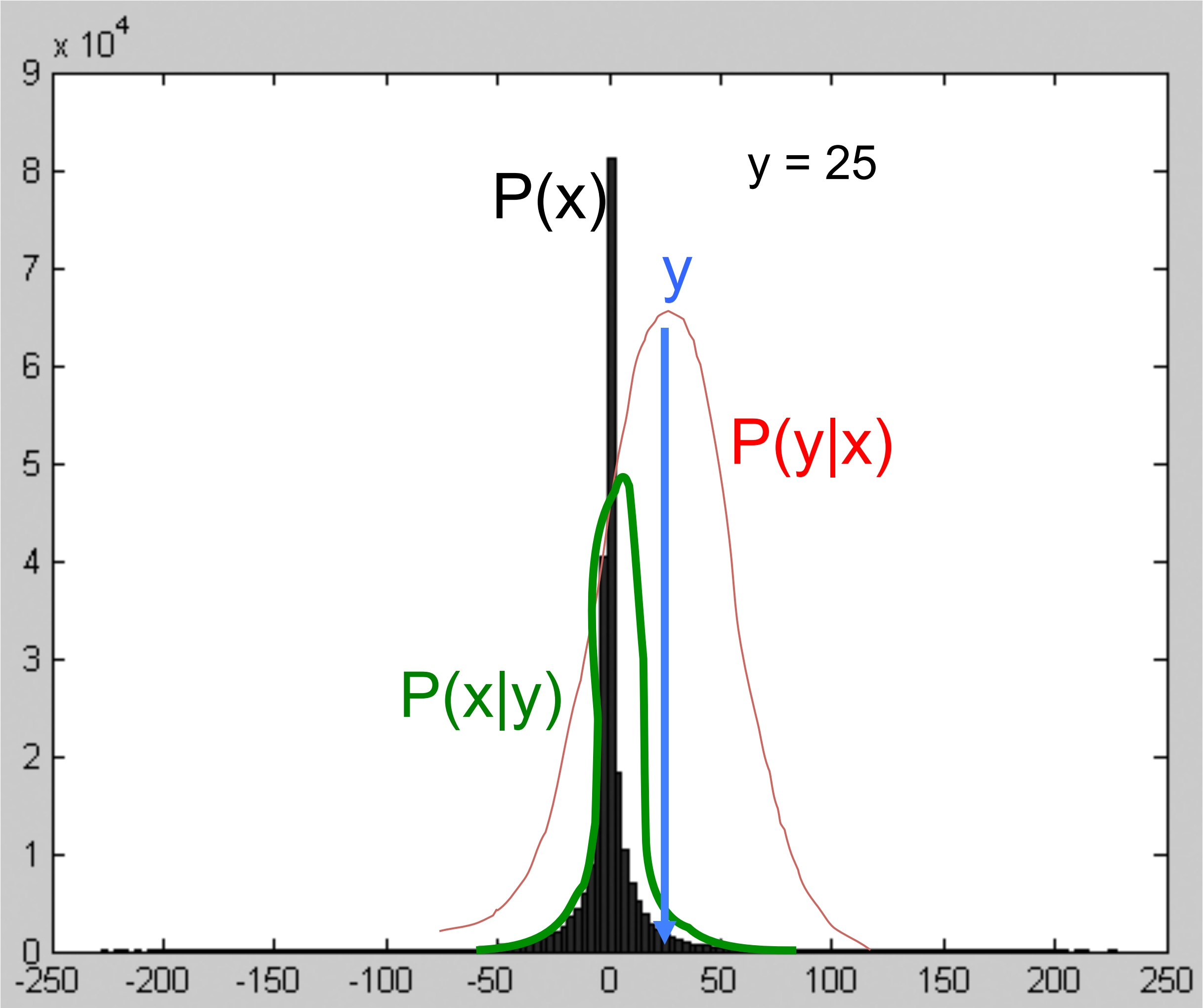


Denoising with the marginal wavelet model

Let x = bandpassed image value before adding noise.
Let y = noise-corrupted observation.

By Bayes theorem

P(x|y) ~ P(y|x) P(x)



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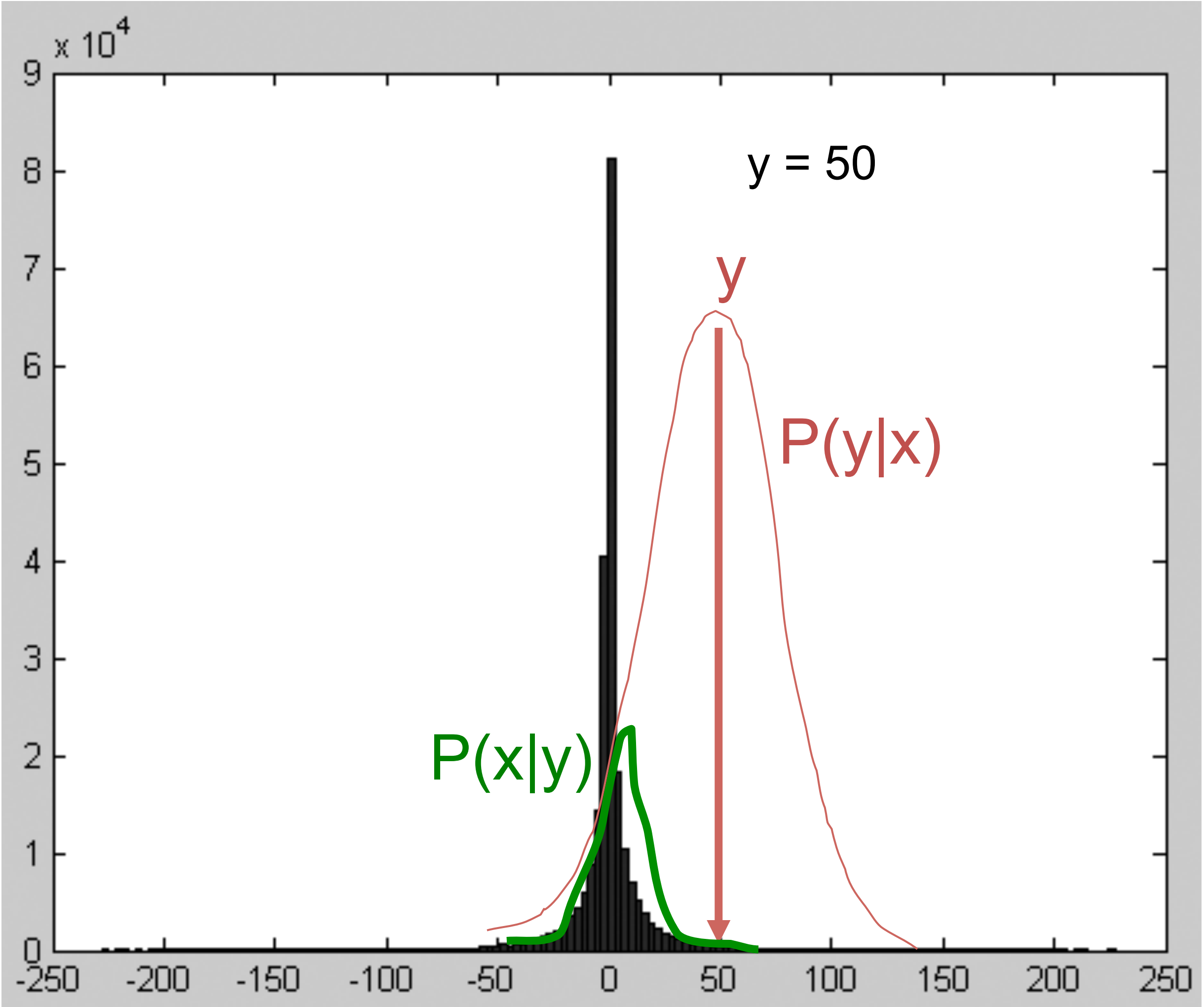

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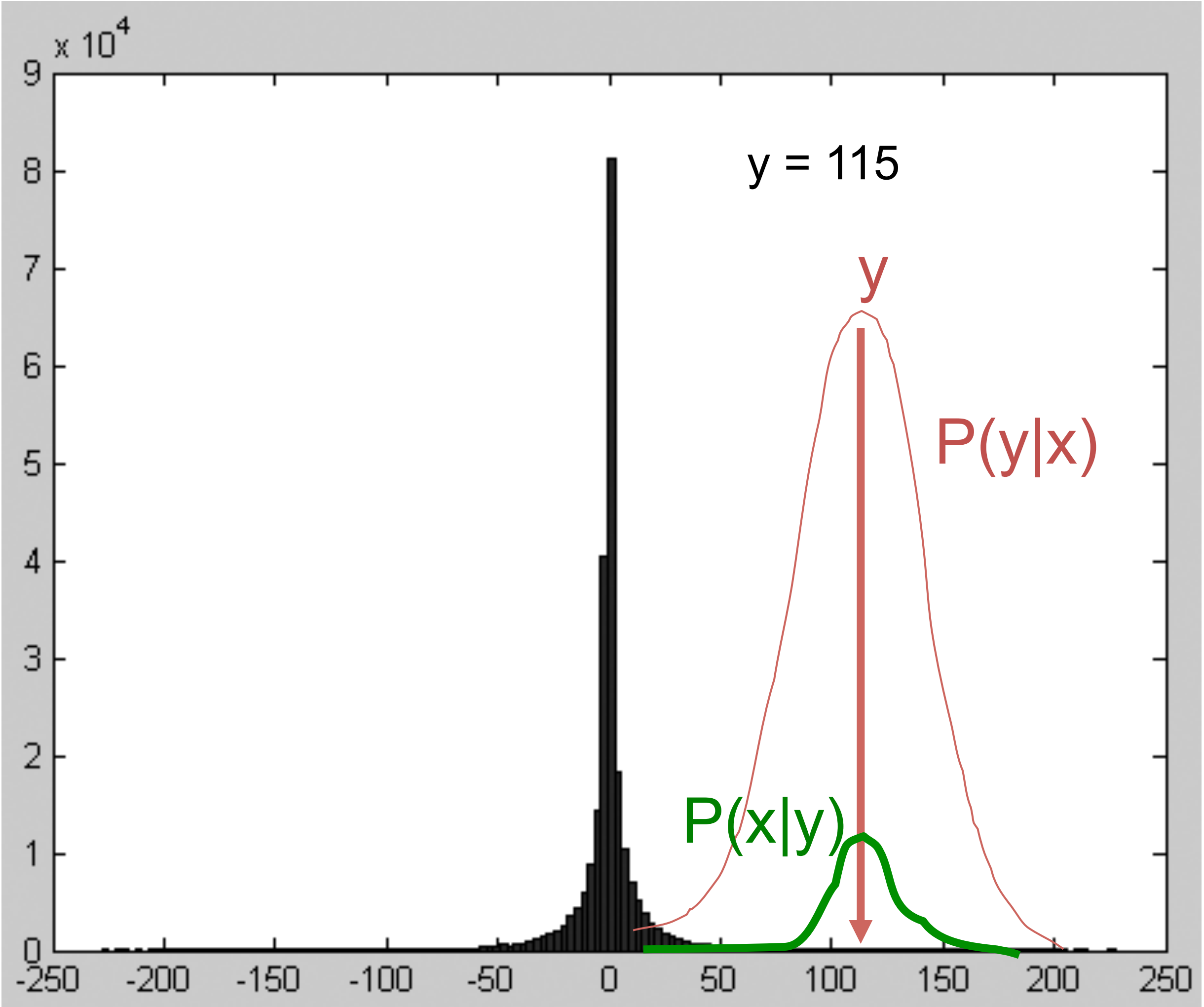
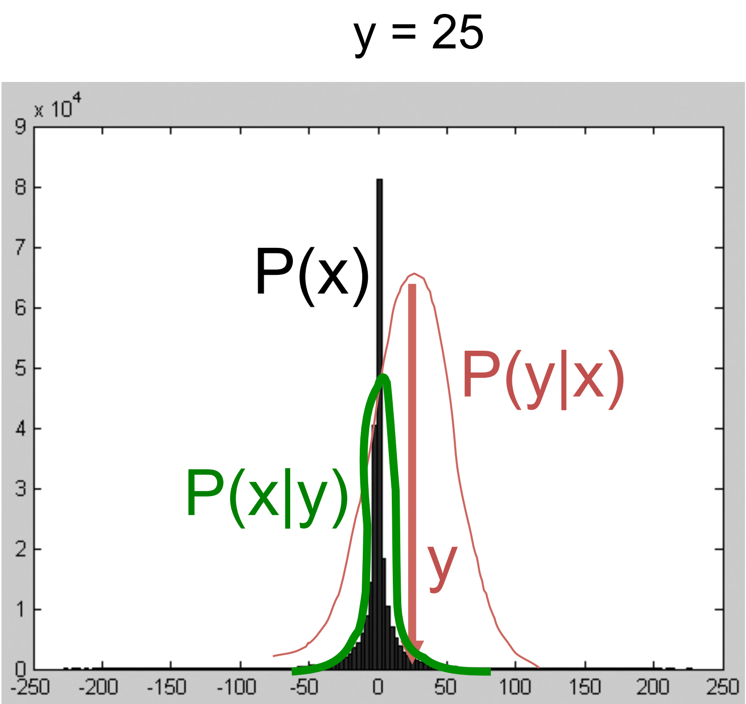
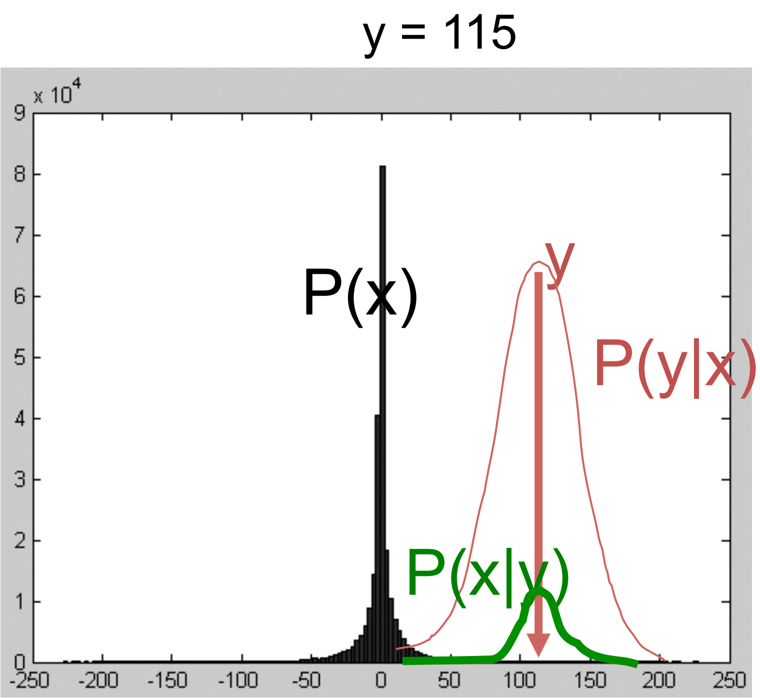


Denoising with the marginal wavelet model





For small y: probably it is due to noise and y should be set to 0
For large y: probably it is due to an image edge and it should be kept untouched

MAP estimate, $\hat{x}$, as function of observed coefficient value, y

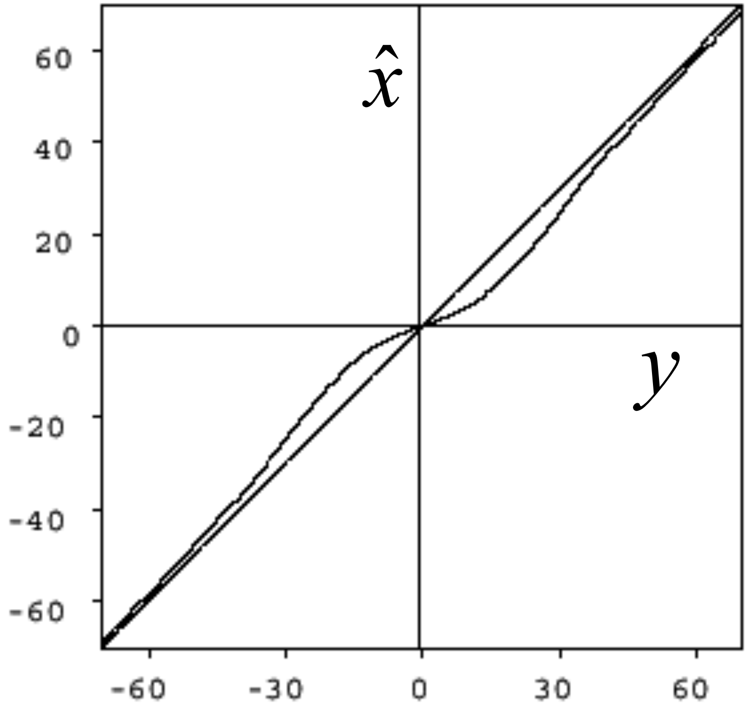
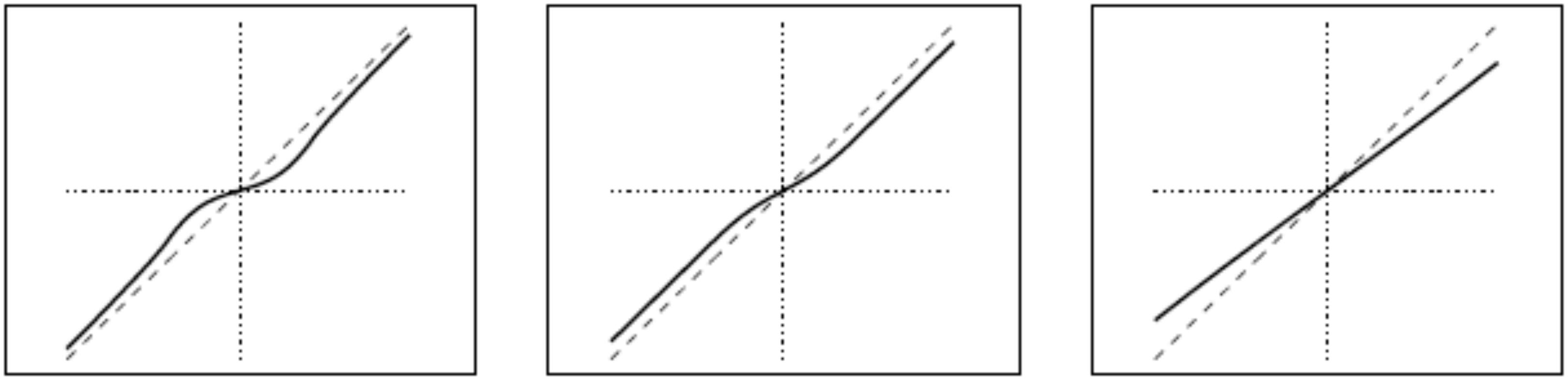


Figure 2: Bayesian estimator (symmetrized) for the signal and noise histograms shown in figure 1. Superimposed on the plot is a straight line indicating the identity function.

http://www-bcs.mit.edu/people/adelson/pub_pdfs/simoncelli_noise.pdf

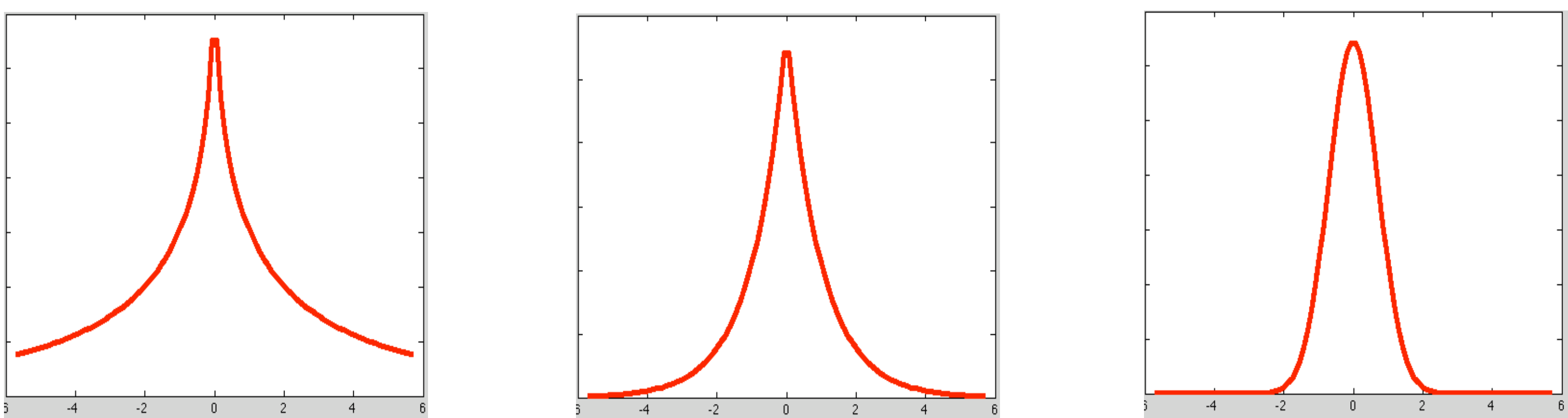
Simoncelli and Adelson, Noise Removal via Bayesian Wavelet Coring



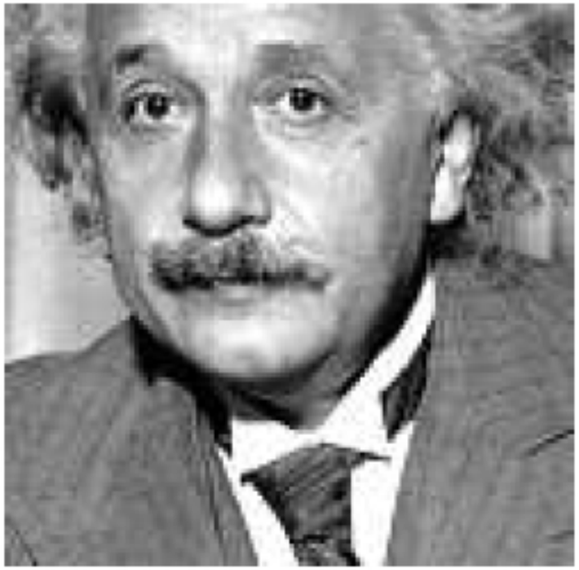
r = 0.5

r = 1
Laplacian distribution

r = 2
Gaussian distribution

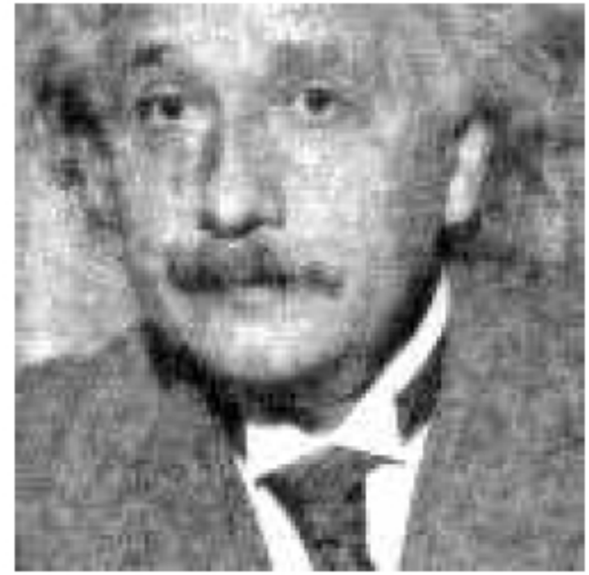


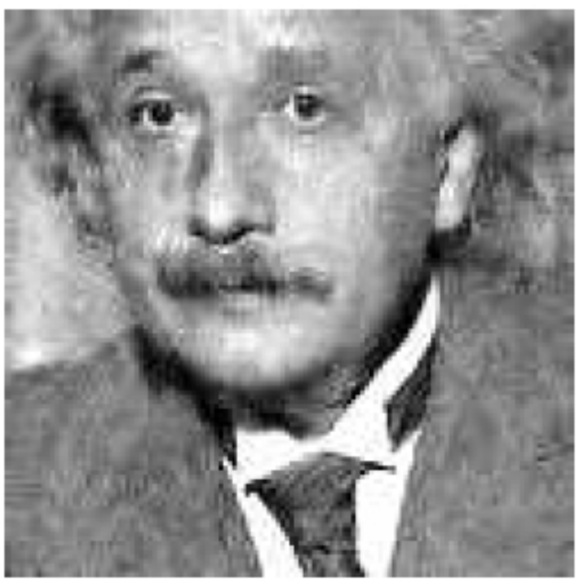
original



With Gaussian noise of std. dev. 21.4 added, giving PSNR=22.06

(1) Denoised with Gaussian model, PSNR=27.87





(2) Denoised with wavelet marginal model, PSNR=29.24

http://www.cns.nyu.edu/pub/eero/simoncelli05a-preprint.pdf

Gaussian scale mixtures

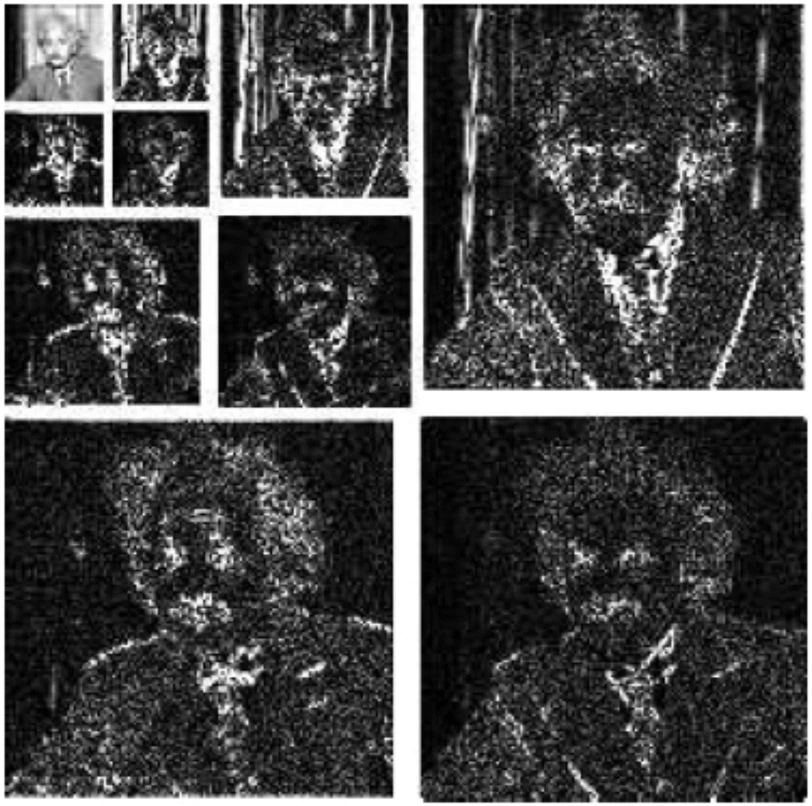


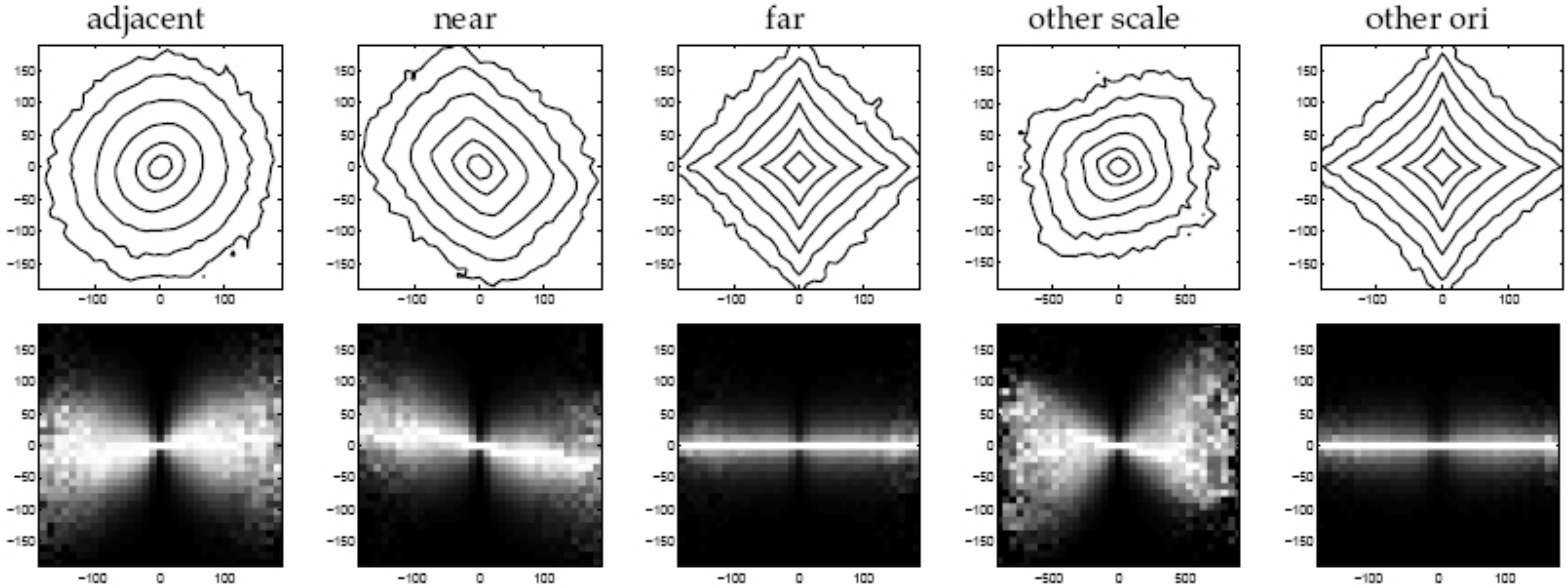
Fig. 7. Amplitudes of multi-scale wavelet coefficients for the "Einstein" image. Each subimage shows coefficient amplitudes of a subband obtained by convolution with a filter of a different scale and orientation, and subsampled by an appropriate factor. Coefficients that are spatially near each other within a band tend to have similar amplitudes. In addition, coefficients at different orientations or scales but in nearby (relative) spatial positions tend to have similar amplitudes.

Note correlations between the amplitudes of each wavelet subband.

http://www.cns.nyu.edu/pub/eero/simoncelli05a-preprint.pdf

Statistics of pairs of wavelet coefficients

Contour plots of the joint histogram of various wavelet coefficient pairs



Conditional distributions of the corresponding wavelet pairs

Fig. 8. Empirical joint distributions of wavelet coefficients associated with different pairs of basis functions, for a single image of a New York City street scene (see Fig. 1 for image description). The top row shows joint distributions as contour plots, with lines drawn at equal intervals of log probability. The three leftmost examples correspond to pairs of basis functions at the same scale and orientation, but separated by different spatial offsets. The next corresponds to a pair at adjacent scales (but the same orientation, and nearly the same position), and the rightmost corresponds to a pair at orthogonal orientations (but the same scale and nearly the same position). The bottom row shows corresponding conditional distributions: brightness corresponds to frequency of occurance, except that each column has been independently rescaled to fill the full range of intensities.

http://www.cns.nyu.edu/pub/eero/simoncelli05a-preprint.pdf

Gaussian scale mixtures

$$P(\vec{x}) = \int \frac{\exp(-\frac{1}{2}\vec{x}^T (z\Lambda)^{-1} \vec{x})}{(2\pi)^{N/2} \, | z\Lambda |^{1/2}} P_z(z)\, dz$$

Wavelet coefficient probability

A mixture of Gaussians of scaled covariances

observed

Gaussian scale mixture model simulation

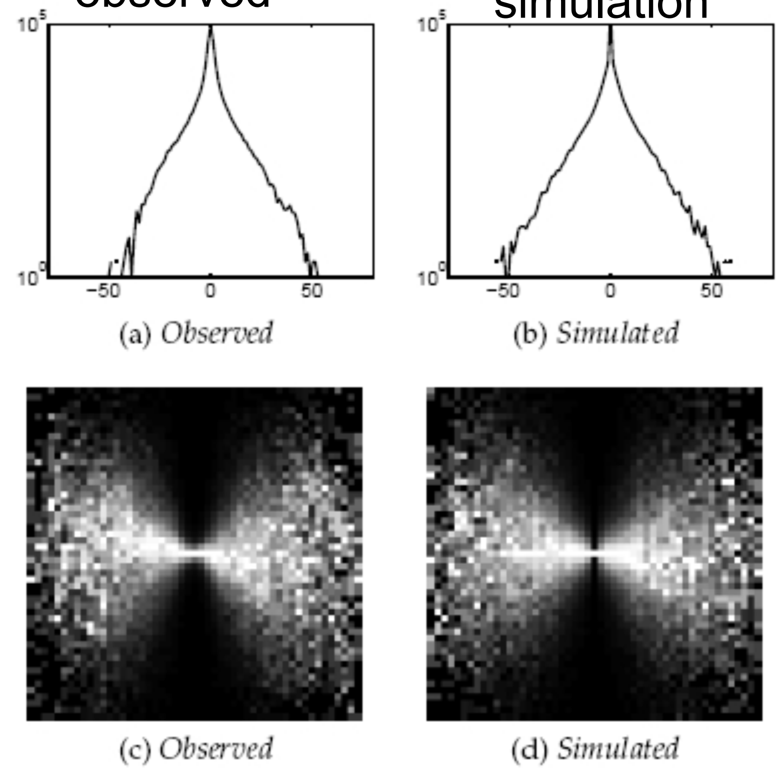
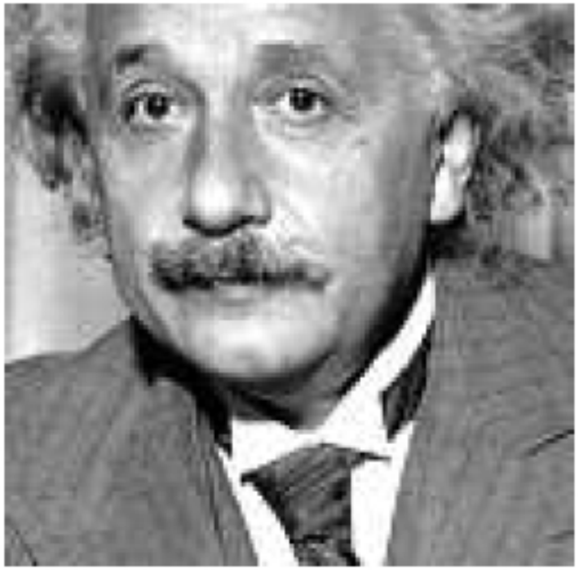


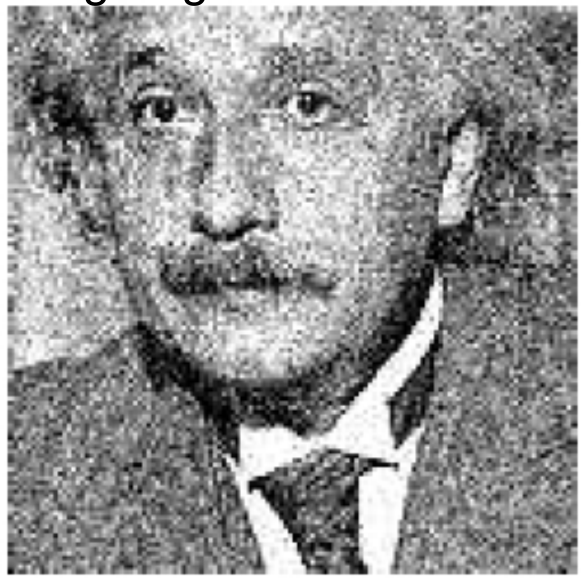
Fig. 9. Comparison of statistics of coefficients from an example image subband (left panels) with those generated by simulation of a local GSM model (right panels).

z is a spatially varying hidden variable that can be used to
(a) Create the non-gaussian histograms from a mixture of Gaussian densities, and
(b) model correlations between the neighboring wavelet coefficients.

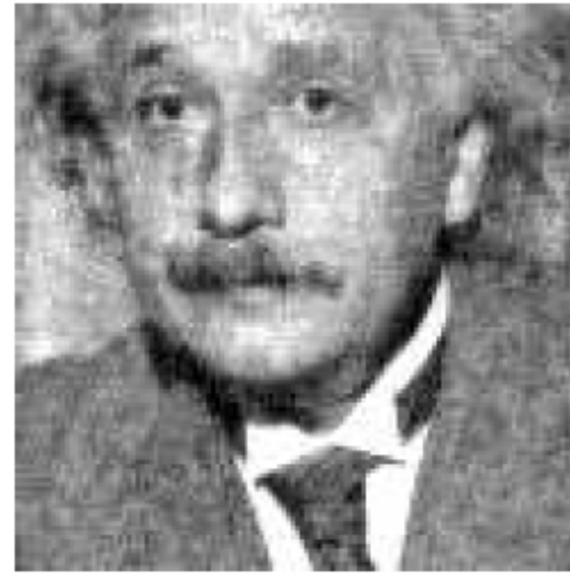
original



With Gaussian noise of std. dev. 21.4 added, giving PSNR=22.06



(1) Denoised with Gaussian model, PSNR=27.87



Separating reflections from a single image using local features

Anat Levin Assaf Zomet Yair Weiss

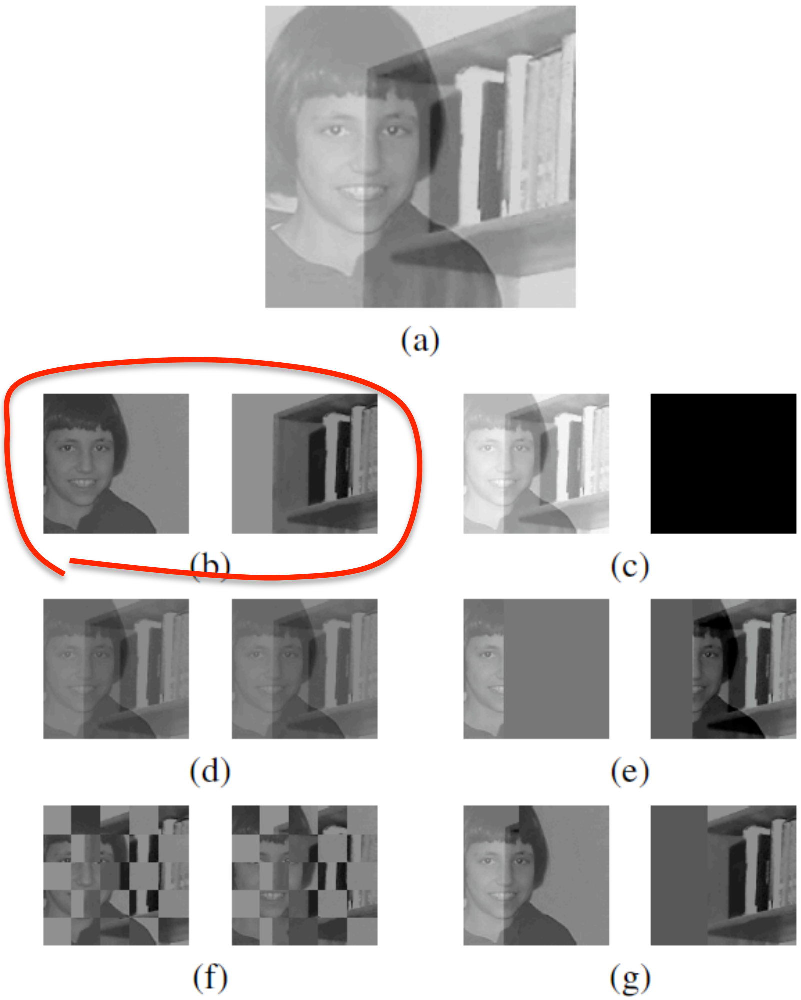


Figure 1: (a) Original input image (constructed by summing the two images in b). (b) the correct decomposition. (c)-(g) alternative possible decompositions. Why should the decomposition in (b) be favored?

very simple cost function: it favors decompositions which have a small number of edges and corners. Surprisingly, this simple cost function gives the "right" decompositions for challenging real images.[1]

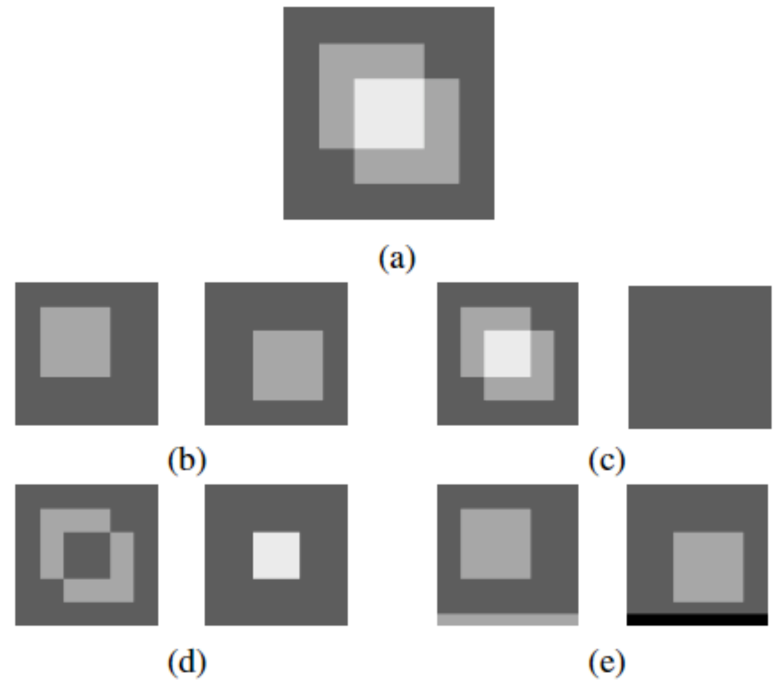


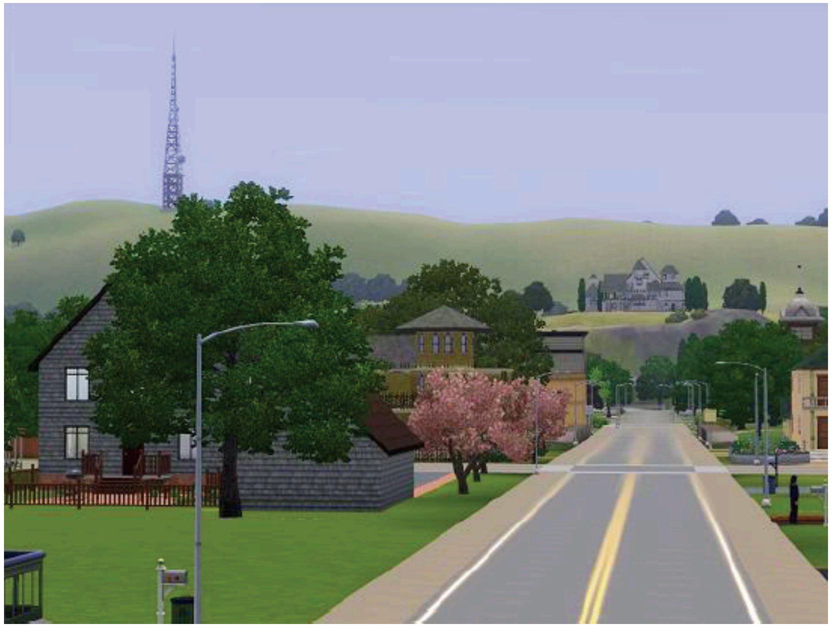
Figure 2: An input image and some decompositions

Applications

- Detecting fake images

- Camera shake removal

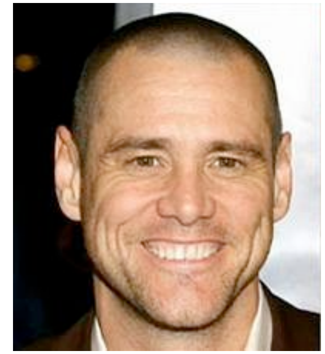
Visual Worlds



Prof. Hany Farid,
Dartmouth University

How do you tell if an image is fake?

http://www.life.com/archive/realfake



+

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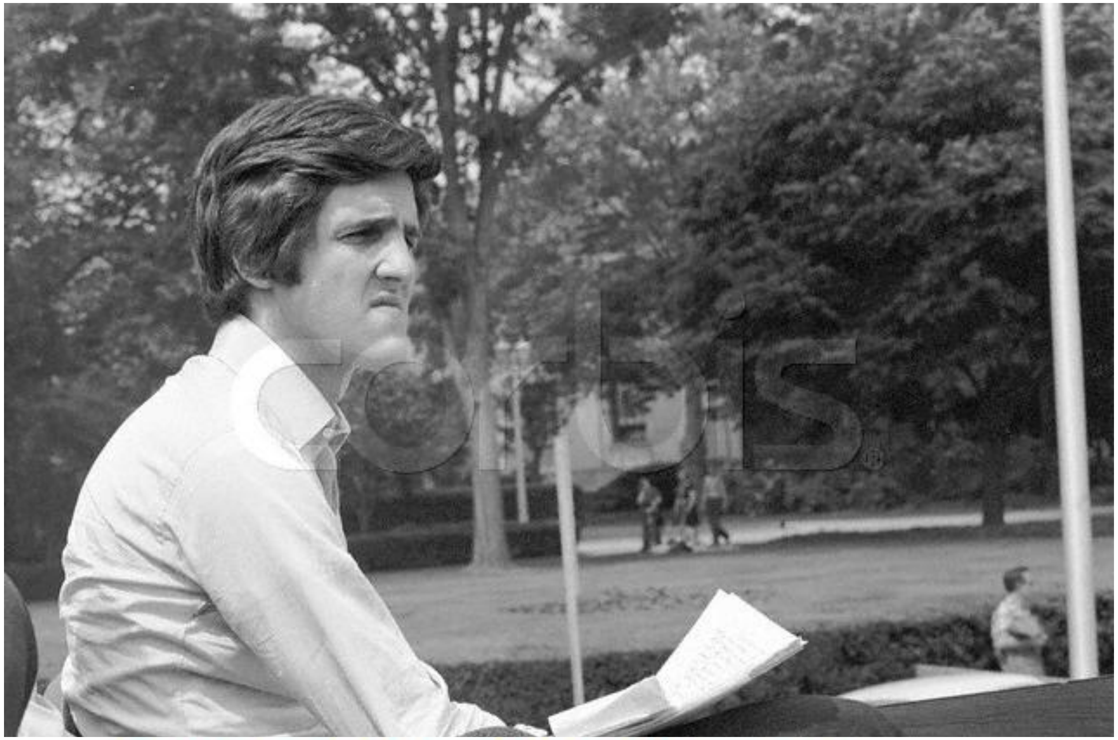
Image circulated on internet


Fonda Speaks To Vietnam Veterans At Anti-War Rally

Actress And Anti-War Activist Jane Fonda Speaks to a crowd of Vietnam Veterans as Activist and former Vietnam Vet John Kerry (LEFT) listens and prepares to speak next concerning the war in Vietnam (AP Photo)

http://www.cs.dartmouth.edu/farid/publications/deception09.pdf
http://www.cs.dartmouth.edu/farid/publications/significance06.pdf

The source images



Update: Fonda, Kerry and Photo Fakery (free reg. required) - Photographer Ken Light describes the experience of discovering his 1970 photograph of John Kerry circulating in altered form on the Internet. "As far as I know, John Kerry never shared a demonstration podium with Jane Fonda, and the fact that a widely circulated photo showed him doing so — until it was exposed in recent weeks as a hoax — tells us more about the troublesome combination of Photoshop and the Internet than it does about the prospective Democratic candidate for president." (*Washington Post*)

IEEE Transactions on Signal Processing, 53(2):845-850, 2005

How Realistic is Photorealistic?

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Hanover, NH 03755
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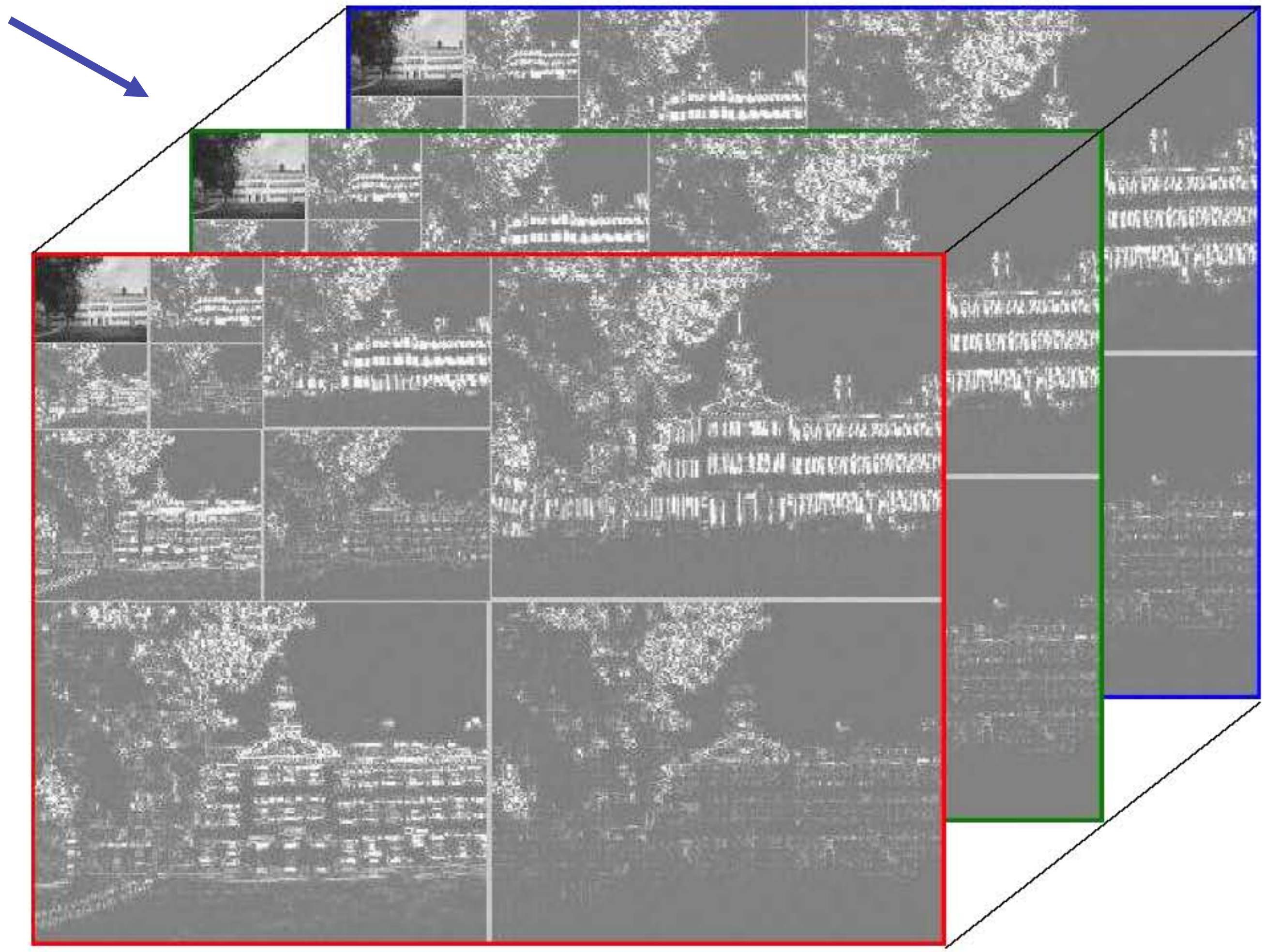
***Abstract*—Computer graphics rendering software is capable of generating highly photorealistic images that can be impossible to differentiate from photographic images. As a result, the unique stature of photographs as a definitive recording of events is being diminished (the ease with which digital images can be manipulated is, of course,**

There has been some work in evaluating the photorealism of computer graphics rendered images from a human perception point of view (e.g., [10], [9], [11]). To our knowledge, however, no computational techniques exist to differentiate between photographic and photorealis-

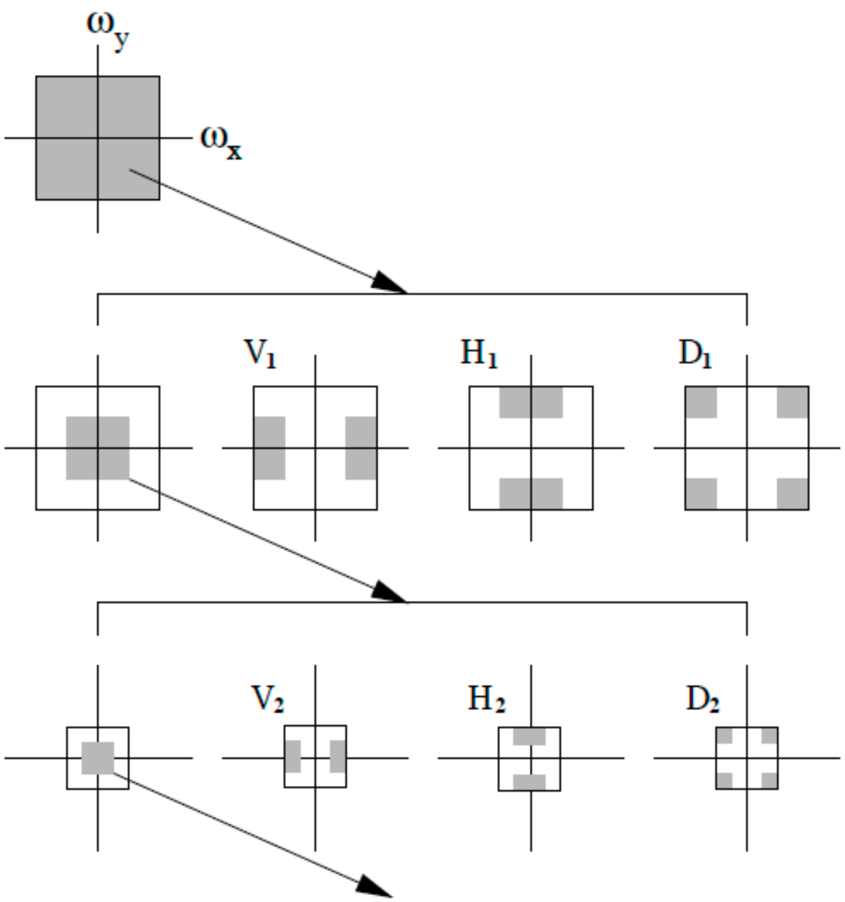
Input image

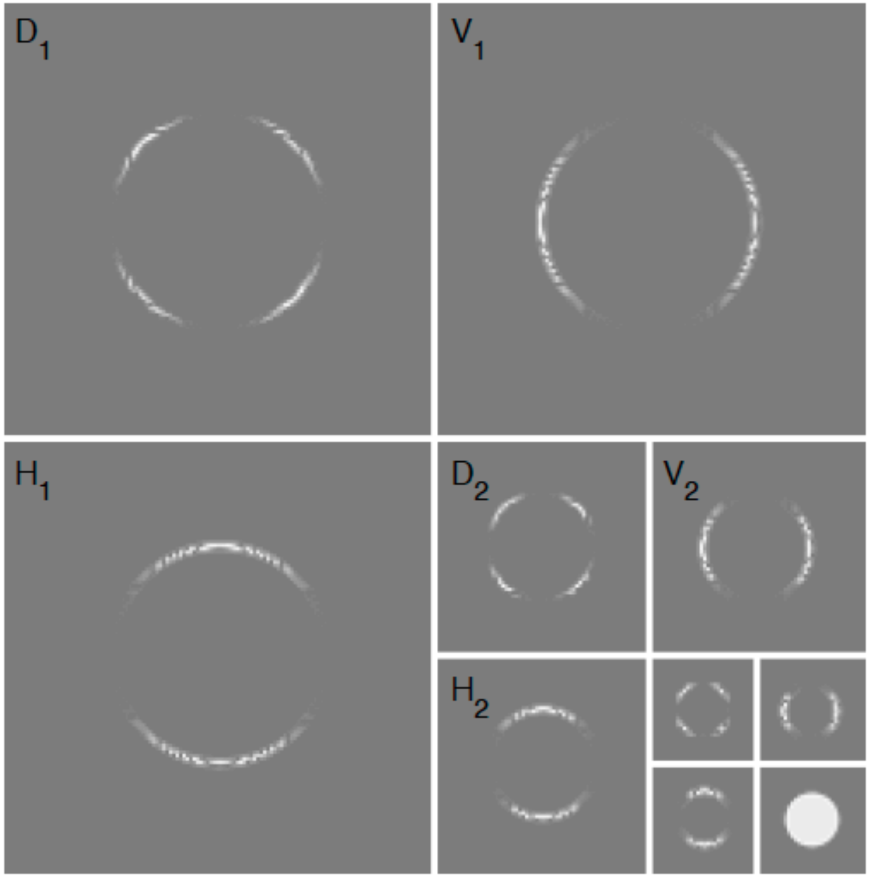
Representation of color input image in wavelet subbands



Filter bank

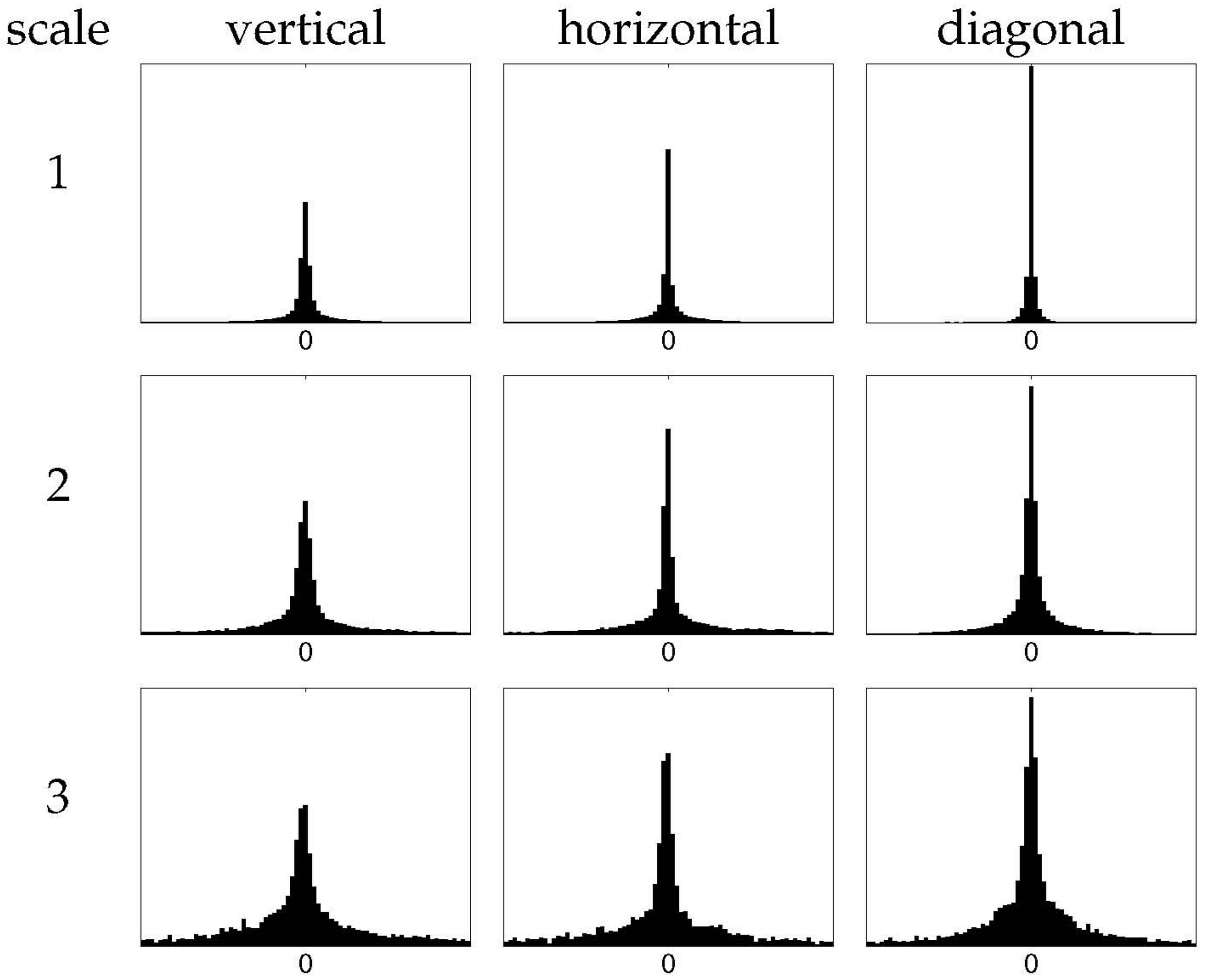
Separable Quadrature Mirror Filters





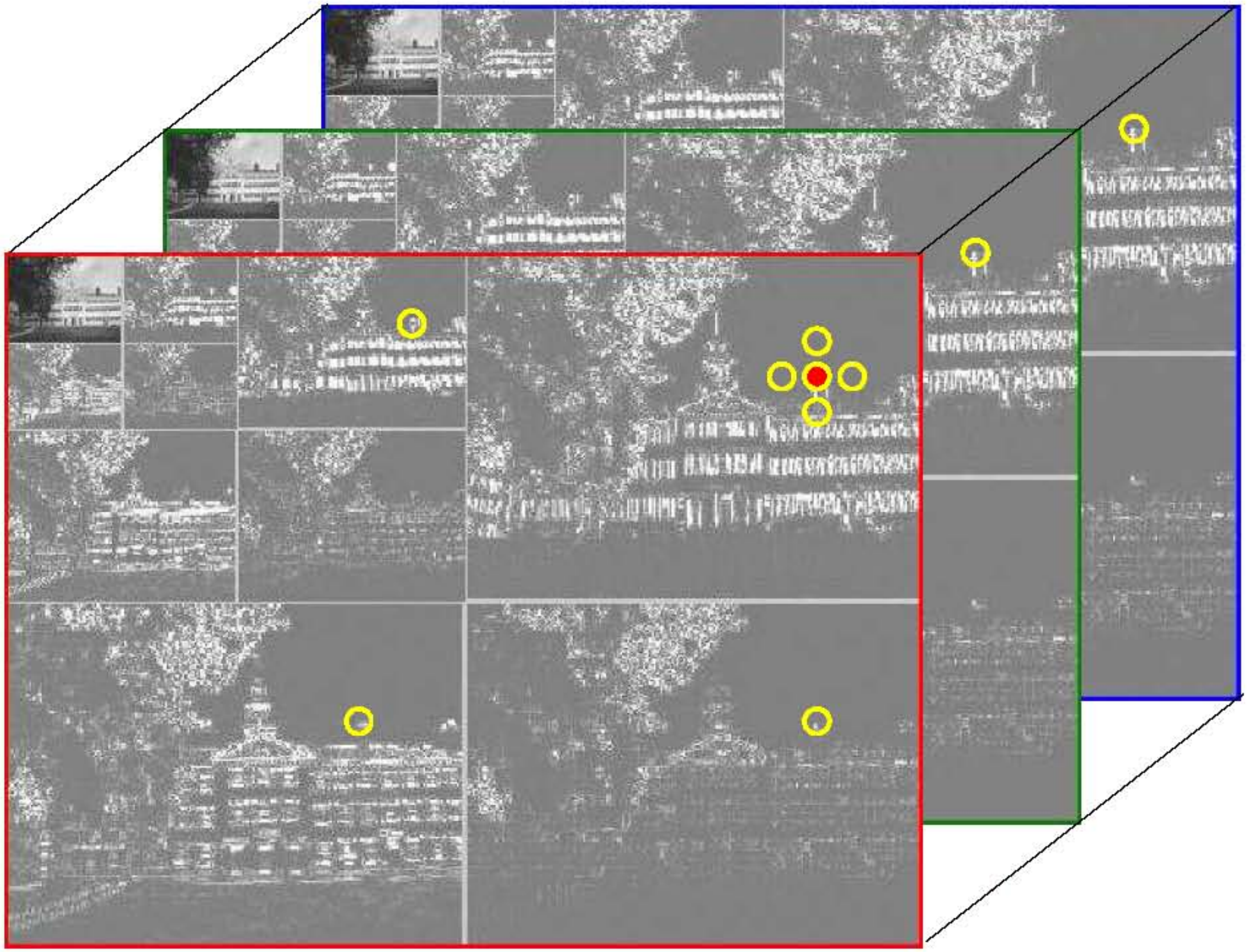
Each output is called *subband*

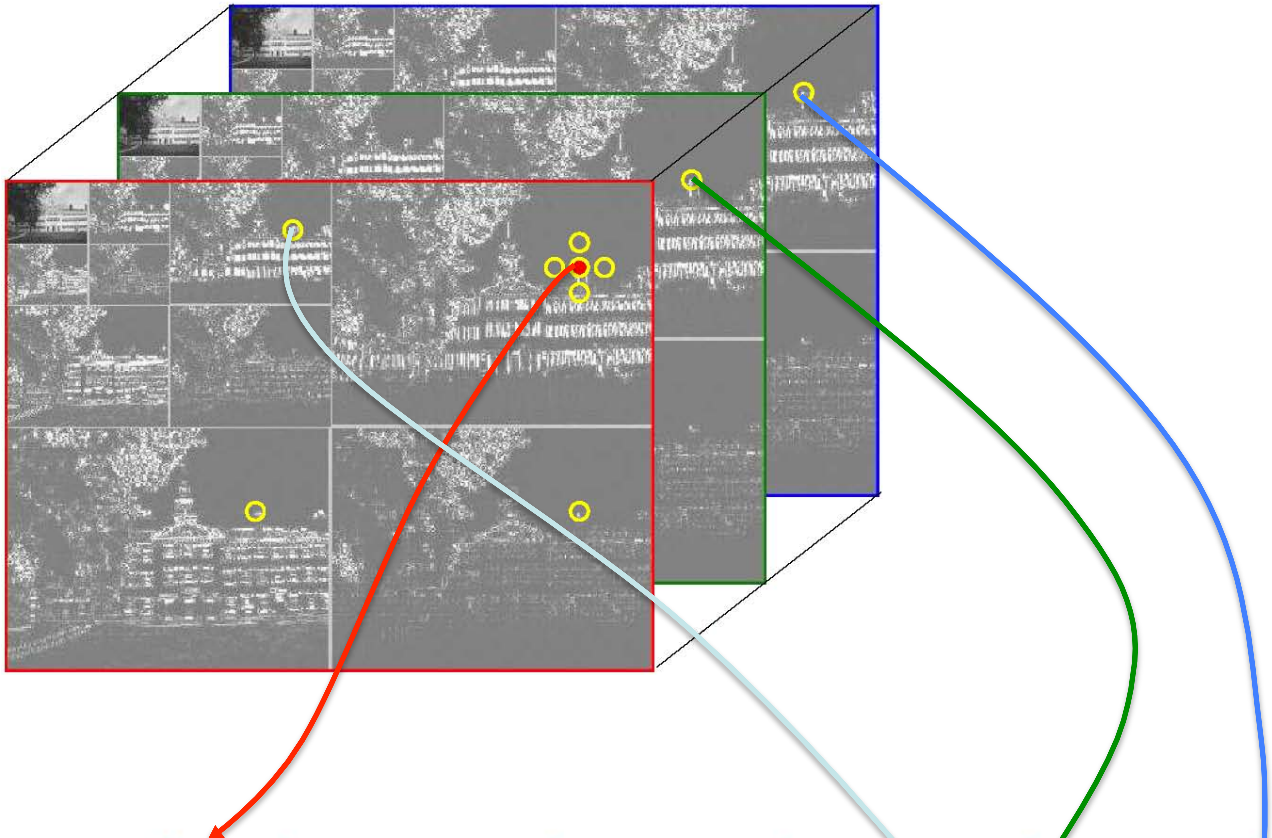
Histograms of wavelet subband coefficients



mean (μ), variance (μ_2), skewness (μ_3/σ^3), kurtosis (μ_4/σ^4)

There are correlations between subband coefficients





$$
\begin{aligned}
V_i(x,y) \;=\;& w_1 V_i(x-1,y) + w_2 V_i(x+1,y) + w_3 V_i(x,y-1) \\
+\;& w_4 V_i(x,y+1) + w_5 V_{i+1}(x/2,y/2) + w_6 H_i(x,y) \\
+\;& w_7 D_i(x,y) + w_8 V_i(x,y) + w_9 V_i(x,y)
\end{aligned}
$$

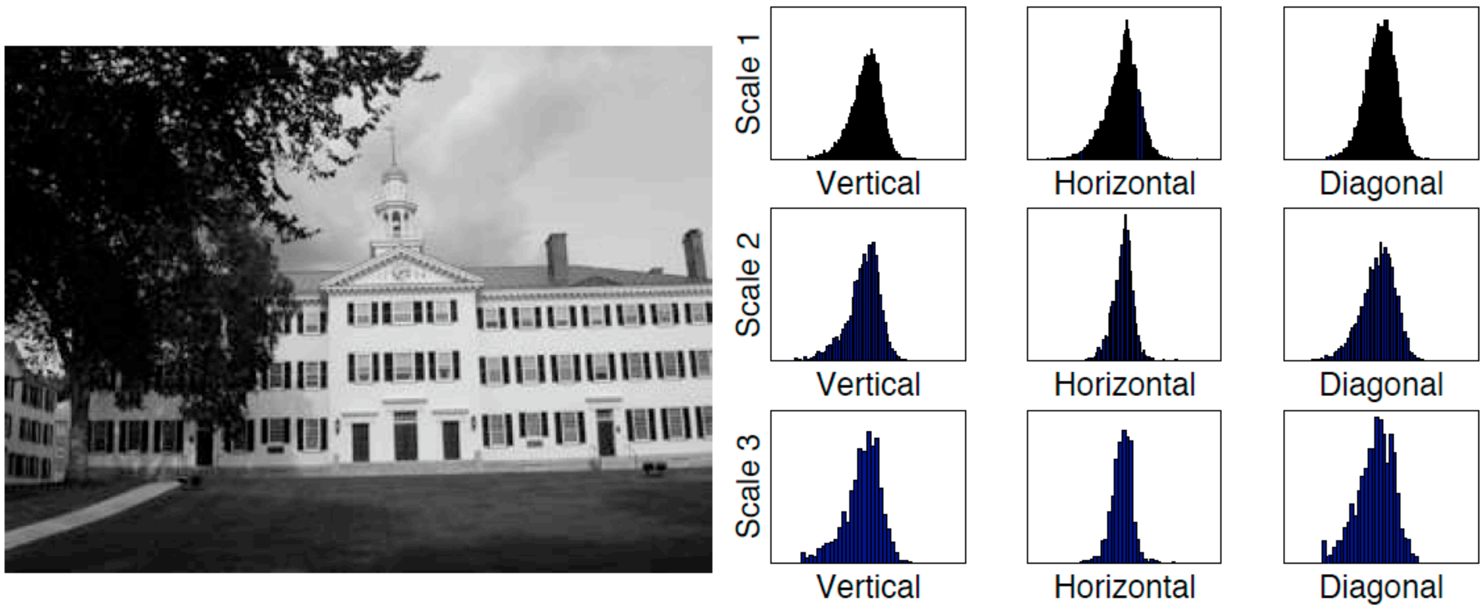
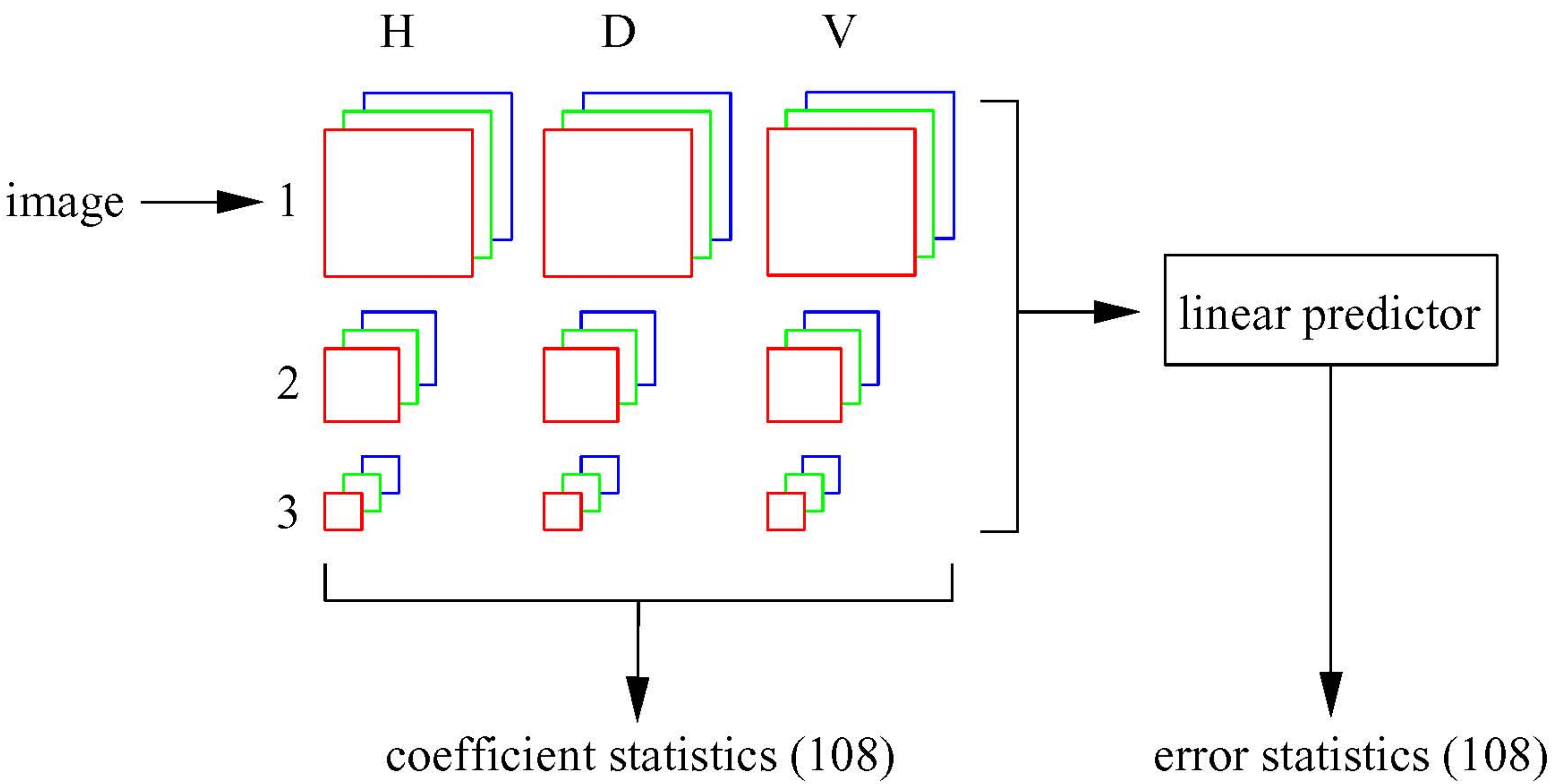
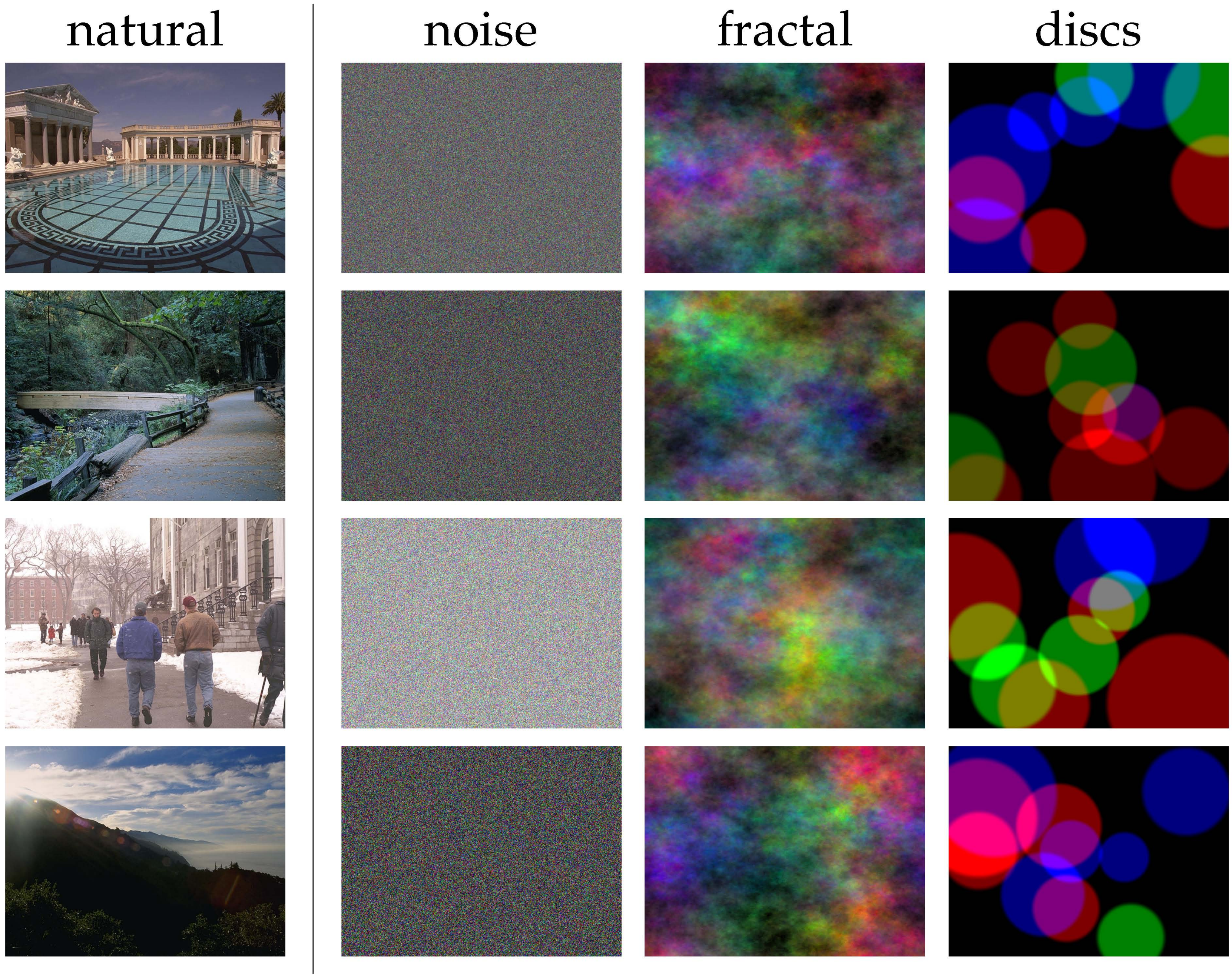


Figure 2.10: A natural image (left) and the histograms of the linear prediction errors of coefficient magnitudes for all subbands in a three-scale QMF pyramid decomposition of the image on the left.

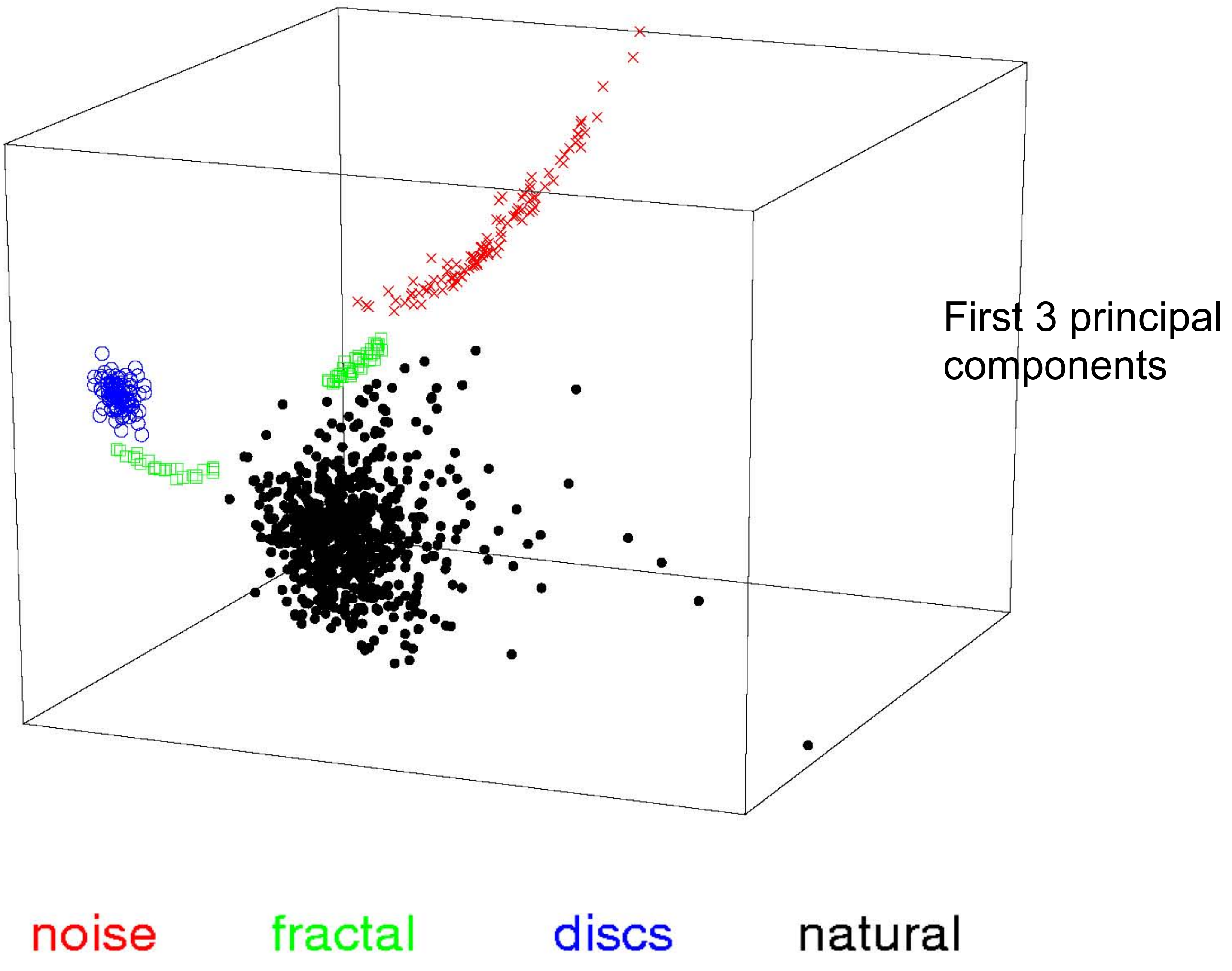
Hypothesis: there is something different in the correlation between wavelet coefficients between real images and computer generated images.

Summary of features used for image classification




natural
noise
fractal
discs

Projection of measured features into a 3-d space: well separated even in that low-dimensional space



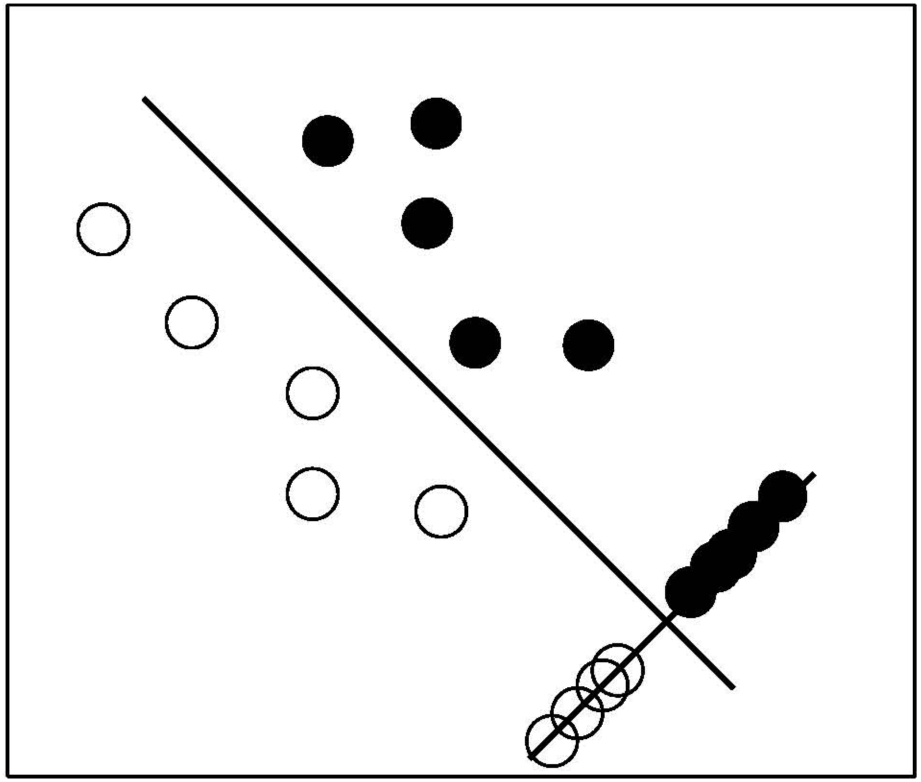
Photographic training set: downloaded from www.freefoto.com

photographic (40,000)

Photorealistic training set: downloaded from www.raph.com and www.irtc.org

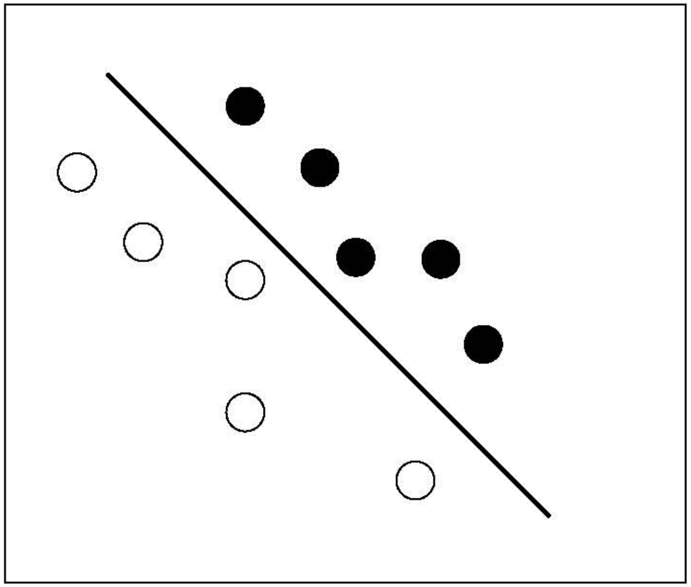
photorealistic $(6,000)$

Classifier 1: LDA. Simple, amenable to analysis

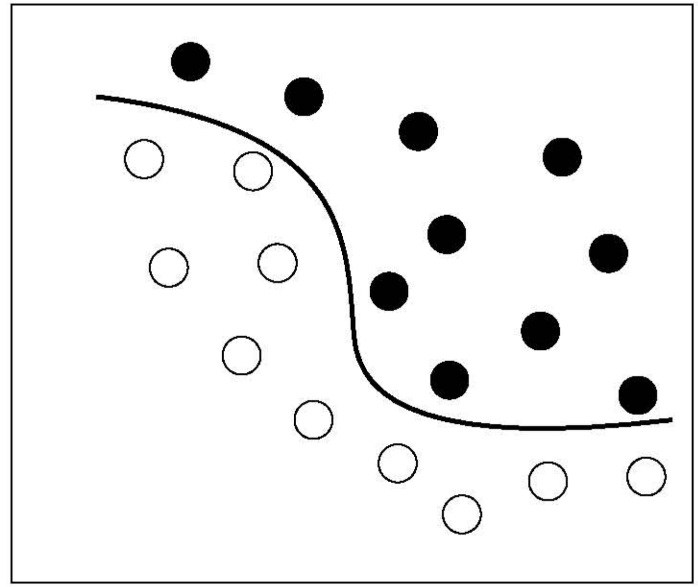


linear discriminant analysis (LDA)

Classifier 2: SVM. State of the art.



linear SVM



non-linear SVM

Easily classified photographic images

Fig. 4: Easily classified photographic images.

Easily classified photorealistic images

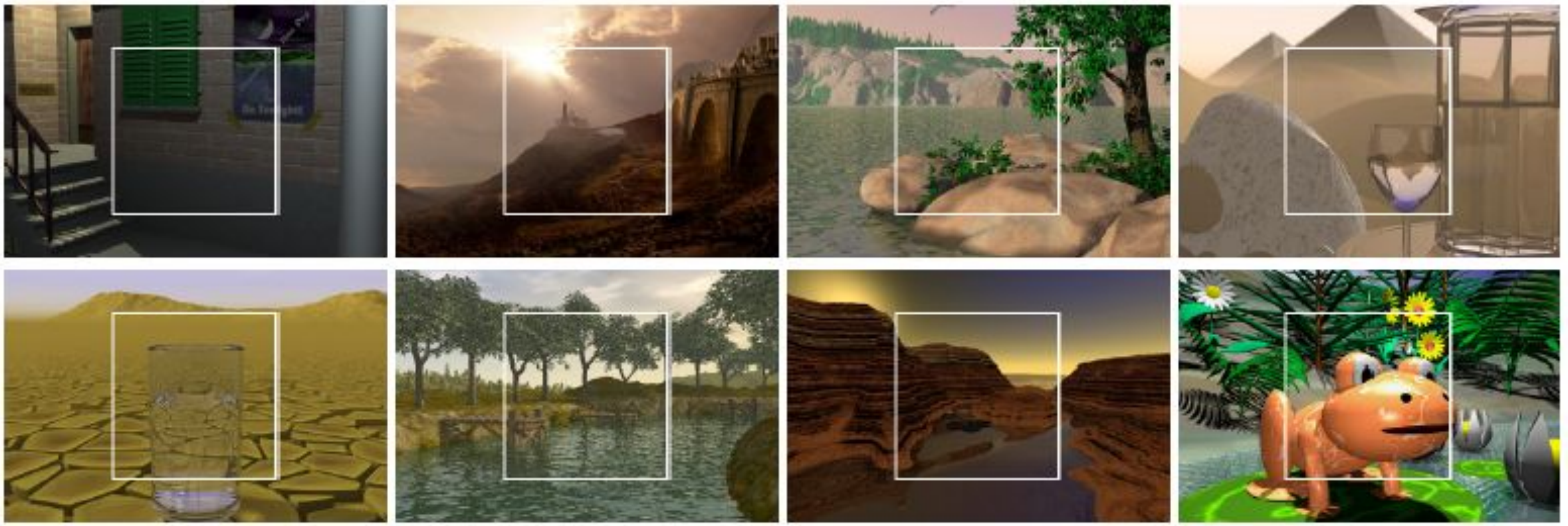


Fig. 5: Easily classified photorealistic images.

Incorrectly classified photographic images

Fig. 6: Incorrectly classified photographic images.

Incorrectly classified photorealistic images

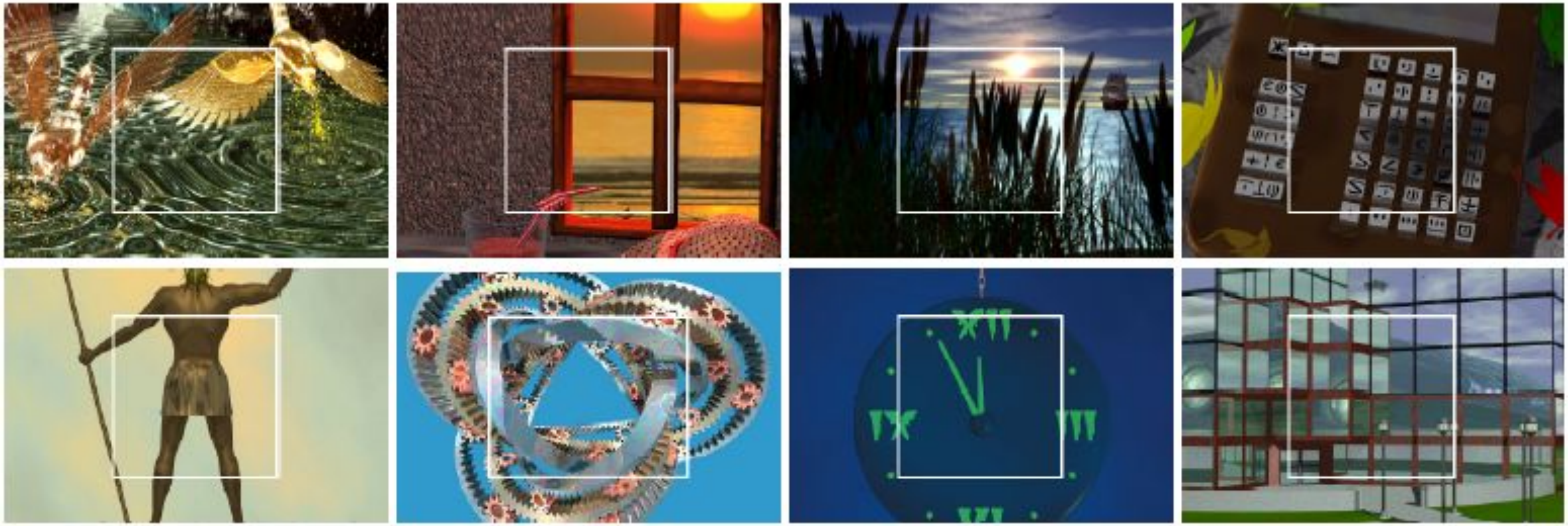
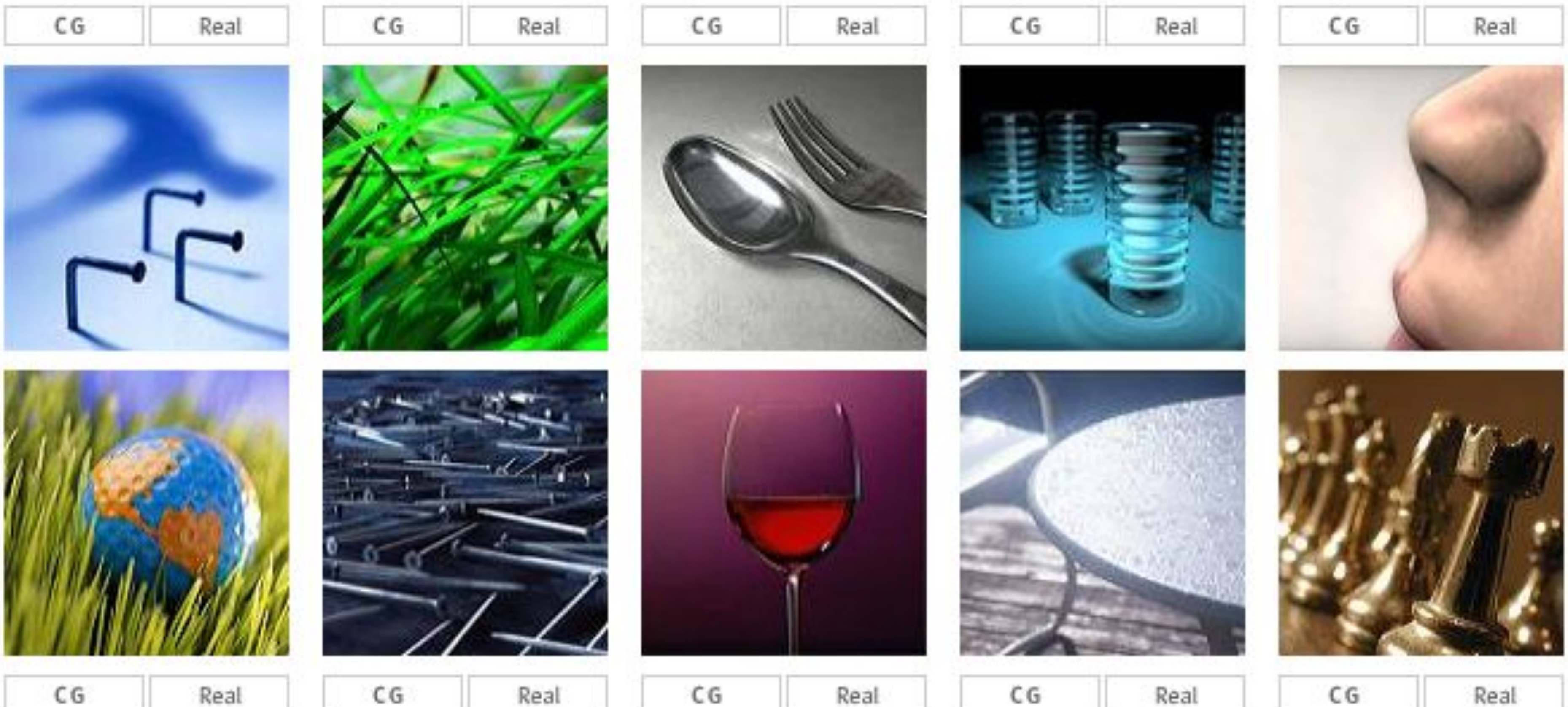


Fig. 7: Incorrectly classified photorealistic images.

www.fakeorfoto.com



Results of algorithm

Photographic images | Photorealistic images

Correct

Incorrect

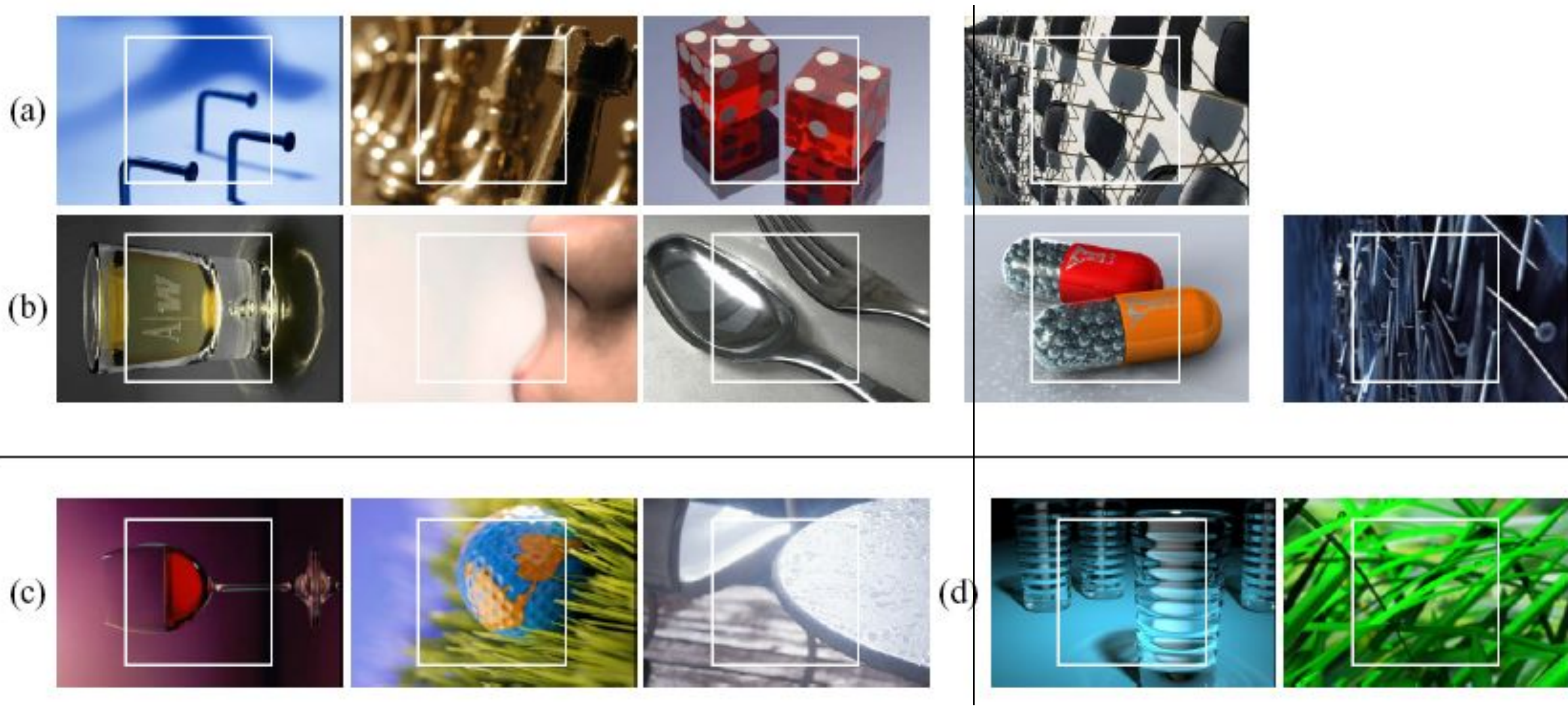


Fig. 9: Images from www.fakeorfoto.com. Shown in (a) and (c) are correctly and incorrectly classified photographic images, respectively. Shown in (b) and (d) are correctly and incorrectly classified photorealistic images, respectively.

Taking a picture…

What the camera give us…

How do we correct this?

Slides R. Fergus

Close-up

Original

Naïve Sharpening

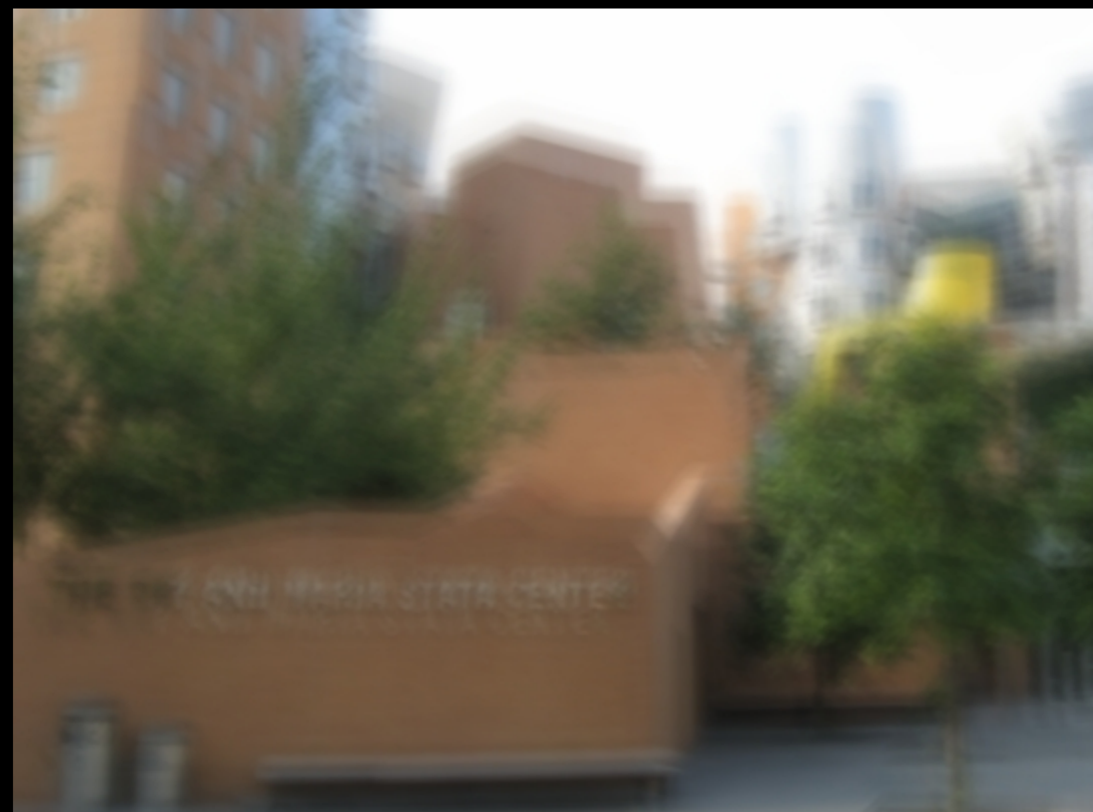
Our algorithm

Slides R. Fergus

Why does picture appear blurry?

Let’s take a photo

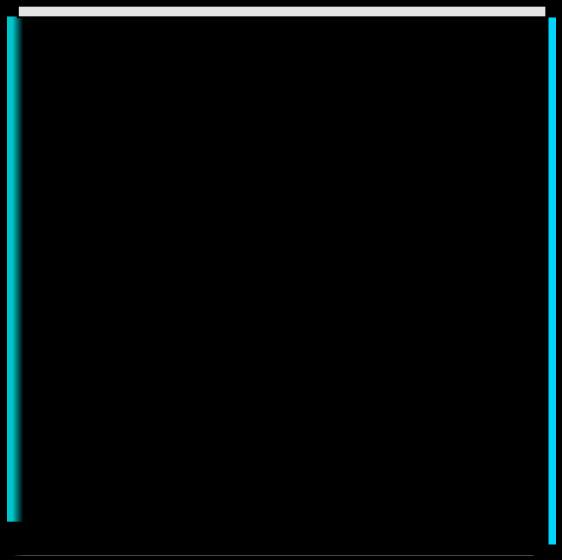
Blurry result



Slow-motion replay

Slides R. Fergus

Slow-motion replay



Motion of camera

Slides R. Fergus

Image formation process

Blurry image

Input to algorithm

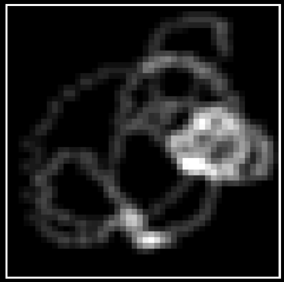
=

Sharp image

Desired output

$\otimes$

Convolution operator



Blur kernel

Model is approximation

Why is this hard?

Simple analogy:

11 is the product of two numbers.

What are they?

No unique solution:

11 = 1 x 11

11 = 2 x 5.5

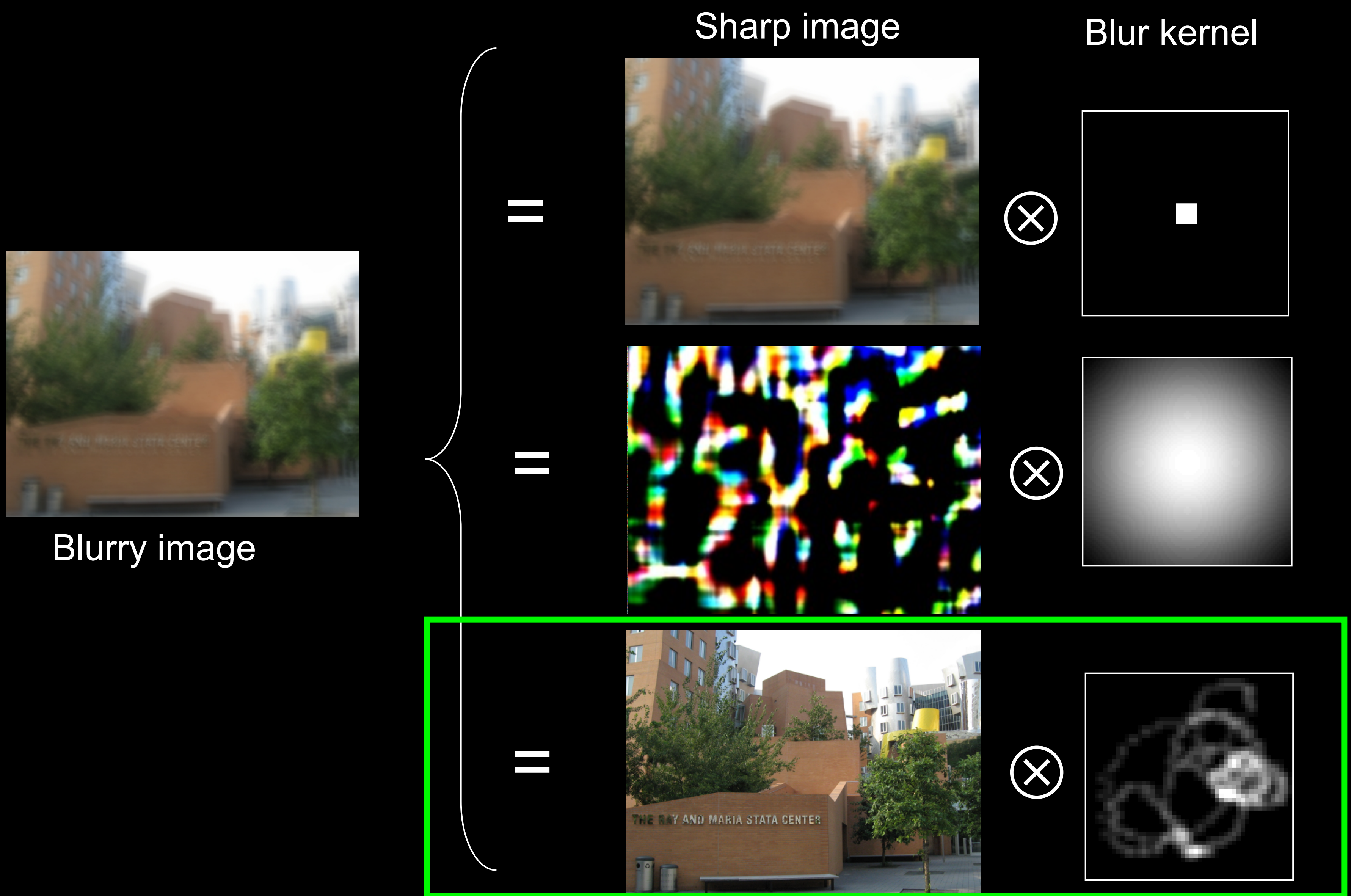
11 = 3 x 3.667

etc…..

Need more information !!!!

Slides R. Fergus

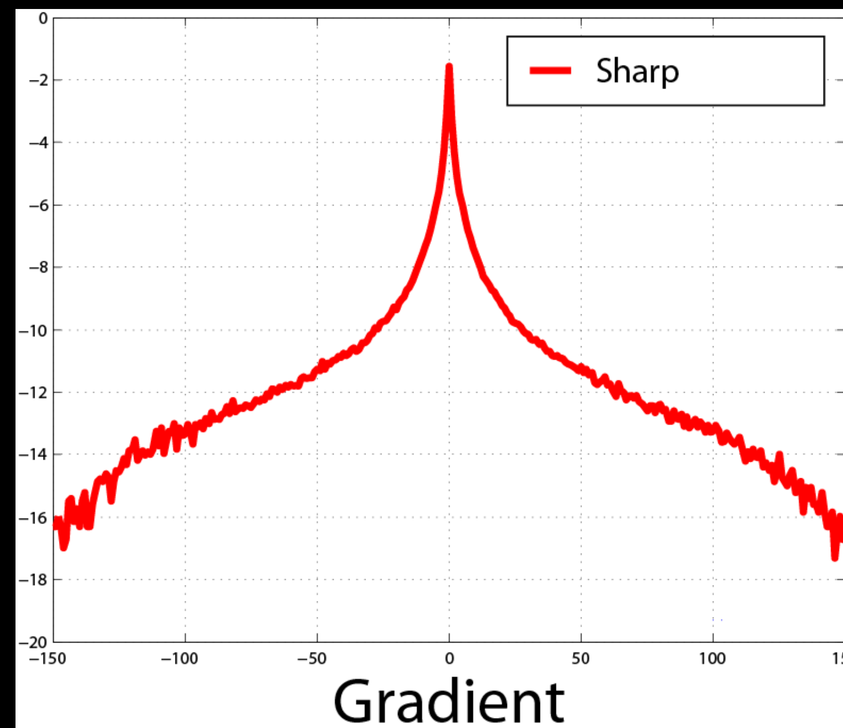
Multiple possible solutions



Natural image statistics

Characteristic distribution with heavy tails

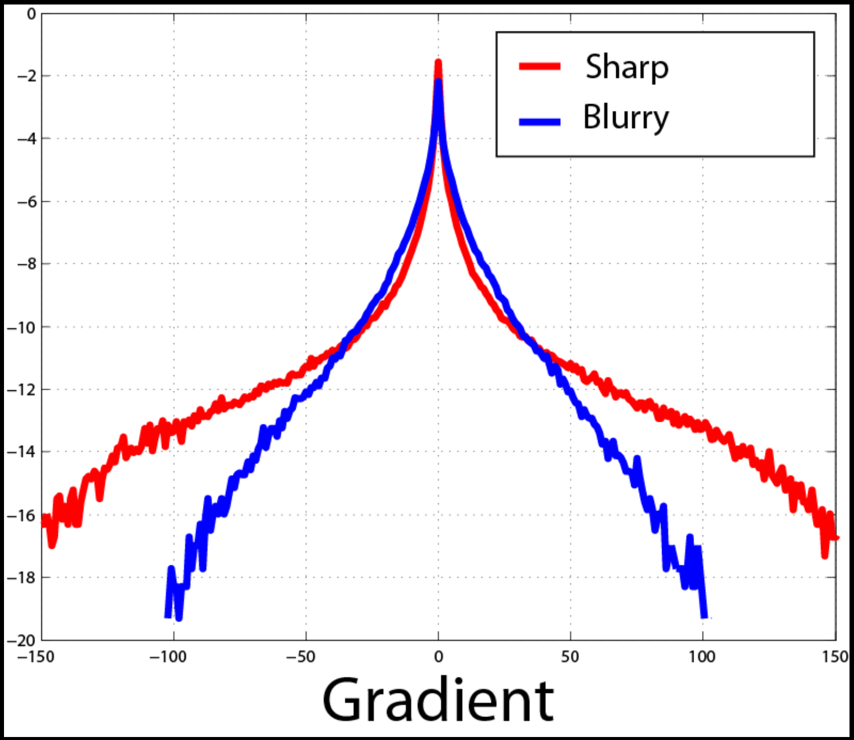
Histogram of image gradients



Slides R. Fergus

Blury images have different statistics

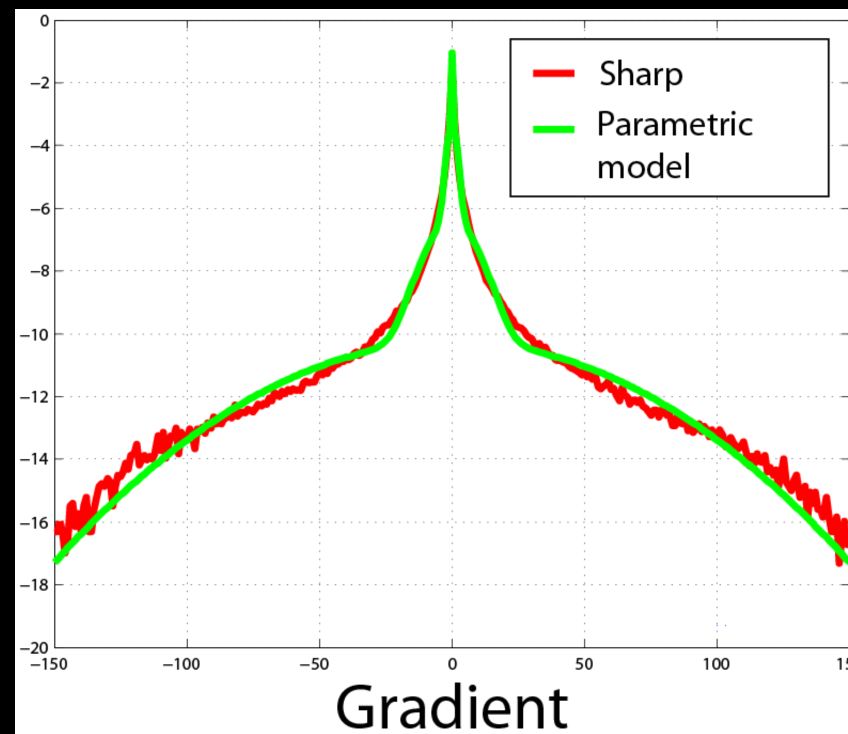
Histogram of image gradients



Slides R. Fergus

Parametric distribution

Histogram of image gradients



Use parametric model of sharp image statistics

Slides R. Fergus

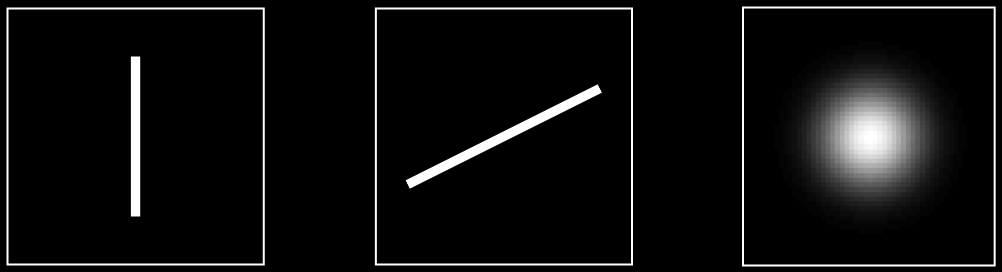
Existing work on image deblurring

Software algorithms:

– Extensive literature in signal processing community

– Mainly Fourier and/or Wavelet based

– Strong assumptions about blur
  → not true for camera shake



Assumed forms of blur kernels

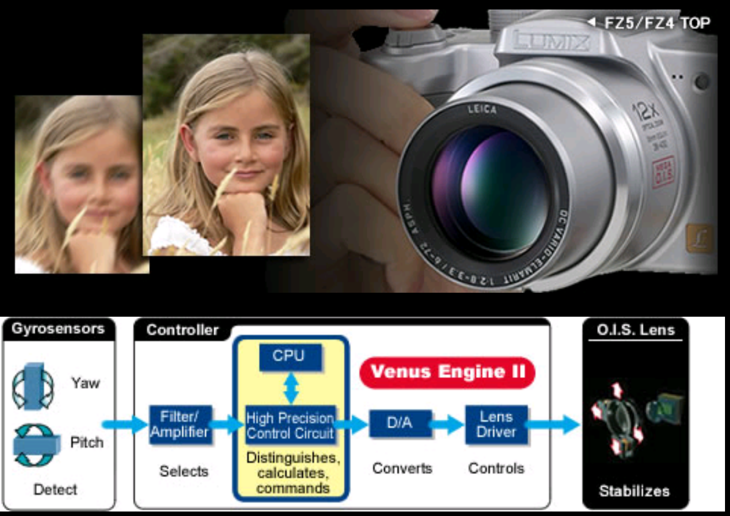
– Image constraints are frequency-domain power-laws

Slides R. Fergus

Existing work on image deblurring

Hardware approaches

Image stabilizers



Dual cameras

Ben-Ezra and Nayar 2004

Coded shutter

Raskar et al. SIGGRAPH 2006

Our approach can be combined with these hardware methods

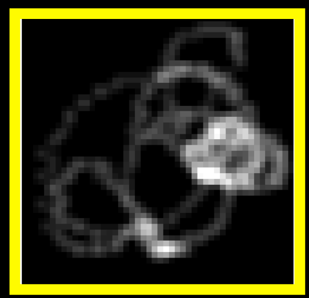
Three sources of information

1. Reconstruction constraint:

(

Estimated sharp image

$\otimes$

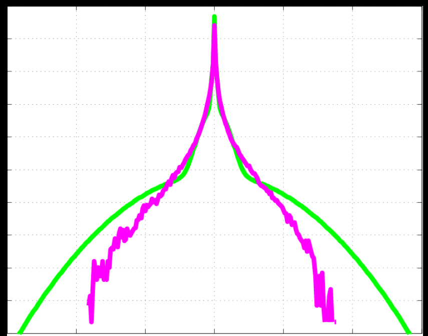


Estimated blur kernel)

$=$

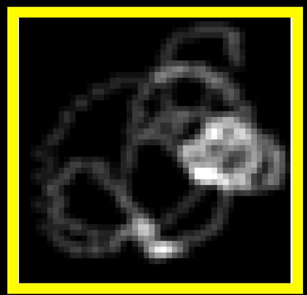
Input blurry image

2. Image prior:



Distribution of gradients

3. Blur prior:



Positive & Sparse

Three sources of information

y = observed image   b = blur kernel   x = sharp image

$$p(b, x|y) = k \; p(y|b, x) \; p(x) \; p(b)$$

Posterior

1. Likelihood (Reconstruction constraint)

2. Image prior

3. Blur prior

1. Likelihood $p(y|b,x)$

y = observed image   b = blur   x = sharp image

Reconstruction constraint:

$$p(y|b,x) = \prod_i \mathcal{N}(y_i|x_i \otimes b, \sigma^2)$$

$$\propto \prod_i e^{-\frac{(x_i \otimes b - y_i)^2}{2\sigma^2}}$$

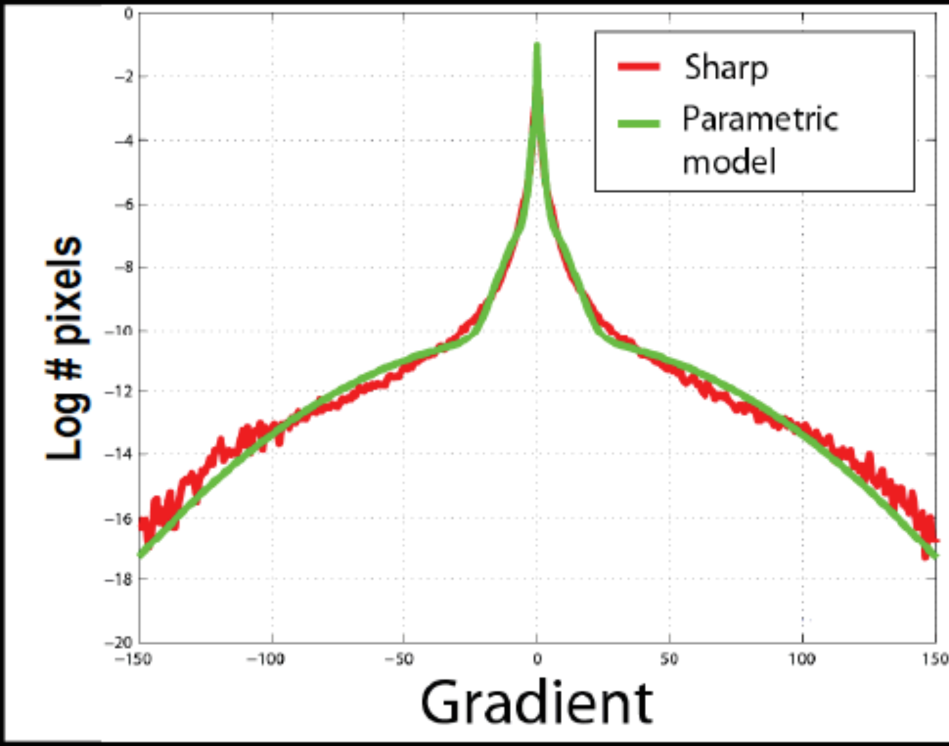
i - pixel index

2. Image prior $p(x)$

y = observed image　　b = blur　　x = sharp image

$$p(x) = \prod_i \sum_{c=1}^{C} \pi_c \mathcal{N}(f(x_i)|0, s_c^2)$$

Mixture of Gaussians fit to empirical distribution of image gradients



i - pixel index

c - mixture component index

f - derivative filter

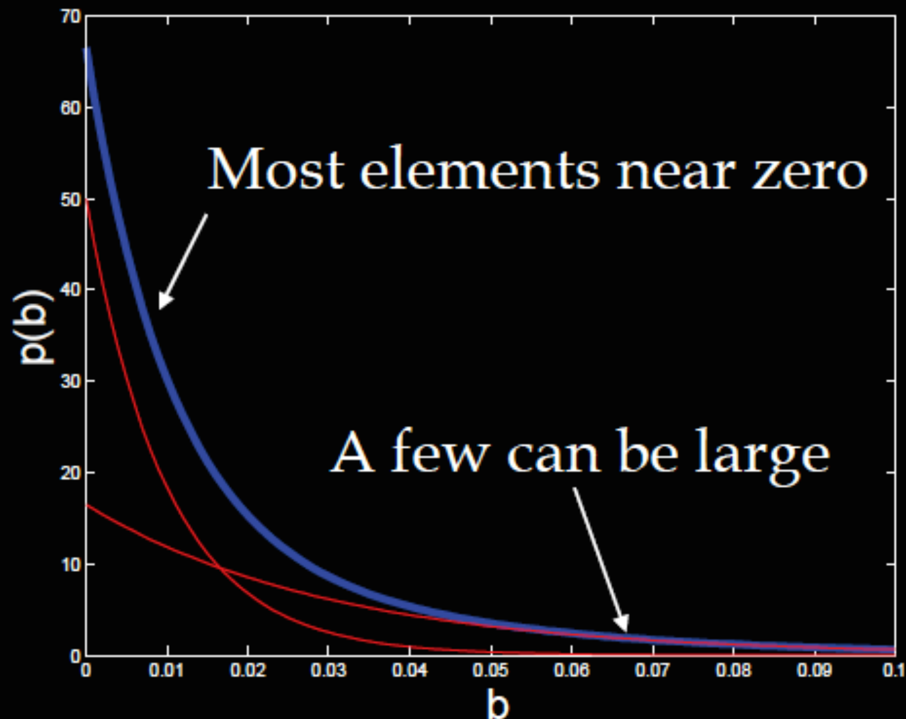
3. Blur prior $p(b)$

y = observed image b = blur x = sharp image

$$p(b) = \prod_j \sum_{d=1}^{D} \pi_d \, \mathcal{E}(b_j | \lambda_d)$$

Mixture of Exponentials

- Positive & sparse
- No connectivity constraint



j - blur kernel element

d - mixture component index

How do we use this information?

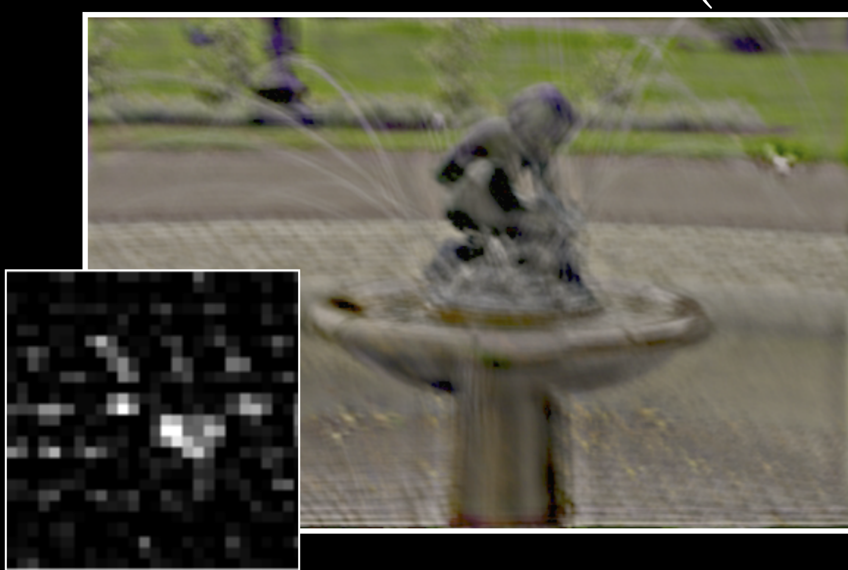
Obvious thing to do:

- Combine 3 terms into an objective function
- Run conjugate gradient descent
- This is Maximum a-Posteriori (MAP)

Results from MAP estimation

Input blurry image

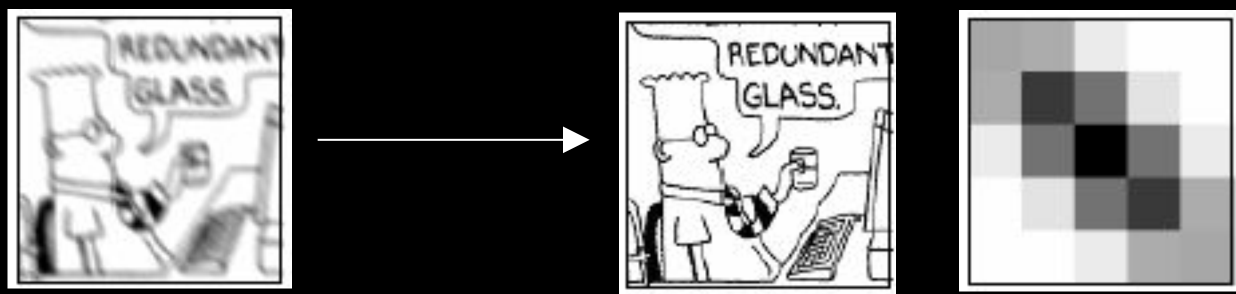
Maximum a-Posteriori (MAP)



Our method: Variational Bayes

Variational Bayesian method

Based on work of Miskin & Mackay 2000



Keeps track of uncertainty in estimates of image and blur by using a distribution instead of a single estimate

Helps avoid local maxima and over-fitting

Overview of algorithm

Input image

1. Pre-processing
2. Kernel estimation
   - Multi-scale approach
3. Image reconstruction
   - Standard non-blind deconvolution routine

Preprocessing

Input image

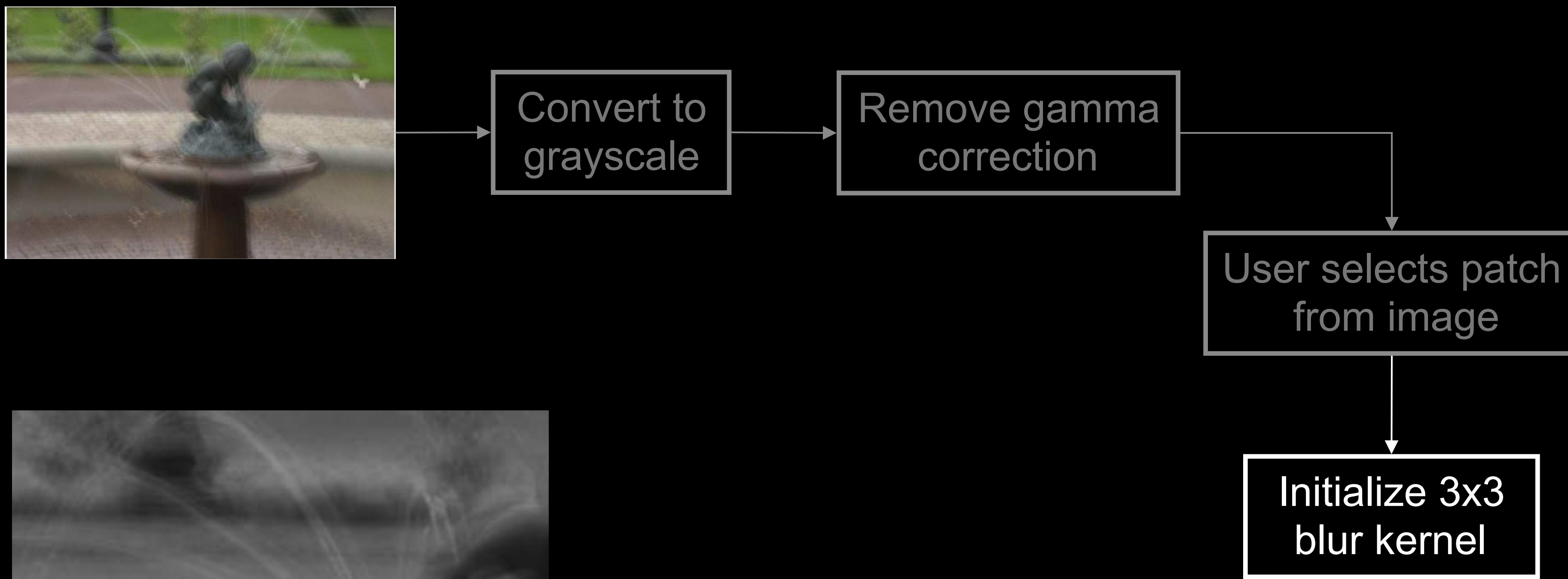
Convert to grayscale → Remove gamma correction → User selects patch from image

Bayesian inference too slow to run on whole image

Infer kernel from this patch

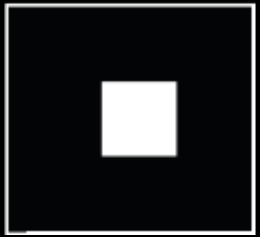
Initialization

Input image



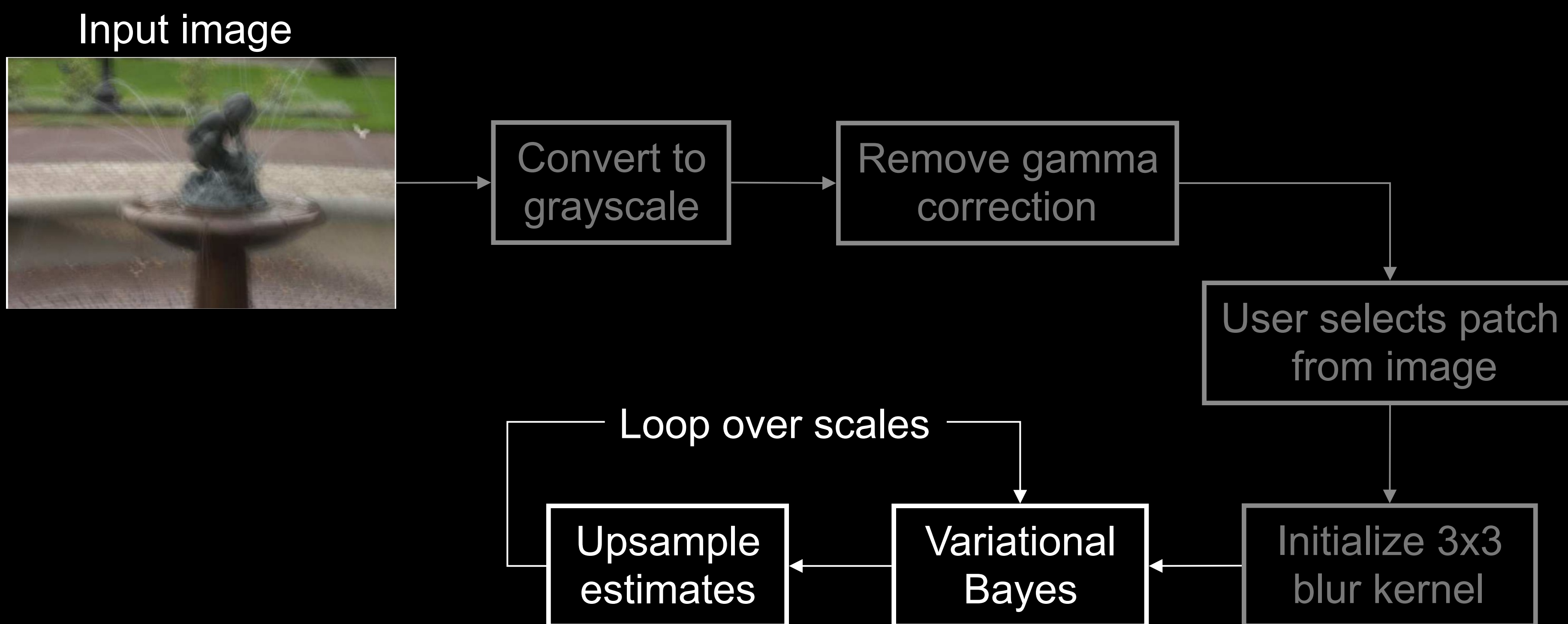
Blurry patch

Initial image estimate



Initial blur kernel

Inferring the kernel: multiscale method



Use multi-scale approach to avoid local minima:

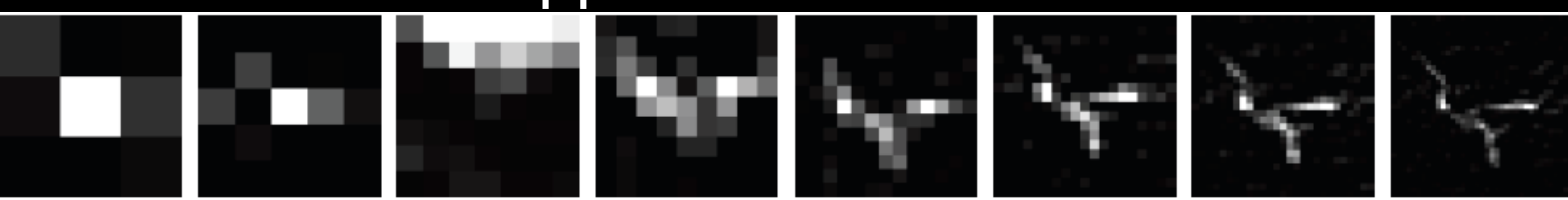
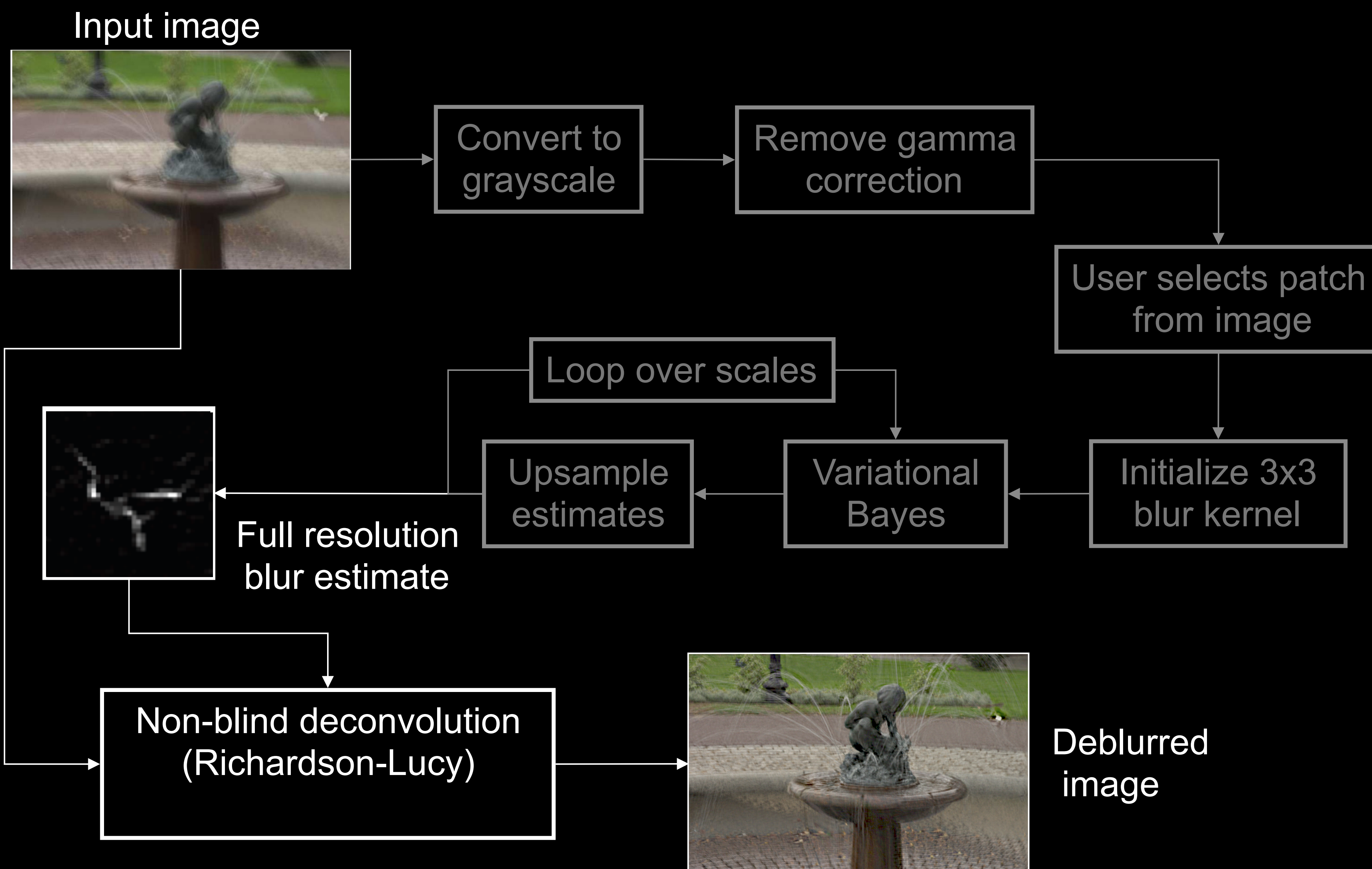


Image Reconstruction



Results on real images

Submitted by people from their own photo collections

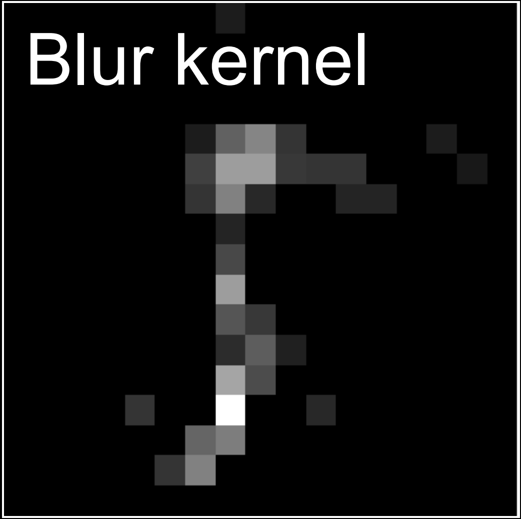
Type of camera unknown

Output does contain artifacts

- Increased noise
- Ringing

Compares well to existing methods

Original photograph


Blur kernel


Our output

Matlab's `deconvblind`

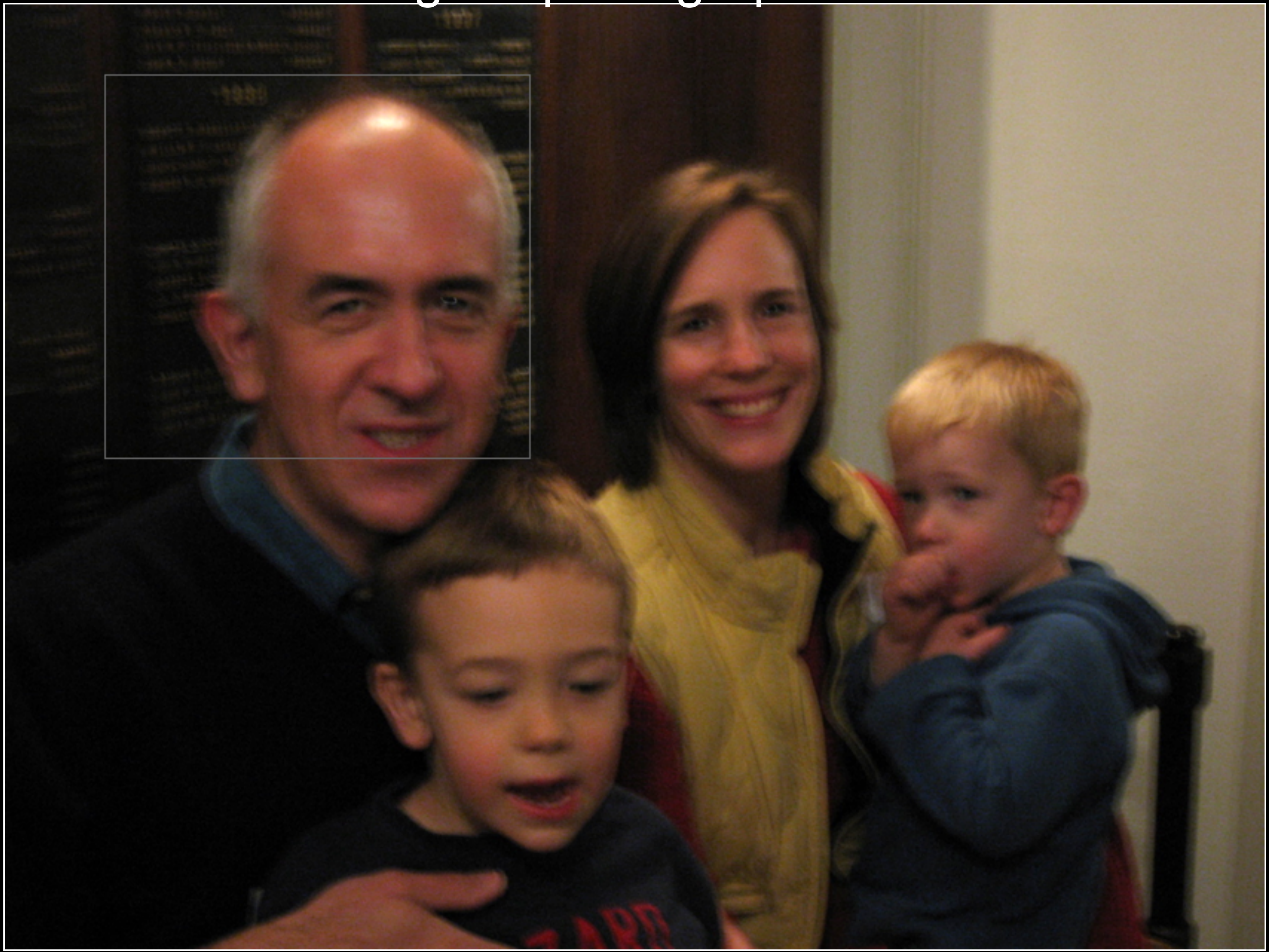
Close-up of garland

Original

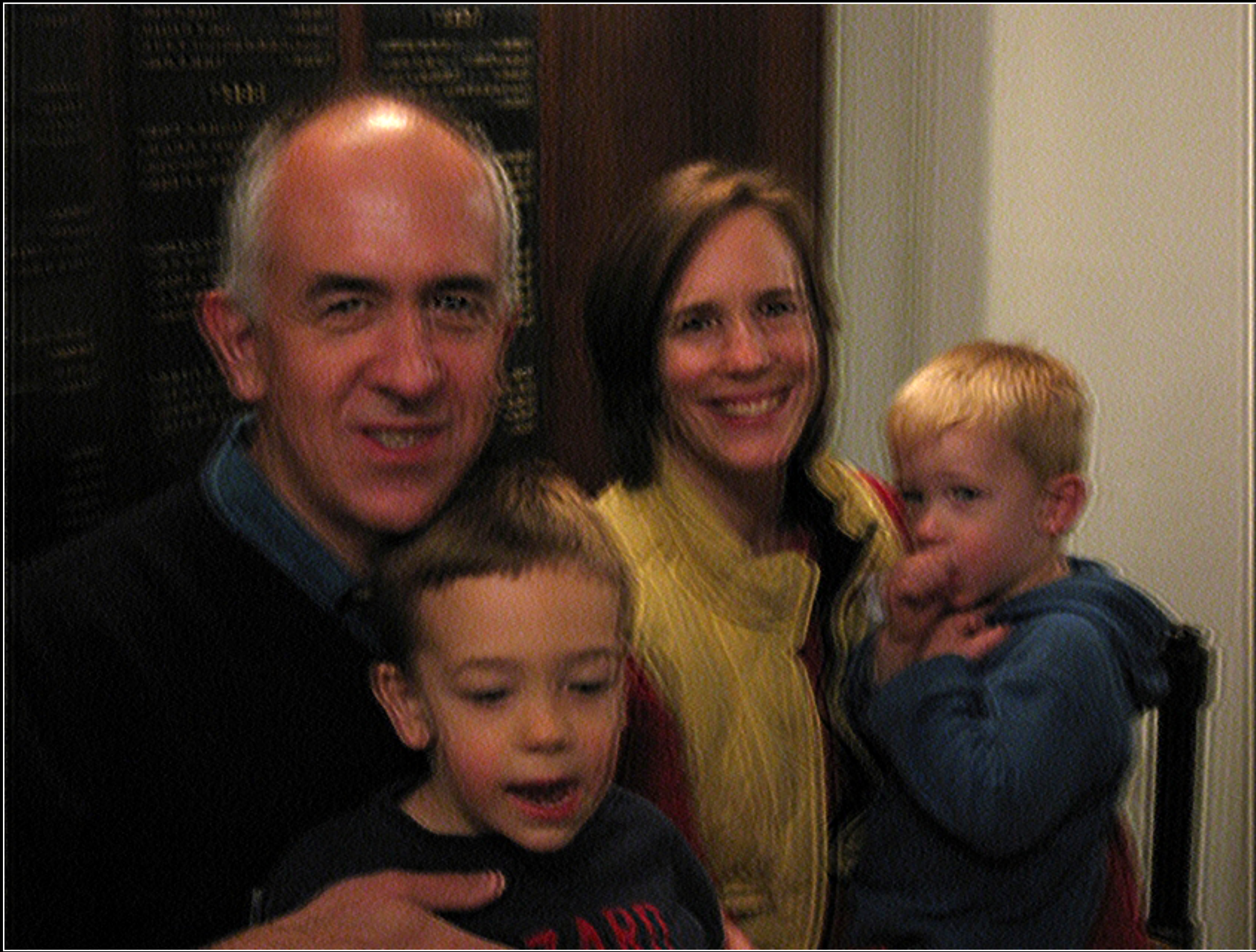
Matlab's `deconvblind`

Our output

Original photograph

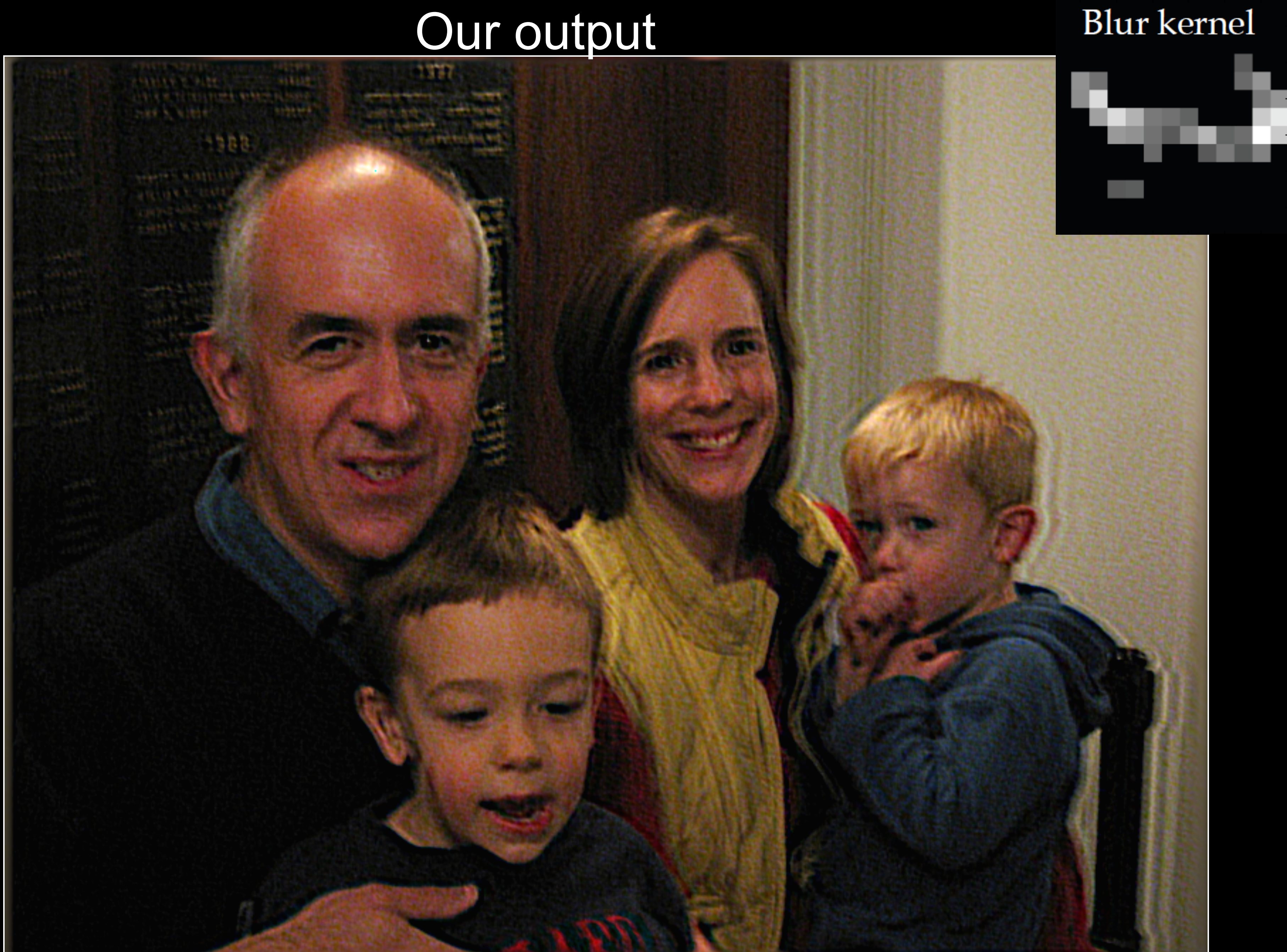


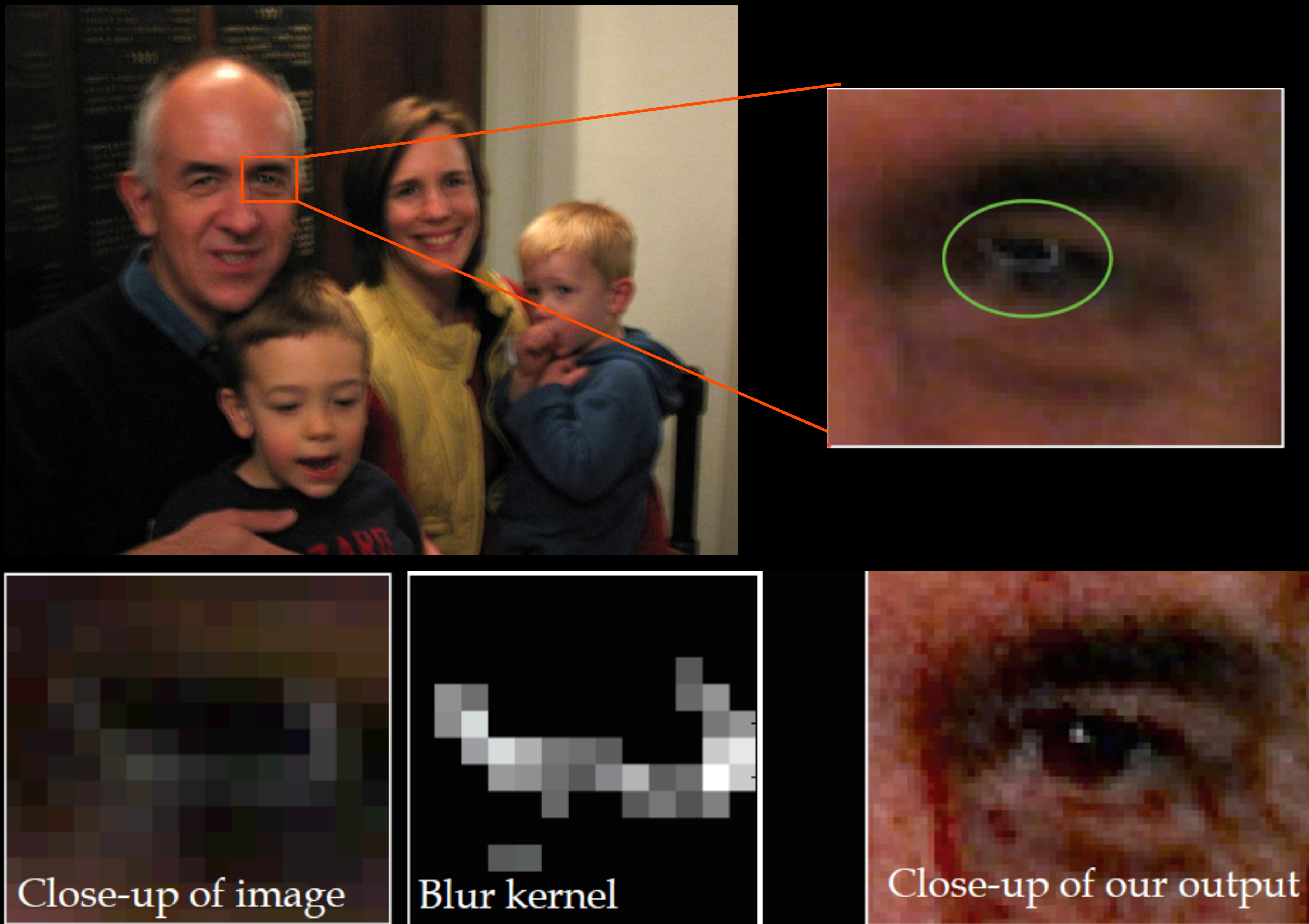
Matlab’s `deconvblind`



Photoshop sharpen more

Our output




Close-up of image
Blur kernel
Close-up of our output

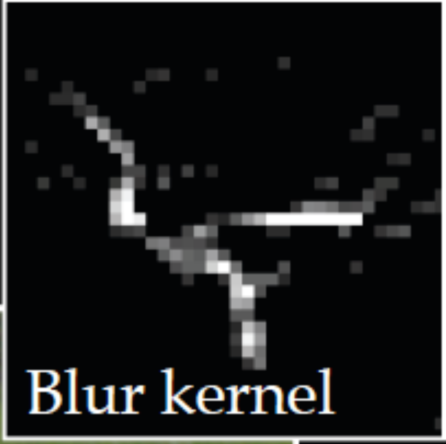
Original photograph

Our output

Blur kernel

Original photograph

Our output



Matlab's `deconvblind`

Original photograph


Our output
Blur kernel

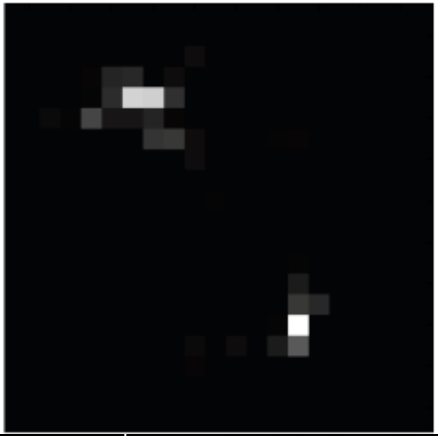
Close-up of bird

Original

Unsharp mask

Our output


Original photograph



Blur kernel

Image artifacts & estimated kernels

Blur kernels

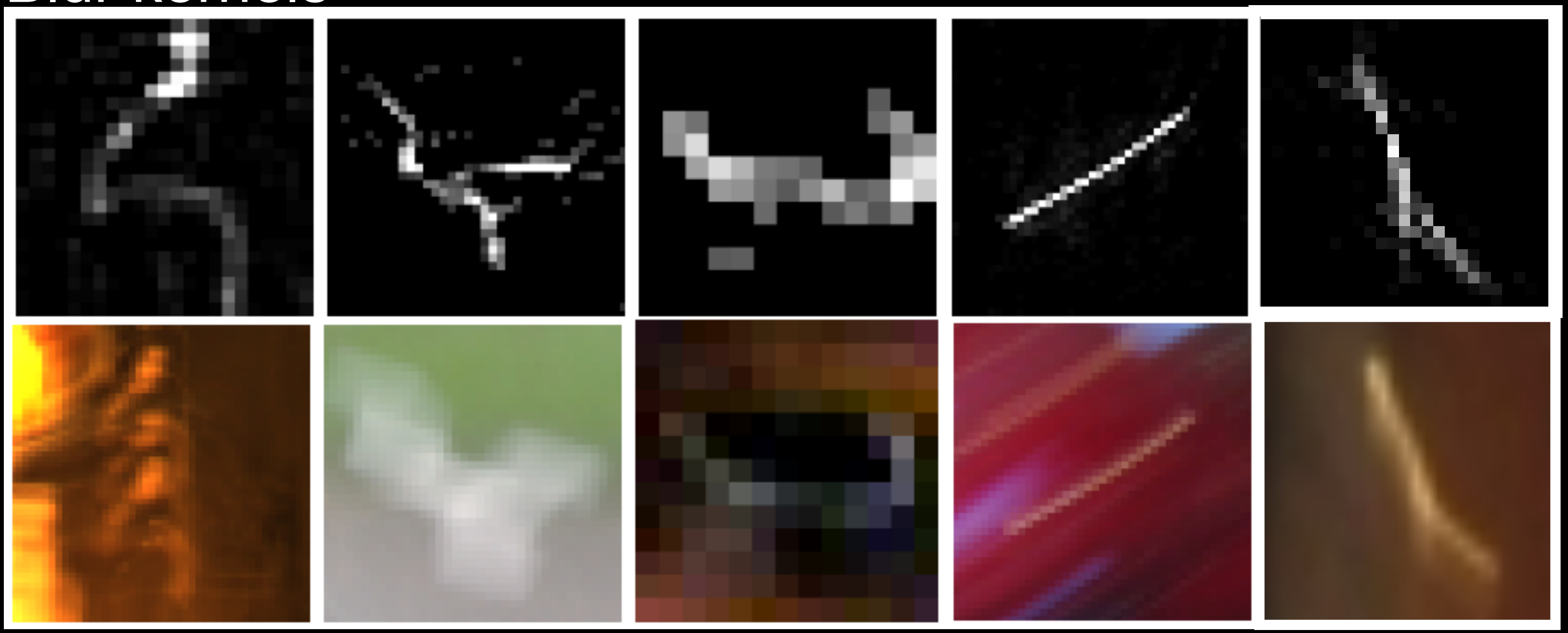


Image patterns

Note: blur kernels were inferred from large image patches, NOT the image patterns shown

Bayesian methods