## Hint for homework 3

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6.869

To clarify some vocabulary:

- perceptual match: two colors are a perceptual match if the eye sees them in the same way (they look like they are the same color).
- primary lights: three lights used to generate colors. For instance, a TV uses a set of three primaries to generate colors.
- color matching functions: in the case of the TV, the manufacturer will release the color matching functions (but not necessarily the spectral properties of the primary lights used by the TV). The color matching functions will tell you, for each light wavelength $\lambda$, how to do you need to combine the primaries in order to produce a light that will be a perceptual match to the light with the single $\lambda$ component.

Let

- $\mathbf{C}=$ eye sensitivity curves $[3, n]$
- $\mathbf{M}=$ color matching from the primaries [3,n]
- $\mathbf{P}=$ unknown primaries $[3, n]$

As discussed in class, the three row vectors in the color matching matrix $\mathbf{M}$ span the same sub-space as the eye sensitivity curves, $\mathbf{C}$.
The primaries, $\mathbf{P}$, associated with a given set of color matching functions, $\mathbf{M}$, are not unique. (Sometimes, for a given set of color matching functions, the primaries can even have negative values.)
To find the primaries associated with a given set of color matching functions, note that

$$
\begin{equation*}
\mathbf{M} \mathbf{P}^{\prime}=\mathbf{I} \tag{1}
\end{equation*}
$$

where $\mathbf{I}$ is the $3 \times 3$ identity matrix. This follows because projecting a spectrum onto the color matching functions tells how much of each primary is needed to perceptually match that spectrum. If the spectrum happens to be exactly that of one of the primaries, then one times that primary, and zero times the others, will match.
Note that $\mathbf{P}^{\prime}$ has a uniquely-determined component that lies in the subpace of the row vectors of $\mathbf{M}$, and an arbitrary component from the null space of $\mathbf{M}$.

