

# Color and color constancy

6.869, MIT

(Bill Freeman)

Antonio Torralba

Sept. 12, 2013

# Why does a visual system need color?



<http://www.hobbyline.com/gr/pll/pll5019.jpg>

Wednesday, September 11, 13

Why does a visual system need color?  
(an incomplete list...)

# Why does a visual system need color? (an incomplete list...)

- To tell what food is edible.

# Why does a visual system need color? (an incomplete list...)

- To tell what food is edible.
- To distinguish material changes from shading changes.

# Why does a visual system need color? (an incomplete list...)

- To tell what food is edible.
- To distinguish material changes from shading changes.
- To group parts of one object together in a scene.

# Why does a visual system need color? (an incomplete list...)

- To tell what food is edible.
- To distinguish material changes from shading changes.
- To group parts of one object together in a scene.
- To find people's skin.

# Why does a visual system need color? (an incomplete list...)

- To tell what food is edible.
- To distinguish material changes from shading changes.
- To group parts of one object together in a scene.
- To find people's skin.
- Check whether a person's appearance looks normal/healthy.



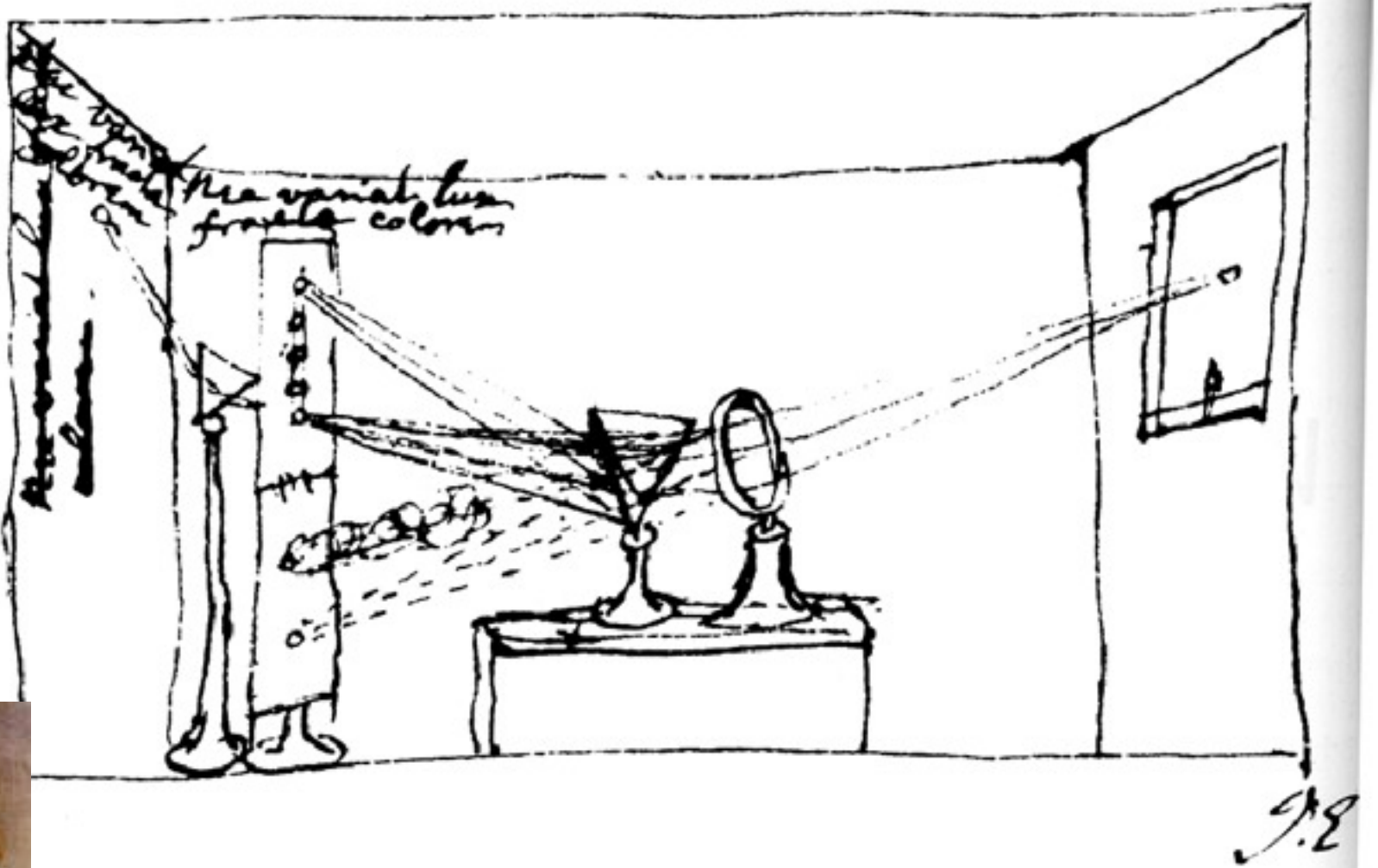
# Lecture outline

- Color physics.
- Color perception.

# Lecture outline

- Color physics.
- Color perception.

# Color

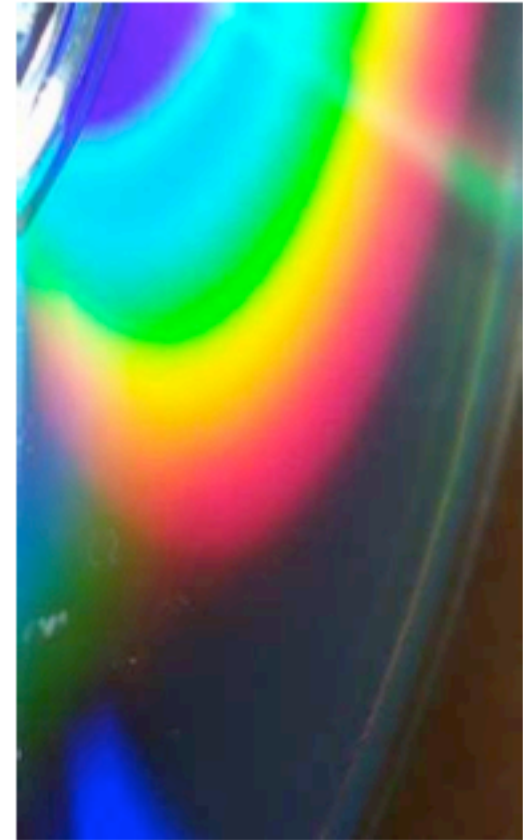


4.1 NEWTON'S SUMMARY DRAWING of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

From Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

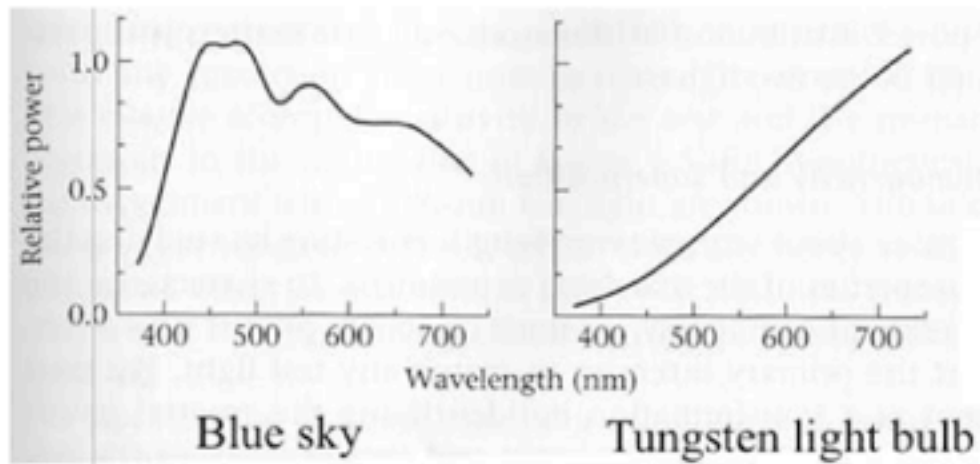


(a)



(b)

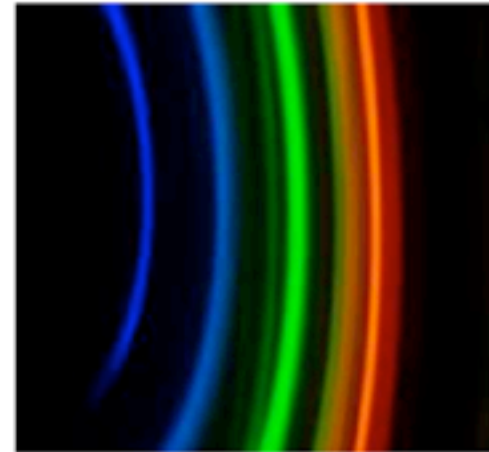
Figure 6.3: (a) A spectrograph constructed using a compact disk (CD). Light enters through a slit and is diffracting from the narrowly spaced lines of the CD. (b) Photograph of diffraction pattern from the CD, seen through hole at bottom left.



Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

(a)

(b)

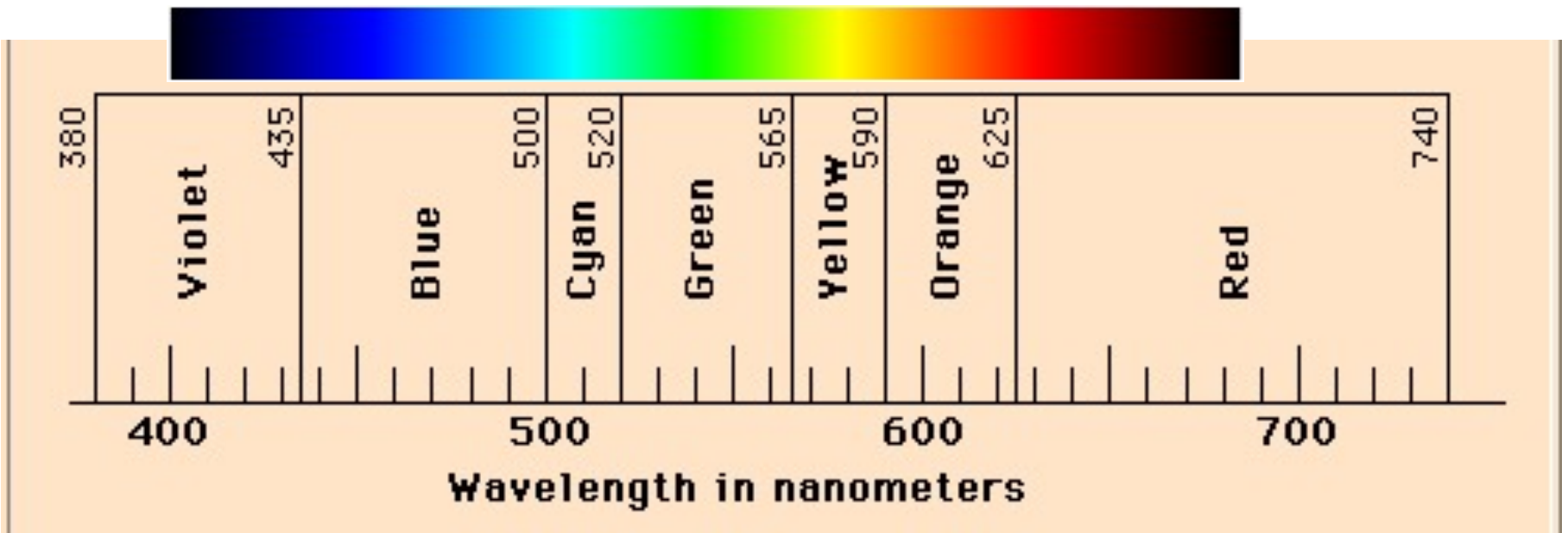


(d)

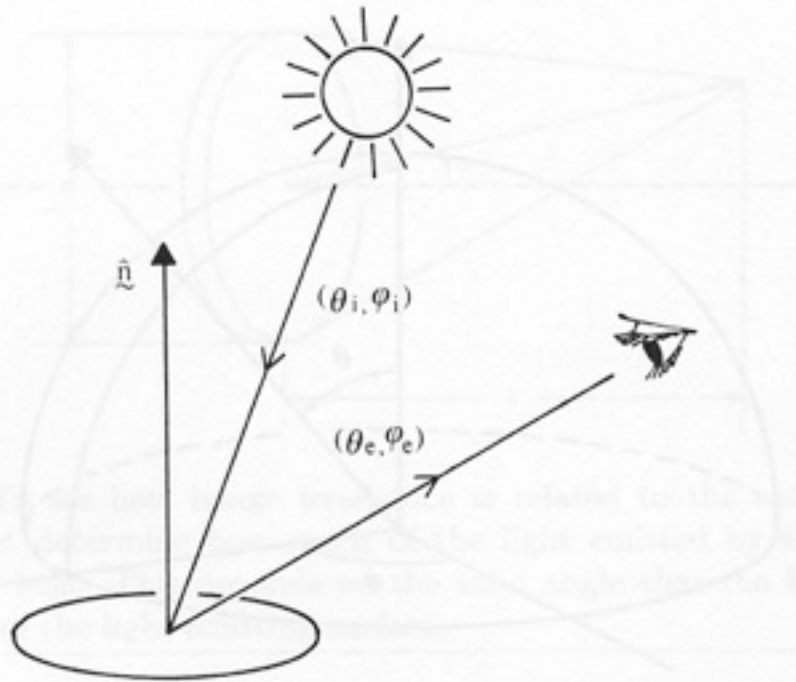
(c)

Figure 6.4: (a) and (b): Plots of the power spectra of blue sky and a tungsten light bulb. Photographs show (c) a fluorescent light and (d) its spectrum as viewed with the spectrograph of Fig. (6.3) (a).

# Spectral colors



<http://hyperphysics.phy-astr.gsu.edu/hbase/vision/specol.html#c2>

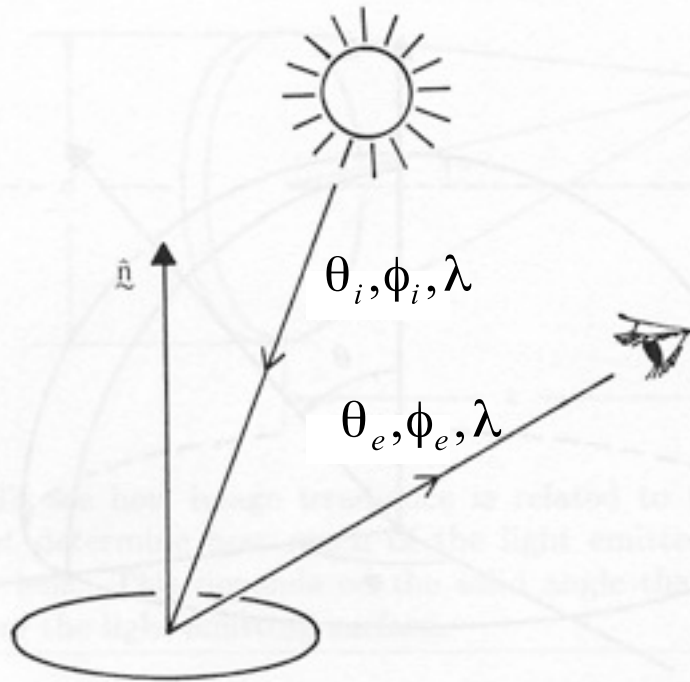


# Radiometry for color

Horn, 1986

**Figure 10-7.** The bidirectional reflectance distribution function is the ratio of the radiance of the surface patch as viewed from the direction  $(\theta_e, \phi_e)$  to the irradiance resulting from illumination from the direction  $(\theta_i, \phi_i)$ .

# Radiometry for color



Horn, 1986

**Figure 10-7.** The bidirectional reflectance distribution function is the ratio of the radiance of the surface patch as viewed from the direction  $(\theta_e, \phi_e)$  to the irradiance resulting from illumination from the direction  $(\theta_i, \phi_i)$ .

**Spectral radiance:** power in a specified direction, per unit area, per unit solid angle, per unit wavelength

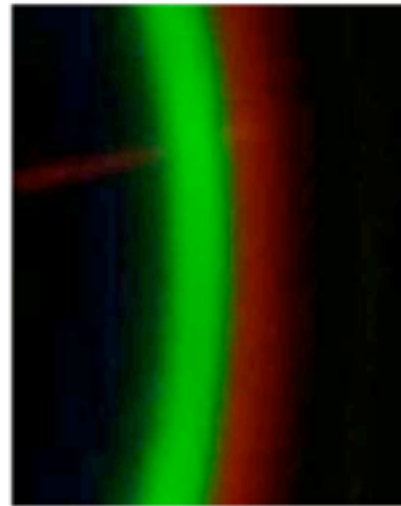
$$BRDF = f(\theta_i, \phi_i, \theta_e, \phi_e, \lambda) = \frac{L(\theta_e, \phi_e, \lambda)}{E(\theta_i, \phi_i, \lambda)}$$

**Spectral irradiance:** incident power per unit area, per unit wavelength





(a)



(b)

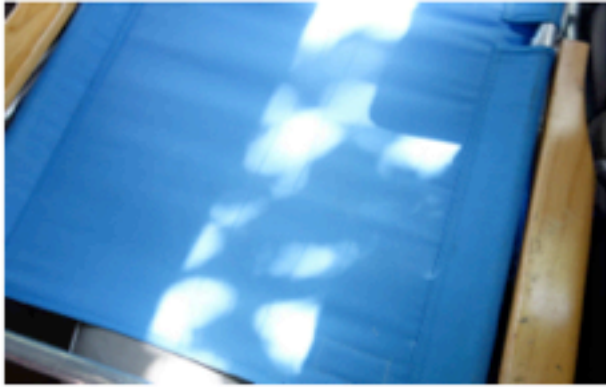


(c)

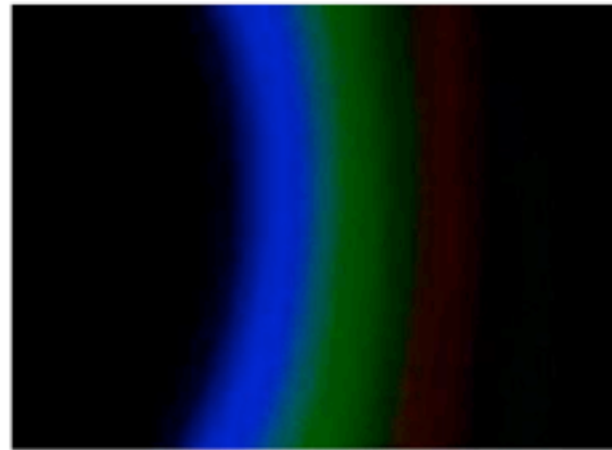


(d)

Figure 6.5: Some real-world objects and the reflected light spectra (photographed using Fig. (6.3) (a)) from outdoor viewing. (a) Leaf and (b) its reflected spectrum. (c) A red door and (d) its reflected



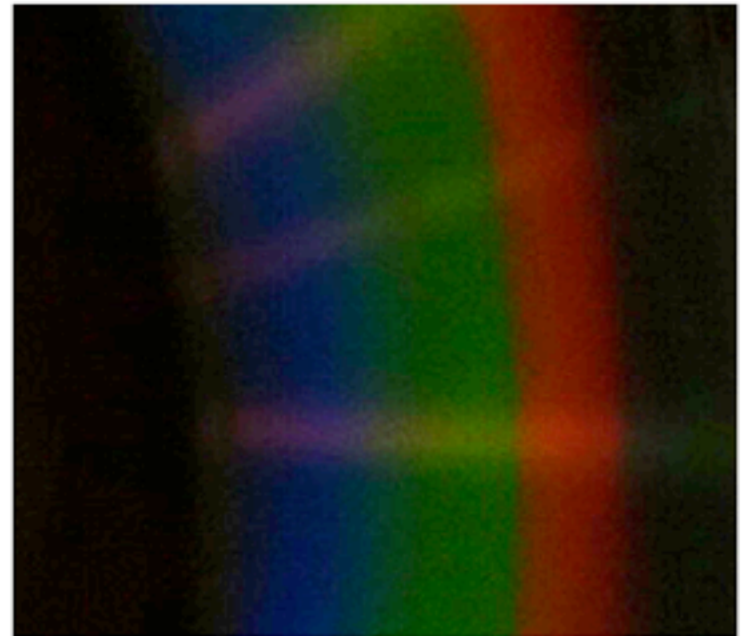
(a)



(b)



(c)



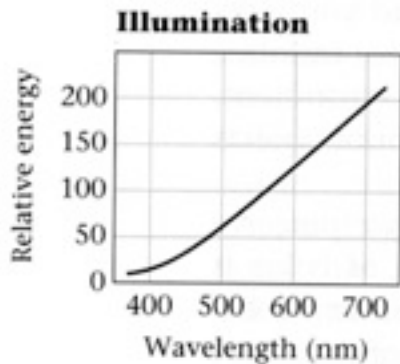
(d)

Figure 6.6: More real-world objects and the reflected light spectra. (a) Blue-green chair and (b) its reflected light. (c) Toby the dog and (d) his reflected spectrum.

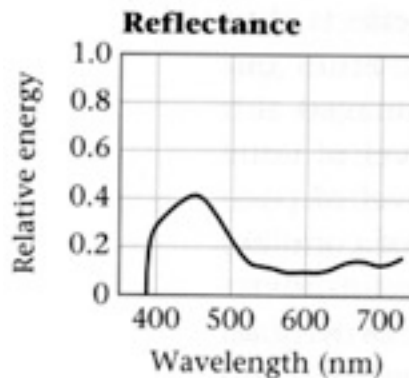
# Simplified rendering models: BRDF $\rightarrow$ reflectance



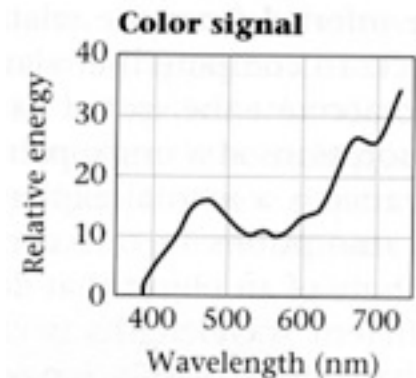
For diffuse reflections, we replace the BRDF calculation with a wavelength-by-wavelength scalar multiplier



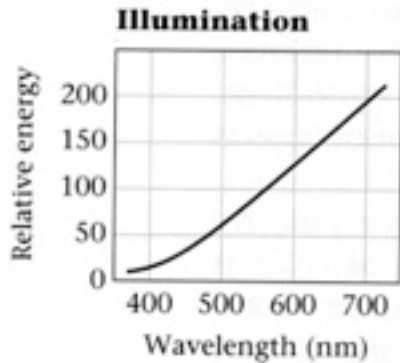
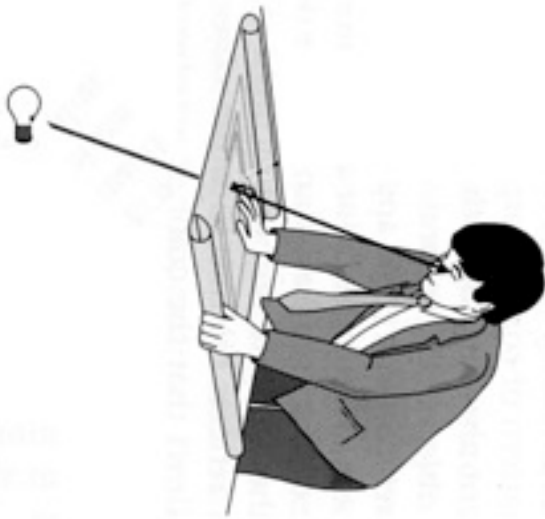
• \*



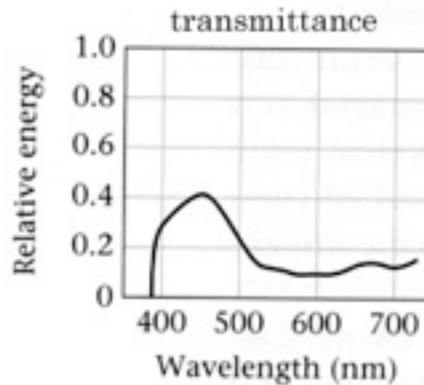
=



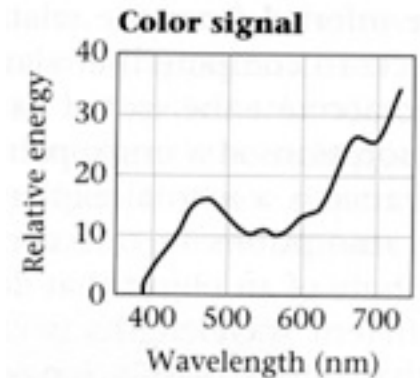
# Simplified rendering models: transmittance



• \*

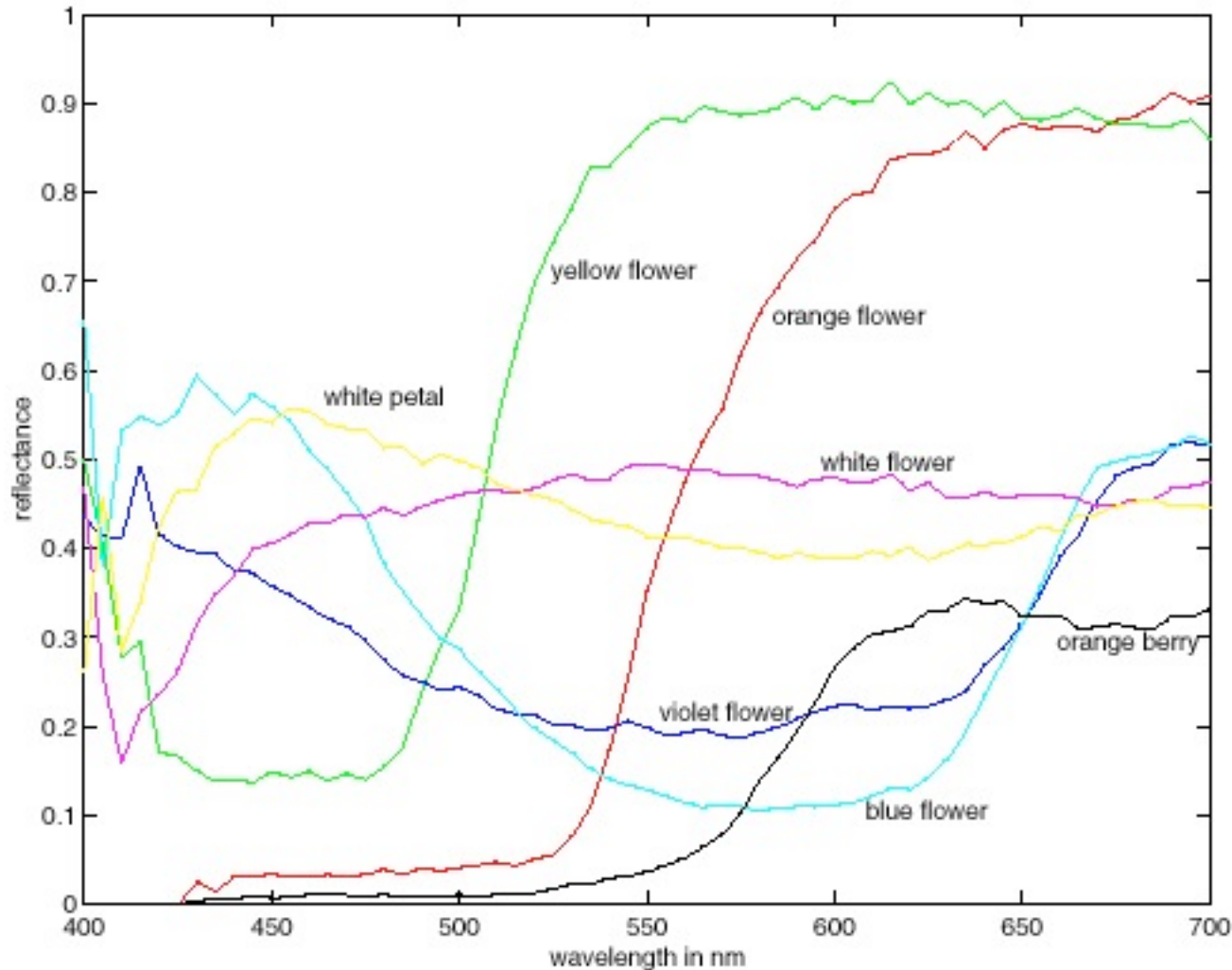


=



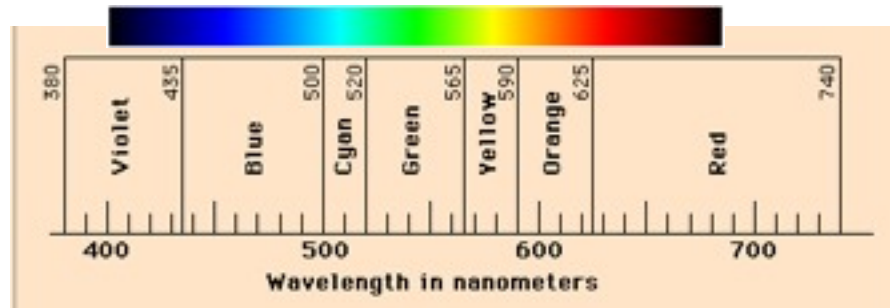
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

# Some reflectance spectra

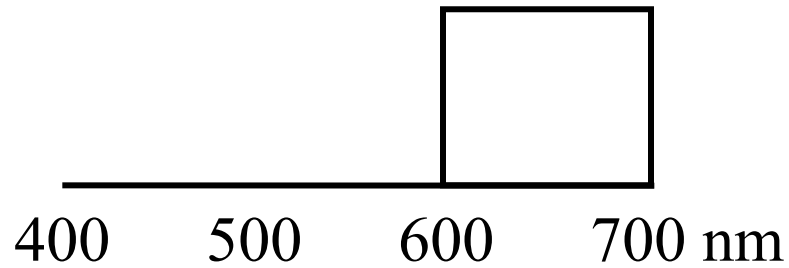
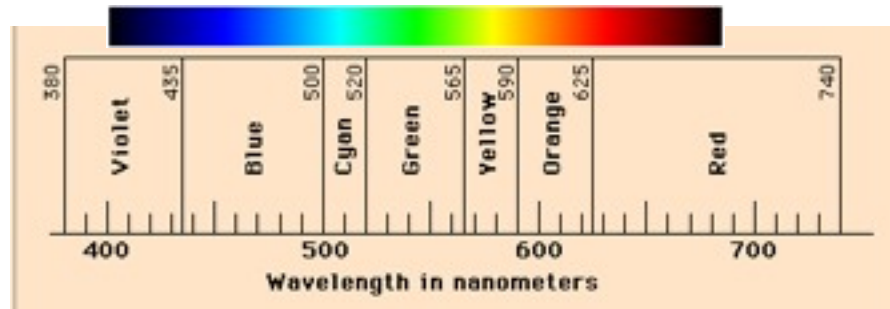


Spectral albedoes for several different leaves, with color names attached. Notice that different colours typically have different spectral albedo, but that different spectral albedoes may result in the same perceived color (compare the two whites). Spectral albedoes are typically quite smooth functions. Measurements by E.Koivisto.

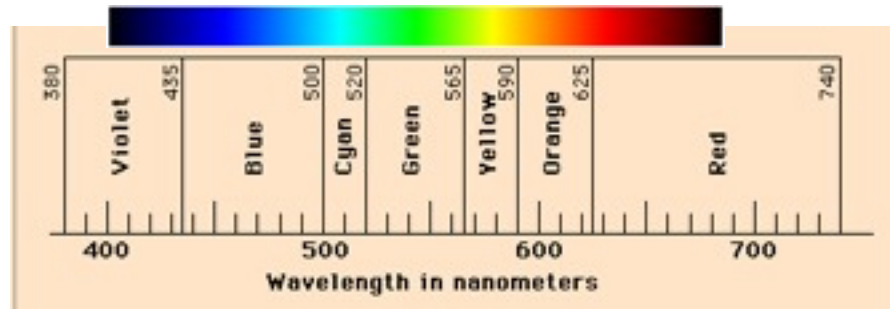
# Color names for cartoon spectra



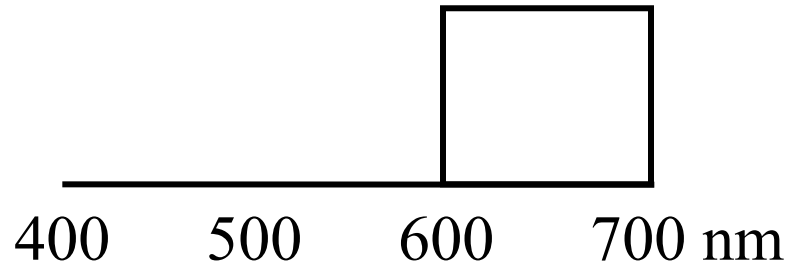
# Color names for cartoon spectra



# Color names for cartoon spectra

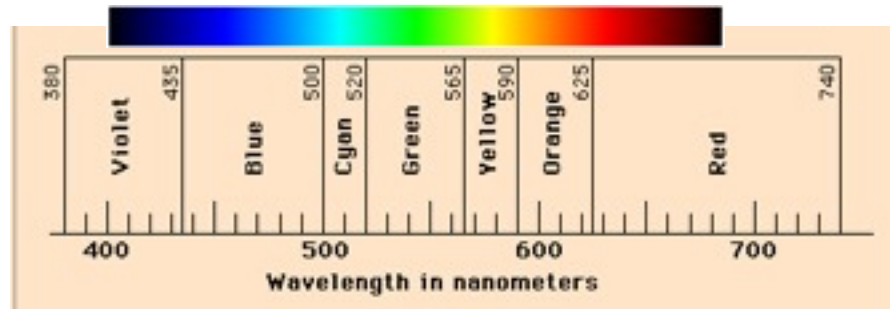


red

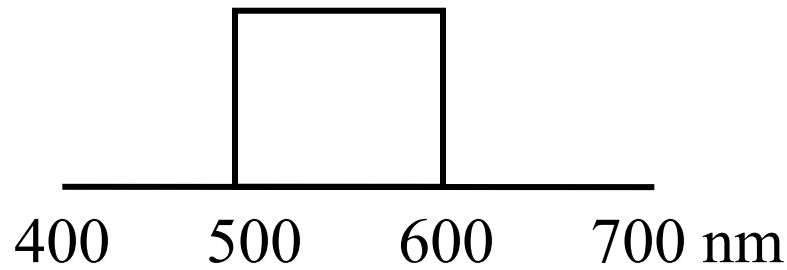
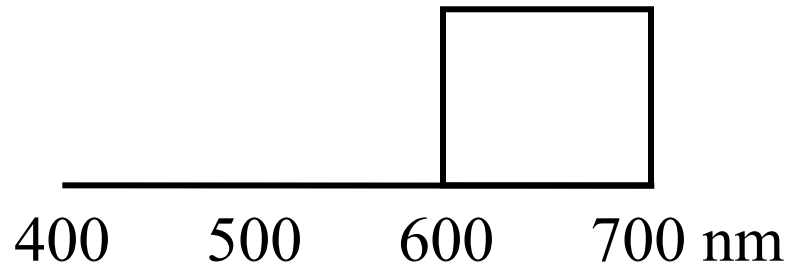




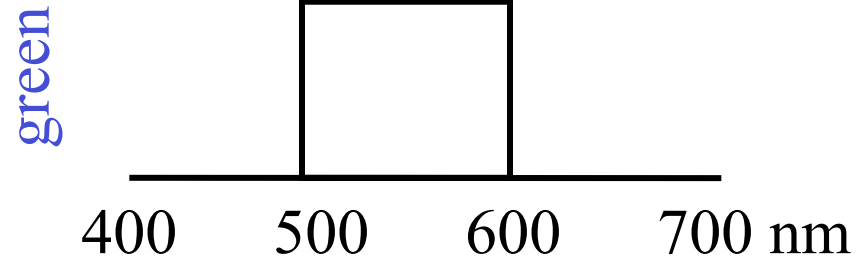
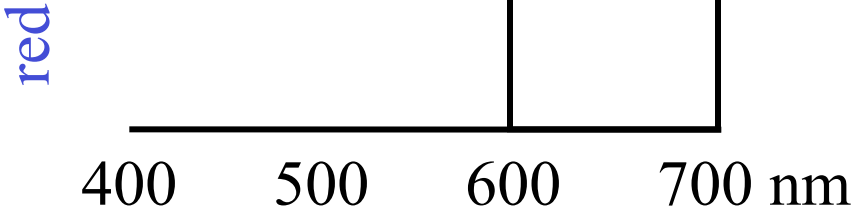
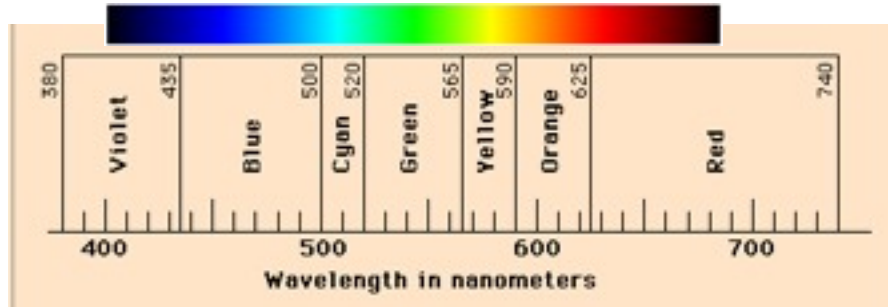
# Color names for cartoon spectra



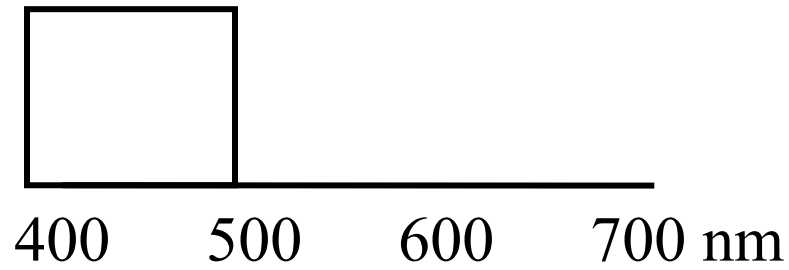
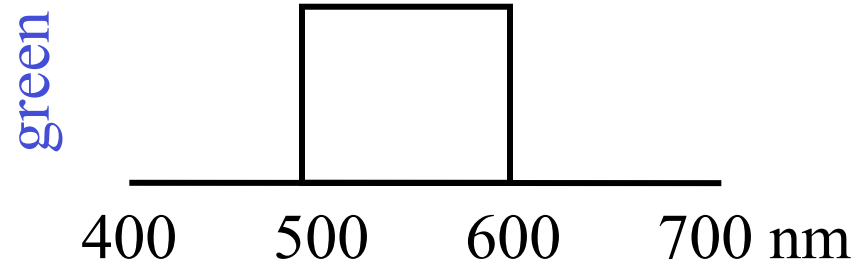
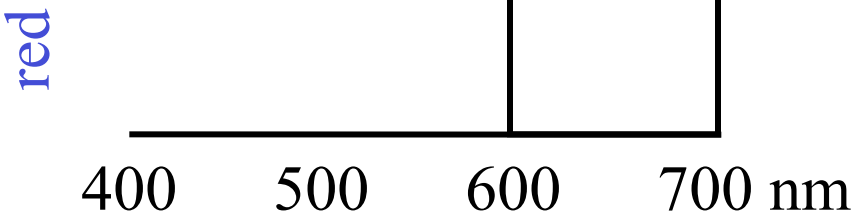
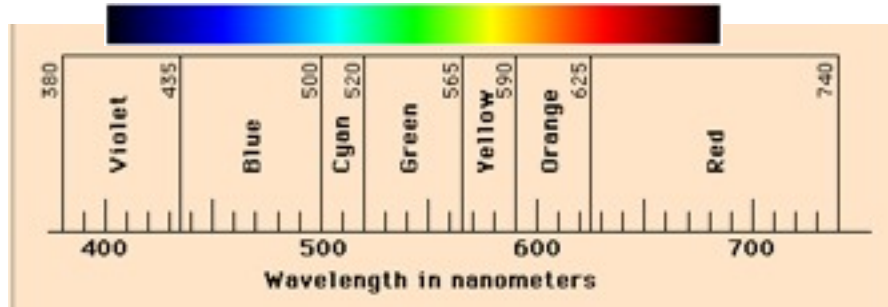
red



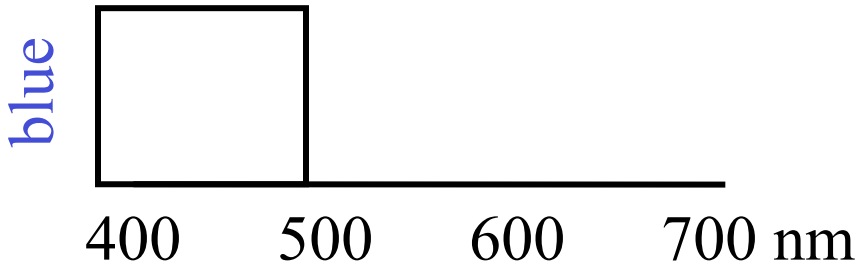
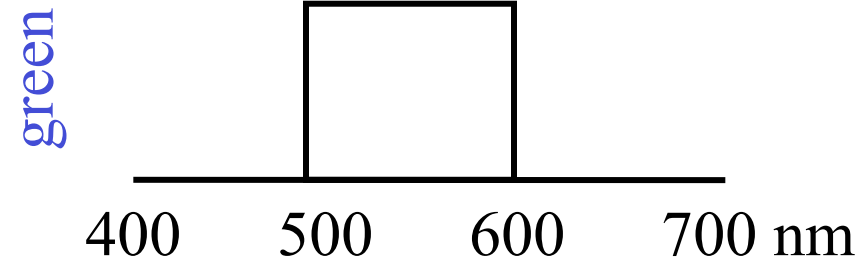
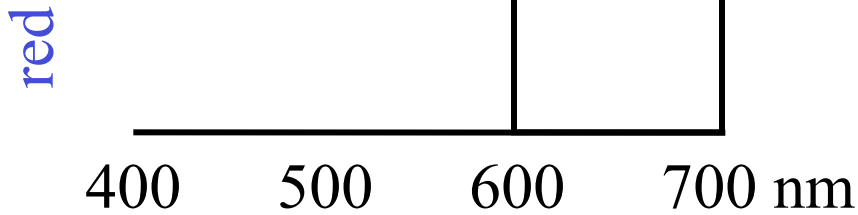
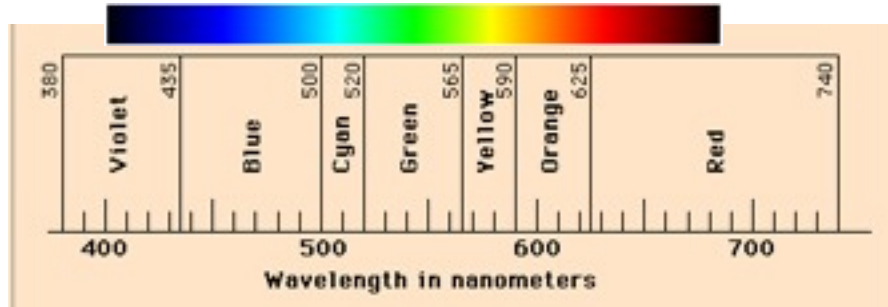
# Color names for cartoon spectra



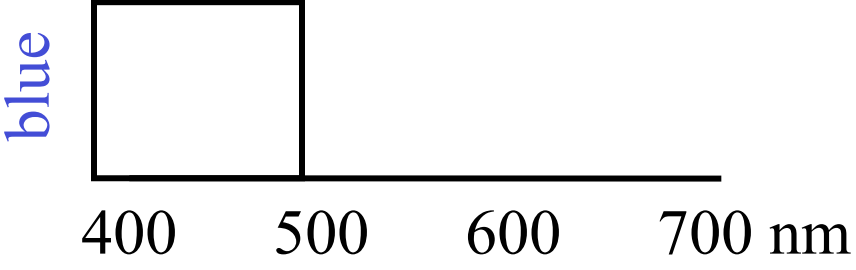
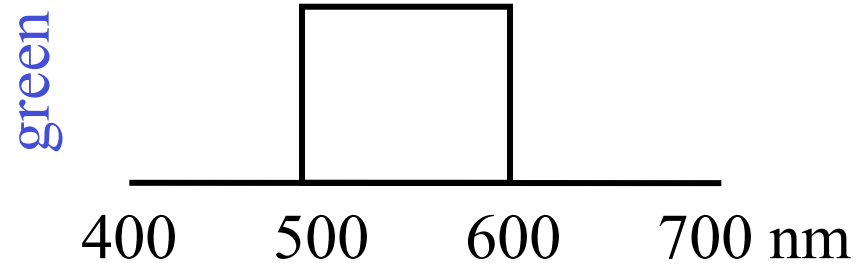
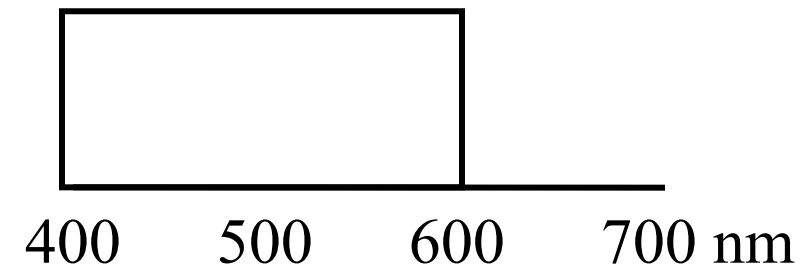
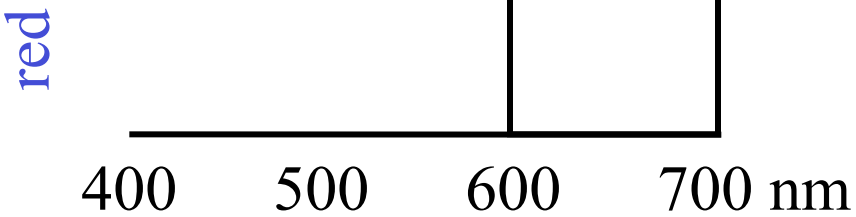
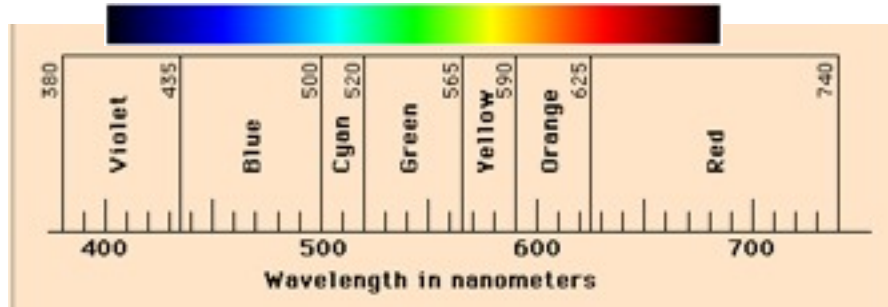
# Color names for cartoon spectra



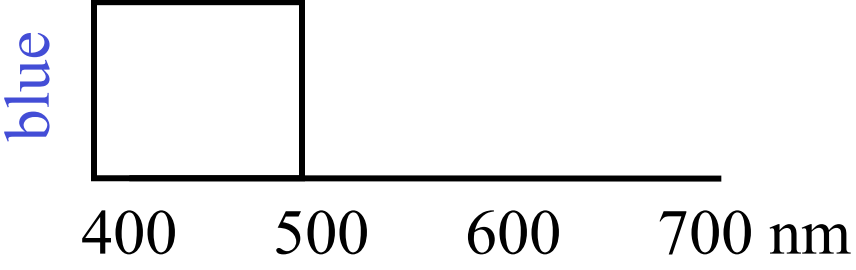
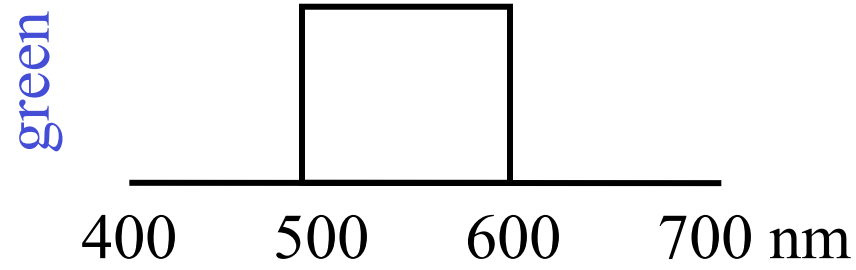
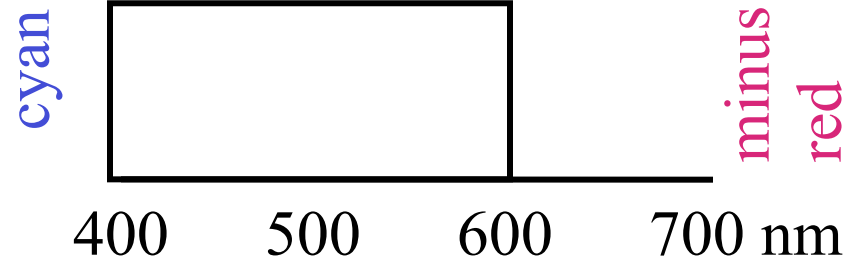
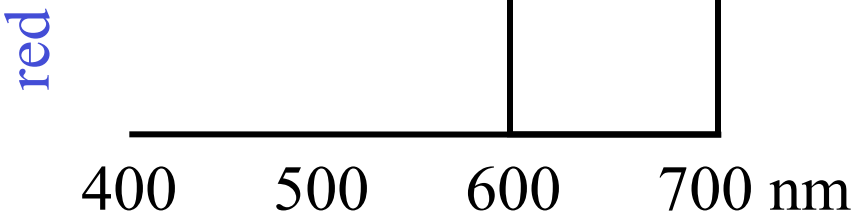
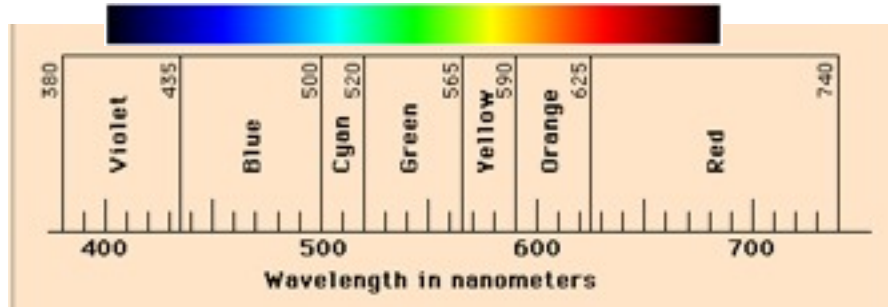
# Color names for cartoon spectra



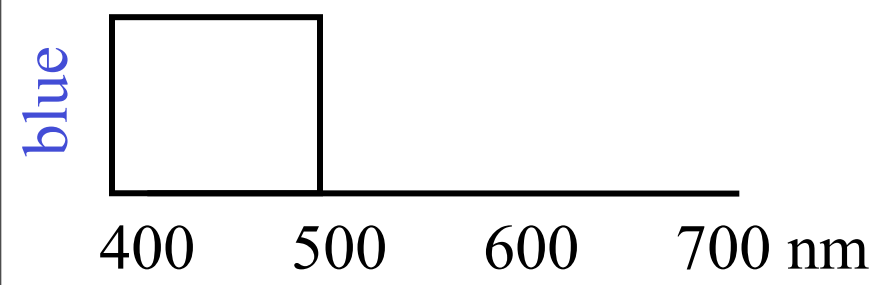
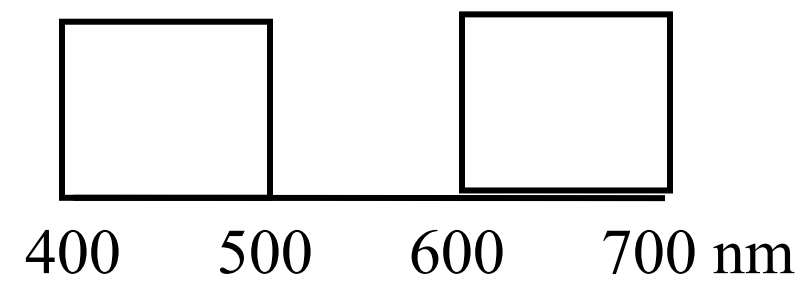
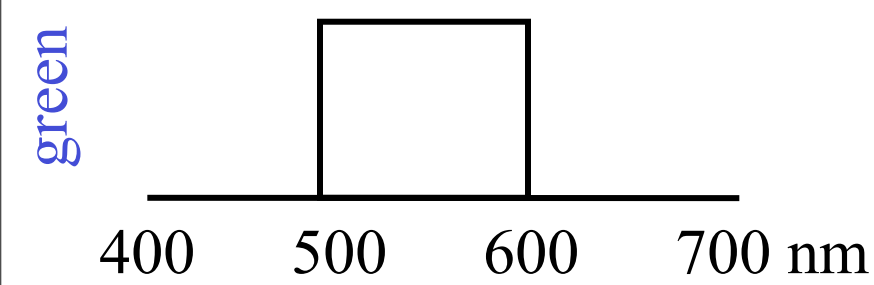
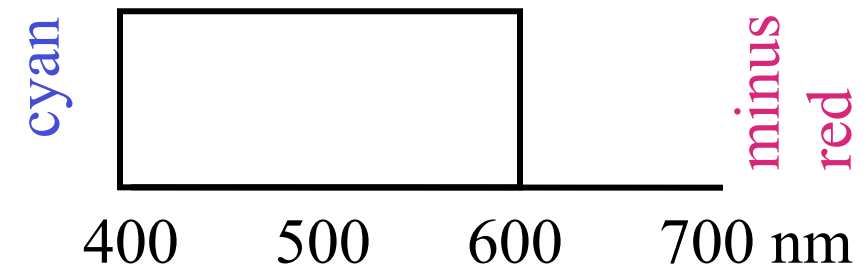
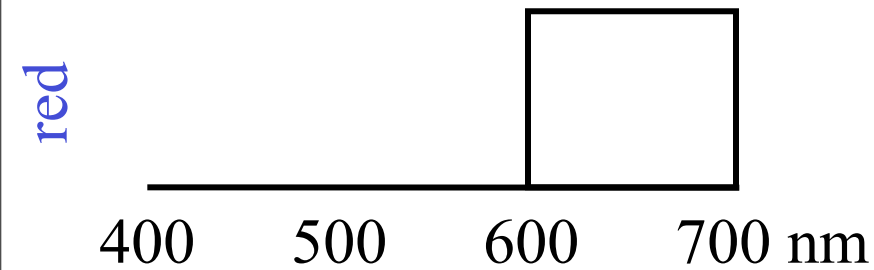
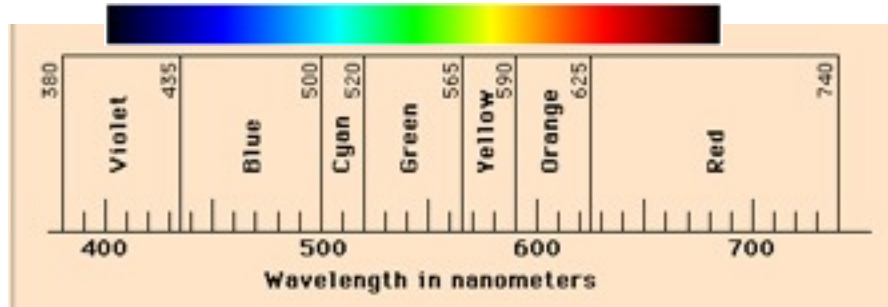
# Color names for cartoon spectra



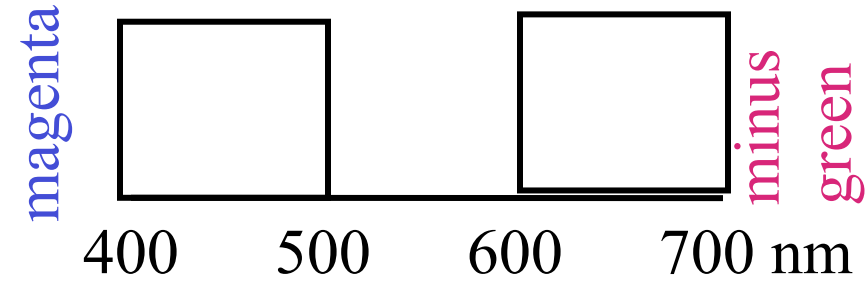
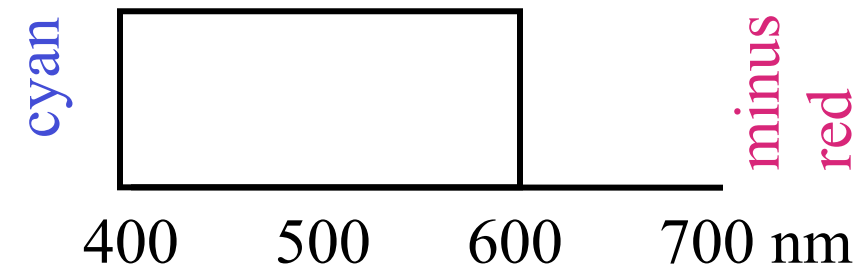
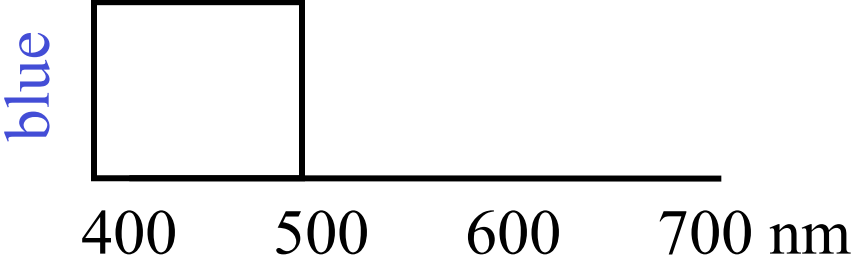
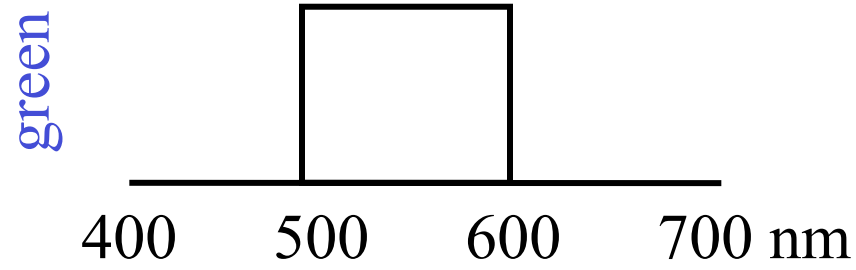
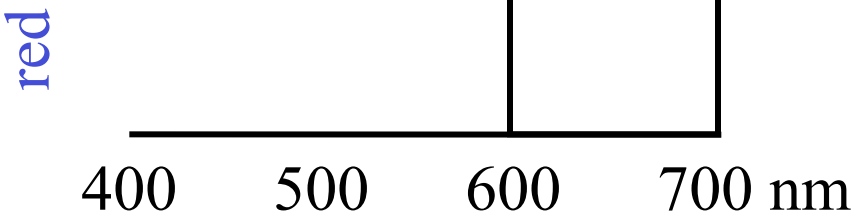
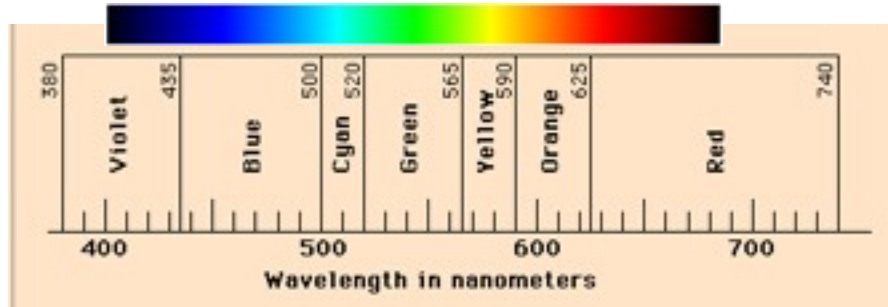
# Color names for cartoon spectra



# Color names for cartoon spectra



# Color names for cartoon spectra

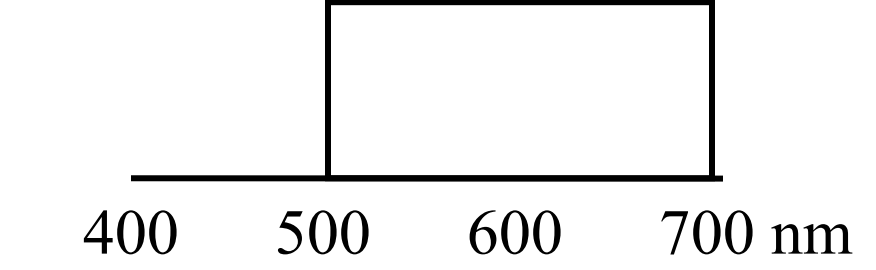
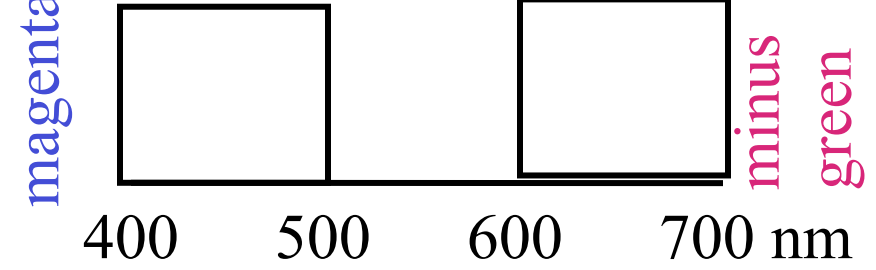
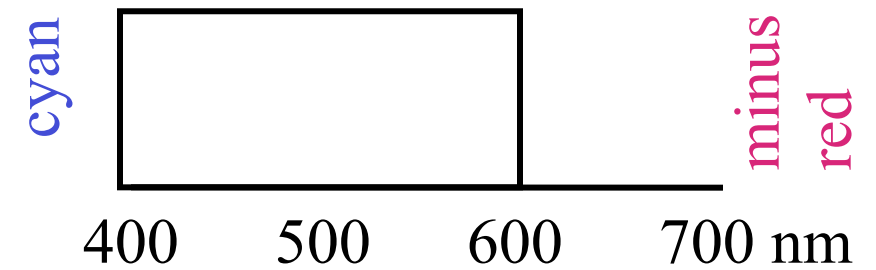
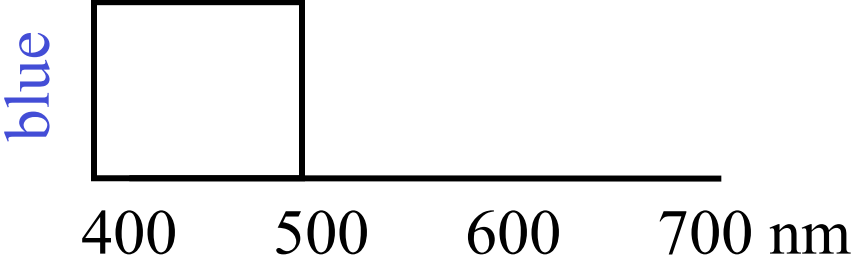
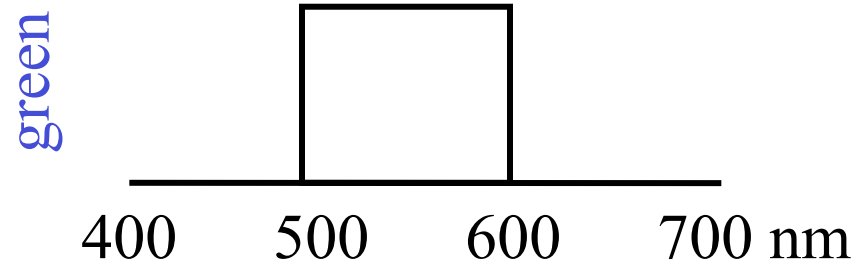
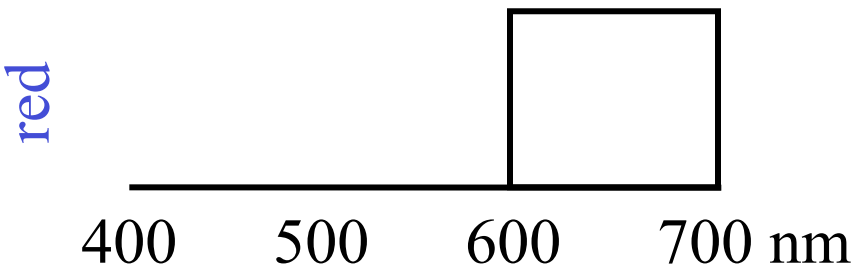
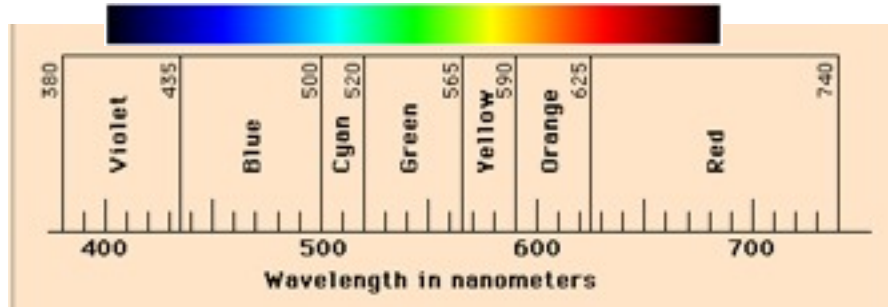


minus red

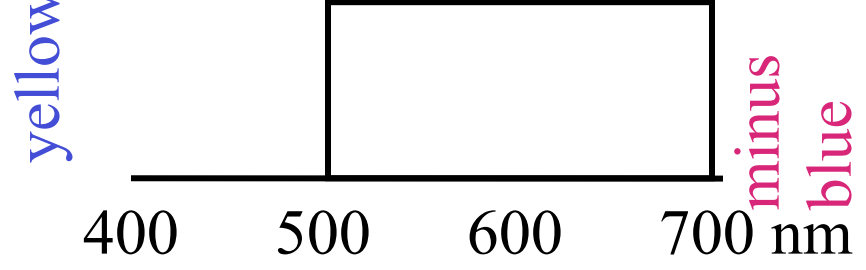
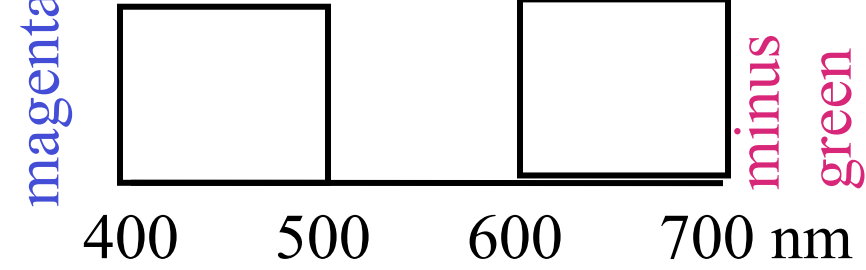
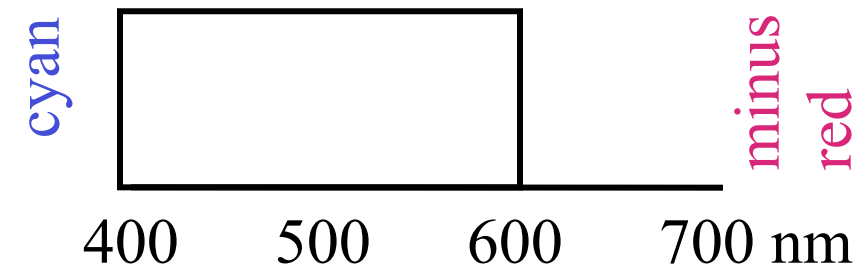
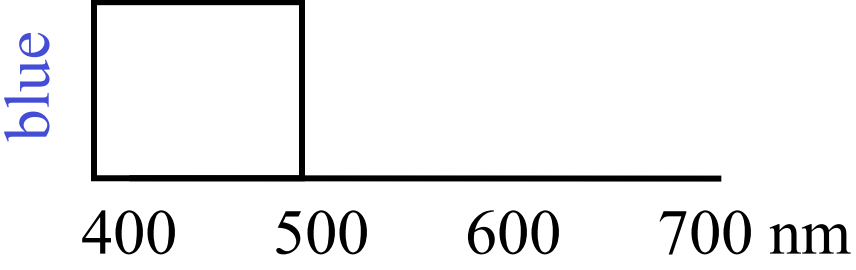
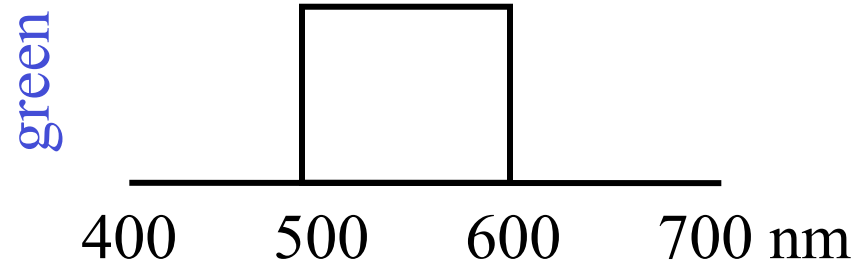
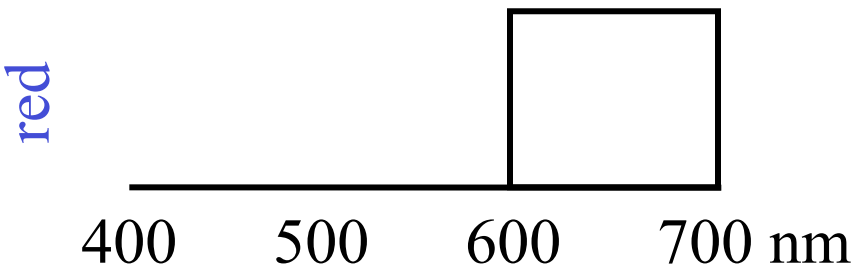
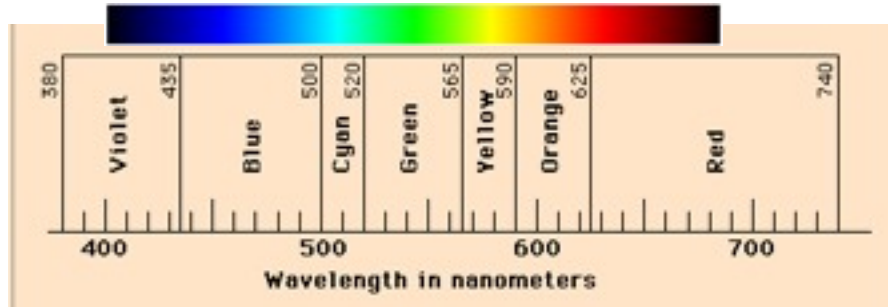
minus green



# Color names for cartoon spectra



# Color names for cartoon spectra



minus  
red

minus  
green

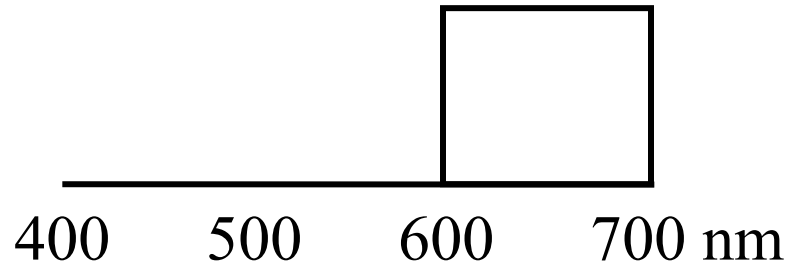
minus  
blue

# Additive color mixing

# Additive color mixing

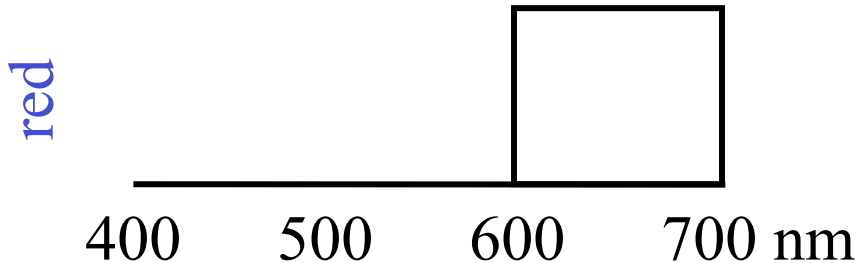
When colors combine by *adding* the color spectra.  
Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.

# Additive color mixing



When colors combine by *adding* the color spectra. Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.

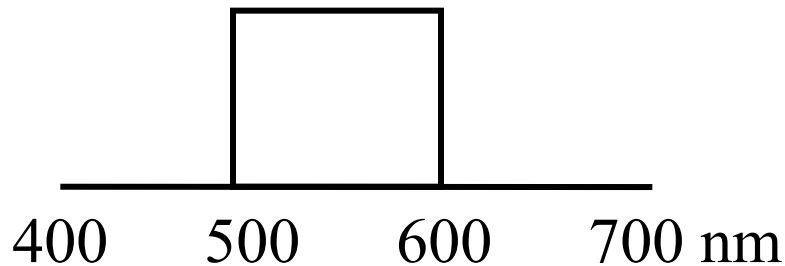
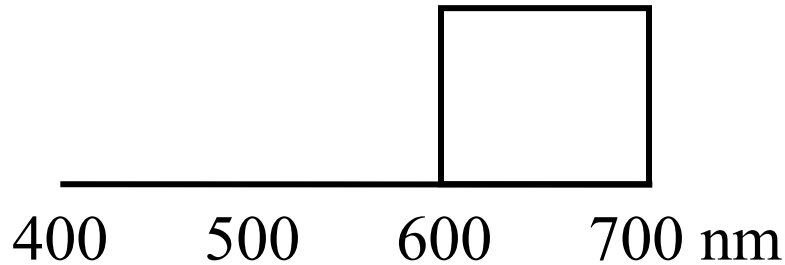
# Additive color mixing



When colors combine by *adding* the color spectra. Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.

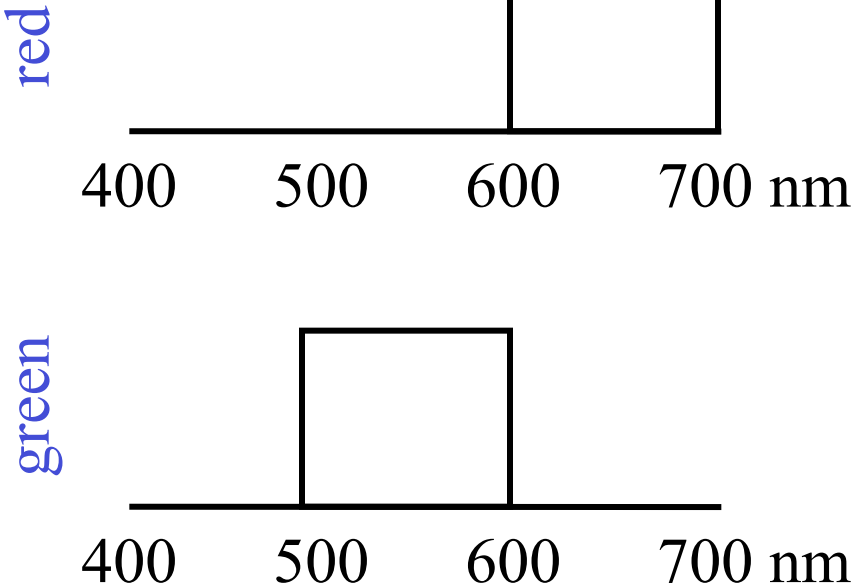
# Additive color mixing

red



When colors combine by *adding* the color spectra. Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.

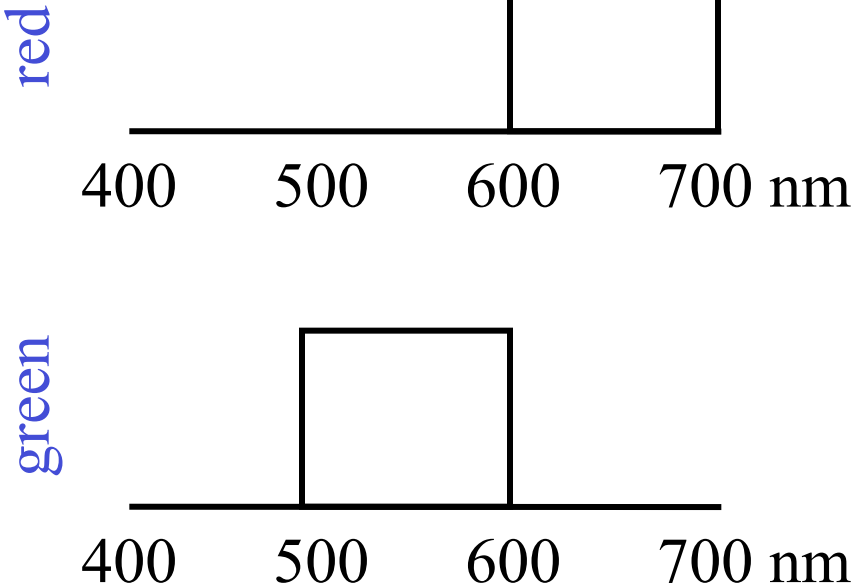
# Additive color mixing



When colors combine by *adding* the color spectra. Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.



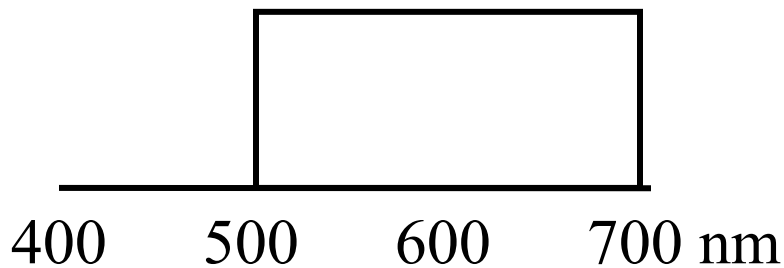
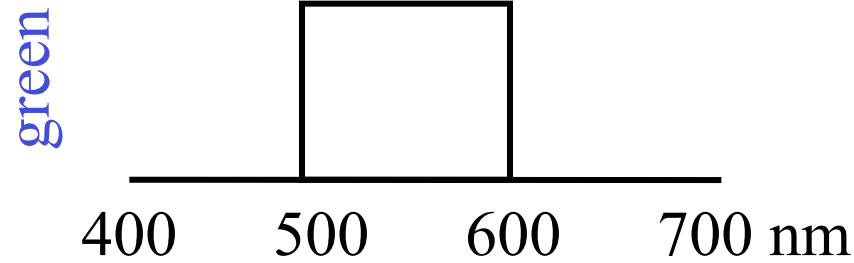
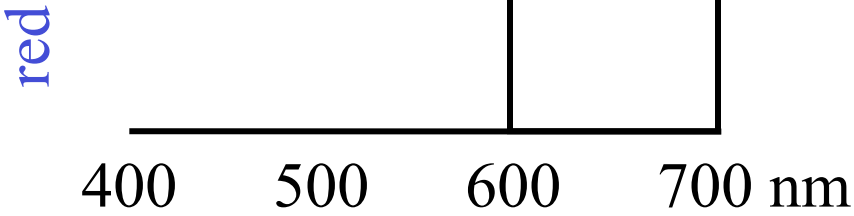
# Additive color mixing



When colors combine by *adding* the color spectra. Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.

Red and green make...

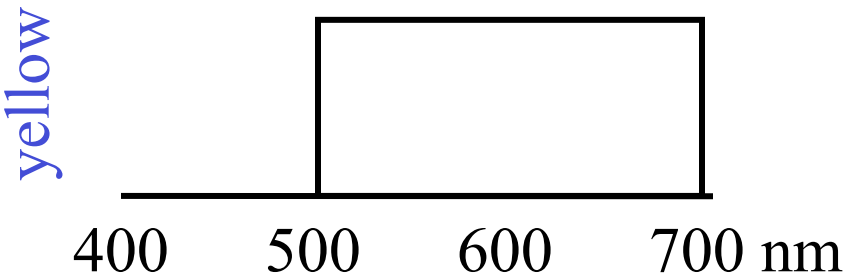
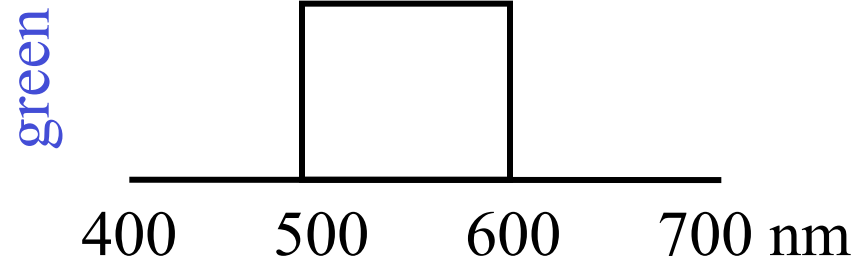
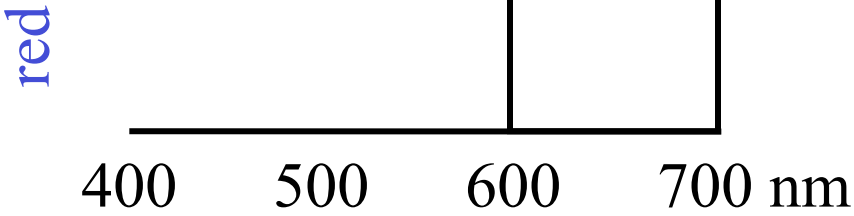
# Additive color mixing



When colors combine by *adding* the color spectra. Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.

Red and green make...

# Additive color mixing



When colors combine by *adding* the color spectra. Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.

Red and green make...

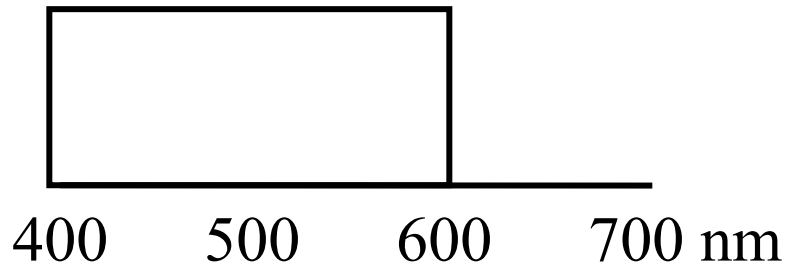
Yellow!

# Subtractive color mixing

# Subtractive color mixing

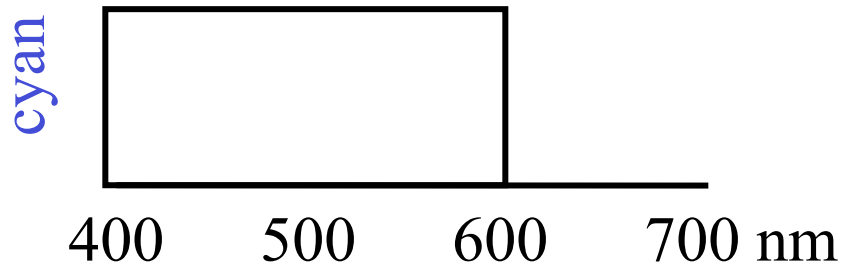
When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

# Subtractive color mixing



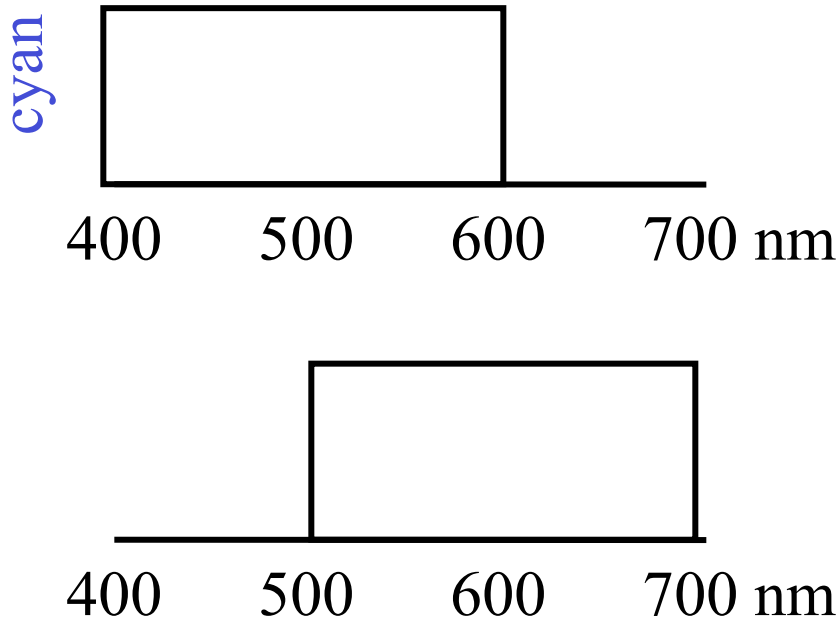
When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

# Subtractive color mixing



When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

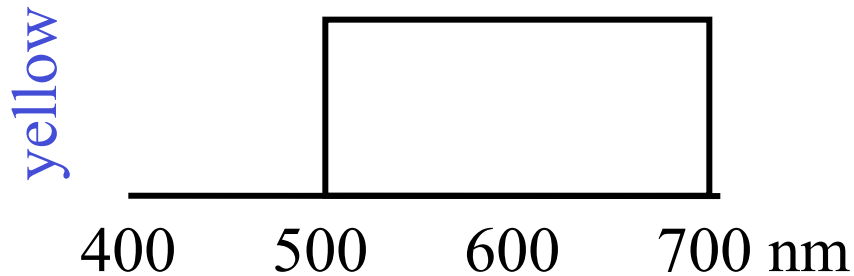
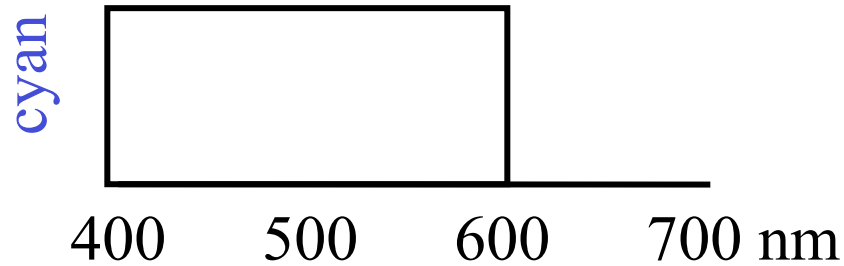
# Subtractive color mixing



When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

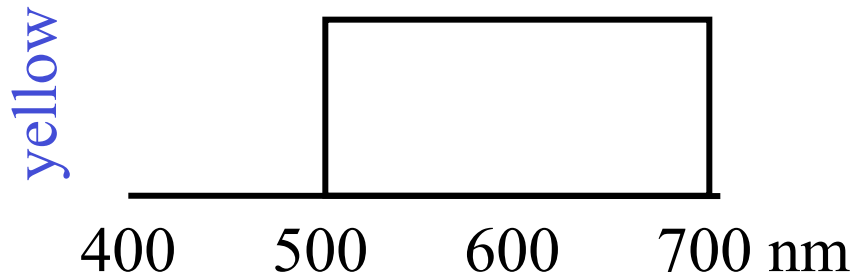
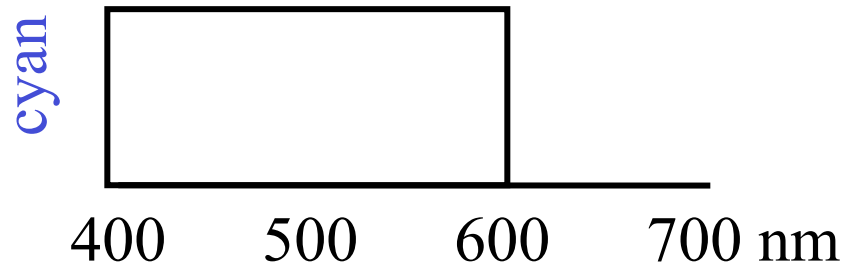


# Subtractive color mixing



When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

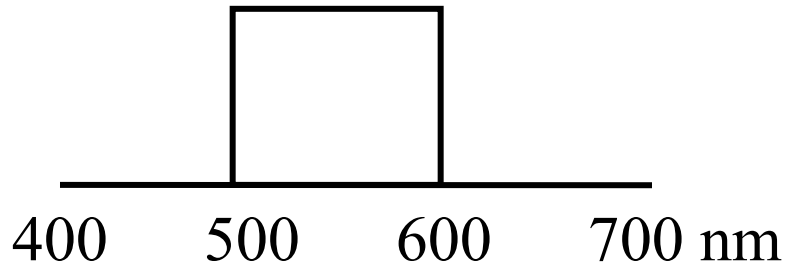
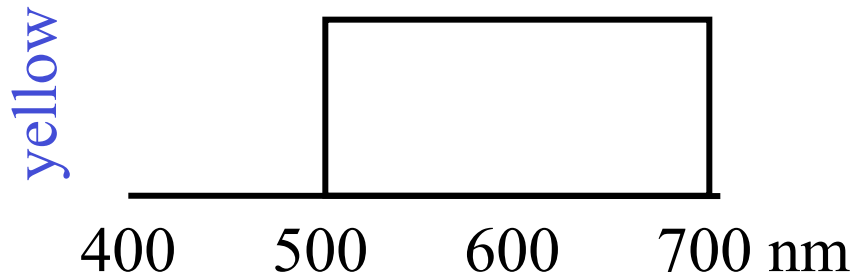
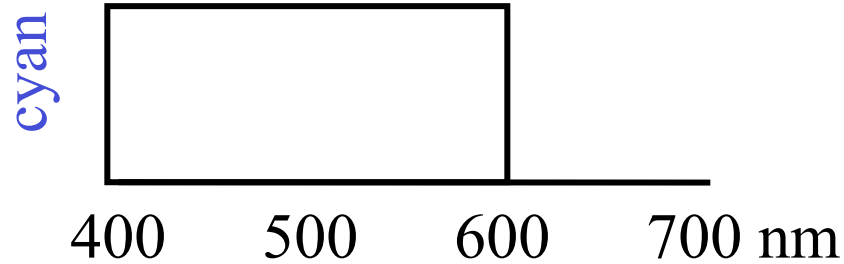
# Subtractive color mixing



When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

Cyan and yellow (in crayons, called “blue” and yellow) make...

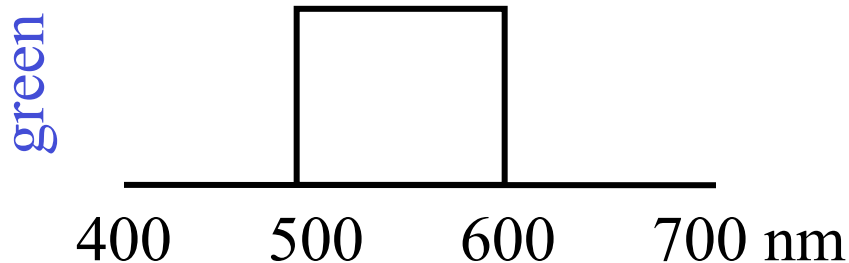
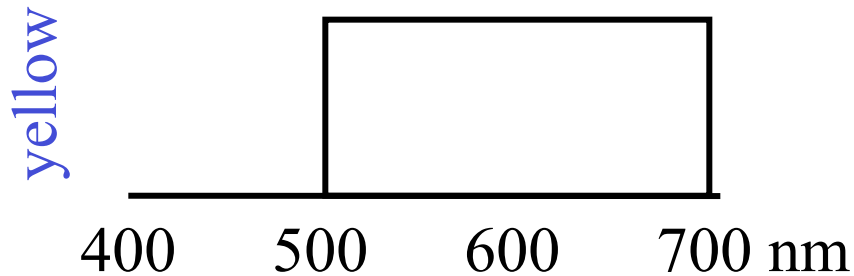
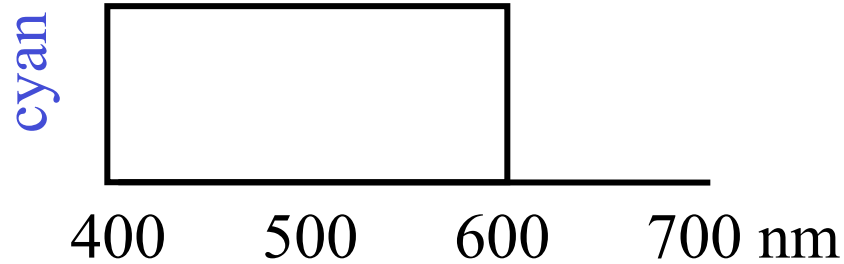
# Subtractive color mixing



When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

Cyan and yellow (in crayons, called “blue” and yellow) make...

# Subtractive color mixing



When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

Cyan and yellow (in crayons, called “blue” and yellow) make...

Green!

# Overhead projector demo

# Overhead projector demo

Subtractive color mixing

# Low-dimensional models for color spectra

$$\begin{pmatrix} \vdots \\ a(\lambda) \\ \vdots \end{pmatrix} \approx \begin{pmatrix} \vdots & \vdots & \vdots \\ a_1(\lambda) & a_2(\lambda) & a_3(\lambda) \\ \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

How to find a linear model for color spectra:

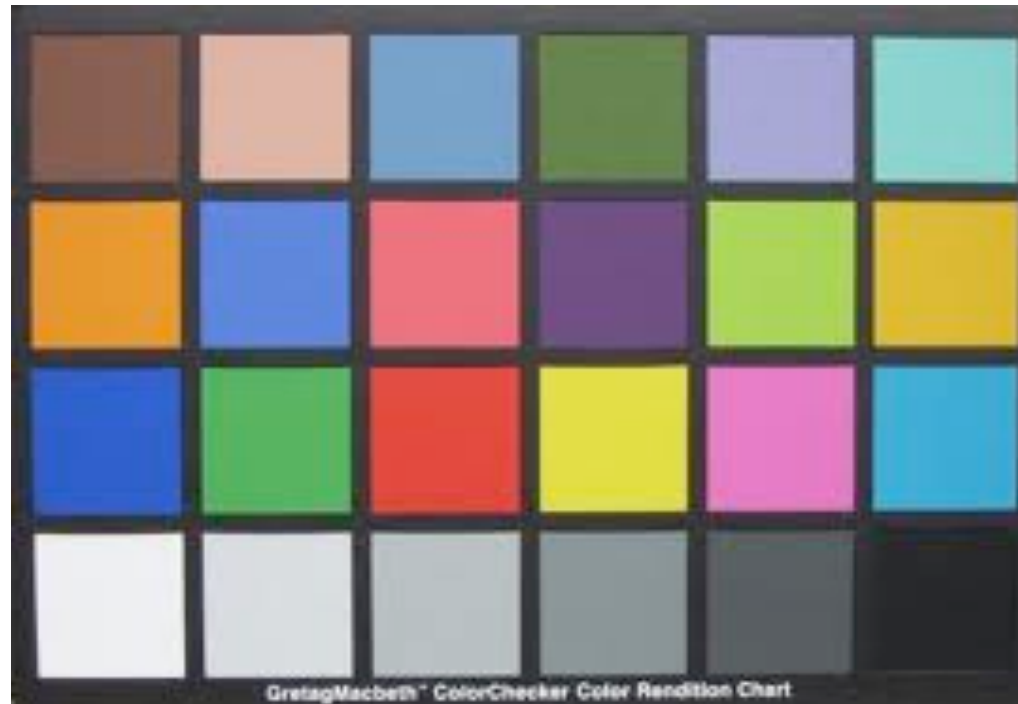
--form a matrix,  $D$ , of measured spectra, 1 spectrum per column.

-- $[u, s, v] = \text{svd}(D)$  satisfies  $D = u*s*v'$

--the first  $n$  columns of  $u$  give the best (least-squares optimal)  
n-dimensional linear bases for the data,  $D$ :

$$D \approx u(:,1:n) * s(1:n,1:n) * v(1:n,:)'$$

# Macbeth Color Checker







My Macbeth Color Checker Tattoo

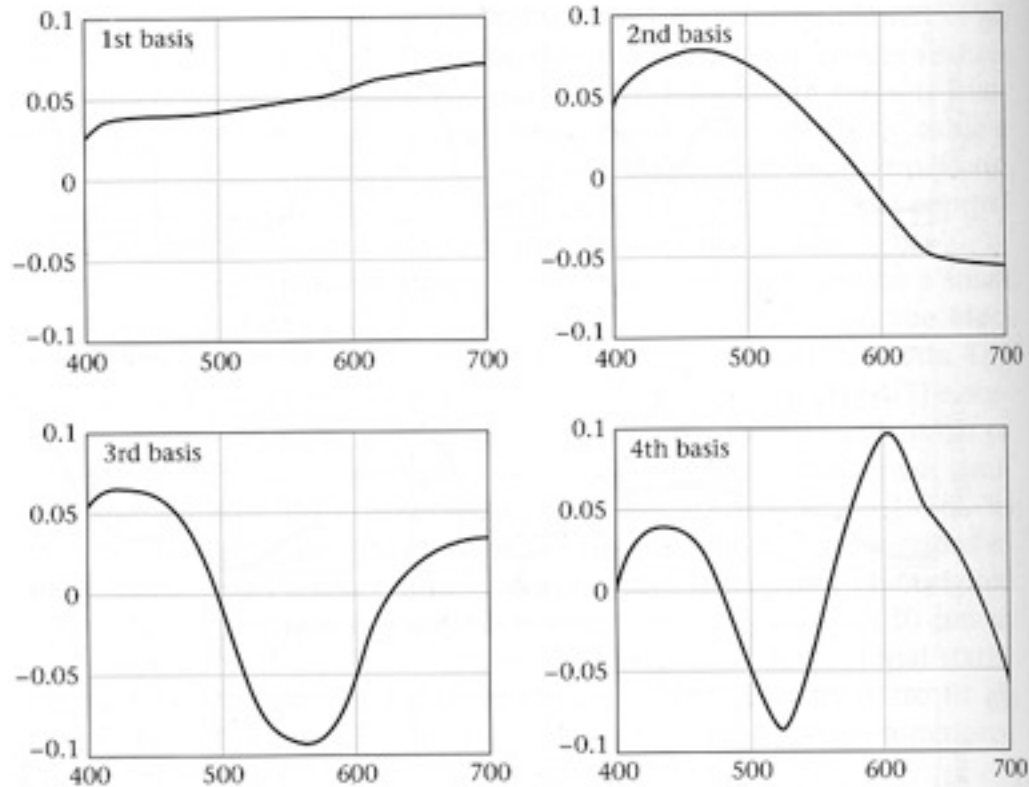
I think I have all the other color checker photos beat...

Yes, the tattoo is real.  
No, it is not a rubik's cube.

THIS PHOTOGRAPH IS COPYRIGHT 2007  
THE X-RITE CORPORATION!

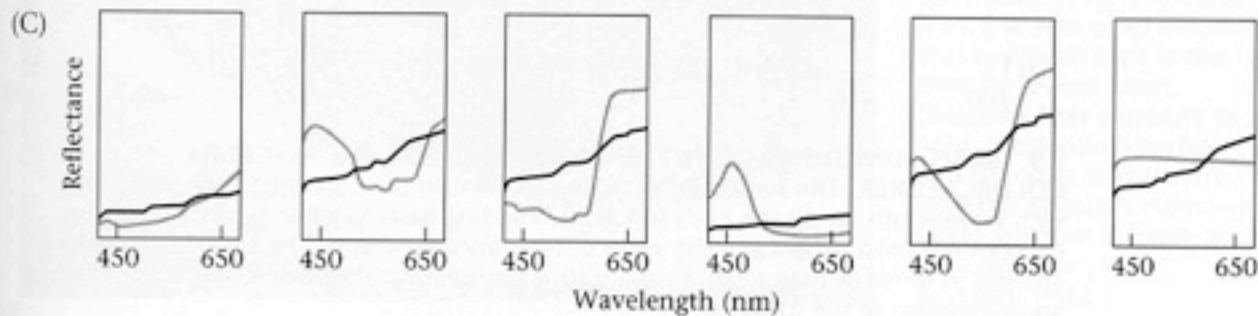
A photograph from this session can be viewed  
on the X-Rite Website: [www.xrite.com/  
top\\_munsell.aspx](http://www.xrite.com/top_munsell.aspx)

# Basis functions for Macbeth color checker



**9.9 BASIS FUNCTIONS OF THE LINEAR MODEL FOR THE MACBETH COLORCHECKER.** The surface-reflectance functions in the collection vary smoothly with wavelength, as do the basis functions. The first basis function is all positive and explains the most variance in the surface-reflectance functions. The basis functions are ordered in terms of their relative significance for reducing the error in the linear-model approximation to the surfaces.

# Fitting color spectra with low-dimensional linear models

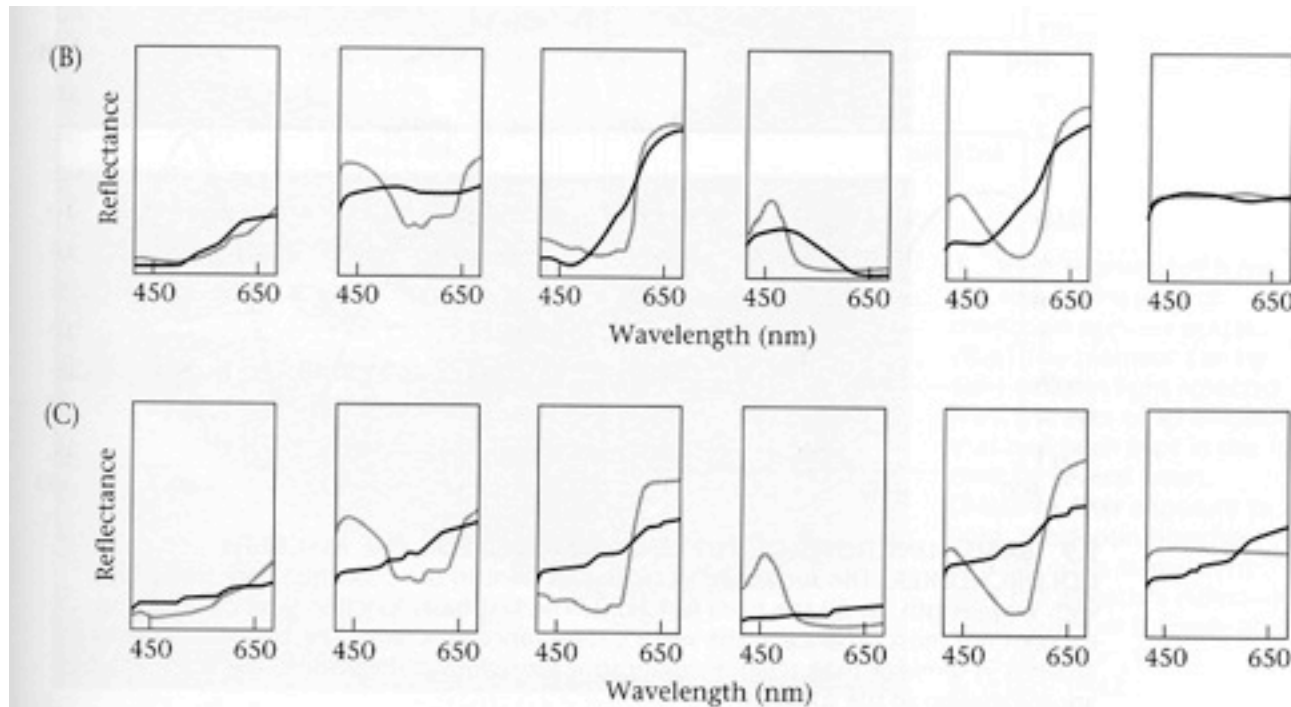


$n = 1$

**9.8 A LINEAR MODEL TO APPROXIMATE THE SURFACE REFLECTANCES IN THE MACBETH COLORCHECKER.** The panels in each row of this figure show the surface-reflectance functions of six colored surfaces (shaded lines) and the approximation to these functions using a linear model (solid lines). The approximations using linear models with (A) three, (B) two, and (C) one dimension are shown.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

# Fitting color spectra with low-dimensional linear models



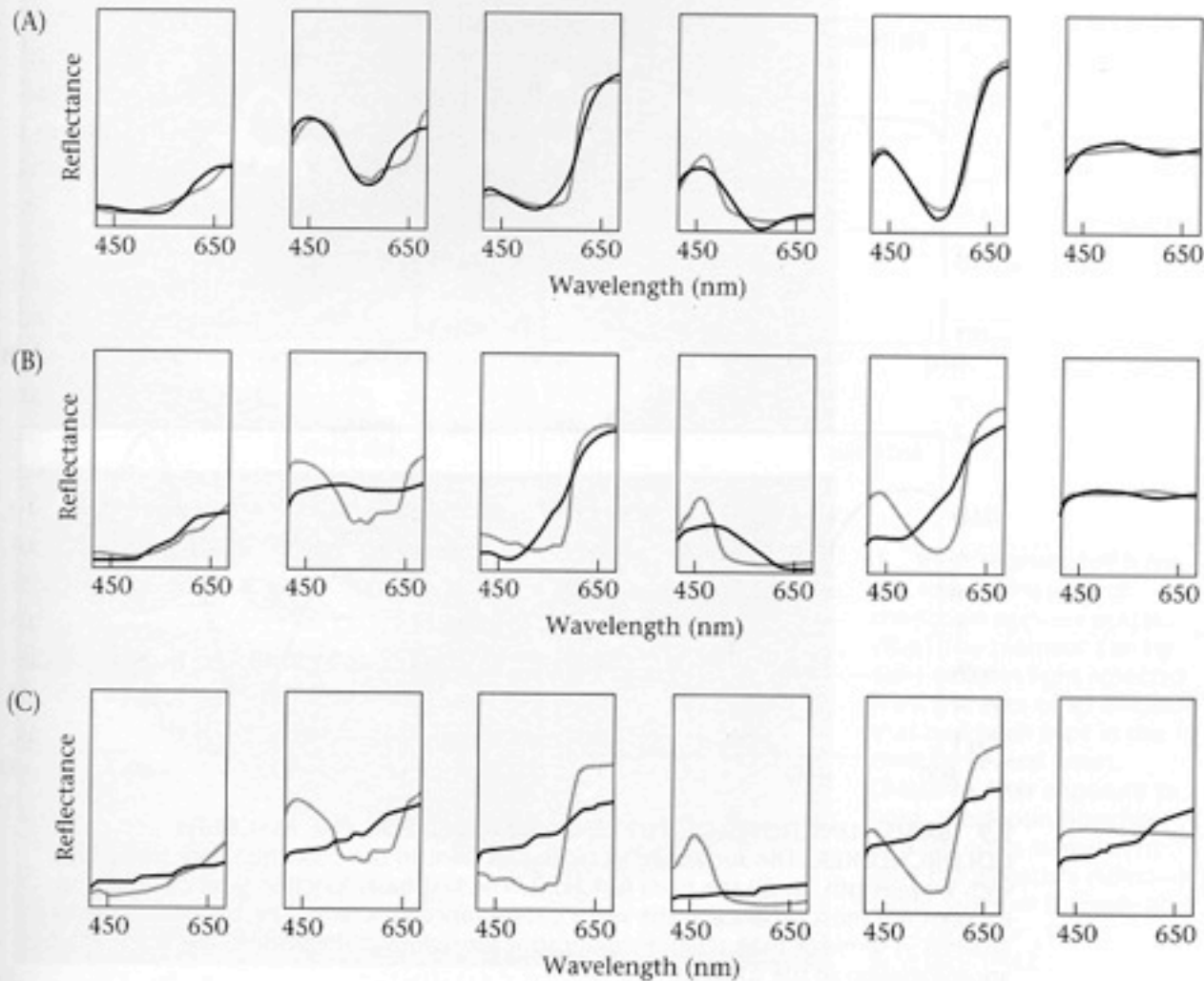
$n = 2$

$n = 1$

**9.8 A LINEAR MODEL TO APPROXIMATE THE SURFACE REFLECTANCES IN THE MACBETH COLORCHECKER.** The panels in each row of this figure show the surface-reflectance functions of six colored surfaces (shaded lines) and the approximation to these functions using a linear model (solid lines). The approximations using linear models with (A) three, (B) two, and (C) one dimension are shown.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

# Fitting color spectra with low-dimensional linear models



**9.8 A LINEAR MODEL TO APPROXIMATE THE SURFACE REFLECTANCES IN THE MACBETH COLORCHECKER.** The panels in each row of this figure show the surface-reflectance functions of six colored surfaces (shaded lines) and the approximation to these functions using a linear model (solid lines). The approximations using linear models with (A) three, (B) two, and (C) one dimension are shown.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

# Lecture outline

- Color physics.
- Color perception.

# Color standards are important in industry

Address <http://www.ams.usda.gov/fv/ppbweb/PPBfilecodes/105a15.htm>



## Fruit and Vegetable Programs

AMS USDA SEARCH

### Processed Products Standards and Quality Certification

#### Visual Aids and Inspection Aids Approved For Use in Ascertaining Grades of Processed Fruits and Vegetables ([Photo](#))

- [Frozen Red Tart Cherries](#)
- [Orange Juice \(Processed\)](#)
- [Canned Tomatoes](#)
- [Frozen French Fried Potatoes](#)
- [Tomato Products](#)
- [Maple Syrup](#)
- [Honey](#)
- [Frozen Lima Beans](#)
- [Canned Mushrooms](#)
- [Peanut Butter](#)
- [Canned Pimientos](#)
- [Frozen Peas](#)
- [Canned Clingstone Peaches](#)
- [Headspace Gauge](#)
- [Canned Applesauce](#)
- [Canned Freestone Peaches](#)
- [Canned Ripe Olives](#)

Return to: [Processed Products Branch](#)



Image of Inspection Aids

UNITED STATES DEPARTMENT OF AGRICULTURE

# COLOR STANDARDS

*for*

FROZEN

FRENCH FRIED POTATOES



FOURTH EDITION, 1988  
© 1988 KOLLMORGEN CORPORATION

MUNSELL COLOR  
BALTIMORE, MARYLAND  
64-1





# Color trademarks

CURRENTLY REGISTERED COLOR TRADEMARKS

<http://blog.patents-tms.com/?p=52>

A color trademark is a non-conventional trademark where at least one color is used to identify the commercial origin of a product or service. A color trademark must meet the same requirements of a conventional trademark. Thus, the color trademark must either be inherently distinctive or have acquired secondary meaning. To be inherently distinctive, the color must be arbitrarily or suggestively applied to a product or service. In contrast, to acquire secondary meaning, consumers must associate the color used on goods or services as originating from a single source. Below is a selection of some currently registered color trademarks in the U.S. Trademark Office:

MARK/COLOR(S)/OWNER:

BANK OF AMERICA 500  
blue, red & grey  
Bank of America Corporation

NATIONAL CAR RENTAL  
green  
NCR Affiliate Servicer, Inc.

FORD  
blue  
Ford Motor Company

VISTEON  
orange  
Ford Motor Company

76  
red & blue  
ConocoPhillips Company

VW  
silver, metallic blue, black and white  
Volkswagen Aktiengesellschaft Corp

THE HOME DEPOT  
orange  
Homer TLC, Inc.

HONDA  
red  
Honda Motor Co., Ltd.

M MARATHON  
brown, orange, yellow  
Marathon Oil Company

M MARATHON  
gray, black & white  
Marathon Oil Company

COSTCO  
red  
Costco Wholesale Membership, Inc.

TEENAGE MUTANT NINJA TURTLES MUTANTS & MONSTERS  
red, green, yellow, black, grey and white  
Mirage Studios, Inc.

TARGET

# What we need from a color measurement system

- Given a color, how do you assign a number to it?
- Given an input power spectrum, what is its numerical color value, and how do we control our printing/projection/cooking system to match it?

What's the machinery in the eye?

# Eye Photoreceptor responses

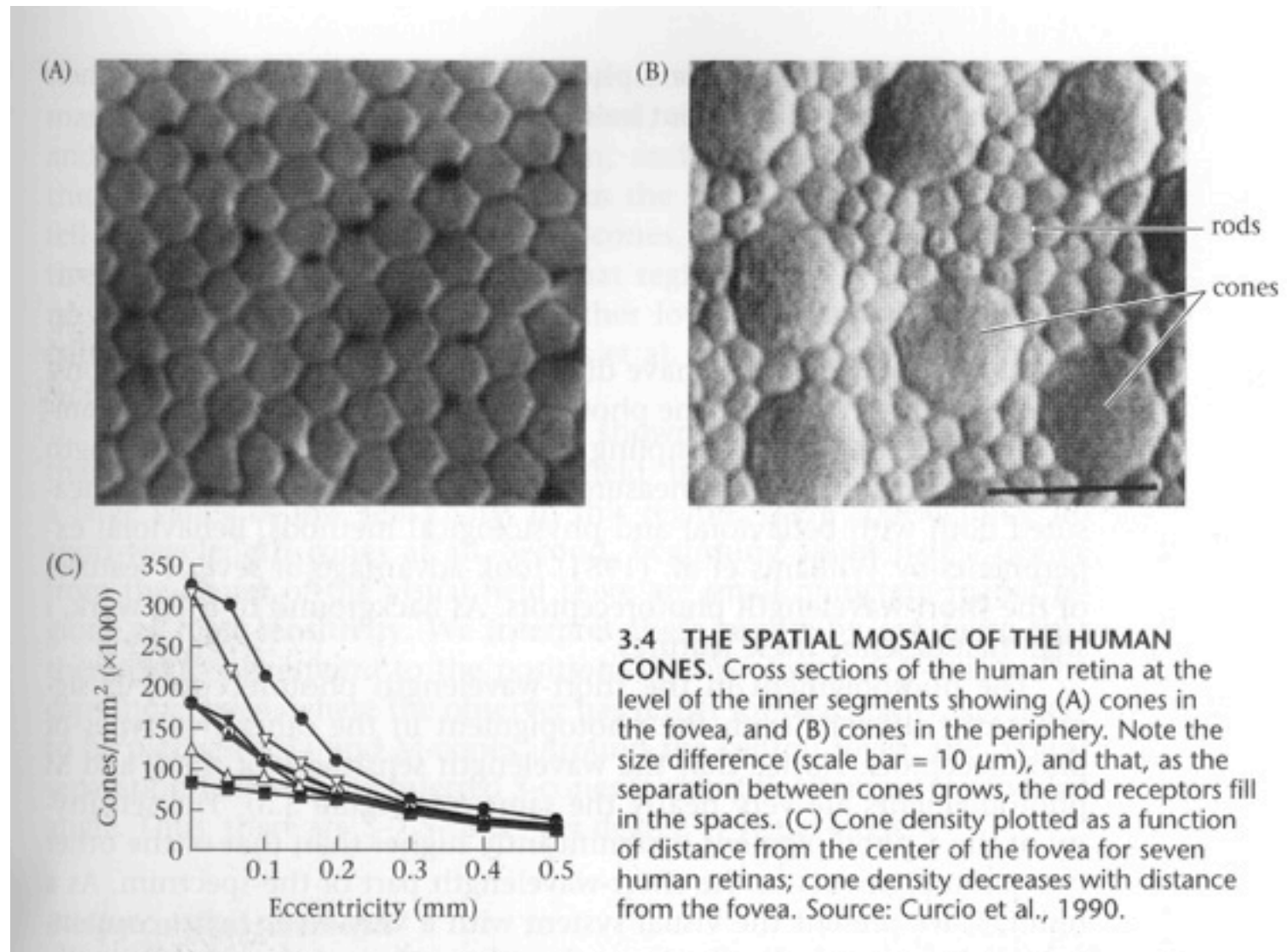


Instituto Cajal. CSIC. Madrid.

The intricate layers and connections of nerve cells in the retina were drawn by the famed Spanish anatomist Santiago Ramón y Cajal around 1900. Rod and cone cells are at the top. Optic nerve fibers leading to the brain may be seen at bottom right.

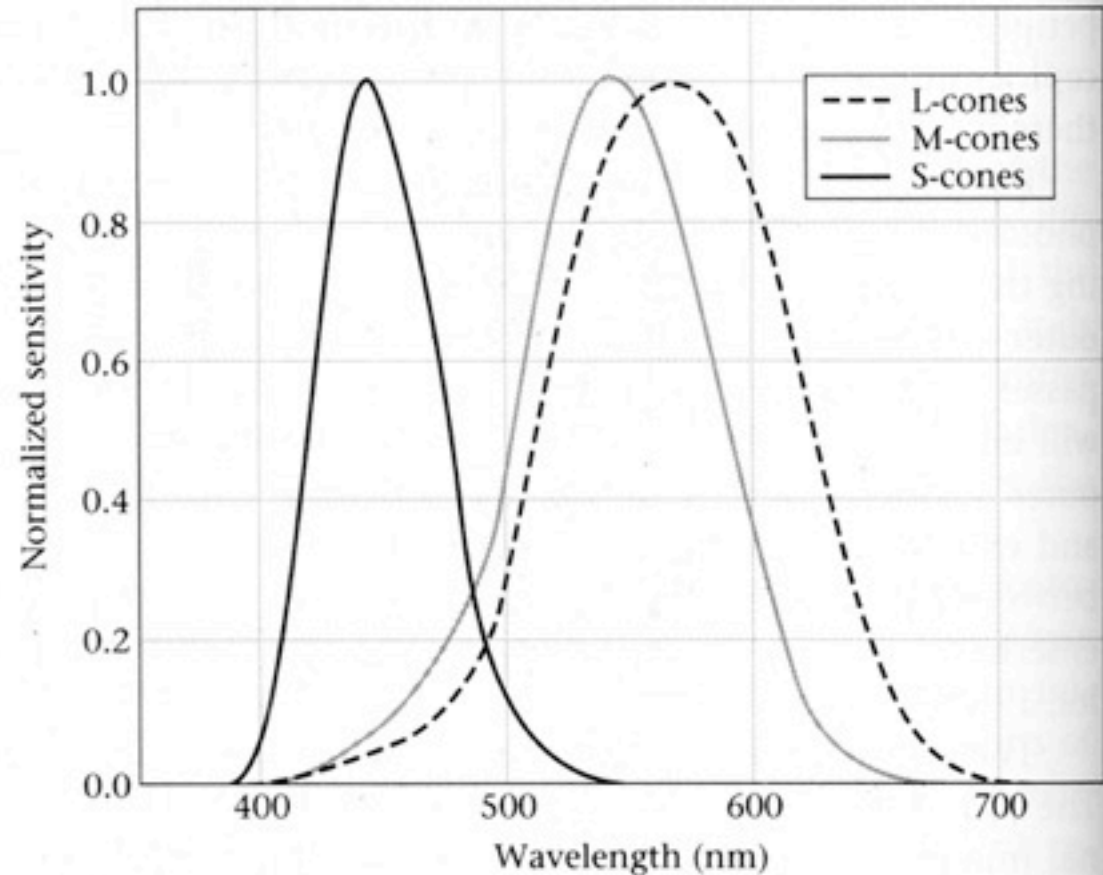
(Where do you think the light comes in?)

# Human Photoreceptors



# Human eye photoreceptor spectral sensitivities

**3.3 SPECTRAL SENSITIVITIES OF THE L-, M-, AND S-CONES** in the human eye. The measurements are based on a light source at the cornea, so that the wavelength loss due to the cornea, lens, and other inert pigments of the eye plays a role in determining the sensitivity. Source: Stockman and MacLeod, 1993.



# Lecture outline

- Color physics.
- Color perception
  - part 1: assume perceived color only depends on light spectrum.
  - part 2: the more general case.

# The assumption for color perception, part 1



# The assumption for color perception, part 1

- We know color appearance really depends on:

# The assumption for color perception, part 1

- We know color appearance really depends on:
  - The illumination

# The assumption for color perception, part 1

- We know color appearance really depends on:
  - The illumination
  - Your eye's adaptation level

# The assumption for color perception, part 1

- We know color appearance really depends on:
  - The illumination
  - Your eye's adaptation level
  - The colors and scene interpretation surrounding the observed color.

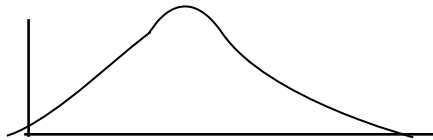
# The assumption for color perception, part 1

- We know color appearance really depends on:
  - The illumination
  - Your eye's adaptation level
  - The colors and scene interpretation surrounding the observed color.

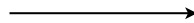
# The assumption for color perception, part 1

- We know color appearance really depends on:
  - The illumination
  - Your eye's adaptation level
  - The colors and scene interpretation surrounding the observed color.
- But for now we will assume that the spectrum of the light arriving at your eye completely determines the perceived color.

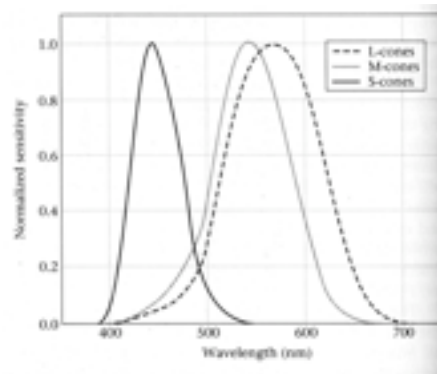
test light



project



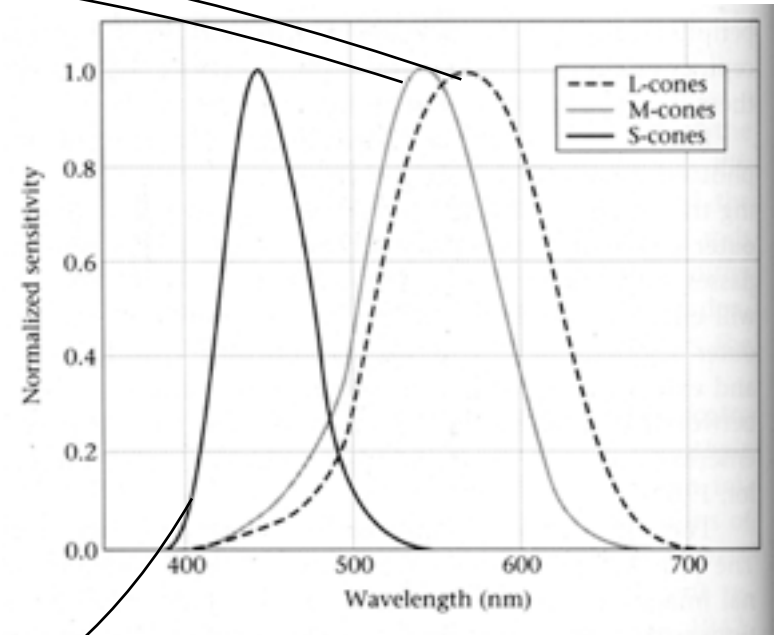
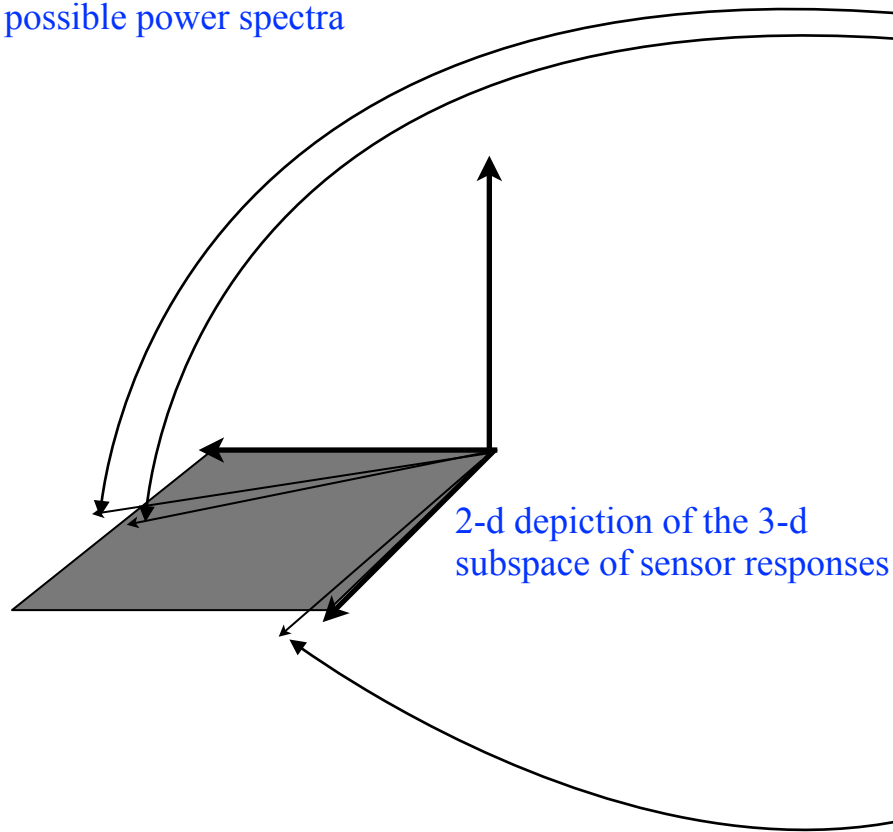
### Cone sensitivities



L, M, S responses

# Cone response curves as basis vectors in a 3-d subspace of light power spectra

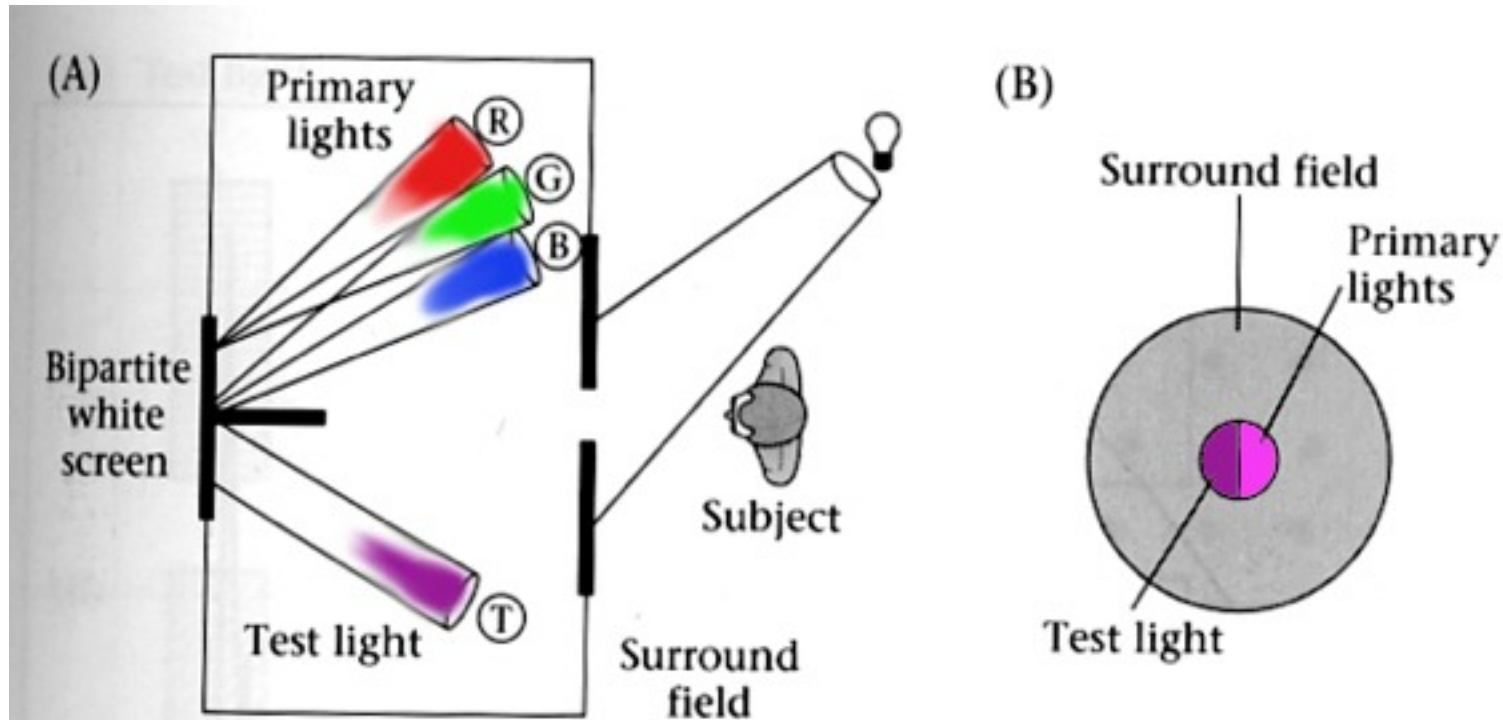
3-d depiction of the high-dimensional space of all possible power spectra



Spectral sensitivities of L, M, and S cones

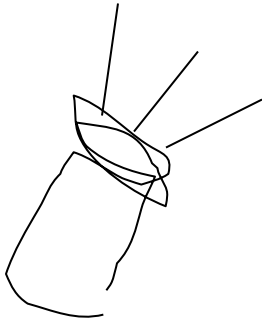
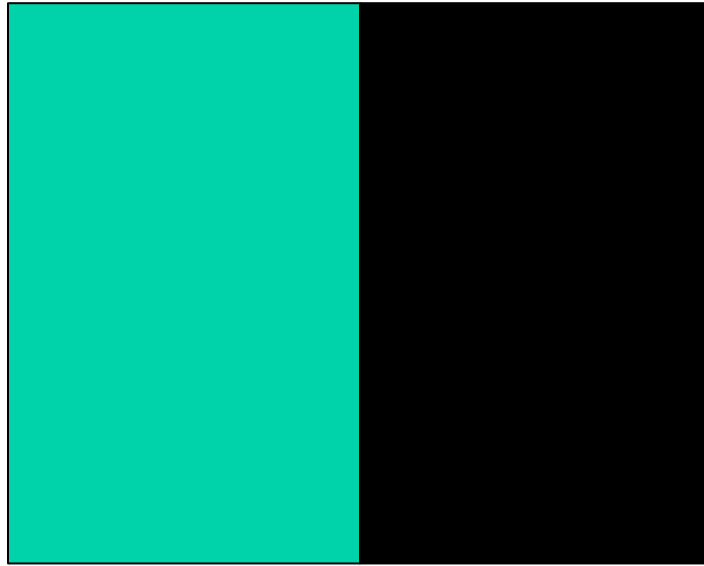


# Color matching experiment

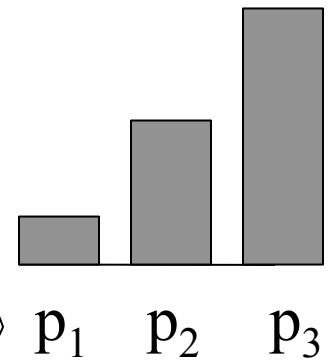
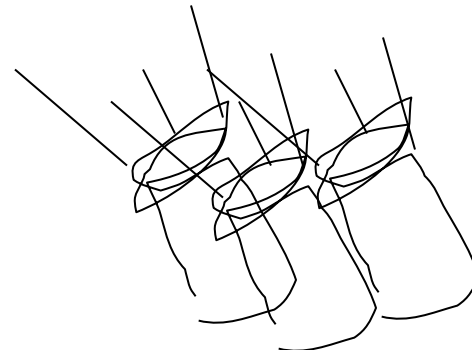
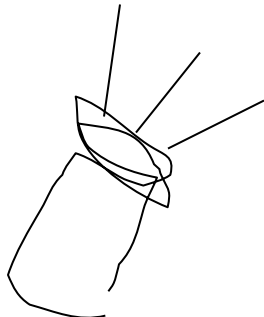
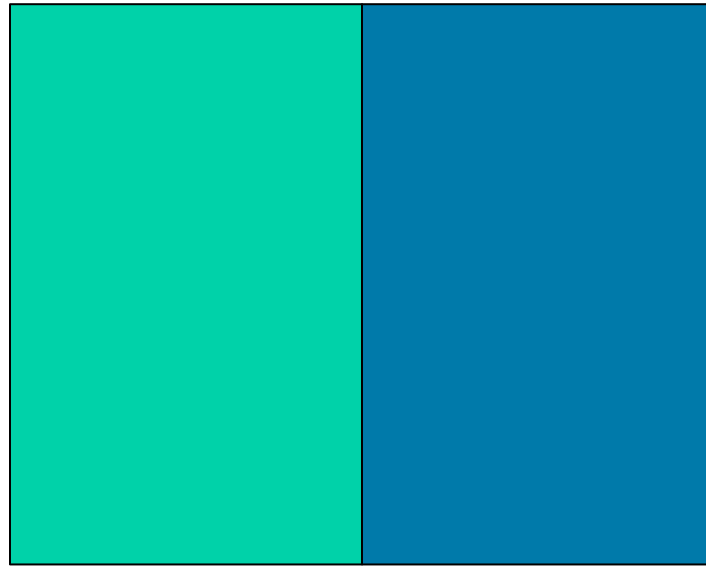


**4.10 THE COLOR-MATCHING EXPERIMENT.** The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.

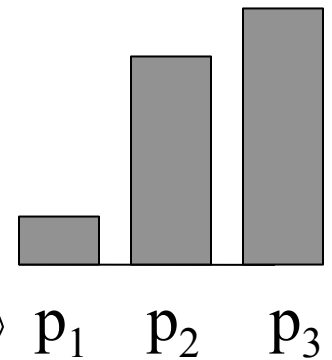
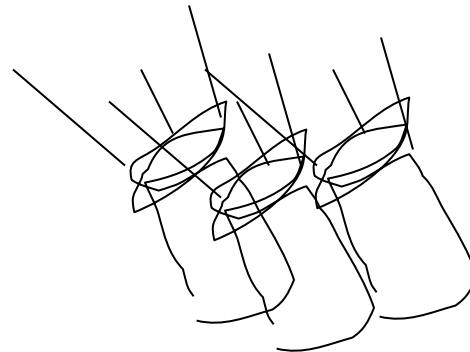
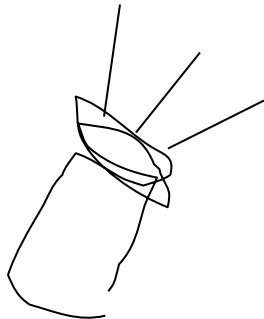
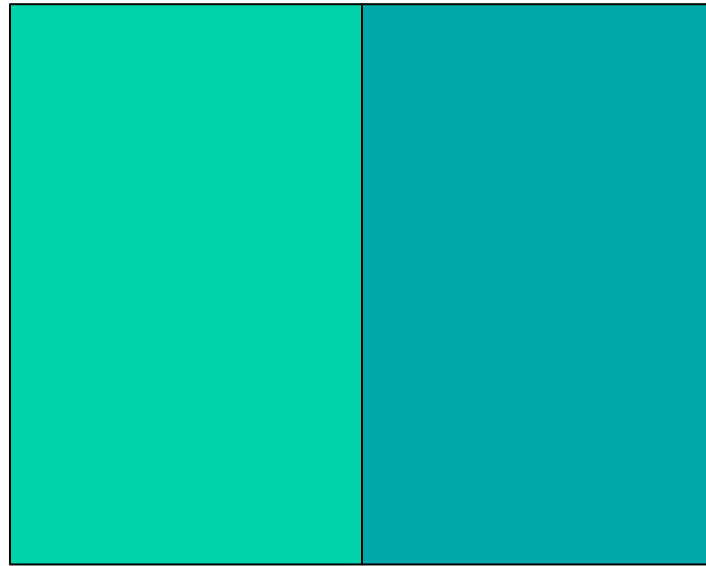
# Color matching experiment 1



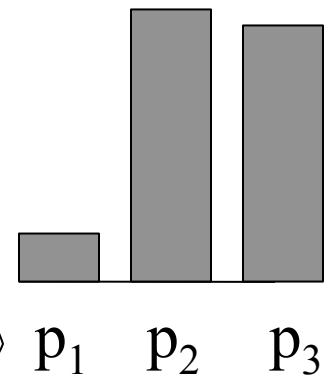
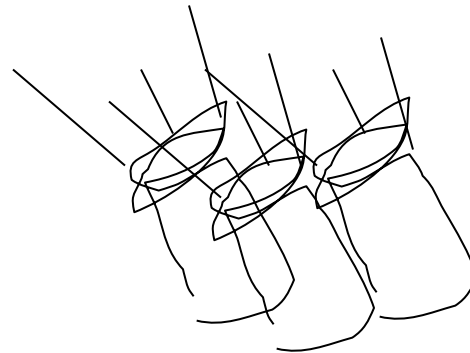
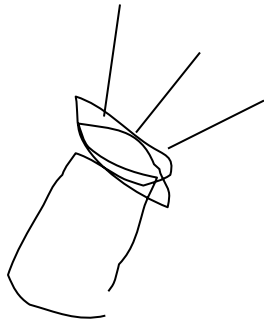
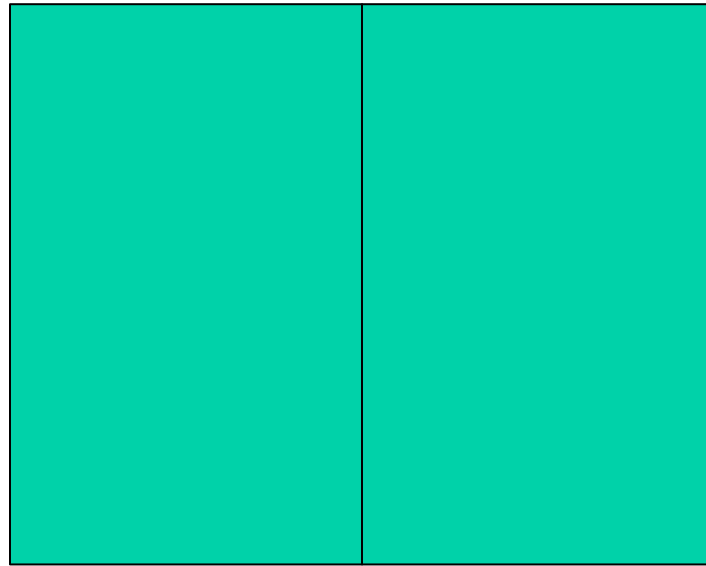
# Color matching experiment 1



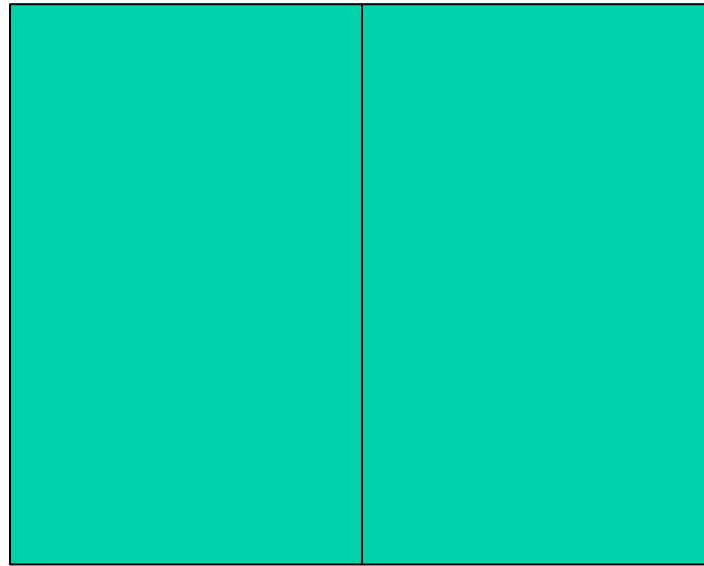
# Color matching experiment 1



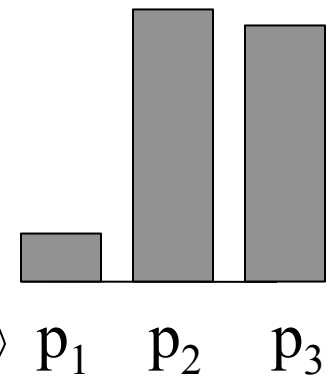
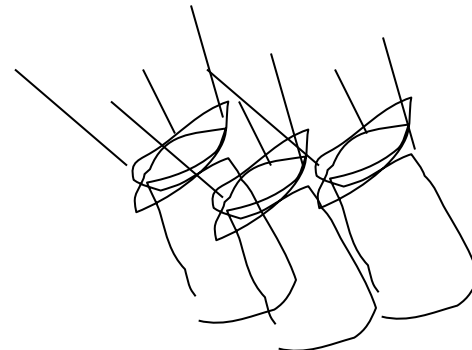
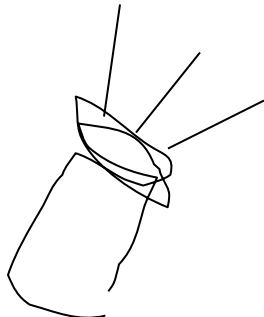
# Color matching experiment 1



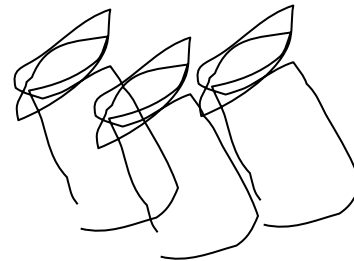
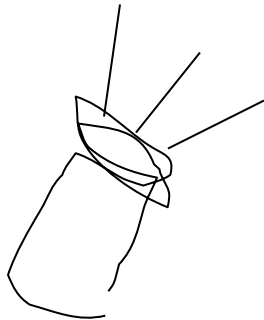
# Color matching experiment 1



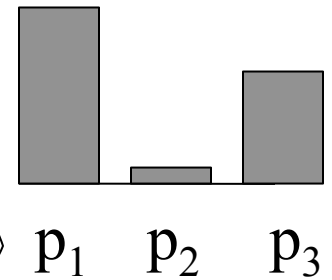
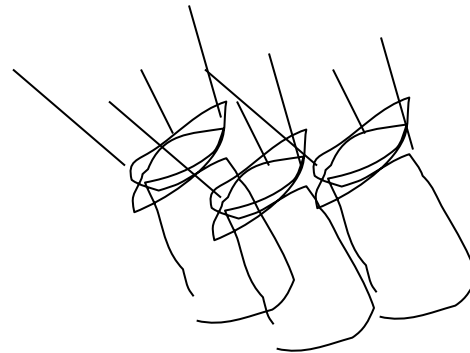
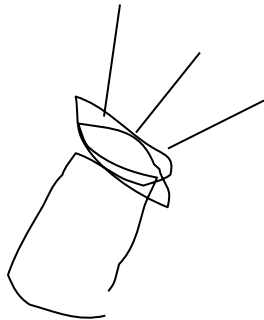
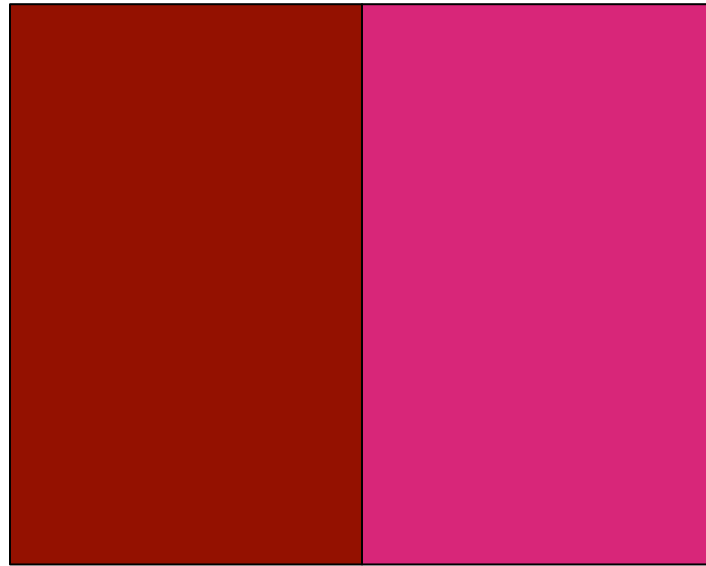
The primary color amounts needed for a match



# Color matching experiment 2

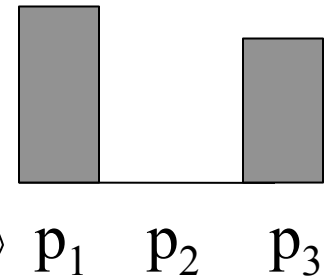
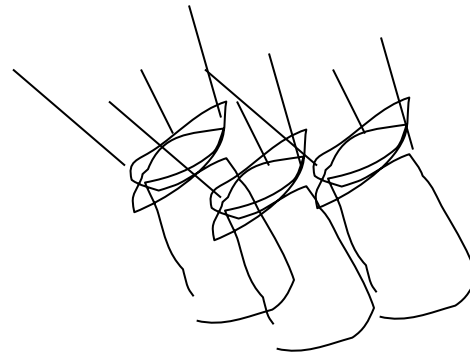
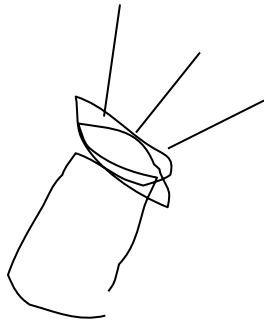
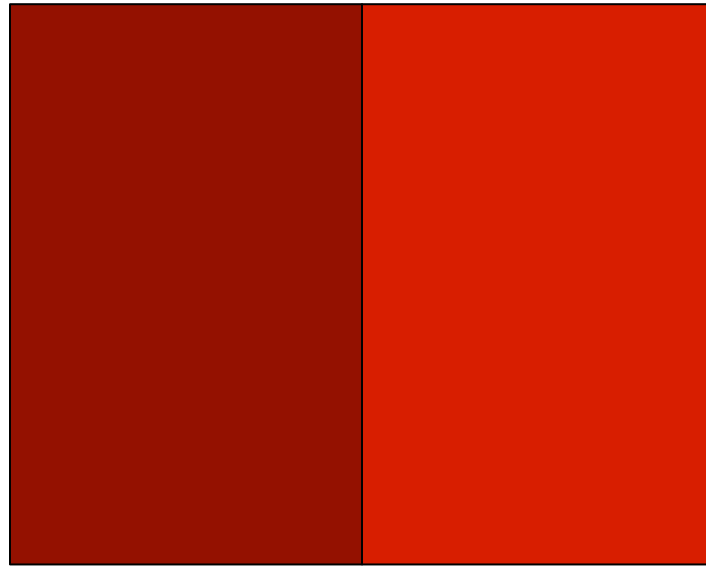


# Color matching experiment 2

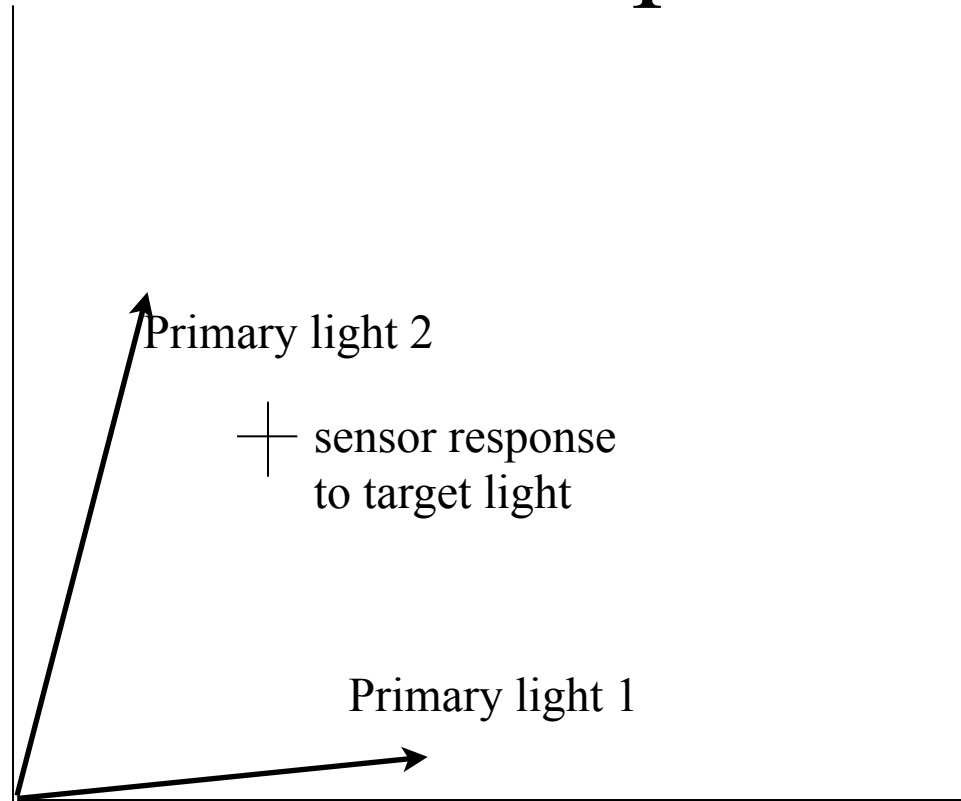




# Color matching experiment 2

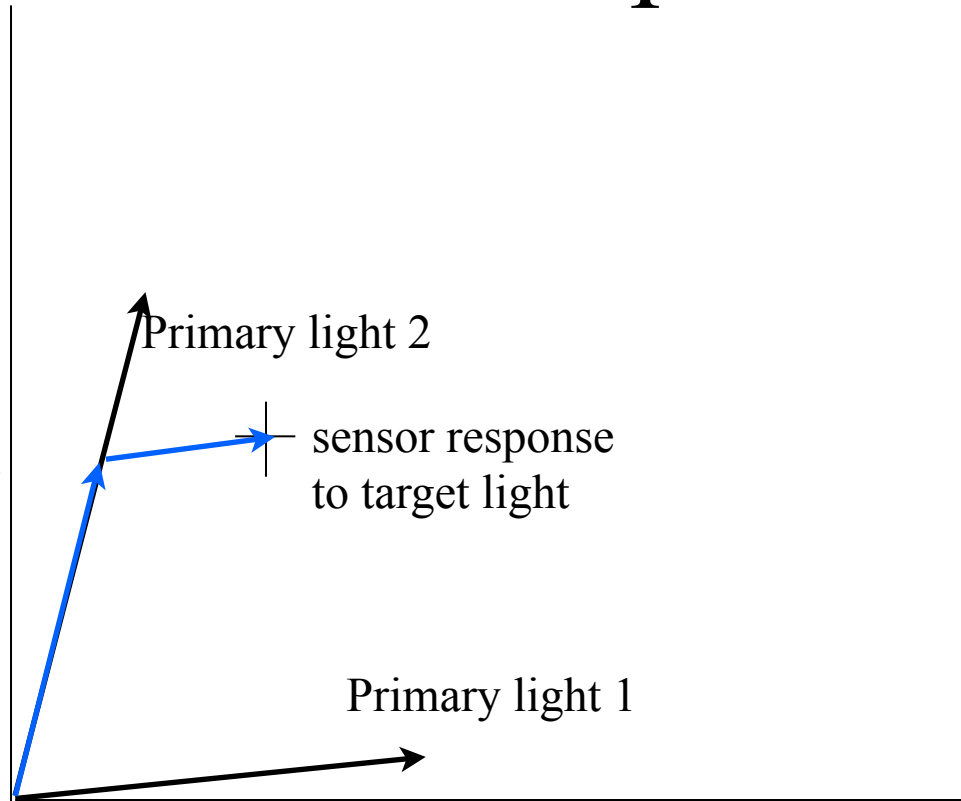


# Color matching with positive amounts of the primaries



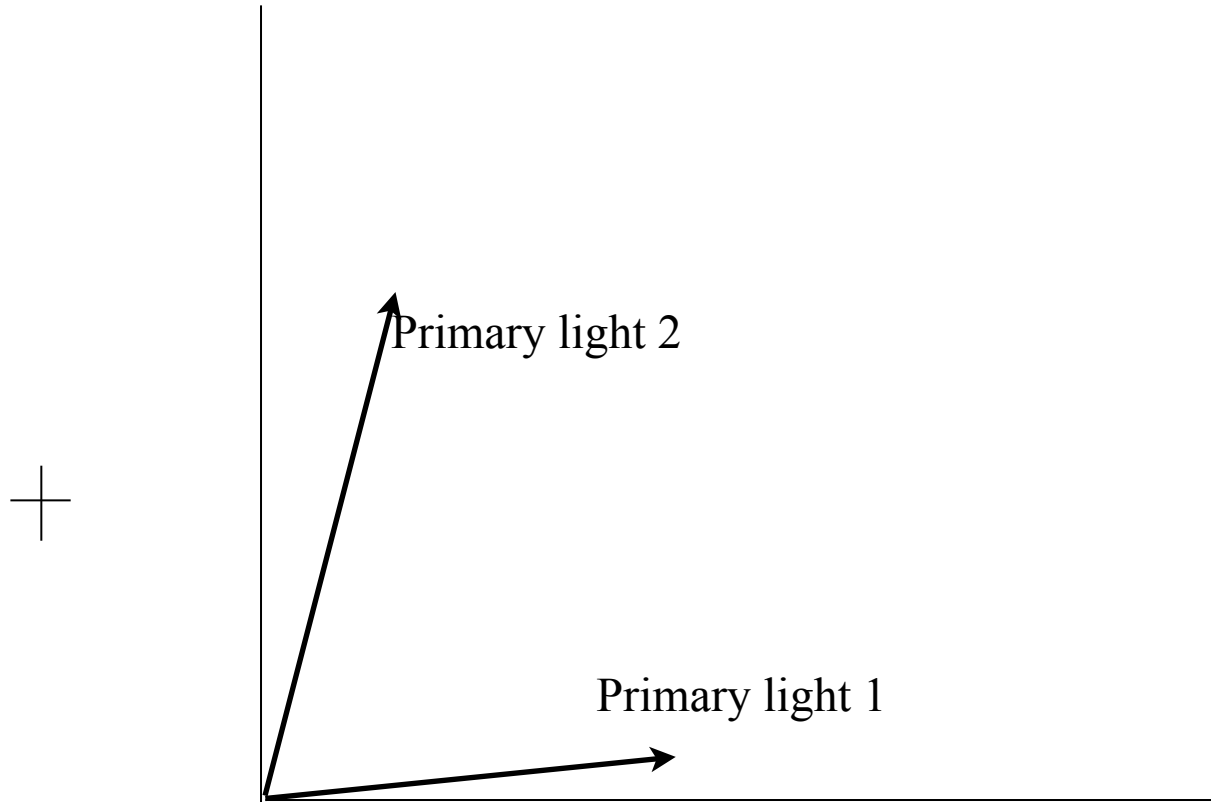
# Color matching with positive amounts of the primaries

Match the sensors' response to the target light to the sum of responses to the primary lights



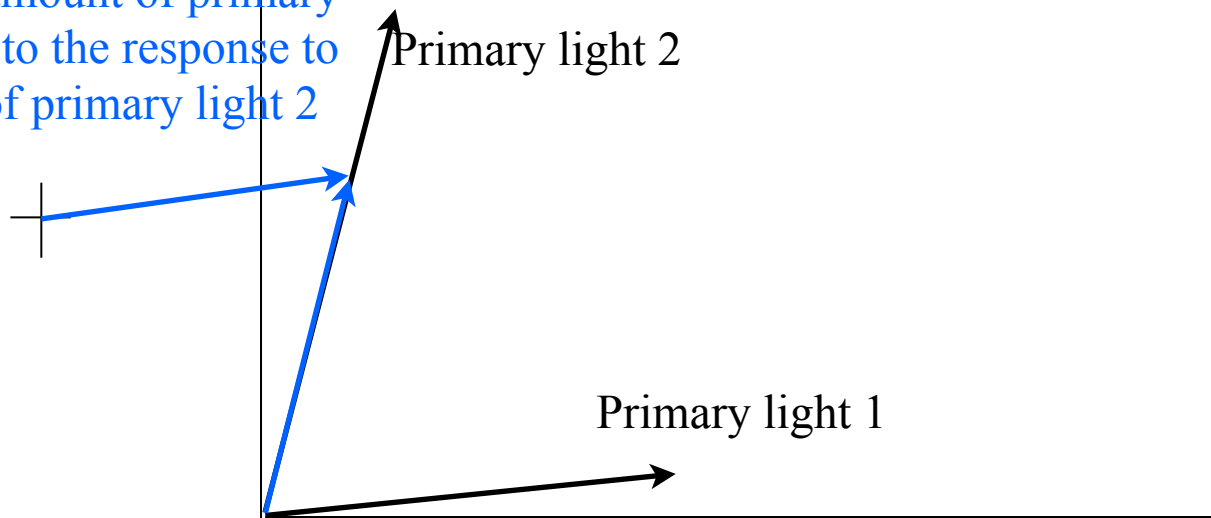
# Color matching with positive amounts of the primaries

# Color matching with a negative amount of primary 1

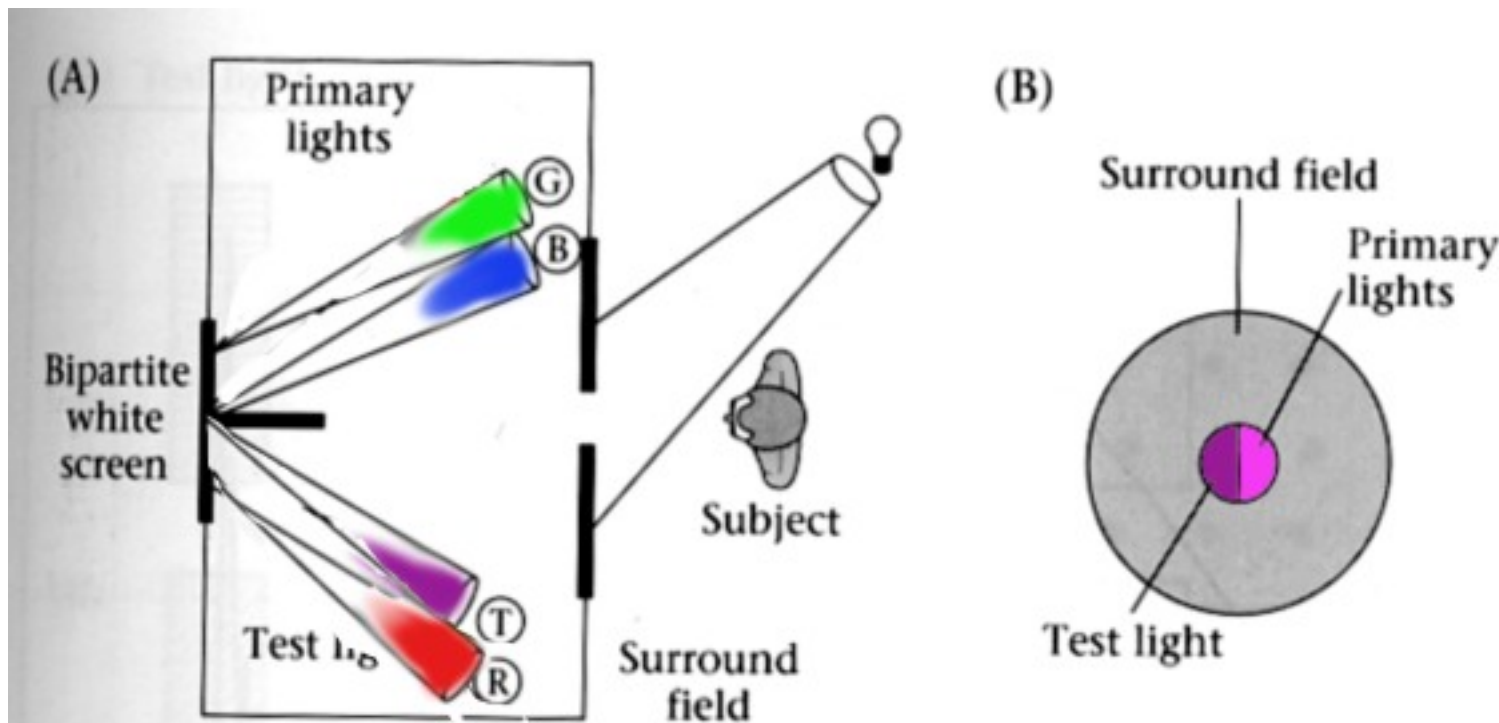


# Color matching with a negative amount of primary 1

Match sensors' response to the target light plus some amount of primary light 1 to the response to some of primary light 2



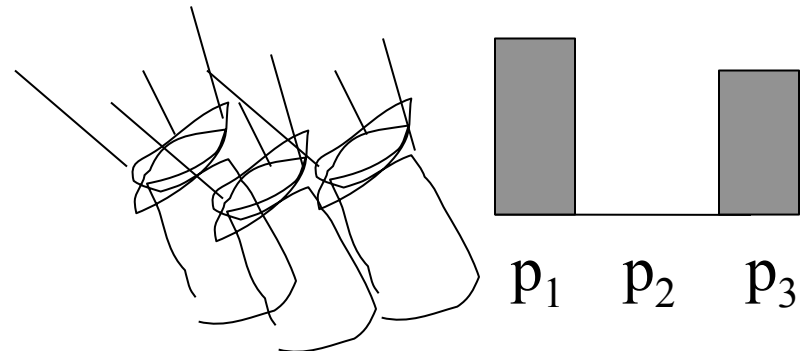
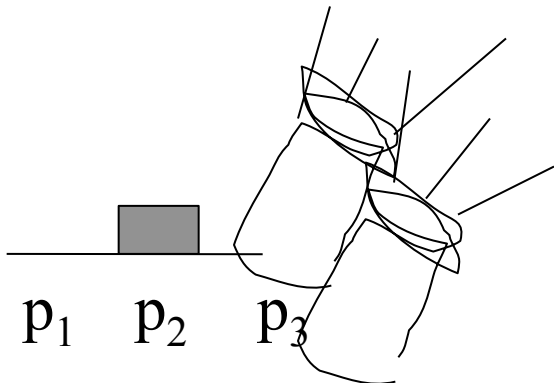
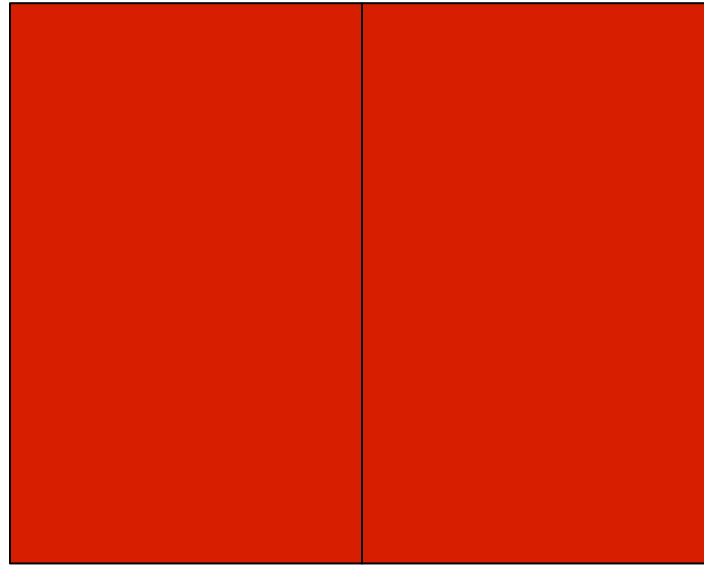
# Color matching experiment--handle negative light by adding light to the test.



**4.10 THE COLOR-MATCHING EXPERIMENT.** The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.

# Color matching experiment 2

We say a “negative” amount of  $p_2$  was needed to make the match, because we added it to the test color’s side.

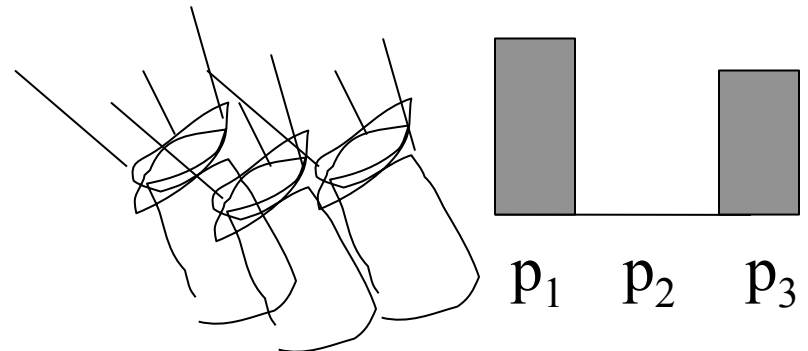
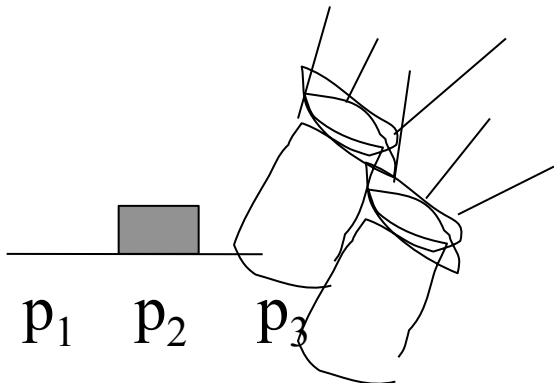
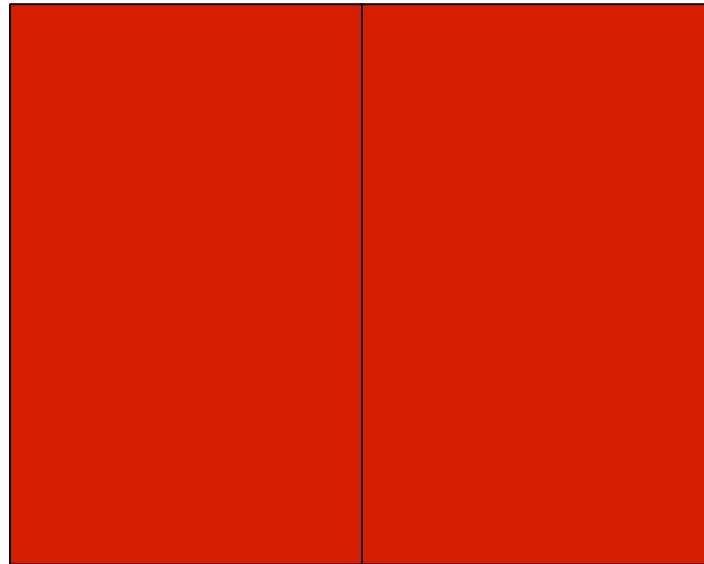




# Color matching experiment 2

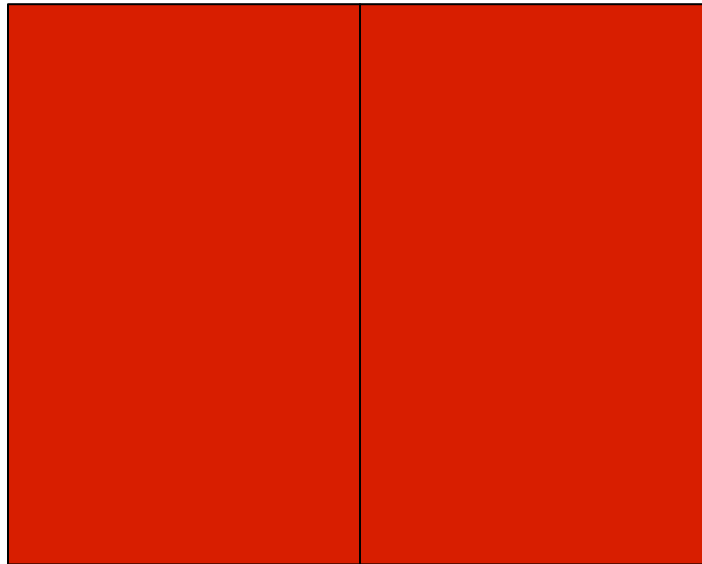
We say a “negative” amount of  $p_2$  was needed to make the match, because we added it to the test color’s side.

The primary color amounts needed for a match:

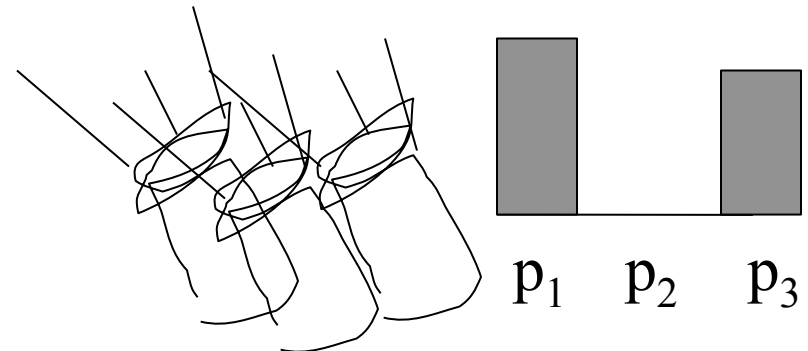
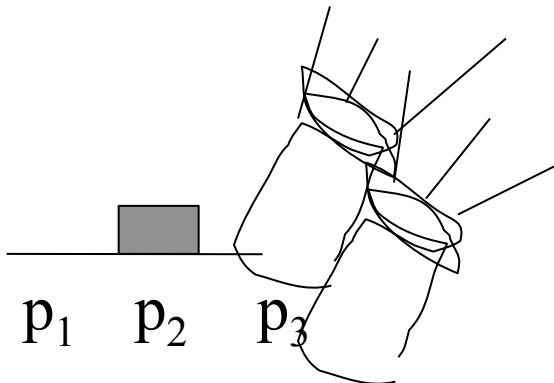
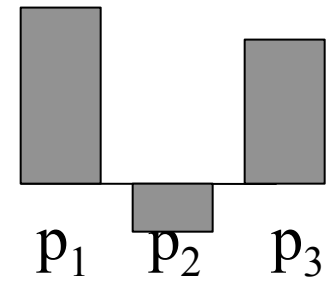


# Color matching experiment 2

We say a “negative” amount of  $p_2$  was needed to make the match, because we added it to the test color’s side.

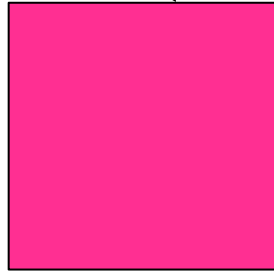


The primary color amounts needed for a match:

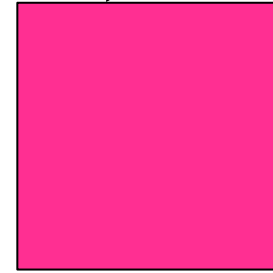


# Color matching superposition (Grassman's laws)

If  $A_1$



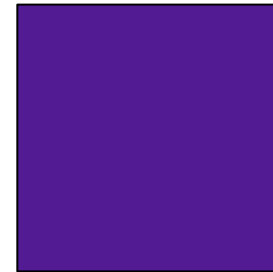
matches  $B_1$



and  $A_2$



matches  $B_2$



then  $A_1 + A_2$

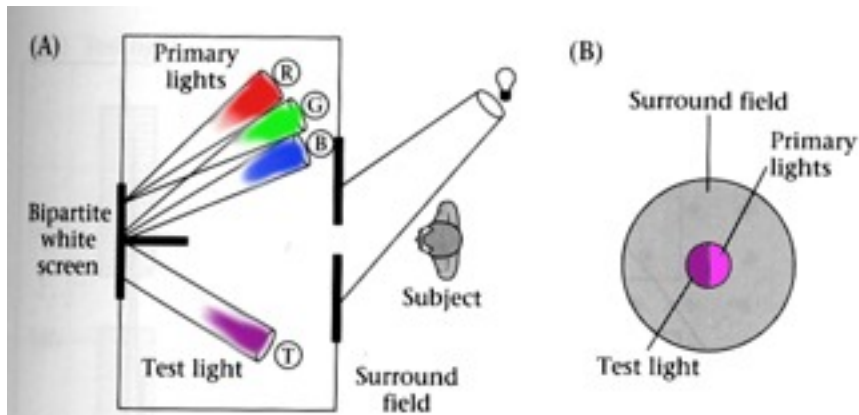


matches  $B_1 + B_2$



# To measure a color

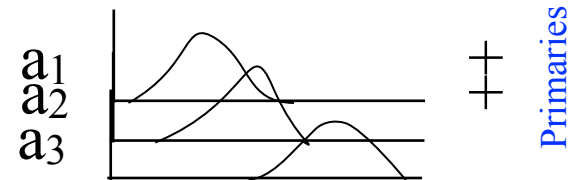
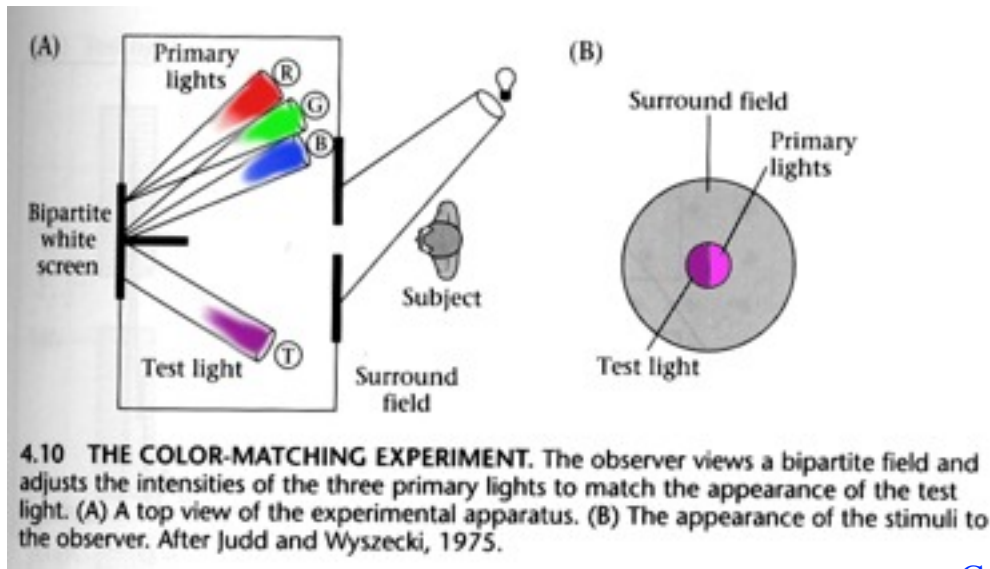
1. Choose a set of 3 primary colors (three power spectra).
2. Determine how much of each primary needs to be added to a probe signal to match the test light.



**4.10 THE COLOR-MATCHING EXPERIMENT.** The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.

# To measure a color

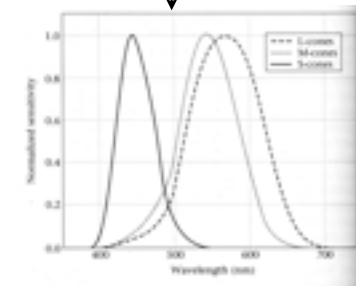
1. Choose a set of 3 primary colors (three power spectra).
2. Determine how much of each primary needs to be added to a probe signal to match the test light.



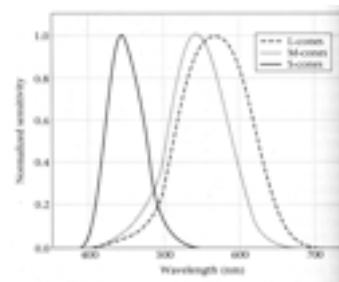
weighted  
sum of  
primaries



project

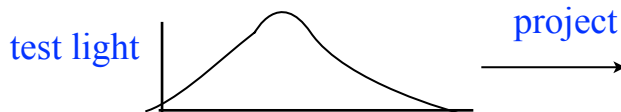


Cone sensitivities



L, M, S responses

52



project

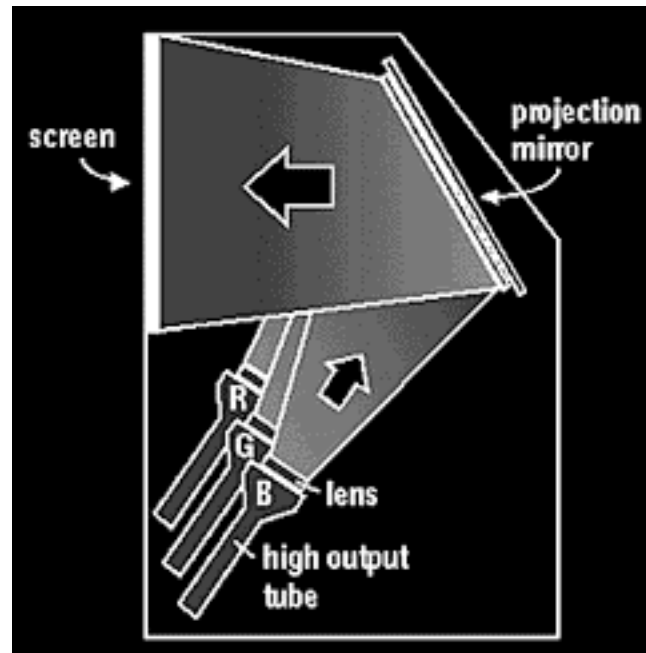
→

# What we need from a color measurement system

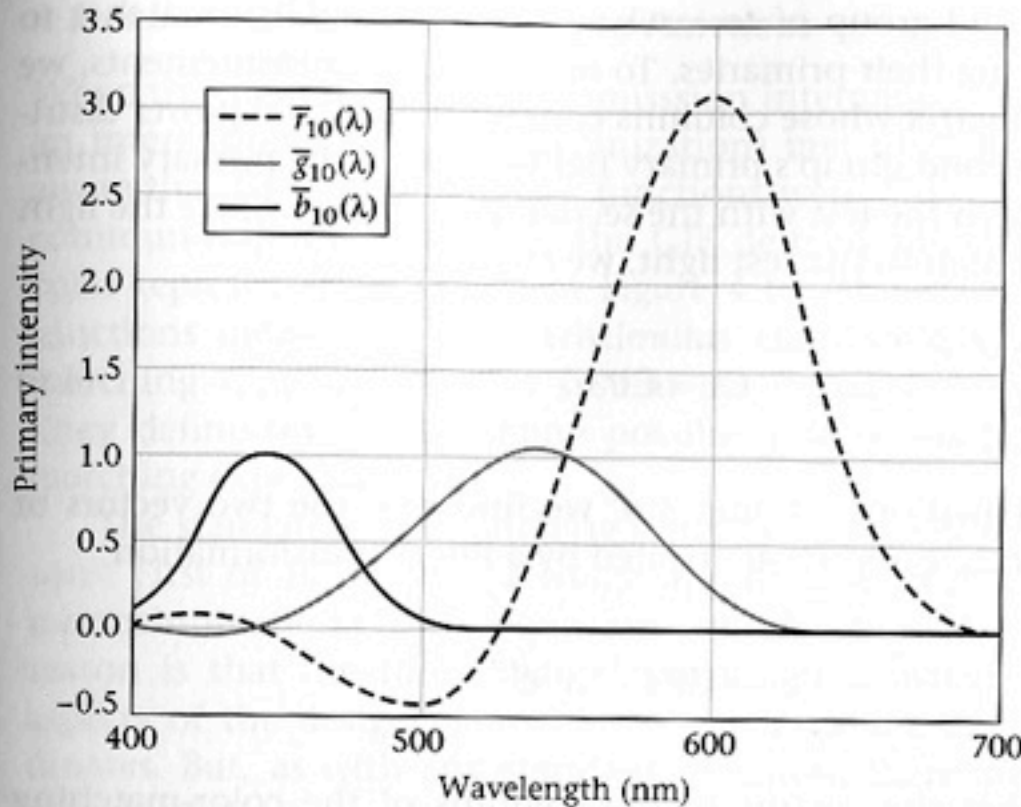
- Given a color, how do you assign a number to it?
- Given an input power spectrum, what is its numerical color value, and how do we control our printing/projection/cooking system to match it?

# What we need from a color measurement system

- Given a color, how do you assign a number to it?
- Given an input power spectrum, what is its numerical color value, and how do we control our printing/projection/cooking system to match it?



“Color matching functions” let us find other basis vectors for the eye response subspace of light power spectra

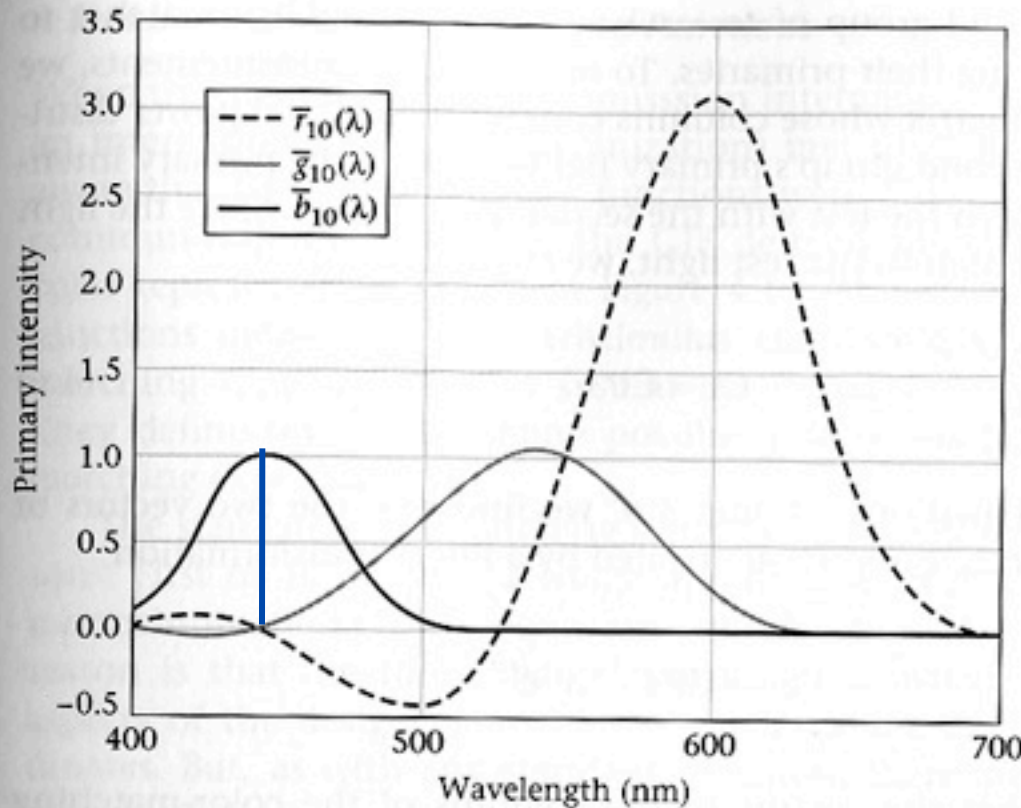


- $p_1 = 645.2 \text{ nm}$
- $p_2 = 525.3 \text{ nm}$
- $p_3 = 444.4 \text{ nm}$

**4.13 THE COLOR-MATCHING FUNCTIONS ARE THE ROWS OF THE COLOR-MATCHING SYSTEM MATRIX.** The functions measured by Stiles and Burch (1959) using a 10-degree bipartite field and primary lights at the wavelengths 645.2 nm, 525.3 nm, and 444.4 nm with unit radiant power are shown. The three functions in this figure are called  $\bar{r}_{10}(\lambda)$ ,  $\bar{g}_{10}(\lambda)$ , and  $\bar{b}_{10}(\lambda)$ .



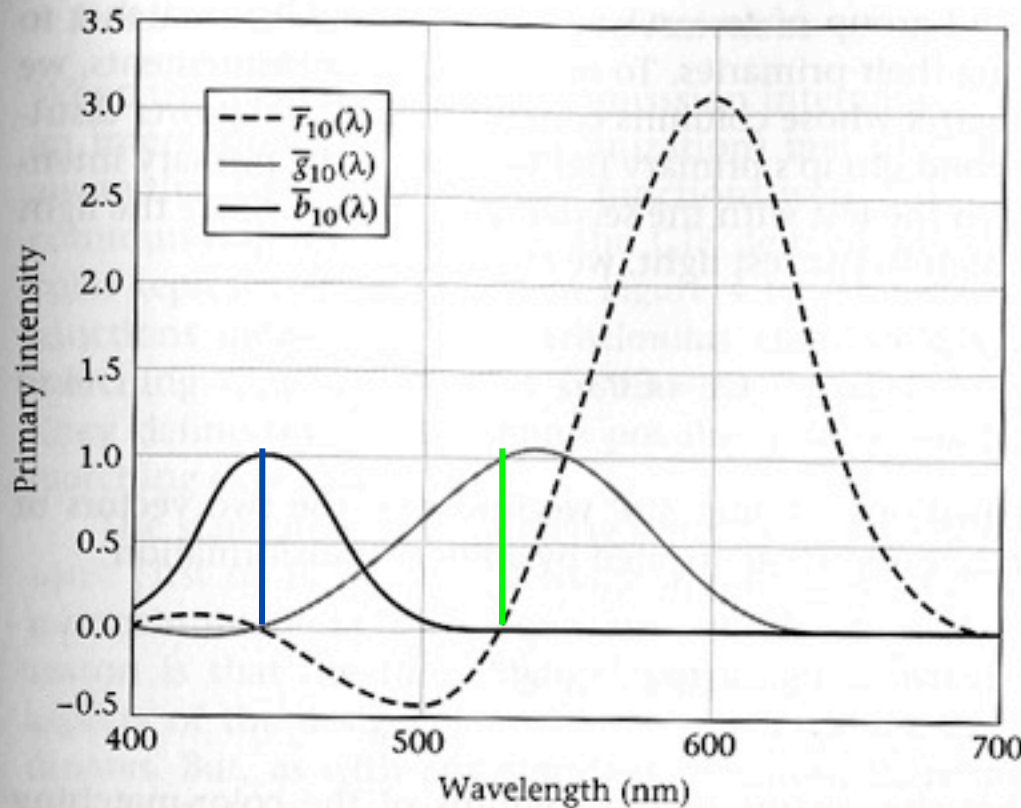
“Color matching functions” let us find other basis vectors for the eye response subspace of light power spectra



- $p_1 = 645.2 \text{ nm}$
- $p_2 = 525.3 \text{ nm}$
- $p_3 = 444.4 \text{ nm}$

**4.13 THE COLOR-MATCHING FUNCTIONS ARE THE ROWS OF THE COLOR-MATCHING SYSTEM MATRIX.** The functions measured by Stiles and Burch (1959) using a 10-degree bipartite field and primary lights at the wavelengths 645.2 nm, 525.3 nm, and 444.4 nm with unit radiant power are shown. The three functions in this figure are called  $\bar{r}_{10}(\lambda)$ ,  $\bar{g}_{10}(\lambda)$ , and  $\bar{b}_{10}(\lambda)$ .

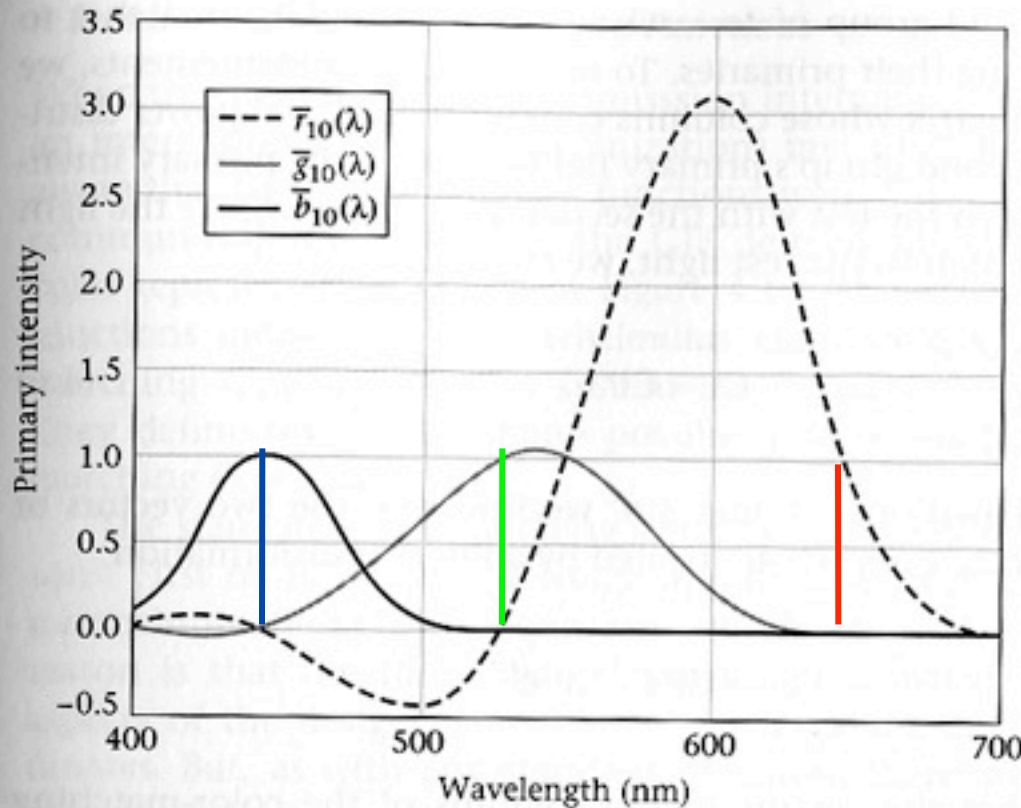
“Color matching functions” let us find other basis vectors for the eye response subspace of light power spectra



- $p_1 = 645.2 \text{ nm}$
- $p_2 = 525.3 \text{ nm}$
- $p_3 = 444.4 \text{ nm}$

**4.13 THE COLOR-MATCHING FUNCTIONS ARE THE ROWS OF THE COLOR-MATCHING SYSTEM MATRIX.** The functions measured by Stiles and Burch (1959) using a 10-degree bipartite field and primary lights at the wavelengths 645.2 nm, 525.3 nm, and 444.4 nm with unit radiant power are shown. The three functions in this figure are called  $\bar{r}_{10}(\lambda)$ ,  $\bar{g}_{10}(\lambda)$ , and  $\bar{b}_{10}(\lambda)$ .

“Color matching functions” let us find other basis vectors for the eye response subspace of light power spectra



- $p_1 = 645.2 \text{ nm}$
- $p_2 = 525.3 \text{ nm}$
- $p_3 = 444.4 \text{ nm}$

**4.13 THE COLOR-MATCHING FUNCTIONS ARE THE ROWS OF THE COLOR-MATCHING SYSTEM MATRIX.** The functions measured by Stiles and Burch (1959) using a 10-degree bipartite field and primary lights at the wavelengths 645.2 nm, 525.3 nm, and 444.4 nm with unit radiant power are shown. The three functions in this figure are called  $\bar{r}_{10}(\lambda)$ ,  $\bar{g}_{10}(\lambda)$ , and  $\bar{b}_{10}(\lambda)$ .

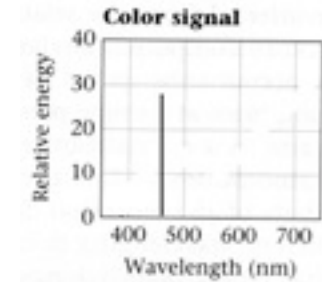
Using the color matching functions to predict  
the primary match to a new spectral signal

# Using the color matching functions to predict the primary match to a new spectral signal

We know that a monochromatic light of wavelength will be matched by the amounts

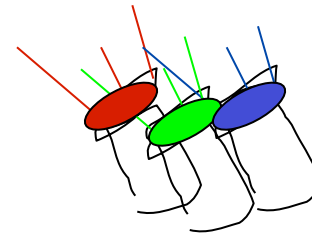
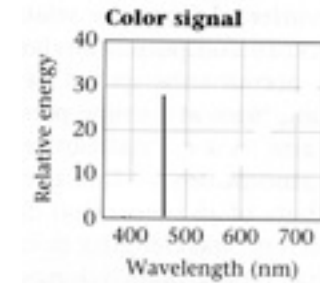
$\lambda_i$

of each primary.



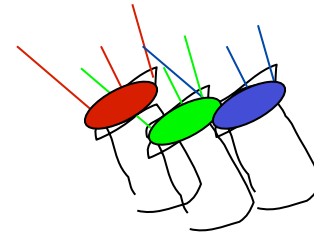
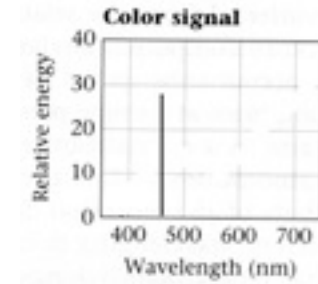
# Using the color matching functions to predict the primary match to a new spectral signal

We know that a monochromatic light of wavelength  $\lambda_i$  will be matched by the amounts  $c_1(\lambda_i), c_2(\lambda_i), c_3(\lambda_i)$  of each primary.



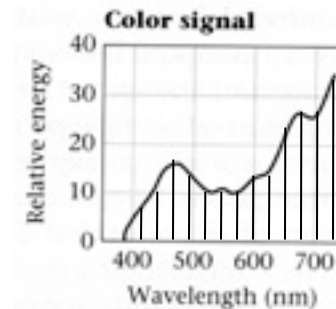
# Using the color matching functions to predict the primary match to a new spectral signal

We know that a monochromatic light of wavelength  $\lambda_i$  will be matched by the amounts  $c_1(\lambda_i), c_2(\lambda_i), c_3(\lambda_i)$  of each primary.



And any spectral signal can be thought of as a linear combination of very many monochromatic lights, with the linear coefficient given by the spectral power at each wavelength.

$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$



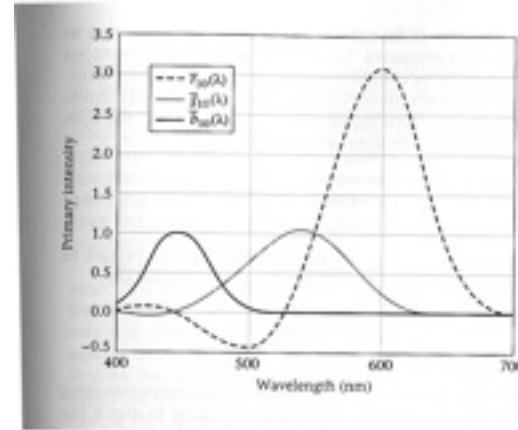
Using the color matching functions to predict  
the primary match to a new spectral signal



# Using the color matching functions to predict the primary match to a new spectral signal

Store the color matching functions in the rows of the matrix,  $C$

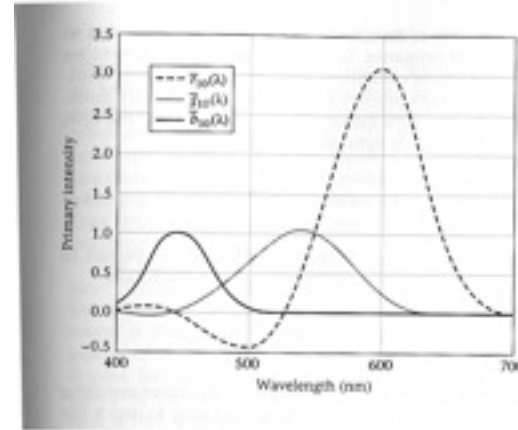
$$C = \begin{pmatrix} c_1(\lambda_1) & \cdots & c_1(\lambda_N) \\ c_2(\lambda_1) & \cdots & c_2(\lambda_N) \\ c_3(\lambda_1) & \cdots & c_3(\lambda_N) \end{pmatrix}$$



# Using the color matching functions to predict the primary match to a new spectral signal

Store the color matching functions in the rows of the matrix,  $C$

$$C = \begin{pmatrix} c_1(\lambda_1) & \cdots & c_1(\lambda_N) \\ c_2(\lambda_1) & \cdots & c_2(\lambda_N) \\ c_3(\lambda_1) & \cdots & c_3(\lambda_N) \end{pmatrix}$$



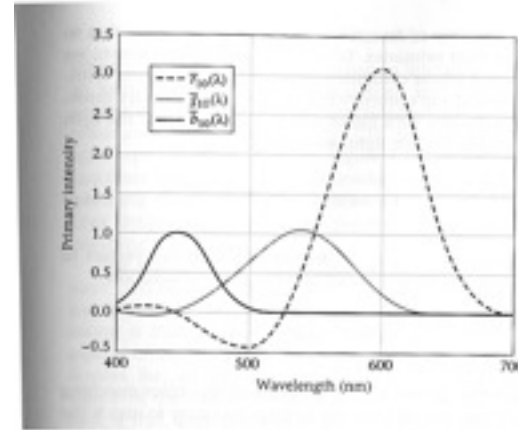
Let the new spectral signal be described by the vector  $t$ .

$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$

# Using the color matching functions to predict the primary match to a new spectral signal

Store the color matching functions in the rows of the matrix,  $C$

$$C = \begin{pmatrix} c_1(\lambda_1) & \cdots & c_1(\lambda_N) \\ c_2(\lambda_1) & \cdots & c_2(\lambda_N) \\ c_3(\lambda_1) & \cdots & c_3(\lambda_N) \end{pmatrix}$$



Let the new spectral signal be described by the vector  $t$ .

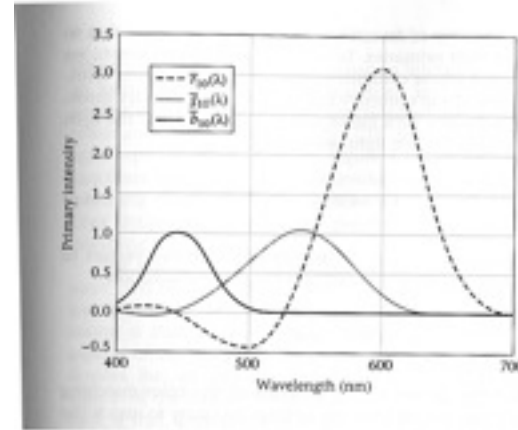
$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$

Then the amounts of each primary needed to match  $t$  are:

# Using the color matching functions to predict the primary match to a new spectral signal

Store the color matching functions in the rows of the matrix,  $C$

$$C = \begin{pmatrix} c_1(\lambda_1) & \cdots & c_1(\lambda_N) \\ c_2(\lambda_1) & \cdots & c_2(\lambda_N) \\ c_3(\lambda_1) & \cdots & c_3(\lambda_N) \end{pmatrix}$$



Let the new spectral signal be described by the vector  $t$ .

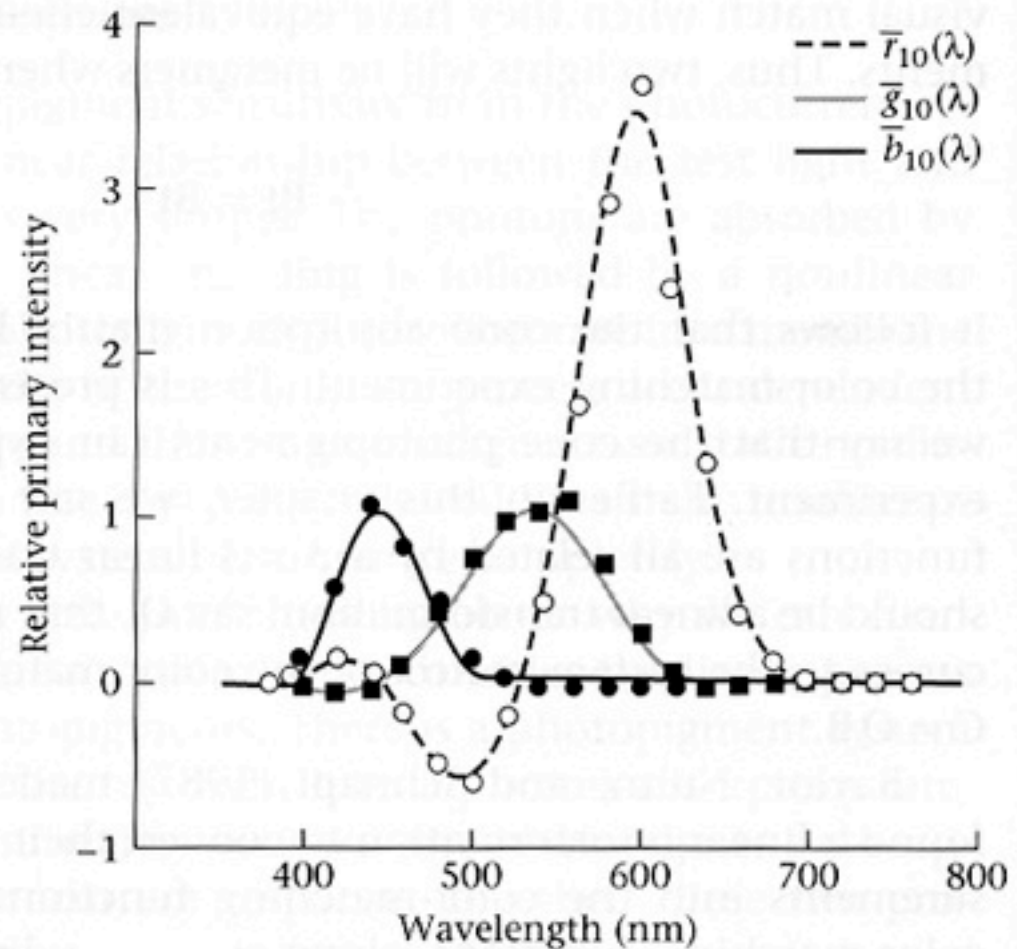
$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$

Then the amounts of each primary needed to match  $t$  are:

$$\sum_j \begin{pmatrix} c_1(\lambda_j)t(\lambda_j) \\ c_2(\lambda_j)t(\lambda_j) \\ c_3(\lambda_j)t(\lambda_j) \end{pmatrix} = C\vec{t}$$

# Comparison of color matching functions with best 3x3 transformation of cone responses

**4.20 COMPARISON OF CONE PHOTOCURRENT RESPONSES AND THE COLOR-MATCHING FUNCTIONS.** The cone photocurrent spectral responsivities are within a linear transformation of the color-matching functions, after a correction has been made for the optics and inert pigments in the eye. The smooth curves show the Stiles and Burch (1959) color-matching functions. The symbols show the matches predicted from the photocurrents of the three types of macaque cones. The predictions included a correction for absorption by the lens and other inert pigments in the eye. Source: Baylor, 1987.



# CIE XYZ color space

# CIE XYZ color space

- Commission Internationale d'Eclairage, 1931  
(International Commission on Illumination).

# CIE XYZ color space

- Commission Internationale d'Eclairage, 1931 (International Commission on Illumination).
- "...as with any standards decision, there are some irritating aspects of the XYZ color-matching functions as well...no set of physically realizable primary lights that by direct measurement will yield the color matching functions."

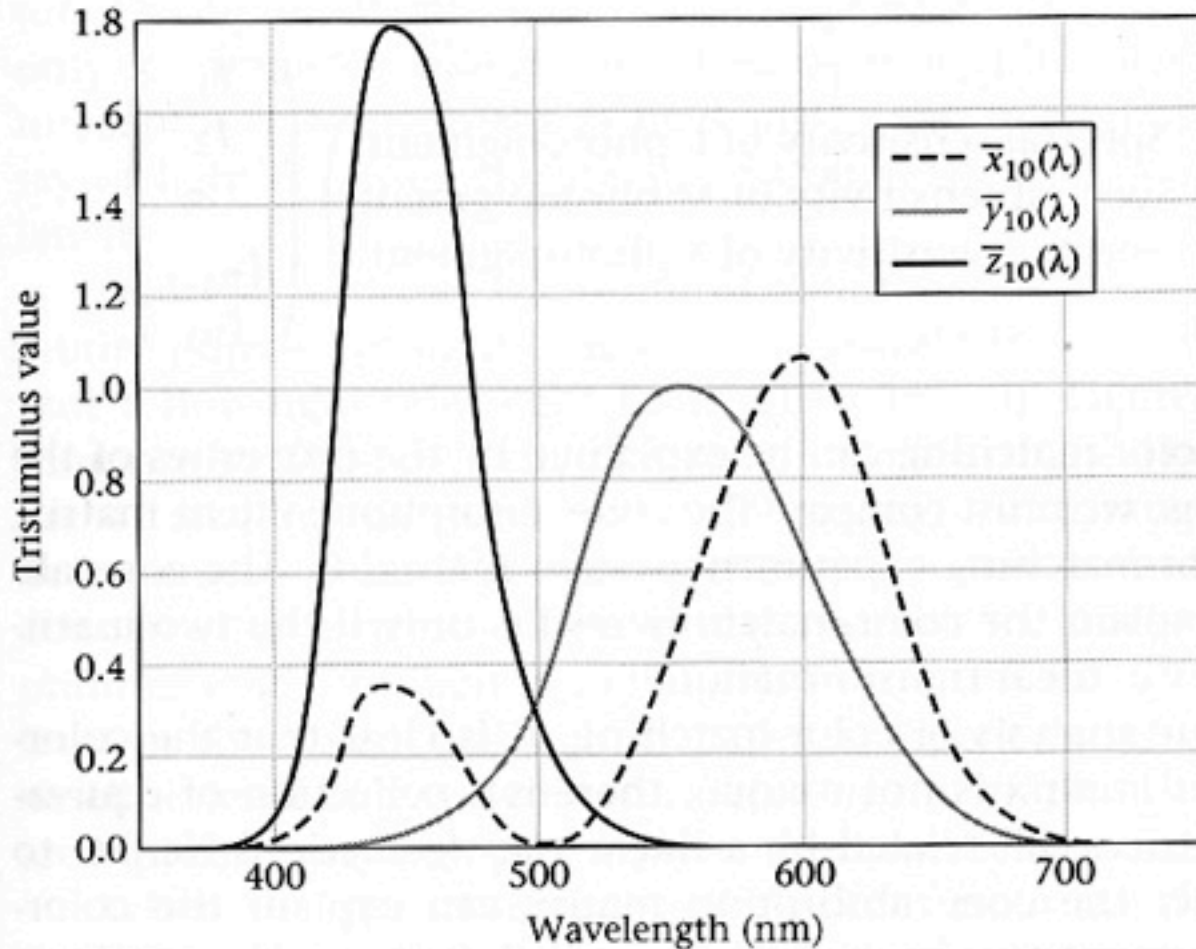


# CIE XYZ color space

- Commission Internationale d'Eclairage, 1931 (International Commission on Illumination).
- "...as with any standards decision, there are some irritating aspects of the XYZ color-matching functions as well...no set of physically realizable primary lights that by direct measurement will yield the color matching functions."
- "Although they have served quite well as a technical standard, and are understood by the mandarins of vision science, they have served quite poorly as tools for explaining the discipline to new students and colleagues outside the field."

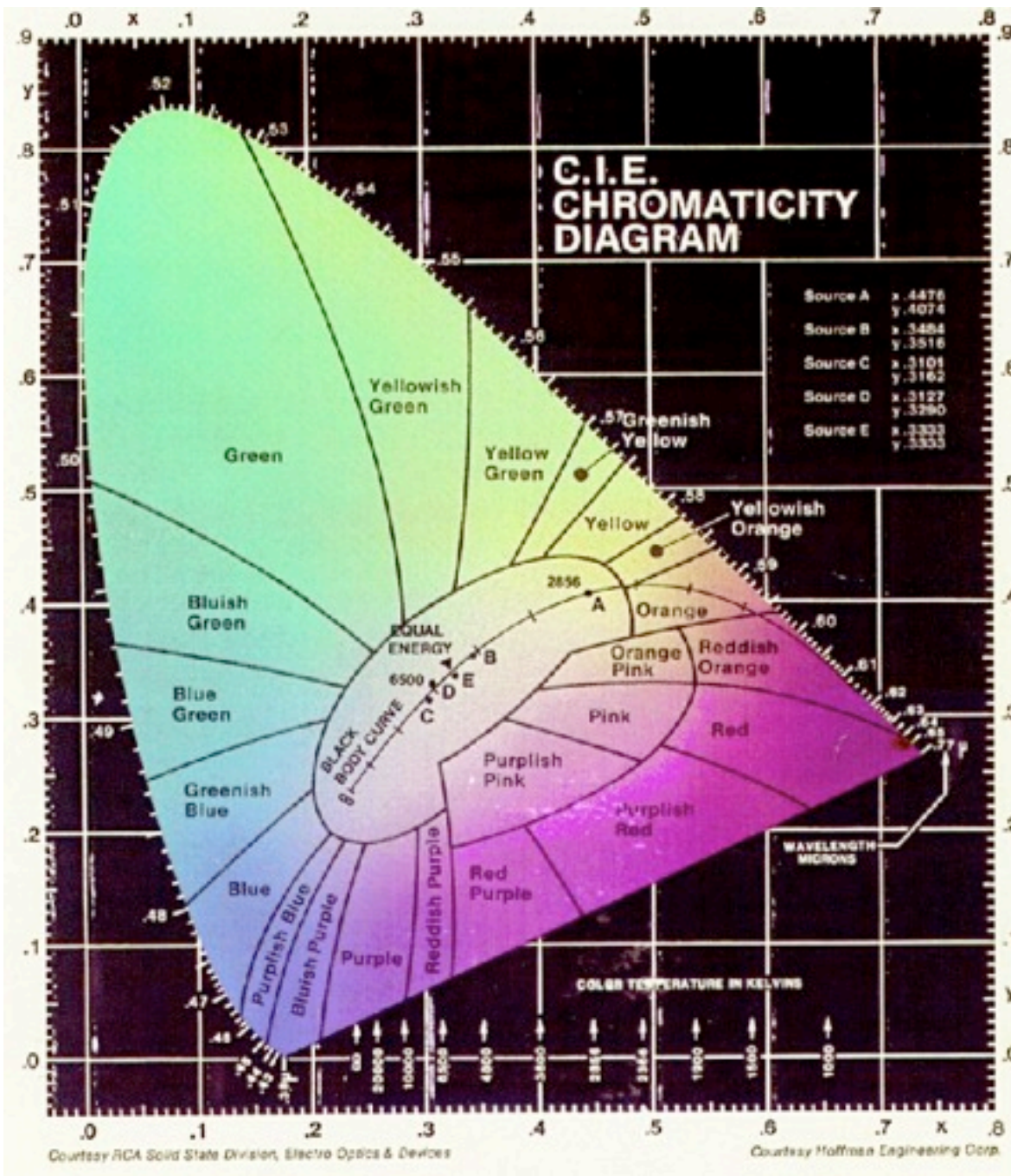
#### 4.14 THE XYZ STANDARD COLOR-MATCHING FUNCTIONS.

In 1931 the CIE standardized a set of color-matching functions for image interchange. These color-matching functions are called  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ , and  $\bar{z}(\lambda)$ . Industrial applications commonly describe the color properties of a light source using the three primary intensities needed to match the light source that can be computed from the XYZ color-matching functions.

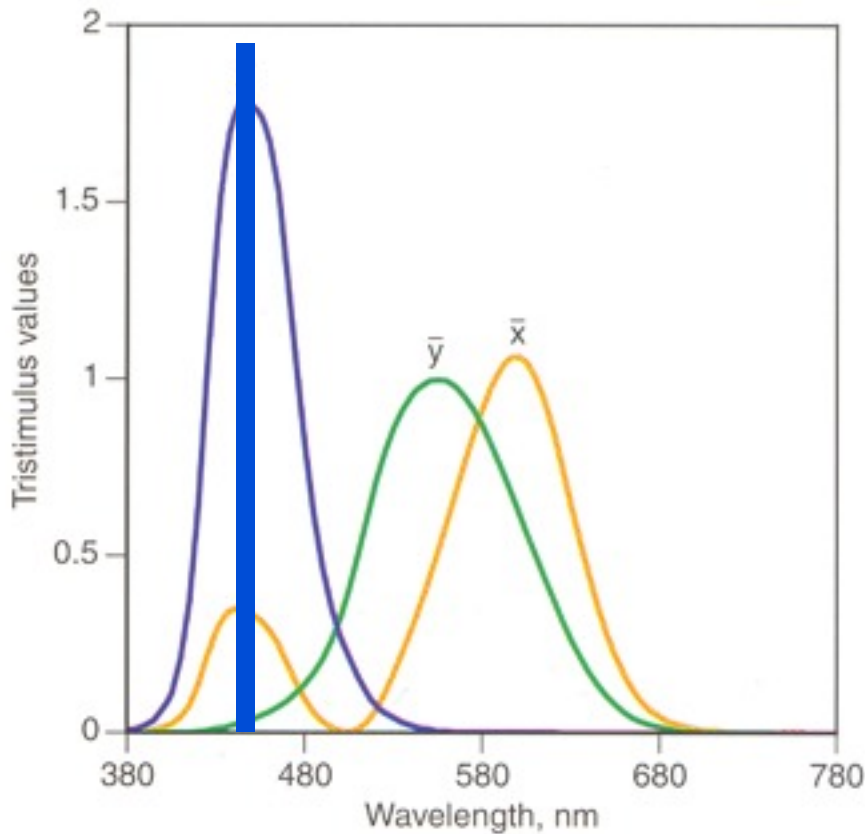


CIE XYZ: Color matching functions are positive everywhere, but primaries are “imaginary” (require adding light to the test color’s side in a color matching experiment). Usually compute  $x, y$ , where  $x = X/(X+Y+Z)$

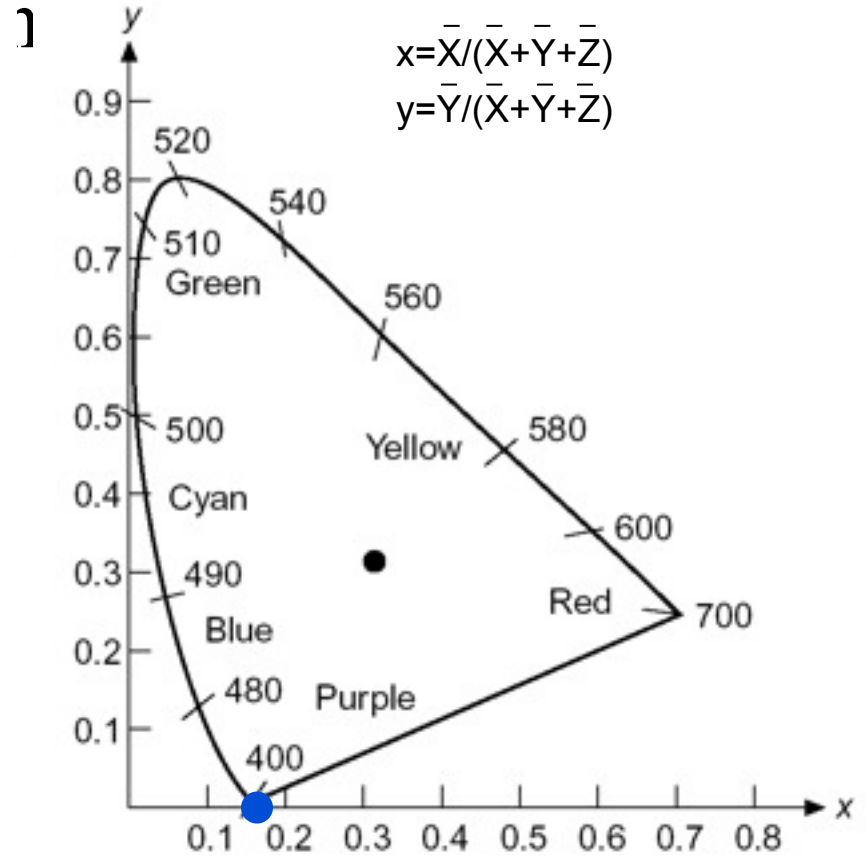
$$y = Y/(X+Y+Z)$$



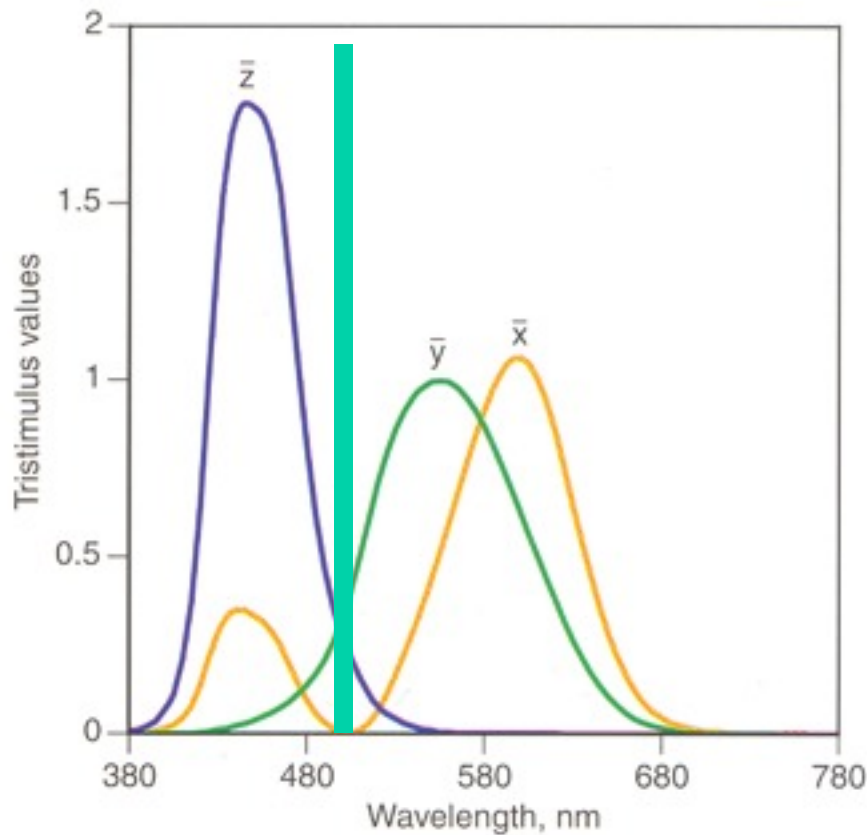
# Pure wavelength in chromaticity diagram



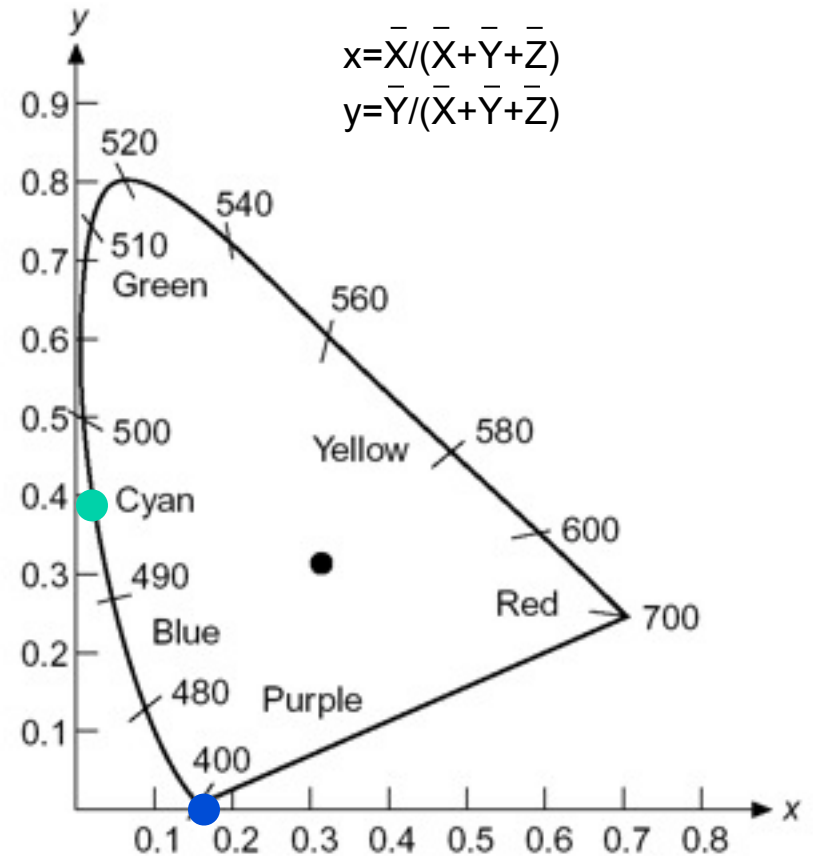
The 1931 standard observer, as it is usually shown.



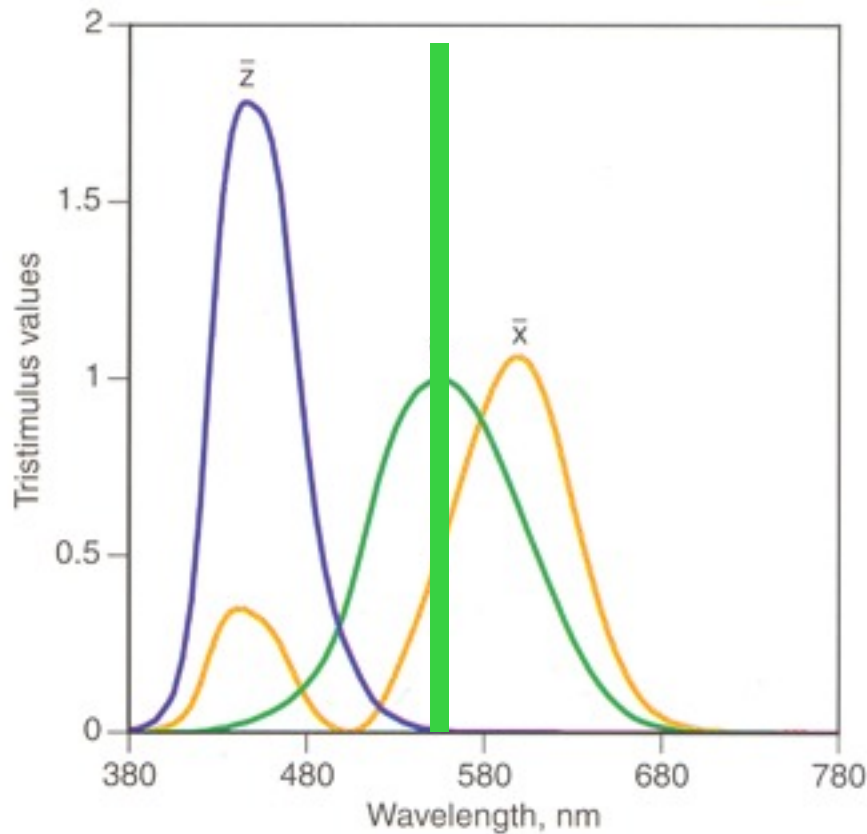
# Pure wavelength in chromaticity diagram



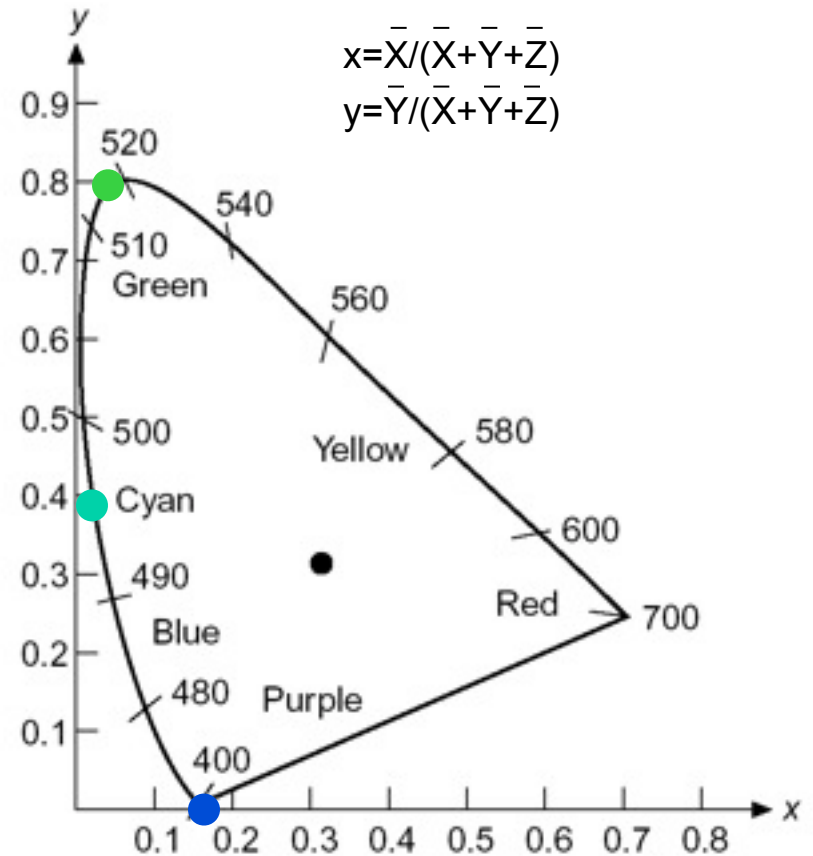
The 1931 standard observer, as it is usually shown.



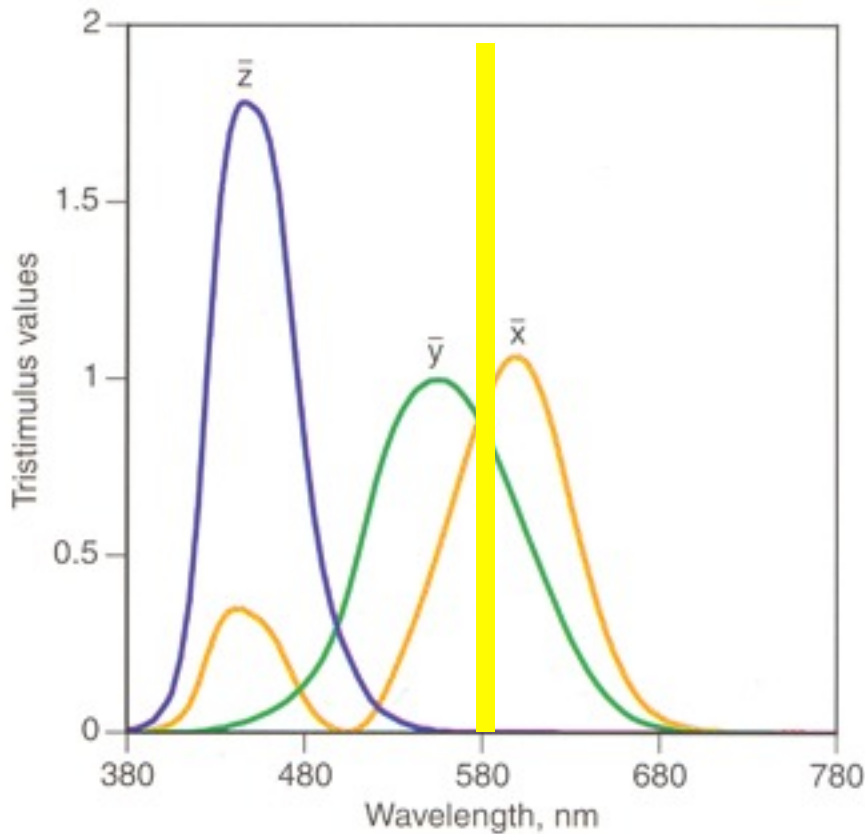
# Pure wavelength in chromaticity diagram



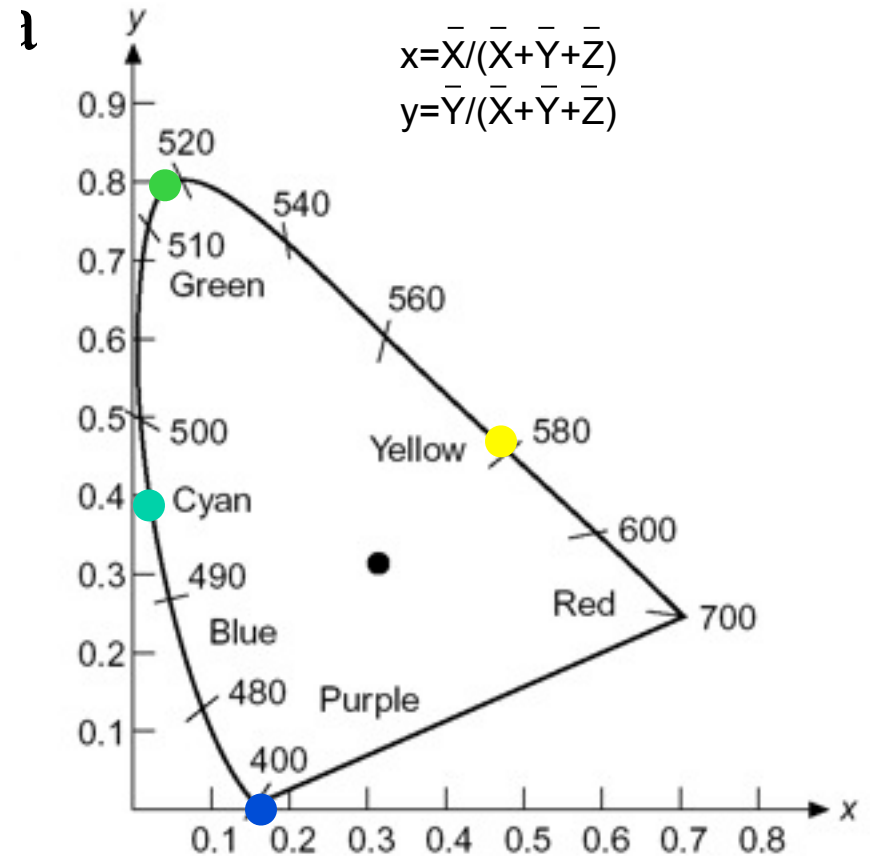
The 1931 standard observer, as it is usually shown.



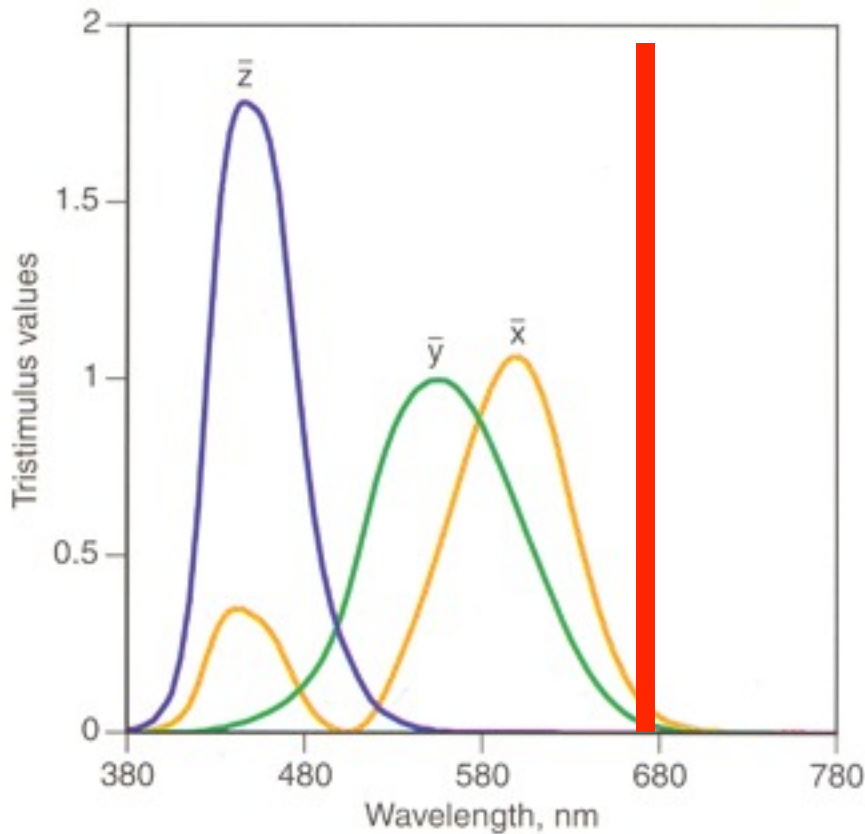
# Pure wavelength in chromaticity diagram



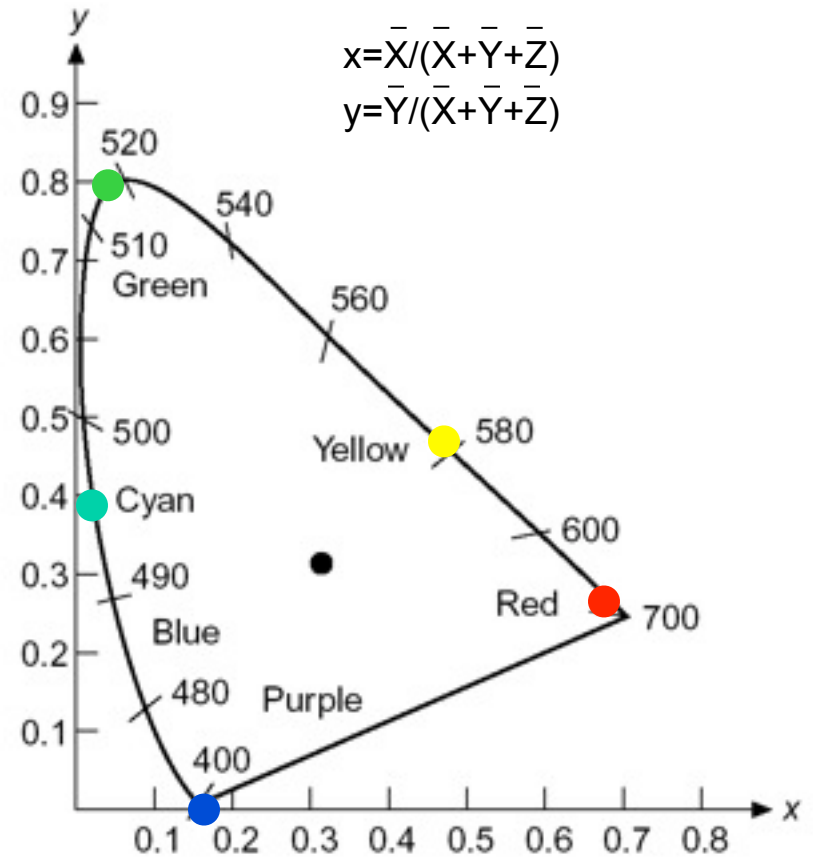
The 1931 standard observer, as it is usually shown.



# Pure wavelength in chromaticity diagram



The 1931 standard observer, as it is usually shown.





# XYZ vs. RGB

- Linear transform
- XYZ is rarely used for storage
- There are tons of flavors of RGB
  - sRGB, Adobe RGB
  - Different matrices!
- XYZ is more standardized
- XYZ can reproduce all colors with positive values
- XYZ is not realizable physically !!
  - What happens if you go “off” the diagram
  - In fact, the orthogonal (synthesis) basis of XYZ requires negative values.

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.24 & -1.54 & -0.50 \\ -0.97 & 1.88 & 0.04 \\ 0.06 & -0.20 & 1.06 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.41 & 0.36 & 0.18 \\ 0.21 & 0.72 & 0.07 \\ 0.02 & 0.12 & 0.95 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

# Color metamerism: different spectra looking the same color

Two spectra,  $t$  and  $s$ , perceptually match when

$$C\vec{t} = C\vec{s}$$

where  $C$  are the color matching functions for some set of primaries.

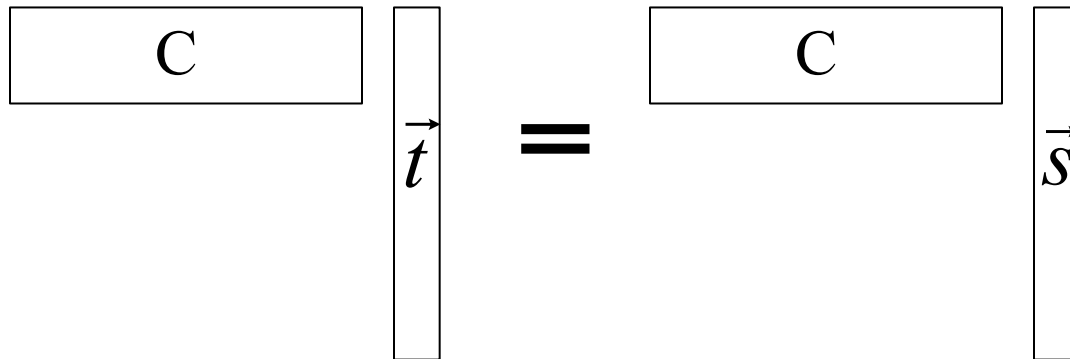
# Color metamerism: different spectra looking the same color

Two spectra,  $t$  and  $s$ , perceptually match when

$$C\vec{t} = C\vec{s}$$

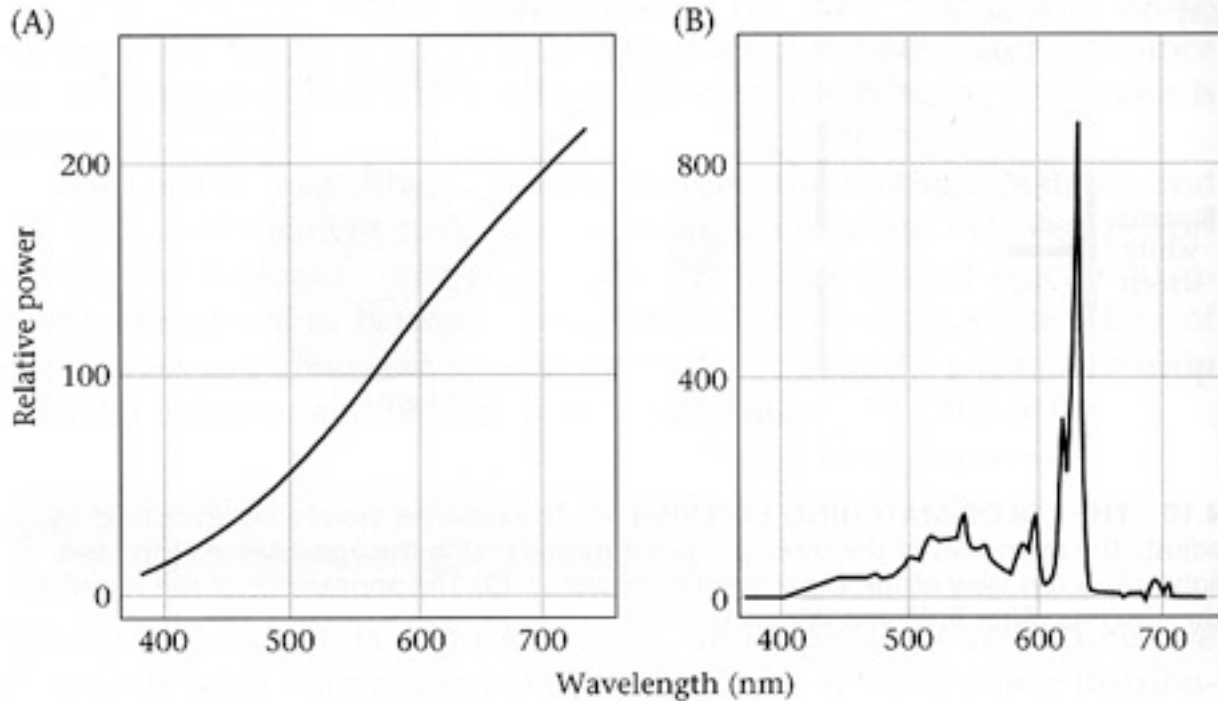
where  $C$  are the color matching functions for some set of primaries.

Graphically,

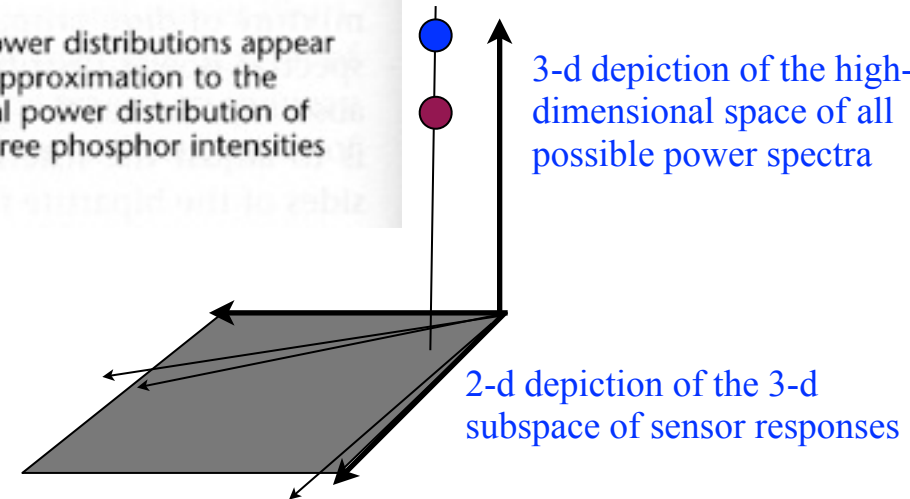


# Metameric lights

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995



**4.11 METAMERIC LIGHTS.** Two lights with these spectral power distributions appear identical to most observers and are called metamers. (A) An approximation to the spectral power distribution of a tungsten bulb. (B) The spectral power distribution of light emitted from a conventional television monitor whose three phosphor intensities were set to match the light in panel A in appearance.



# Concepts in color measurement

- What are colors?
  - Arise from power spectrum of light.
- How represent colors:
  - Pick primaries
  - Measure color matching functions (CMF's)
  - Matrix mult power spectrum by CMF's to find color as the 3 primary color values.
- How share color descriptions between people?
  - Standardize on a few sets of primaries.
  - Translate colors between systems of primaries.

Another psychophysical fact:  
luminance and chrominance  
channels in the brain

From W. E.  
Glenn, in  
Digital  
Images and  
Human  
Vision, MIT  
Press, edited  
by Watson,  
1993

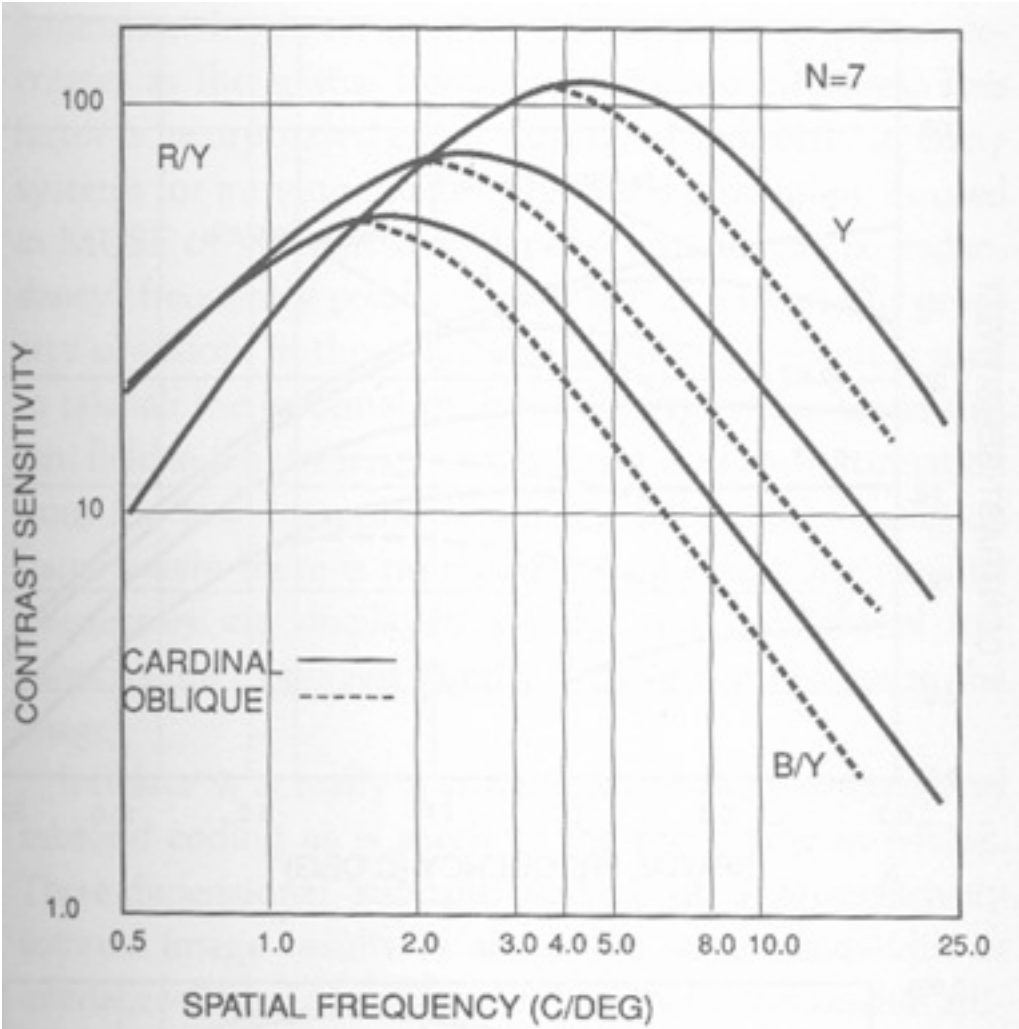
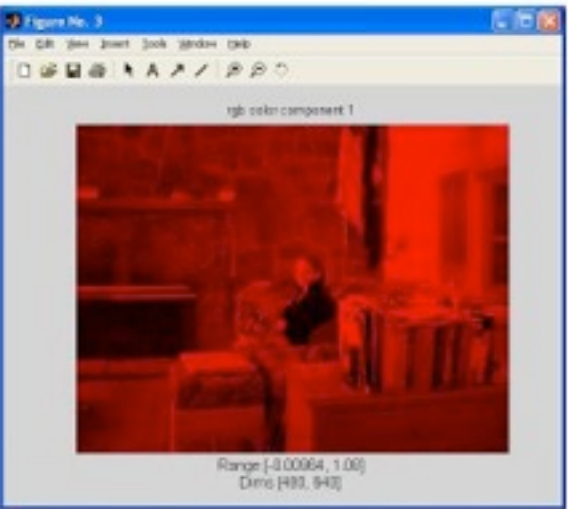


Figure 6.1  
Contrast sensitivity threshold functions for static luminance gratings (Y) and isoluminance chromaticity gratings (R/Y,B/Y) averaged over seven observers.

# NTSC color components: Y, I, Q

$$\begin{pmatrix} Y \\ I \\ Q \end{pmatrix} = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

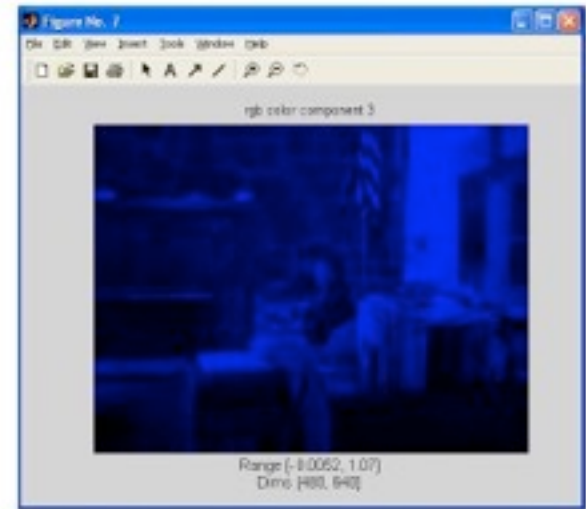
# NTSC - RGB



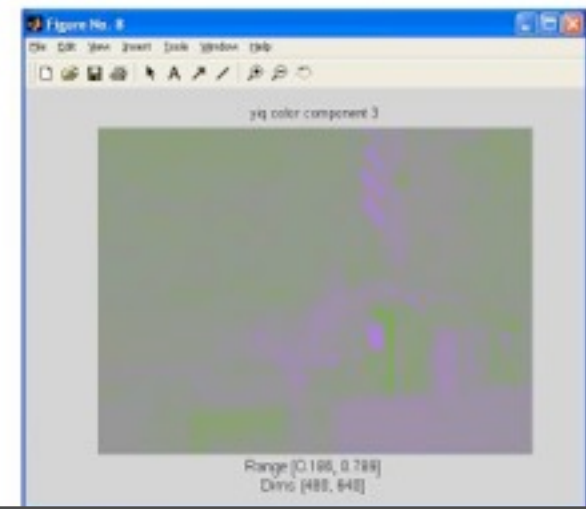
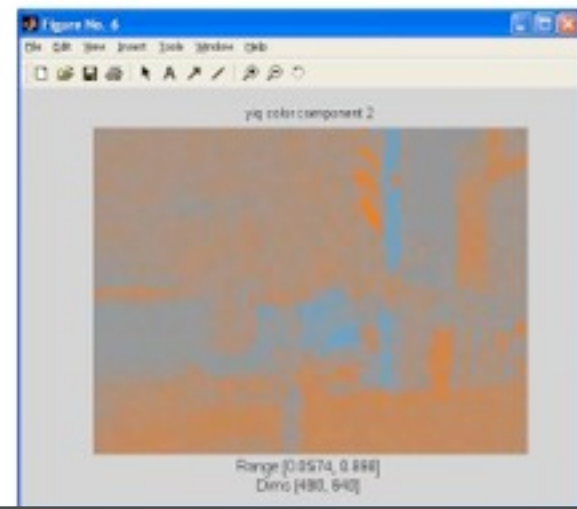
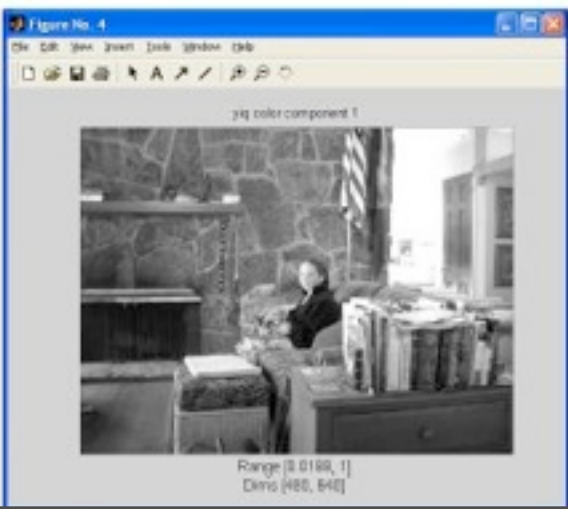
R



G



B





# Spatial resolution and color



original



R



G



B

# Blurring the G component



original



processed

# Blurring the G component



original



processed



R



G



B

# Blurring the R component



original



R



G



B

processed

# Blurring the R component



original



processed



R



G



B

# Blurring the B component



original



R



G



B

# Blurring the B component



original



processed



R



G



B

From W. E.  
Glenn, in  
Digital  
Images and  
Human  
Vision, MIT  
Press, edited  
by Watson,  
1993

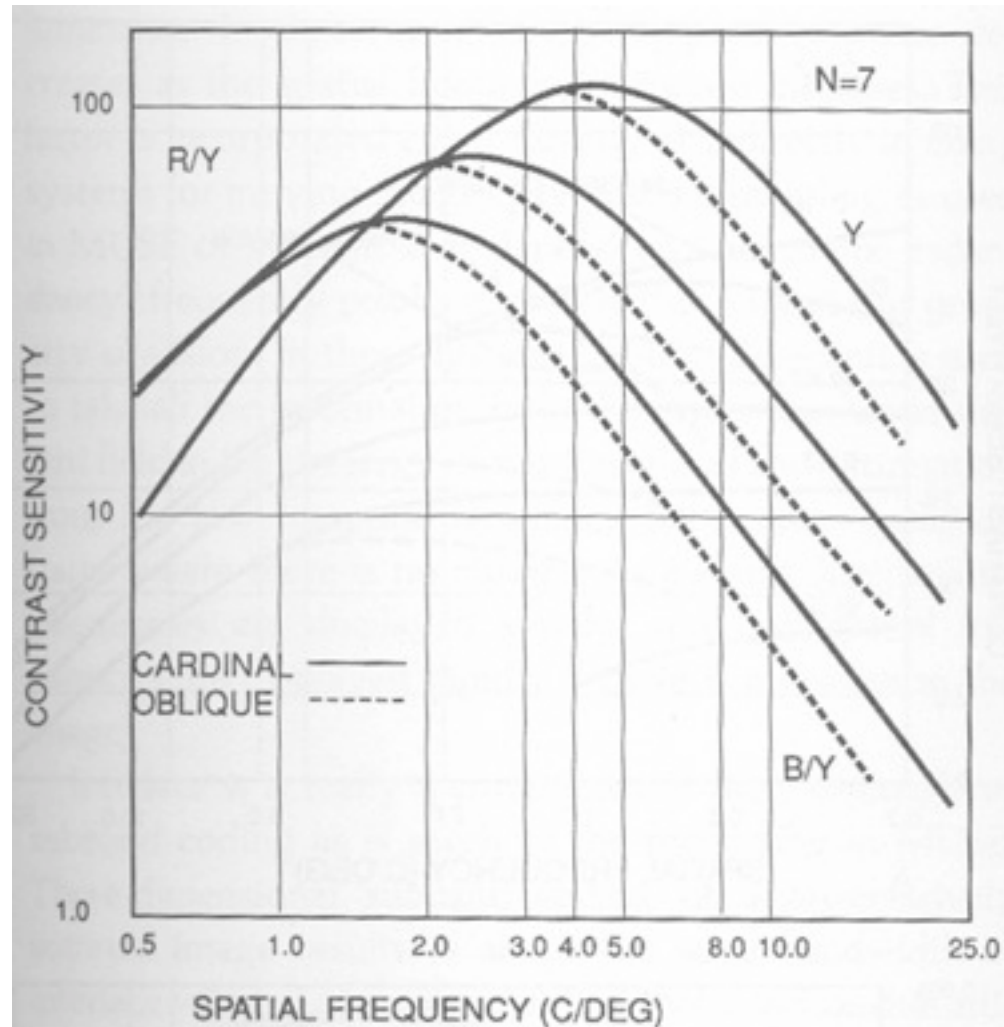


Figure 6.1

Contrast sensitivity threshold functions for static luminance gratings (Y) and isoluminance chromaticity gratings (R/Y, B/Y) averaged over seven observers.



# Lab color components



L      A rotation of the  
color  
coordinates into  
directions that  
are more  
perceptually  
meaningful:  
L: luminance,  
a: red-green,  
b: blue-yellow

a

b

# Blurring the L Lab component



original



L



a



b

# Blurring the L Lab component



original



processed



L



a



b

# Blurring the a Lab component



original



L



a



b

# Blurring the a Lab component



original



processed



L



a



b

# Blurring the b Lab component



original



L



a



b

# Blurring the b Lab component



original



processed



L



a



b

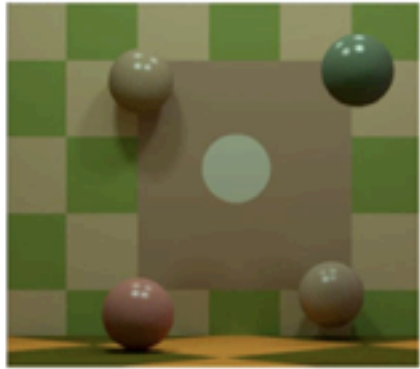
# Lecture outline

- Color physics.
- Color perception
  - part 1: assume perceived color only depends on light spectrum.
  - part 2: the more general case.



# Color constancy demo

- We assumed that the spectrum impinging on your eye determines the object color. That's often true, but not always. Here's a counter-example...



[Optical Society of America](#)

[Fall Vision Meeting](#)

**Spectrum Recovery Competition, 2011**

[David H. Brainard](#) and [Alex R. Wade](#)

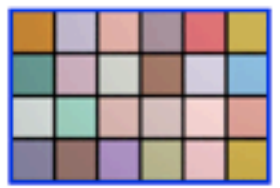


- [The Contest](#)
- [The Prize](#)
- [Entering and The Rules](#)
- [Image Generation](#)
- [Calibration Image](#)
- [Wavelength Sampling](#)
- [Data File Format](#)
- [Illuminant Spectral Power Distributions](#)
- [Surface Reflectance Functions](#)
- [Cone Coordinates](#)
- [Error Measure](#)
- [Sample Program](#)
- [Image Data](#)
- [Frequently Asked Questions \(FAQ\)](#)
- [References](#)

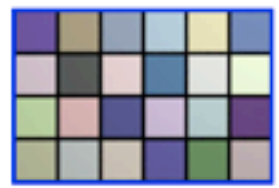
[Download zip archive containing all of the image data as well as the .mat files described above \(~50 MB\).](#)

Each image, we provide the LMS cone coordinates in a MATLAB .mat file. Each of these images contains a N by M by 3 matrix called theImage. The \$ planes provide the L, M, and S cone coordinates at each pixel respectively. We also provide an JPEG rendering of each image. These are simply f... lization and should not be used as actual image data for the contest - these images were scaled and/or tone-mapped by hand to produce reasonable es for display. The archive also contains the calibration image and its illuminant spectrum. For fun, the jpeg images are shown below.

*Calibration Image:*



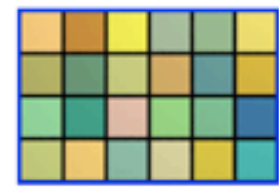
*Image 1:*



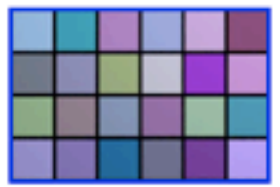
*Image 2:*



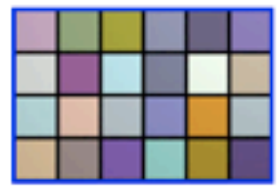
*Image 3:*



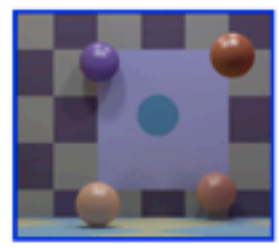
*Image 4:*



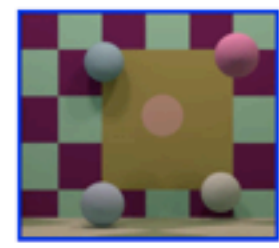
*Image 5:*



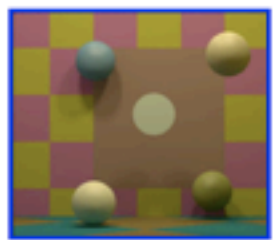
*Image 6:*



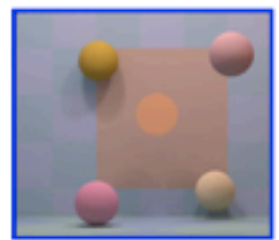
*Image 7:*



*Image 8:*



*Image 9:*



*Image 10:*





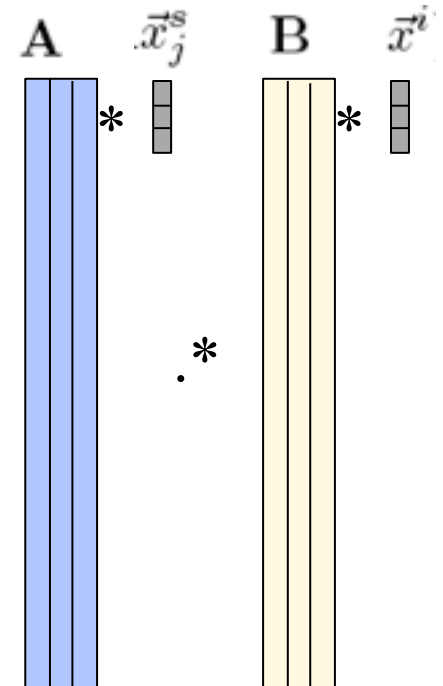
# Color constancy solution 1: find white in the scene

Let the  $k$ th patch be the white one, with surface coefficients assumed to be  $\vec{x}^W$   
 Then we can solve for the illuminant coefficient,  $\vec{x}^i$

$$\begin{pmatrix} L_k \\ M_k \\ S_k \end{pmatrix} = \mathbf{E}^T (\mathbf{A} \vec{x}^W \cdot * \mathbf{B} \vec{x}^i)$$

a 3x3 matrix

$$\begin{pmatrix} L_j \\ M_j \\ S_j \end{pmatrix} = \mathbf{E}^T \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} *$$





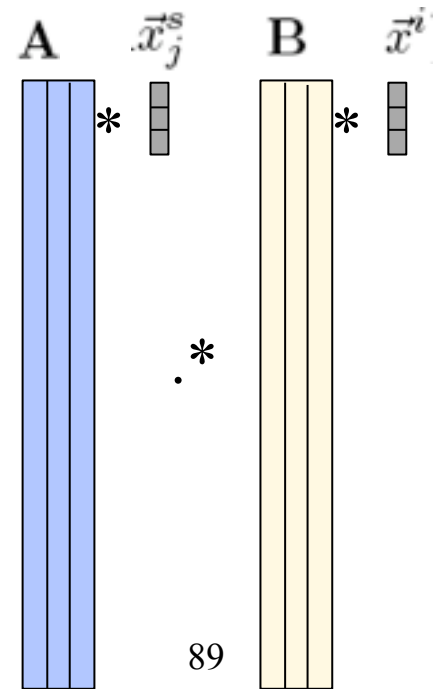
# Color constancy solution 2: assume scene colors average to grey

$$\frac{1}{N} \sum_j \begin{pmatrix} L_j \\ M_j \\ S_j \end{pmatrix} = \mathbf{E}^T \left( \mathbf{A} \frac{1}{N} \sum_j \vec{x}_j^s \cdot * \mathbf{B} \vec{x}^i \right)$$

$$= \mathbf{E}^T \left( \mathbf{A} \vec{x}^G \cdot * \mathbf{B} \vec{x}^i \right)$$

a 3x3 matrix

$$\begin{pmatrix} L_j \\ M_j \\ S_j \end{pmatrix} = \mathbf{E}^T \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} *$$



an image that violates both assumptions



[http://1.bp.blogspot.com/\\_vsiS4vPB35s/S9FiRzKmyEI/AAAAAAAAAAUc/TSb5RVWDM9Q/s1600/NATURE-GreenForest\\_1024x768.jpeg](http://1.bp.blogspot.com/_vsiS4vPB35s/S9FiRzKmyEI/AAAAAAAAAAUc/TSb5RVWDM9Q/s1600/NATURE-GreenForest_1024x768.jpeg)

Wednesday, September 11, 13



# Bayesian approach

Bayes rule

$$P(\vec{x}|\vec{y}) = kP(\vec{y}|\vec{x})P(\vec{x})$$

Likelihood

$$P(\vec{y}_j|\vec{x}^i, \vec{x}_j^s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{-|\vec{y}_j - \vec{f}(\vec{x}^i, \vec{x}_j^s)|^2}{2\sigma^2},$$

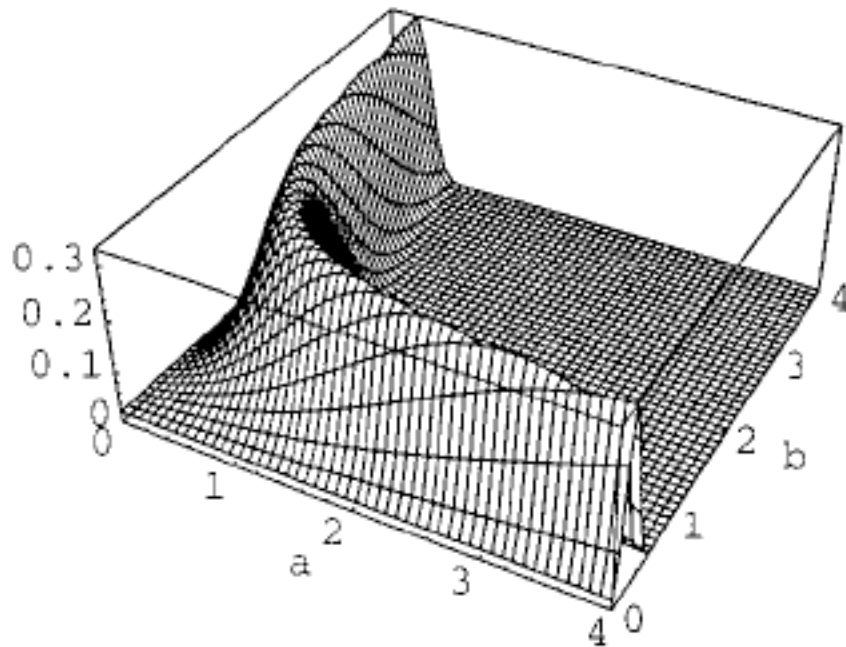
$$\vec{y}_j = \begin{pmatrix} L_j \\ M_j \\ S_j \end{pmatrix}$$

$$\vec{f}(\vec{x}^i, \vec{x}_j^s) = \mathbf{E}^T (\mathbf{A}\vec{x}_j^s \cdot * \mathbf{B}\vec{x}^i).$$

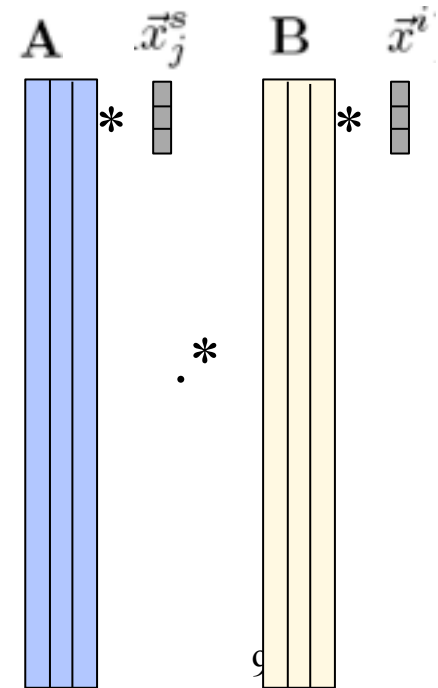
Posterior

$$P(\vec{x}|\vec{y}) = P(\vec{x}^i) \prod_j P(\vec{y}_j|\vec{x}^i, \vec{x}_j^s)P(\vec{x}_j^s)$$

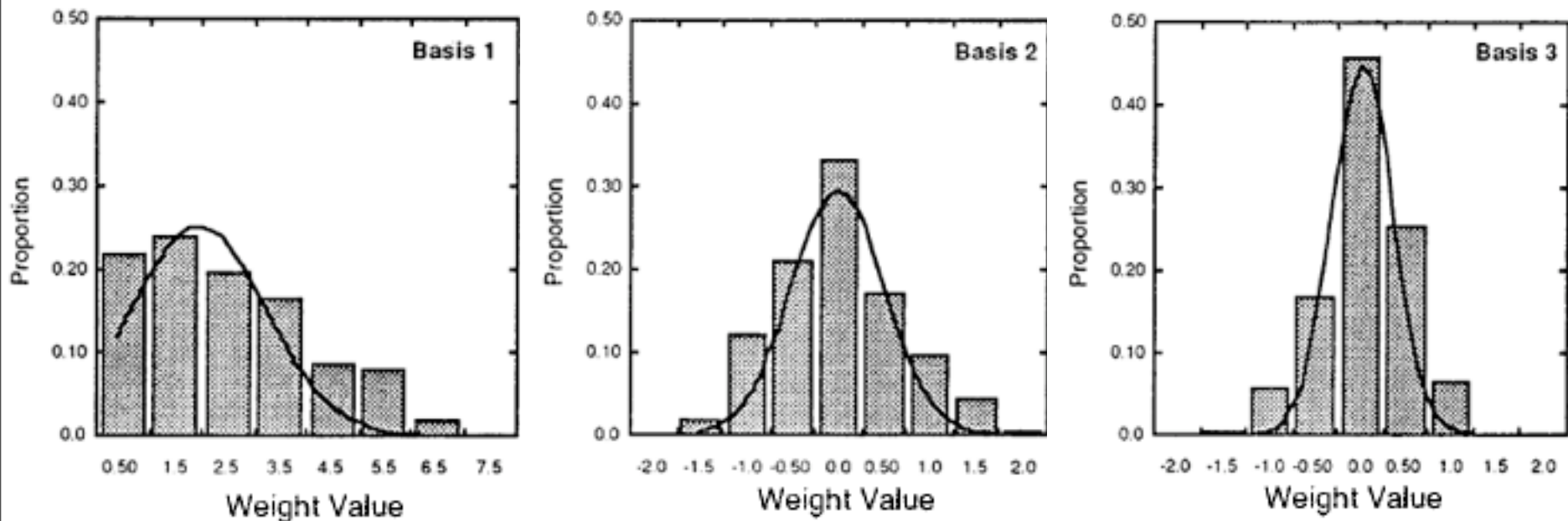
# Likelihood term for a $b = 1$ problem



$$\begin{matrix} L_j \\ M_j \\ S_j \end{matrix} = \mathbf{E}^T \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} *$$



# Bayesian approach: priors on surfaces and illuminants



Distribution of surface weights. The histograms show the distribution of linear model weights derived from the measurements of Kelly *et al.*<sup>68</sup> and Nickerson.<sup>69</sup> Each histogram corresponds to one basis vector. The solid curves show the fit of a truncated trivariate normal distribution to the weights.

# Picking a single best $\mathbf{x}$

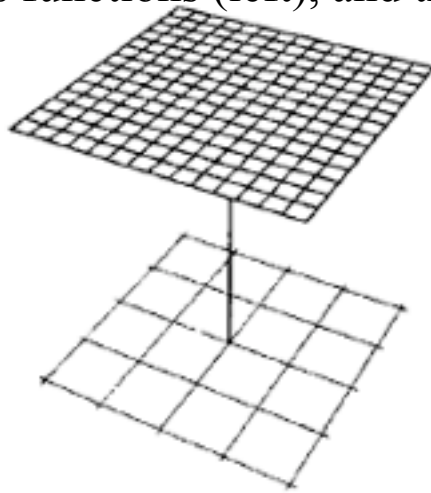
From the supplementary notes for this lecture:

with the *loss function*, which specifies the penalty for guessing wrong. Let  $\hat{\mathbf{x}}$  be your estimate of the parameters,  $\mathbf{x}$ . Then  $L(\hat{\mathbf{x}}, \mathbf{x})$  is the loss incurred by guessing  $\hat{\mathbf{x}}$  when the true value was  $\mathbf{x}$ . With the posterior probability, we can calculate the expected loss,  $\bar{L}(\hat{\mathbf{x}}, \mathbf{x})$

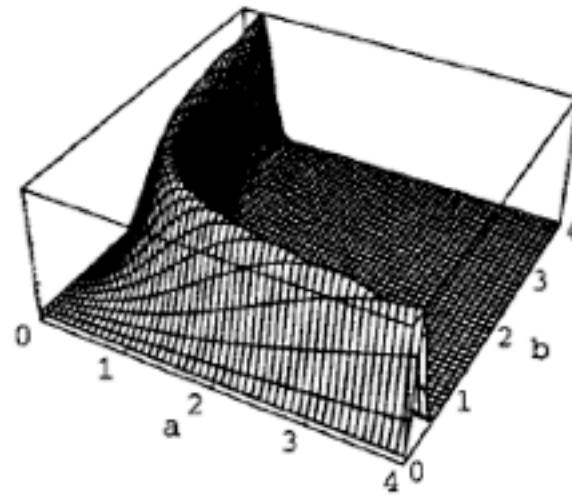
$$\bar{L}(\hat{\mathbf{x}}, \mathbf{x}) = \int_{\mathbf{x}} L(\hat{\mathbf{x}}, \mathbf{x}) P(\mathbf{x}|\mathbf{y}) \quad (6.20)$$

We often use a loss function which is only a function of  $\hat{\mathbf{x}} - \mathbf{x}$ .

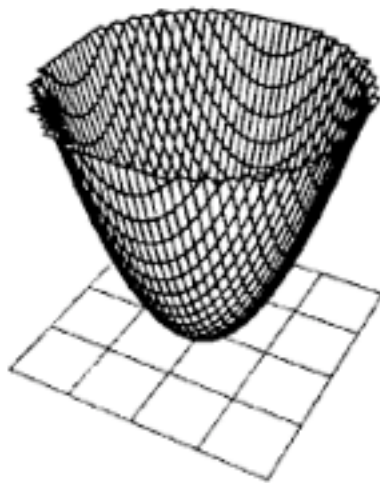
Two loss functions (left), and the (minus) expected losses for the  $1=ab$  problem



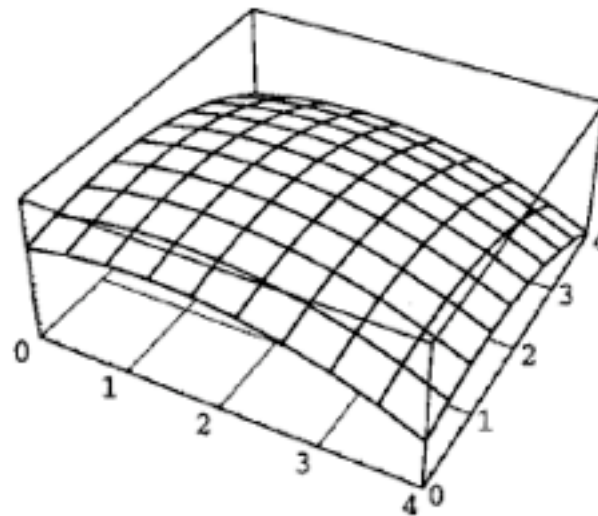
(a) MAP loss function



(d) (minus) MAP expected loss



(b) MMSE loss function



(e) (minus) MMSE expected loss

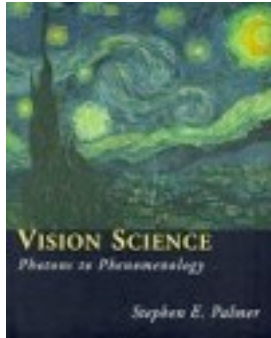
# MAP estimate of illumination spectrum

- Start from some illuminant candidate.
- Find the surface colors that would best explain the observed data.
  - Evaluate the corresponding likelihood and prior probability terms.
- Move to another illuminant choice.

# MMSE estimate of illumination spectrum

For the MMSE estimate, we will use a Monte Carlo method (averaging many different trials). We will take many random draws of candidate illuminant spectra, and the corresponding surface colors that would explain the observed image data, and then check how probable that set of surface colors would be. We'll use that probability as a weight to form a weighted average of the sampled illumination spectra, which will be the MMSE estimate.

# Selected Bibliography



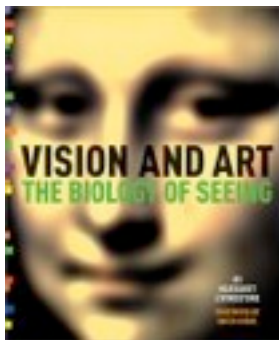
## **Vision Science**

by Stephen E. Palmer  
MIT Press; ISBN: 0262161834  
760 pages (May 7, 1999)



## **Billmeyer and Saltzman's Principles of Color Technology, 3rd Edition**

by Roy S. Berns, Fred W. Billmeyer, Max Saltzman  
Wiley-Interscience; ISBN: 047119459X  
304 pages 3 edition (March 31, 2000)

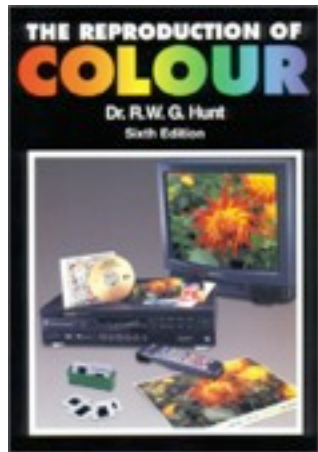


## **Vision and Art : The Biology of Seeing**

by Margaret Livingstone, David H. Hubel  
Harry N Abrams; ISBN: 0810904063  
208 pages (May 2002)

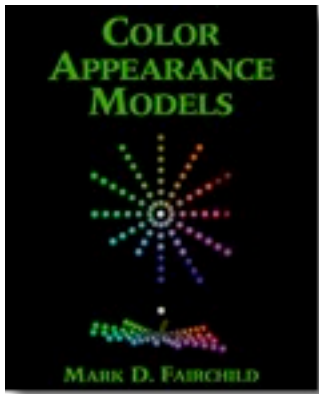


# Selected Bibliography



## **The Reproduction of Color**

by R. W. G. Hunt  
Fountain Press, 1995



## **Color Appearance Models**

by Mark Fairchild  
Addison Wesley, 1998

# Other color references

- Reading:
  - Chapter 6, Forsyth & Ponce
  - Chapter 4 of Wandell, Foundations of Vision, Sinauer, 1995 has a good treatment of this.