MIT CSAIL

6.819 / 6.869: Advances in Computer Vision

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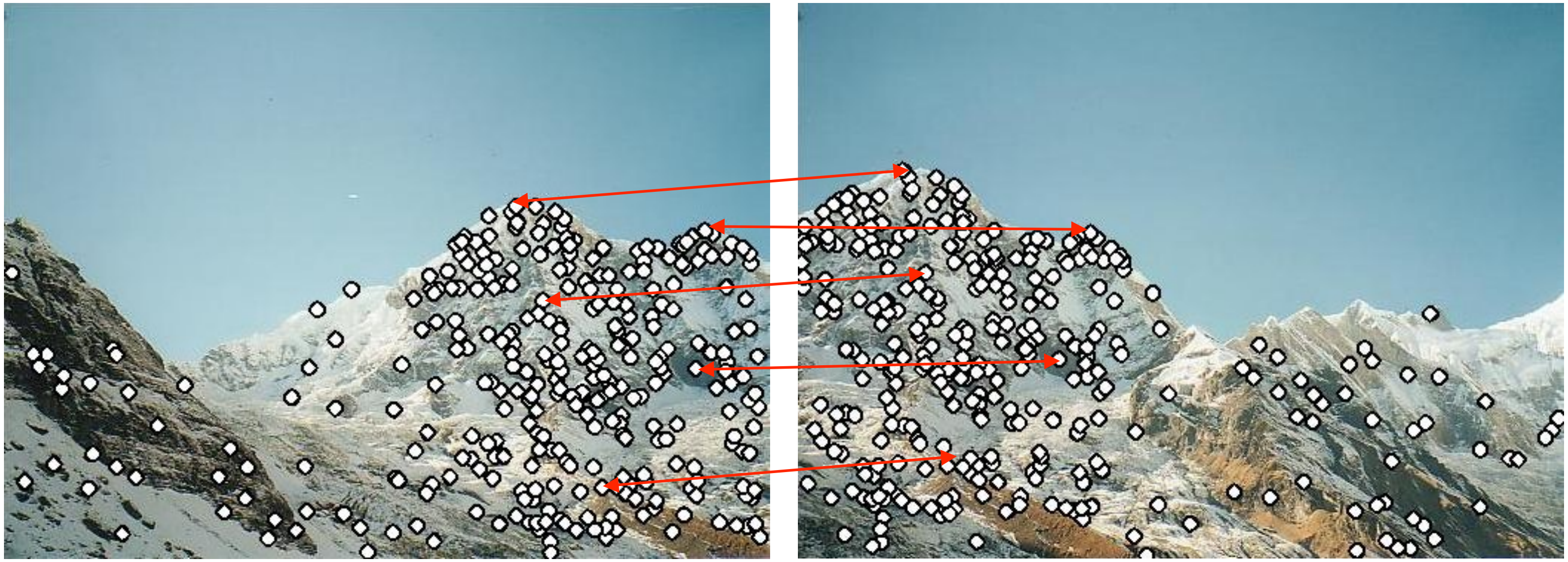
Lecture 13

Image features, SIFT
Homographies, RANSAC and panoramas

Matching with Features

- Detect feature points in both images
- Find corresponding pairs

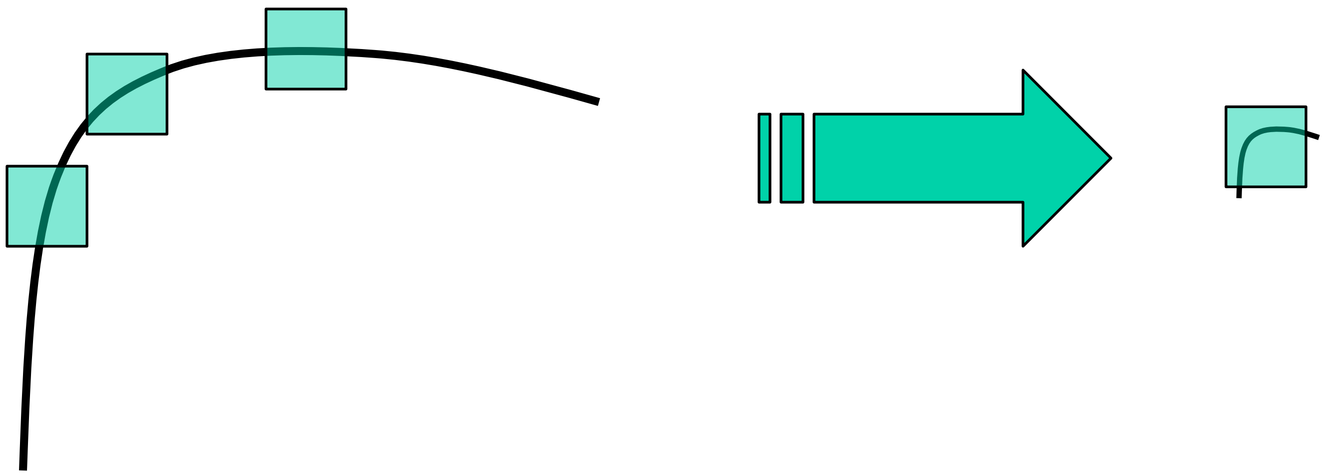


Outline

- Feature point detection
  - Harris corner detector
  - finding a characteristic scale: DoG or Laplacian of Gaussian
- Local image description
  - SIFT features

Harris Detector: Some Properties

- Not invariant to image scale!



All points will be classified as edges

Corner !

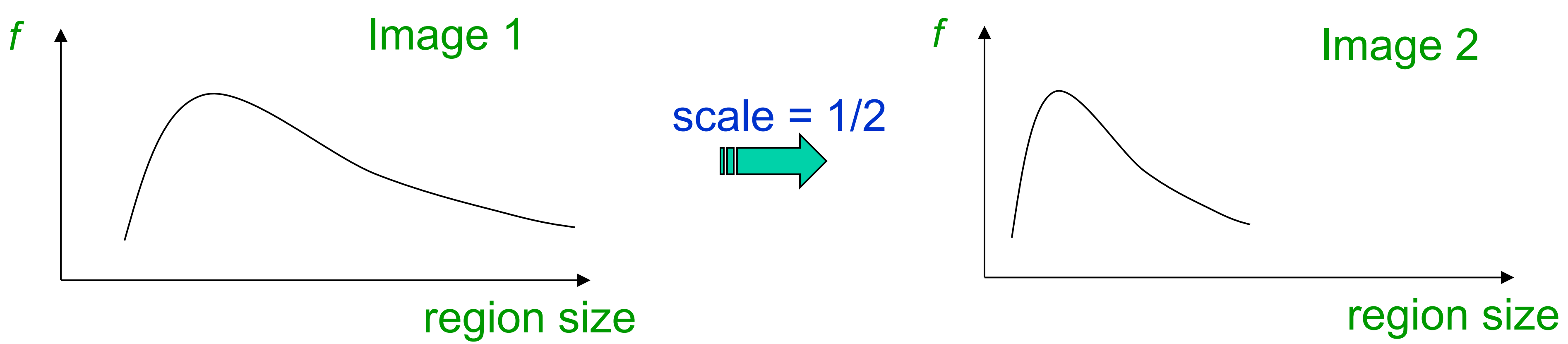
http://www.wisdom.weizmann.ac.il/~deniss/vision_spring04/files/InvariantFeatures.ppt
Darya Frolova, Denis Simakov The Weizmann Institute of Science

Scale Invariant Detection

- Solution:
  – Design a function on the region (circle), which is "scale invariant" (the same for corresponding regions, even if they are at different scales)

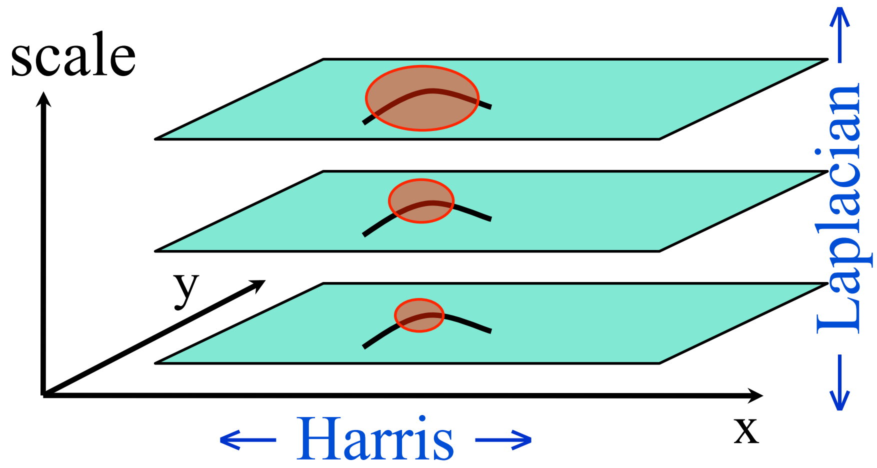
    > Example: average intensity. For corresponding regions (even of different sizes) it will be the same.

  – For a point in one image, we can consider it as a function of region size (circle radius)

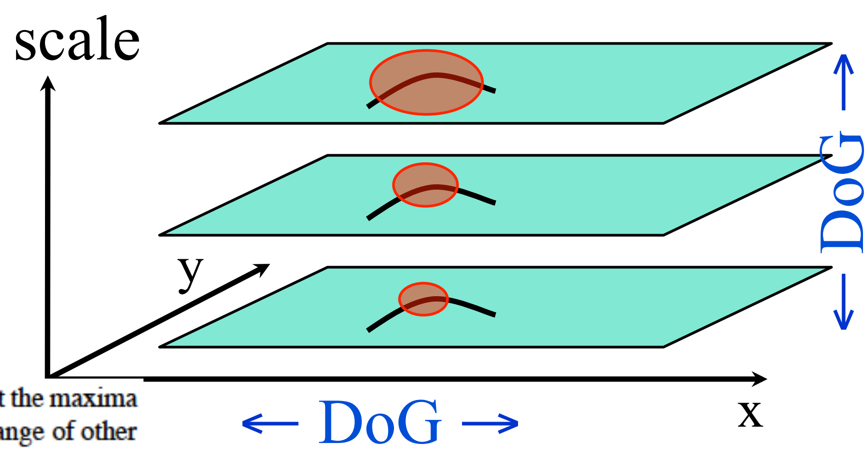


Scale Invariant Detectors

- Harris-Laplacian[1]
  Find local maximum of:
  - Harris corner detector in space (image coordinates)
  - Laplacian in scale



- SIFT (Lowe)[2]
  Find local maximum (minimum) of:
- Difference of Gaussians in space and scale



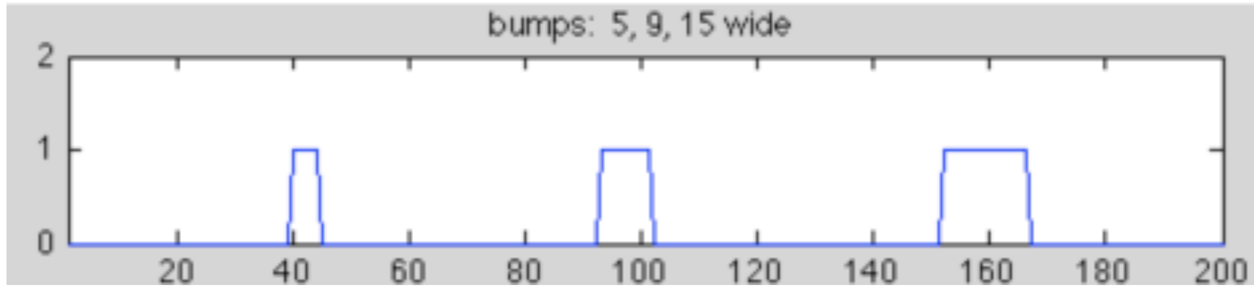
In detailed experimental comparisons, Mikolajczyk (2002) found that the maxima and minima of $\sigma^2\nabla^2 G$ produce the most stable image features compared to a range of other possible image functions, such as the gradient, Hessian, or Harris corner function.

[1] K.Mikolajczyk, C.Schmid. "Indexing Based on Scale Invariant Interest Points". ICCV 2001

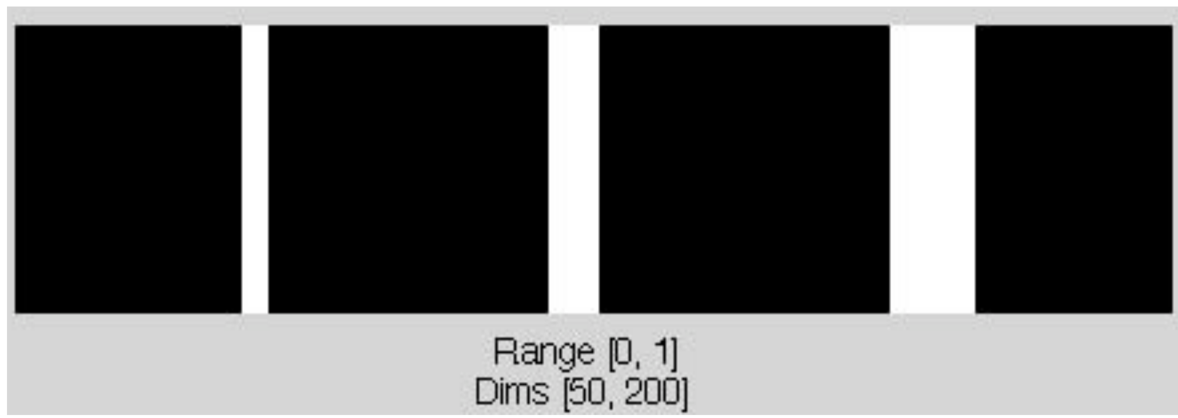
[2] D.Lowe. "Distinctive Image Features from Scale-Invariant Keypoints". Accepted to IJCV 2004

Scale-space example: 3 bumps of different widths.

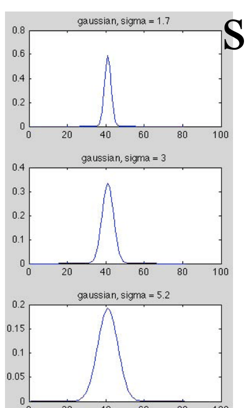
1-d bumps



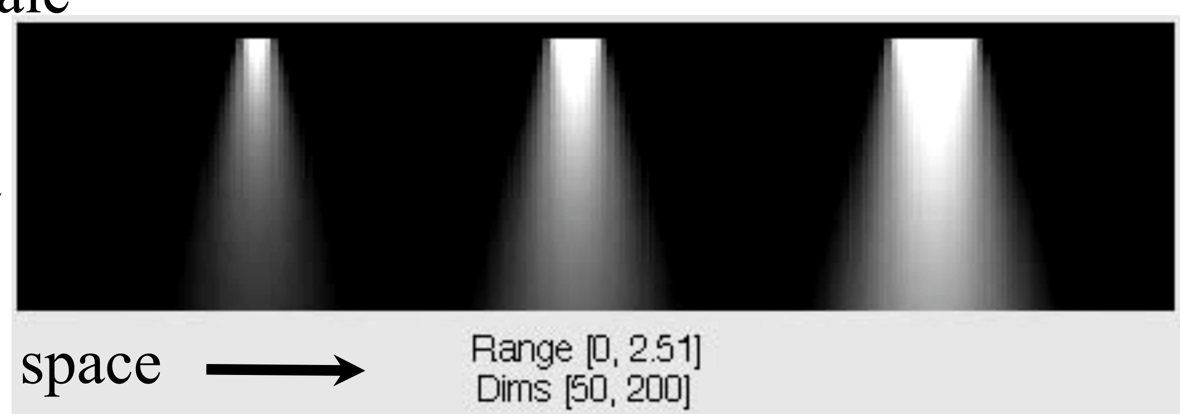
display as an image



blur with Gaussians of increasing width

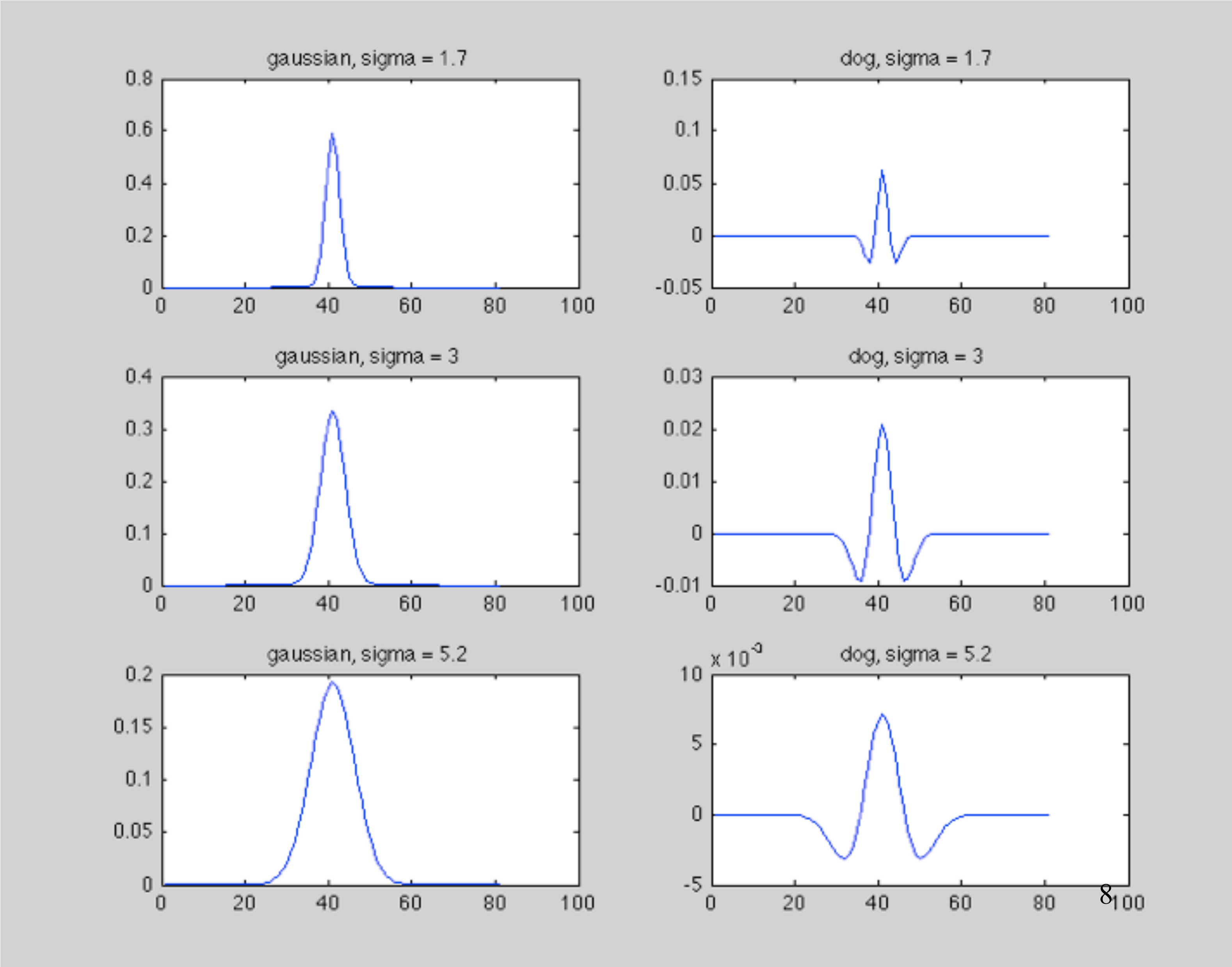


scale

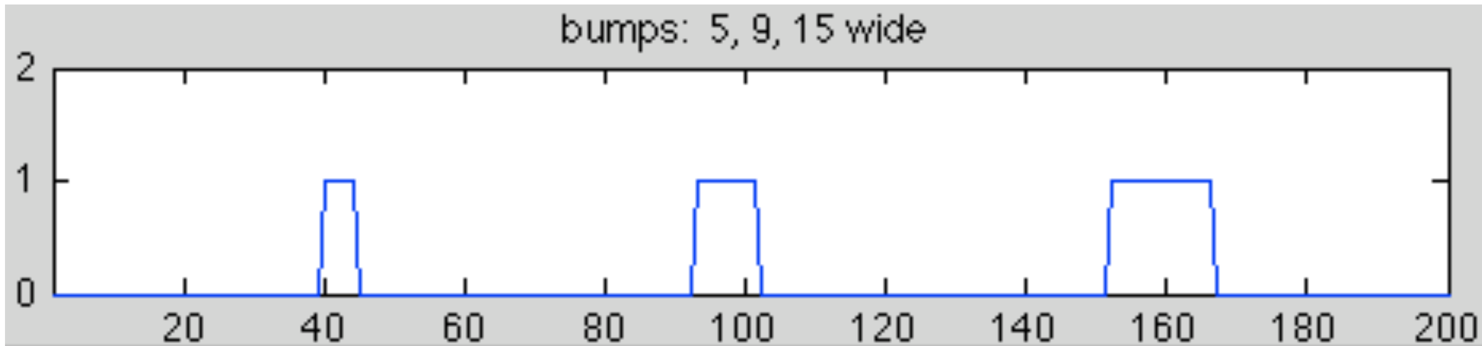


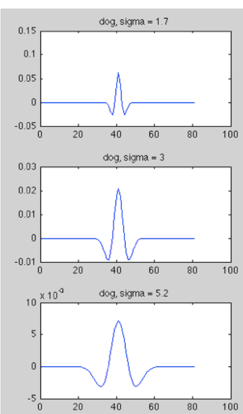
space

Gaussian and difference-of-Gaussian filters

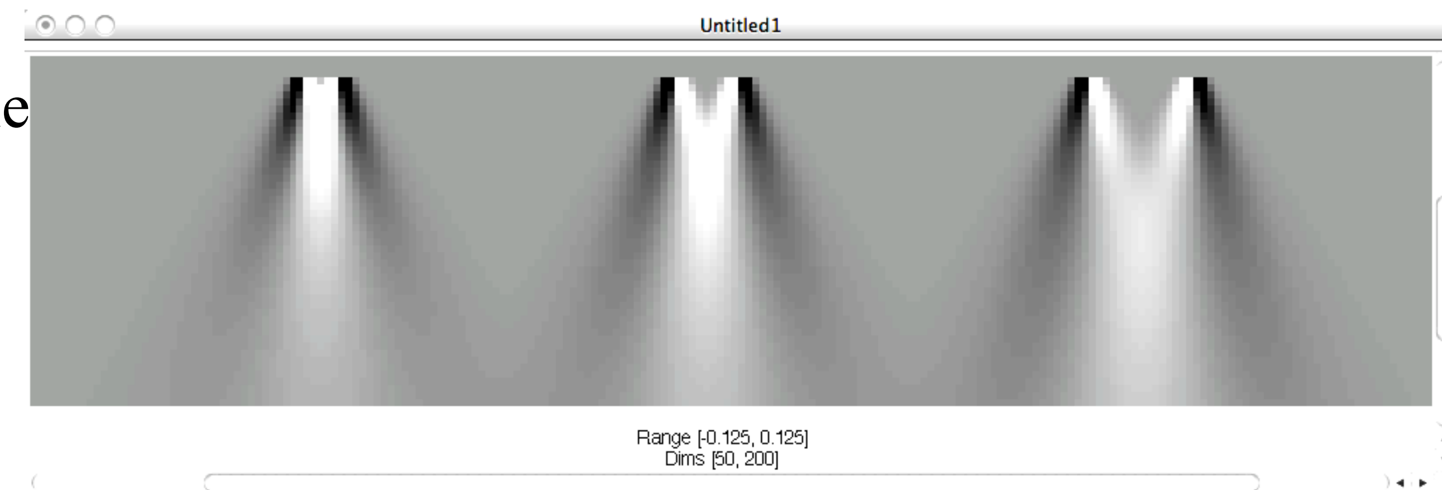


The bumps, filtered by difference-of-Gaussian filters



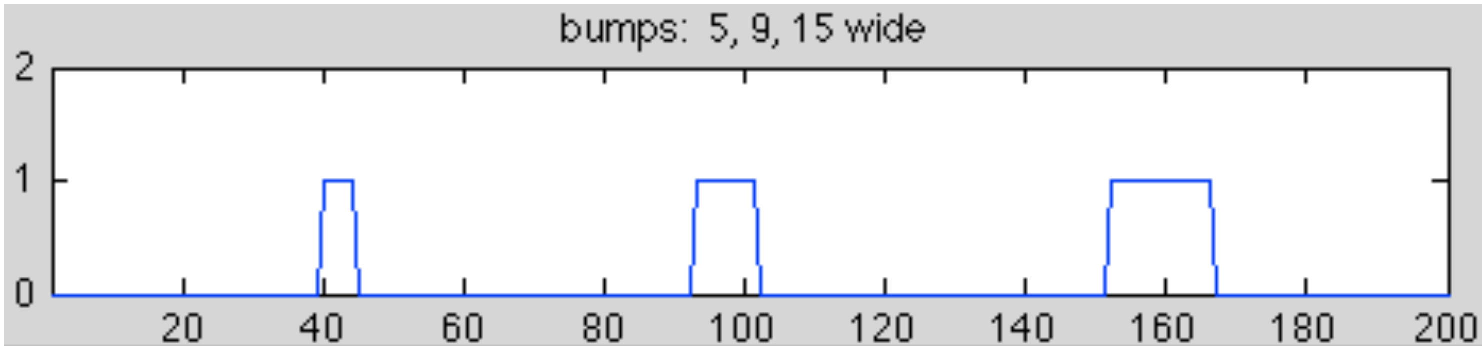


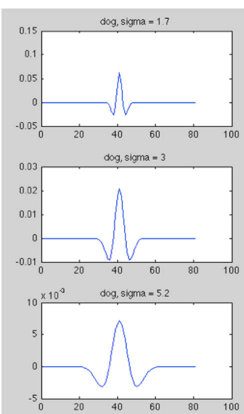
scale

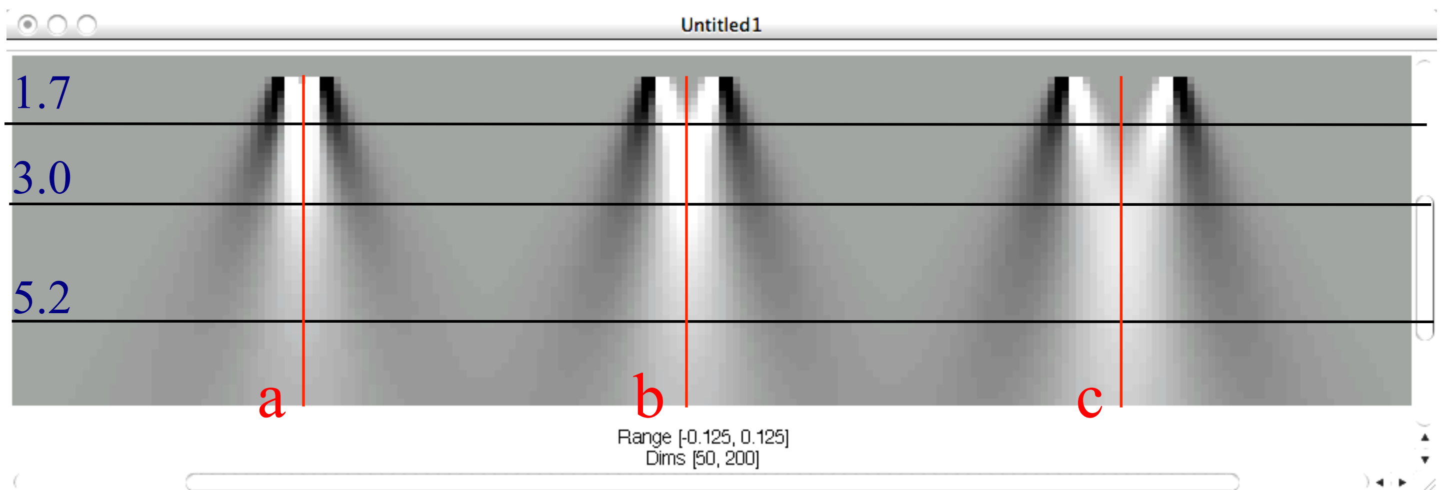


space

The bumps, filtered by difference-of-Gaussian filters



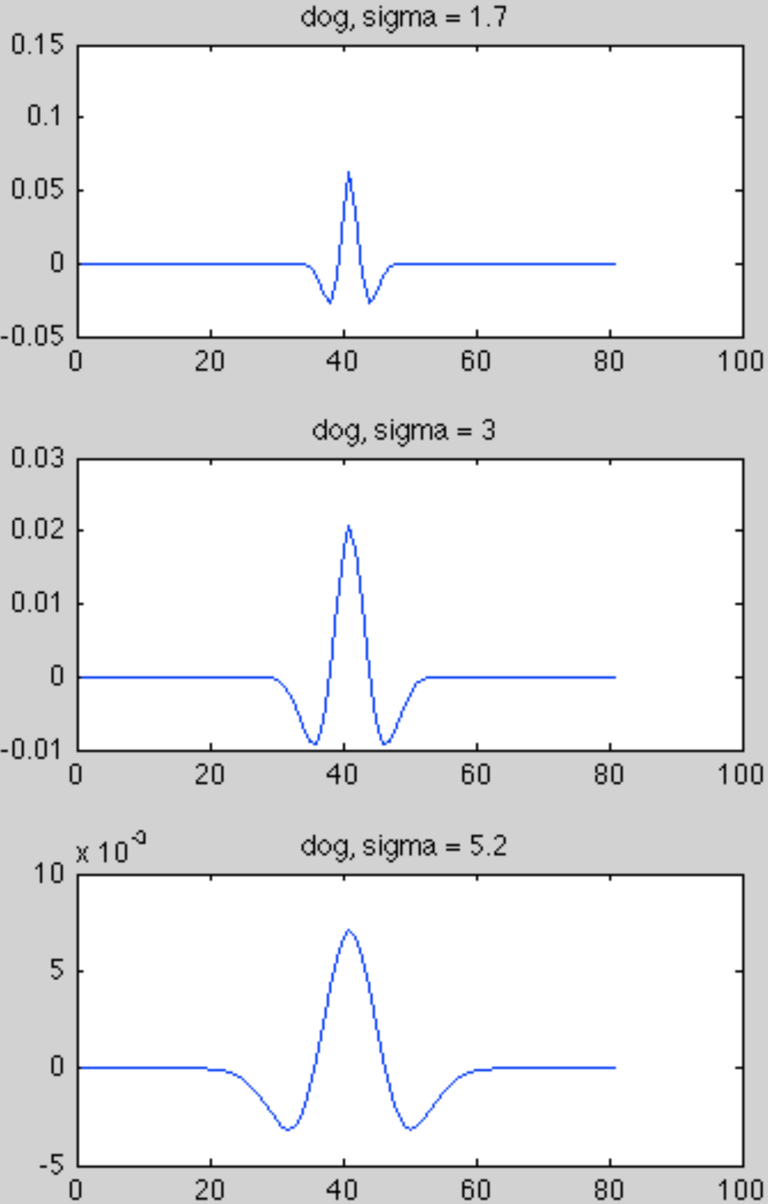


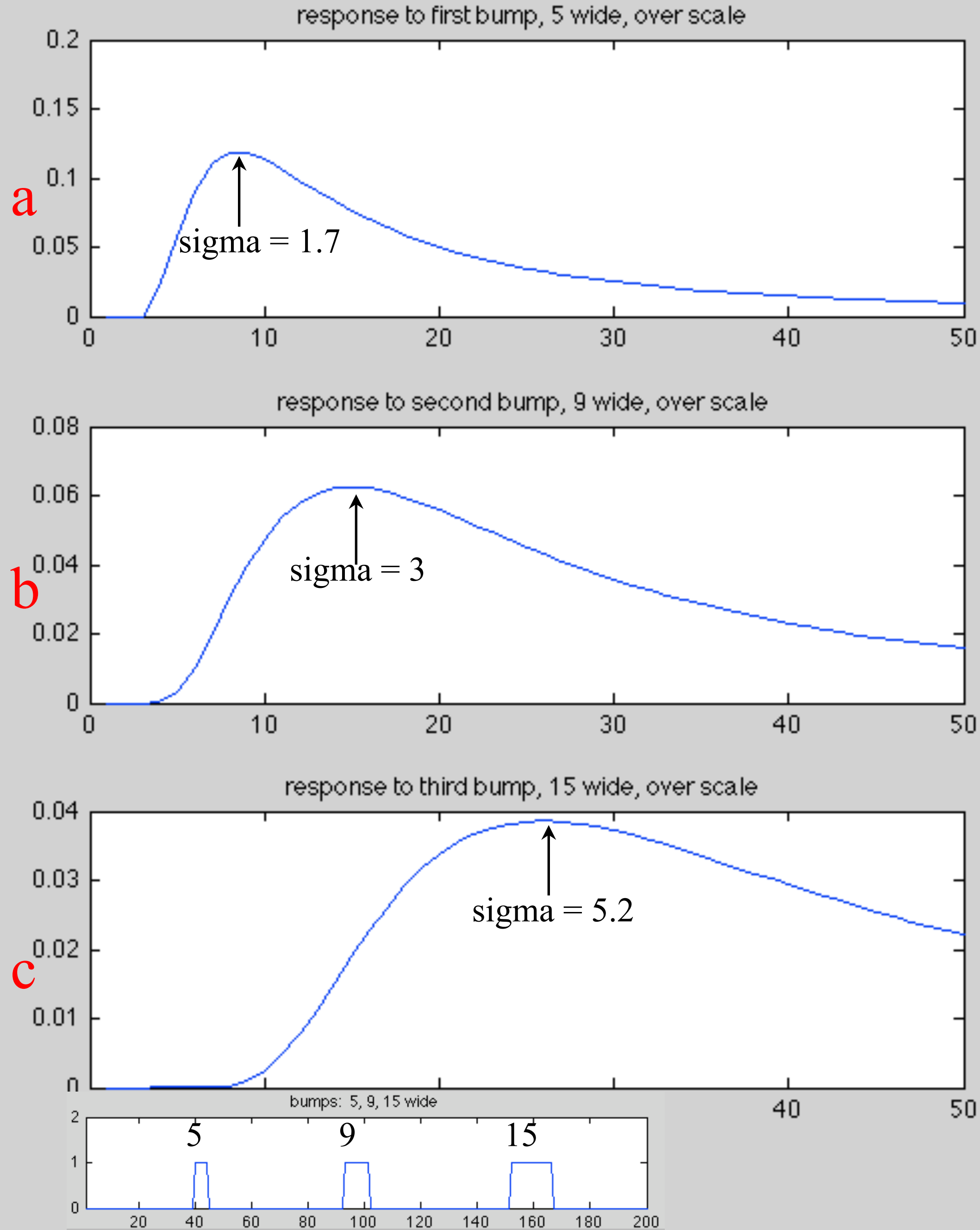


cross-sections along red lines plotted next slide

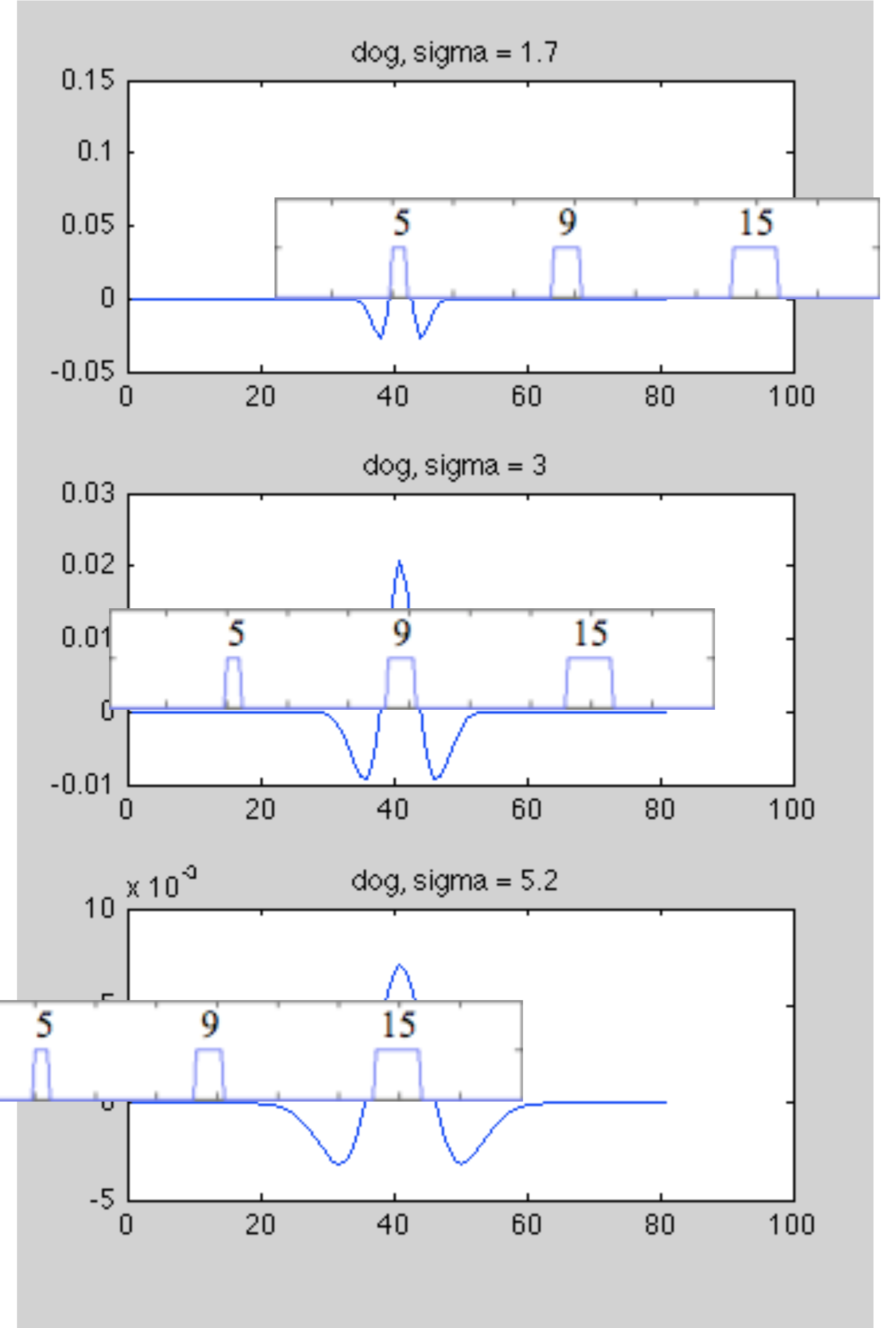
Scales of peak responses are proportional to bump width (the characteristic scale of each bump):

$[1.7, 3, 5.2] ./ [5, 9, 15] = 0.3400 \quad 0.3333 \quad 0.3467$





Diff of Gauss filter giving peak response



Scales of peak responses are proportional to bump width (the characteristic scale of each bump):

[1.7, 3, 5.2] ./ [5, 9, 15] = 0.3400 0.3333 0.3467

Note that the max response filters each has the same relationship to the bump that it favors (the zero crossings of the filter are about at the bump edges). So the scale space analysis correctly picks out the "characteristic scale" for each of the bumps.

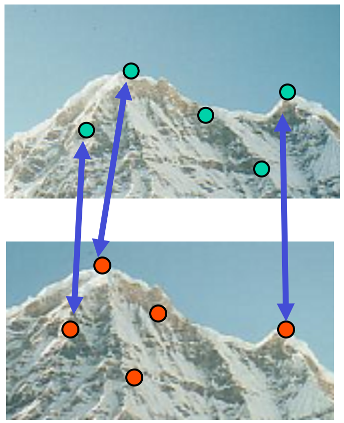
More generally, this happens for the features of the images we analyze.

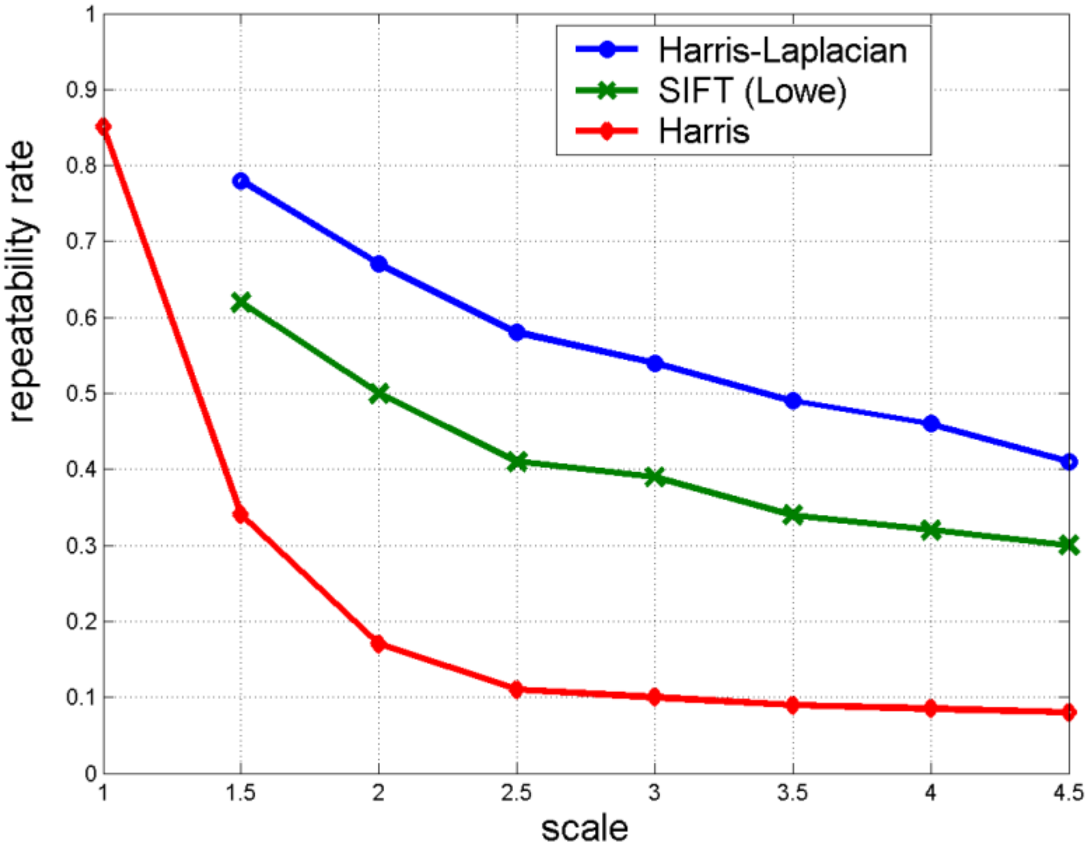
Scale Invariant Detectors

- Experimental evaluation of detectors w.r.t. scale change

Repeatability rate:

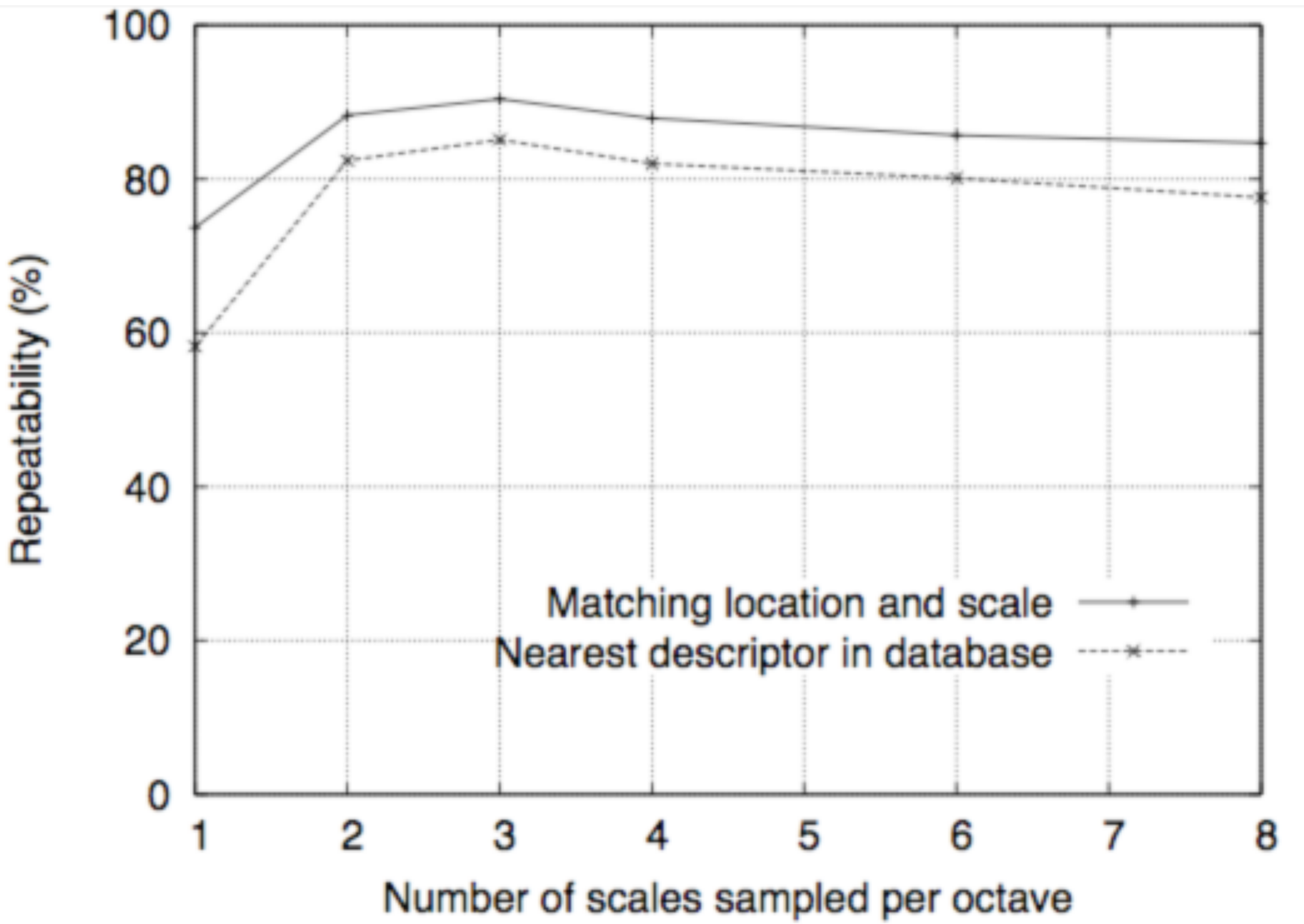
$$\frac{\#\ \text{correspondences}}{\#\ \text{possible correspondences}}$$





K.Mikolajczyk, C.Schmid. "Indexing Based on Scale Invariant Interest Points". ICCV 2001

Repeatability vs number of scales sampled per octave



David G. Lowe, "Distinctive image features from scale-invariant keypoints," International Journal of Computer Vision, 60, 2 (2004), pp. 91-110

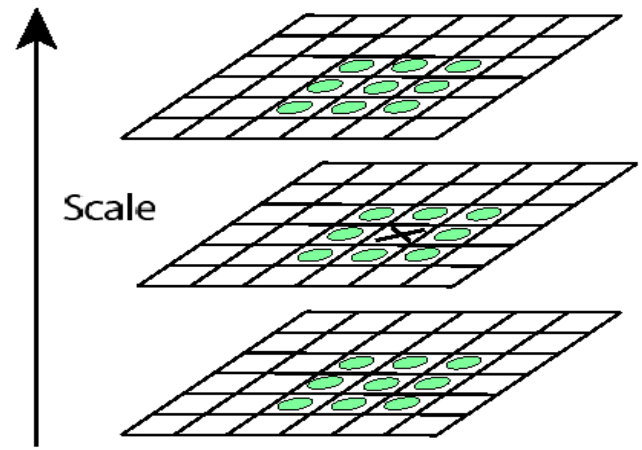
Some details of key point localization over scale and space

- Detect maxima and minima of difference-of-Gaussian in scale space
- Fit a quadratic to surrounding values for sub-pixel and sub-scale interpolation (Brown & Lowe, 2002)
- Taylor expansion around point:

$$D(\mathbf{x}) = D + \frac{\partial D}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^{\mathbf{T}} \frac{\partial^2 D}{\partial \mathbf{x}^2} \mathbf{x}$$

- Offset of extremum (use finite differences for derivatives):

$$\hat{\mathbf{x}} = -\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}}$$



Scale and Rotation Invariant Detection: Summary

- Given: two images of the same scene with a large scale difference and/or rotation between them
- Goal: find the same interest points independently in each image
- Solution: search for maxima of suitable functions in scale and in space (over the image). Also, find characteristic orientation.

Methods:

1. Harris-Laplacian [Mikolajczyk, Schmid]: maximize Laplacian over scale, Harris' measure of corner response over the image
2. SIFT [Lowe]: maximize Difference of Gaussians over scale and space

Example of keypoint detection

(c)

Figure 12. Robust matching: Harris-Laplace detects 190 and 213 points in the left and right images, respectively (a). 58 points are initially matched (b). There are 32 inliers to the estimated homography (c), all of which are correct. The estimated scale factor is 4.9 and the estimated rotation angle is 19 degrees.

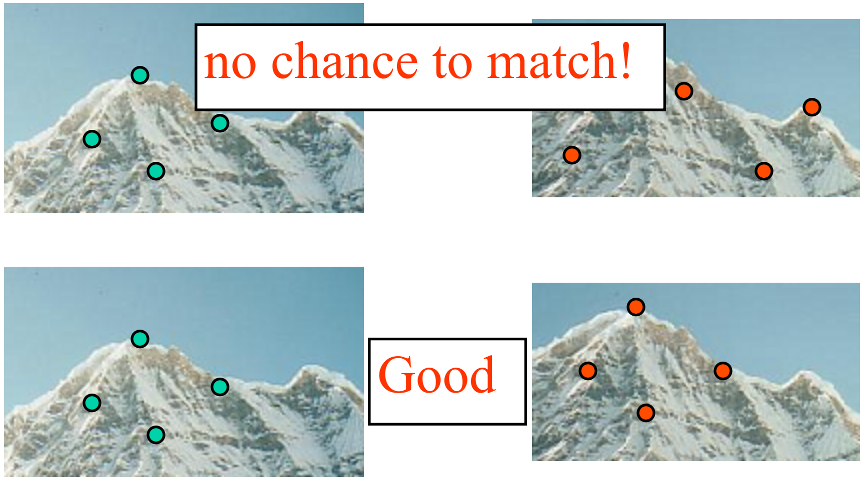
http://www.robots.ox.ac.uk/~vgg/research/affine/det_eval_files/mikolajczyk_ijcv2004.pdf

Outline

- Feature point detection
  - Harris corner detector
  - finding a characteristic scale
- Local image description
  - SIFT features

Recall: Matching with Features

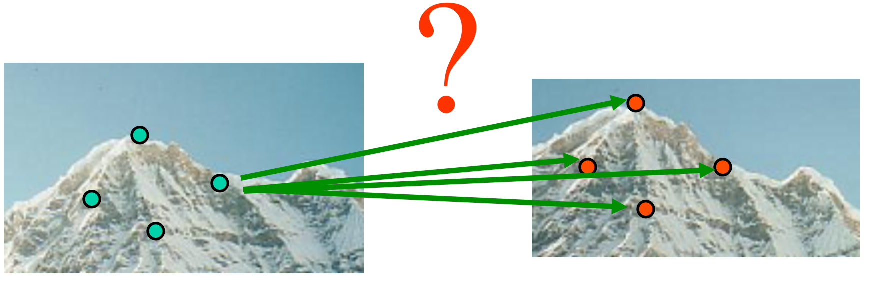
- Problem 1:
  - Detect the same point independently in both images



We need a repeatable detector

Recall: Matching with Features

- Problem 2:
  - For each point correctly recognize the corresponding one



We need a reliable and distinctive descriptor

CVPR 2003 Tutorial

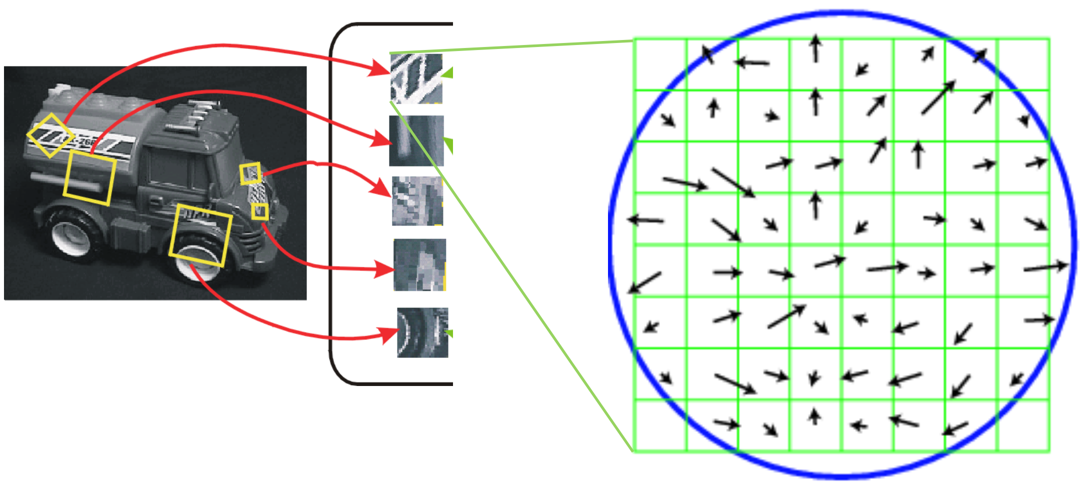
Recognition and Matching Based on Local Invariant Features

David Lowe
Computer Science Department
University of British Columbia

http://www.cs.ubc.ca/~lowe/papers/ijcv04.pdf

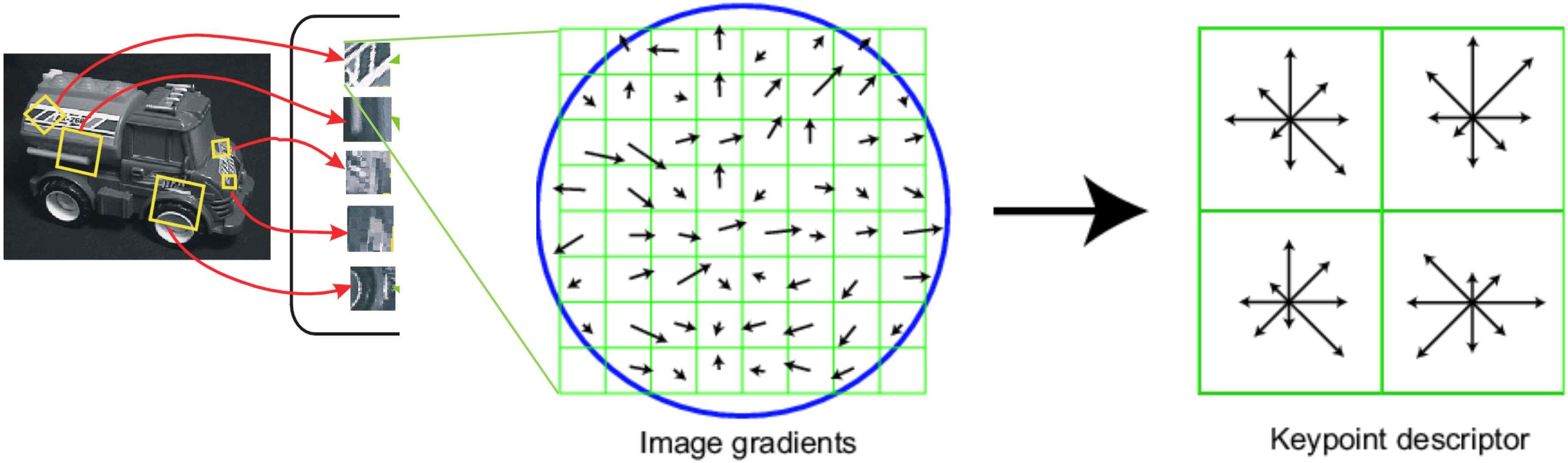
SIFT vector formation

- Computed on rotated and scaled version of window according to computed orientation & scale
  - resample the window
- Based on gradients weighted by a Gaussian of variance half the window (for smooth falloff)



SIFT vector formation

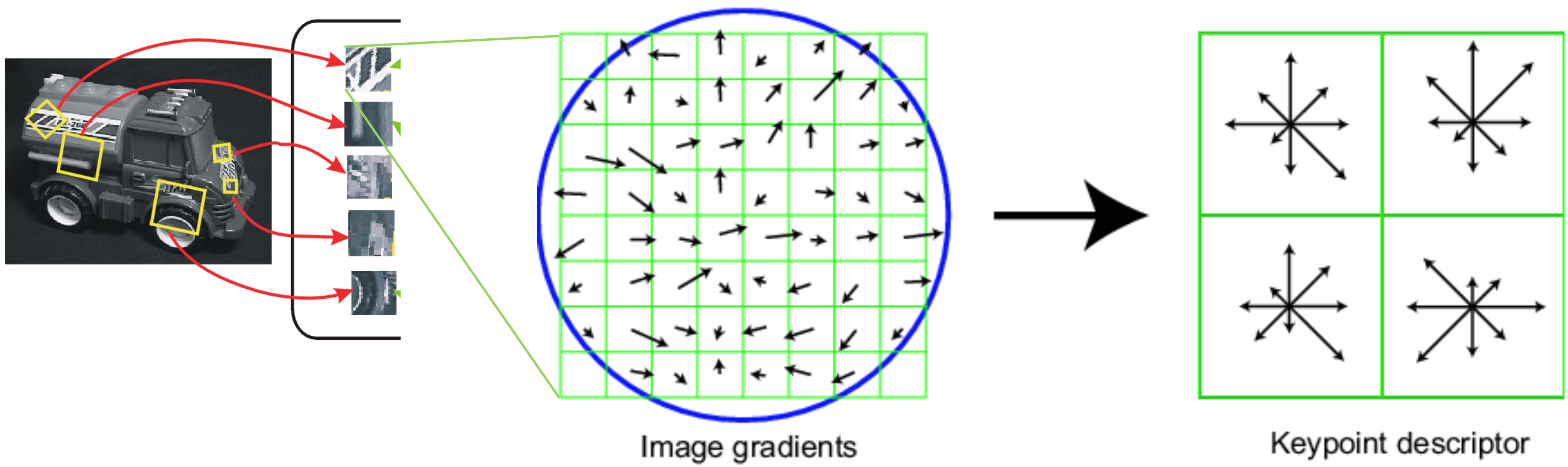
- 4x4 array of gradient orientation histograms
  - – not really histogram, weighted by magnitude
- 8 orientations x 4x4 array = 128 dimensions
- Motivation: some sensitivity to spatial layout, but not too much.



showing only 2x2 here but is 4x4

Reduce effect of illumination

- 128-dim vector normalized to 1
- Threshold gradient magnitudes to avoid excessive influence of high gradients
  - after normalization, clamp gradients >0.2
  - renormalize



Tuning and evaluating the SIFT descriptors

Database images were subjected to rotation, scaling, affine stretch, brightness and contrast changes, and added noise. Feature point detectors and descriptors were compared before and after the distortions, and evaluated for:

- Sensitivity to number of histogram orientations and subregions.
- Stability to noise.
- Stability to affine change.
- Feature distinctiveness

Sensitivity to number of histogram orientations and subregions (n)

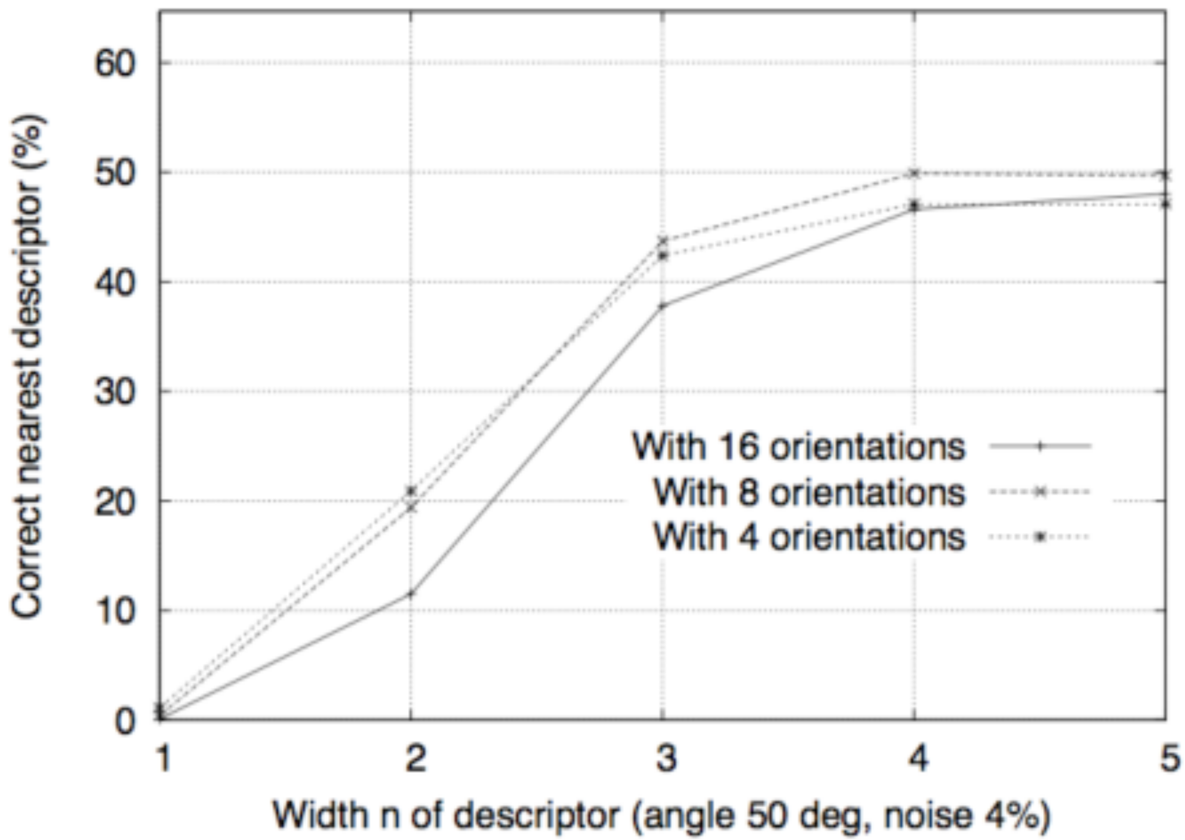
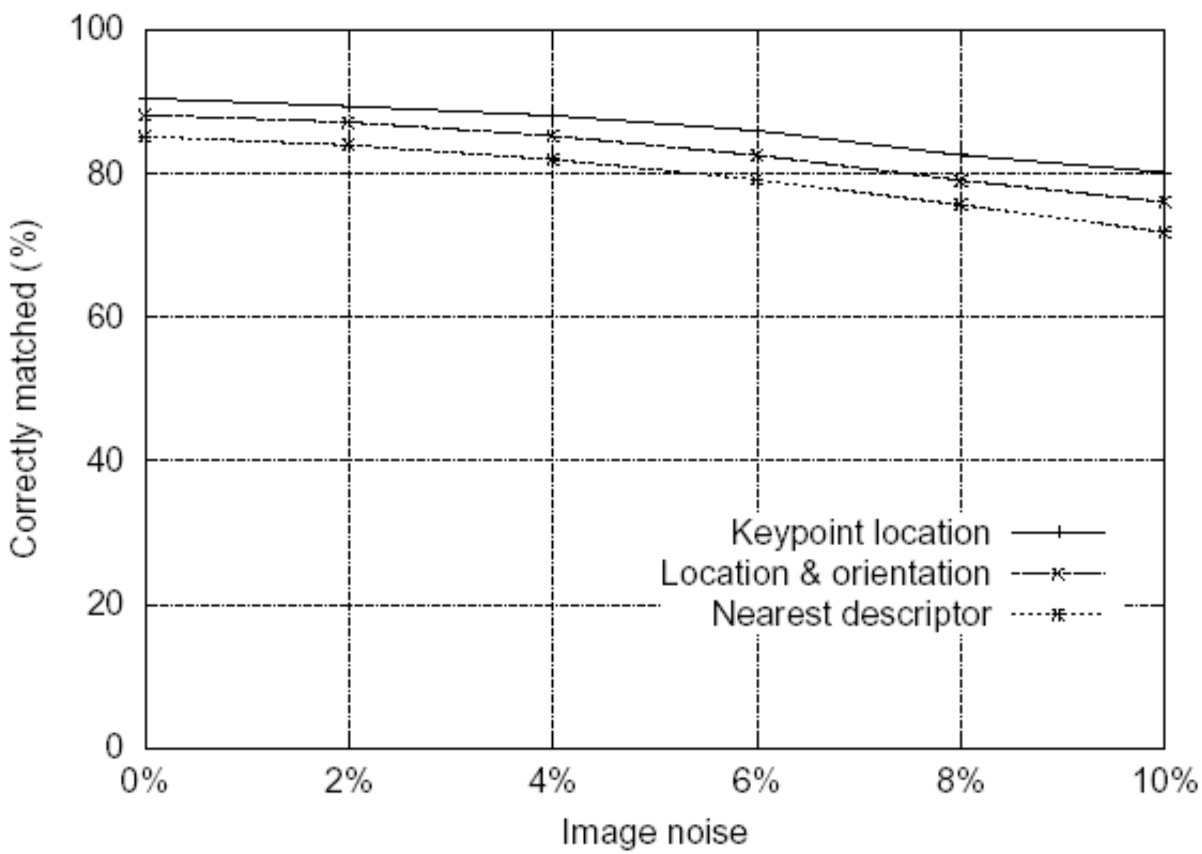


Figure 8: This graph shows the percent of keypoints giving the correct match to a database of 40,000 keypoints as a function of width of the $n \times n$ keypoint descriptor and the number of orientations in each histogram. The graph is computed for images with affine viewpoint change of 50 degrees and addition of 4% noise.

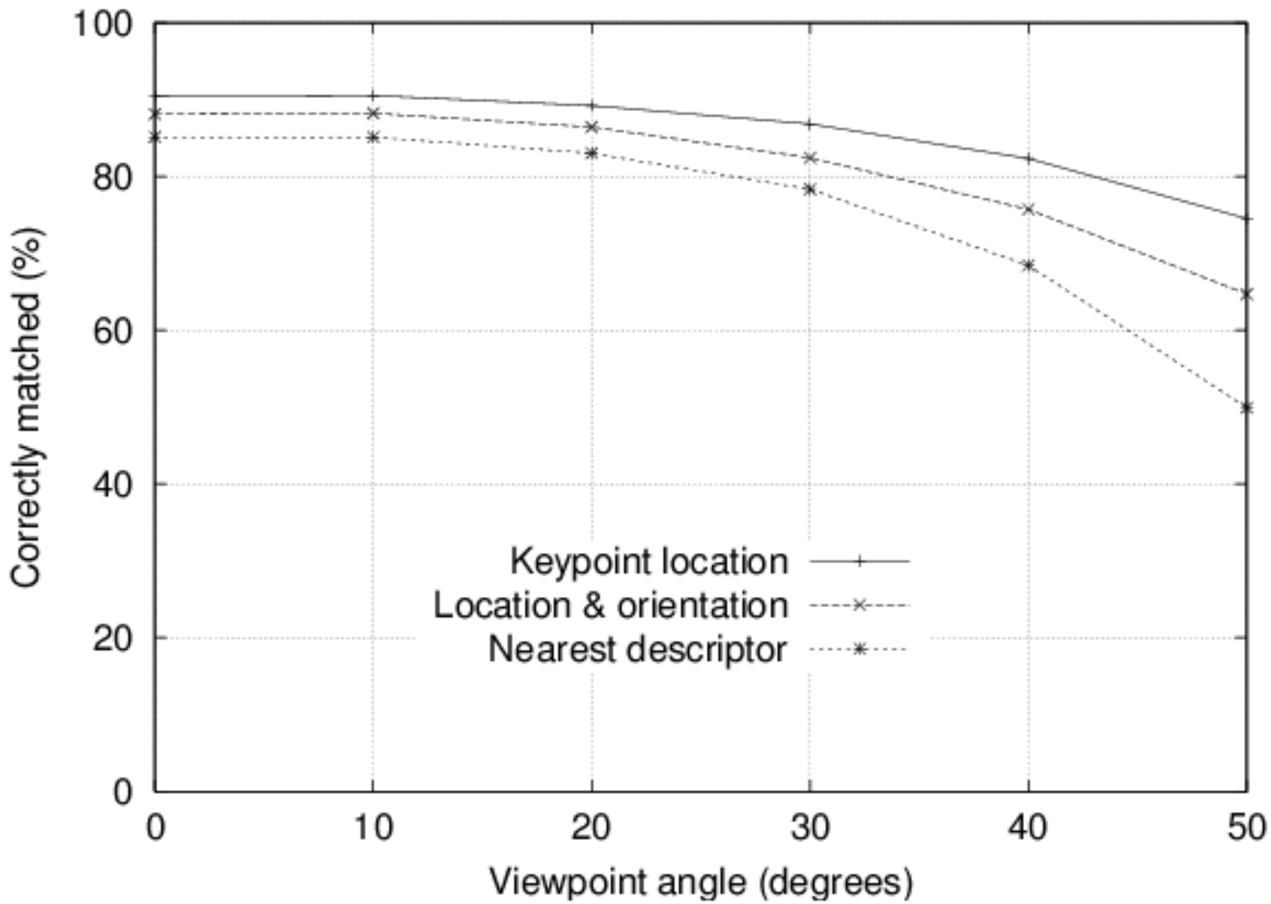
Feature stability to noise

- Match features after random change in image scale & orientation, with differing levels of image noise
- Find nearest neighbor in database of 30,000 features



Feature stability to affine change

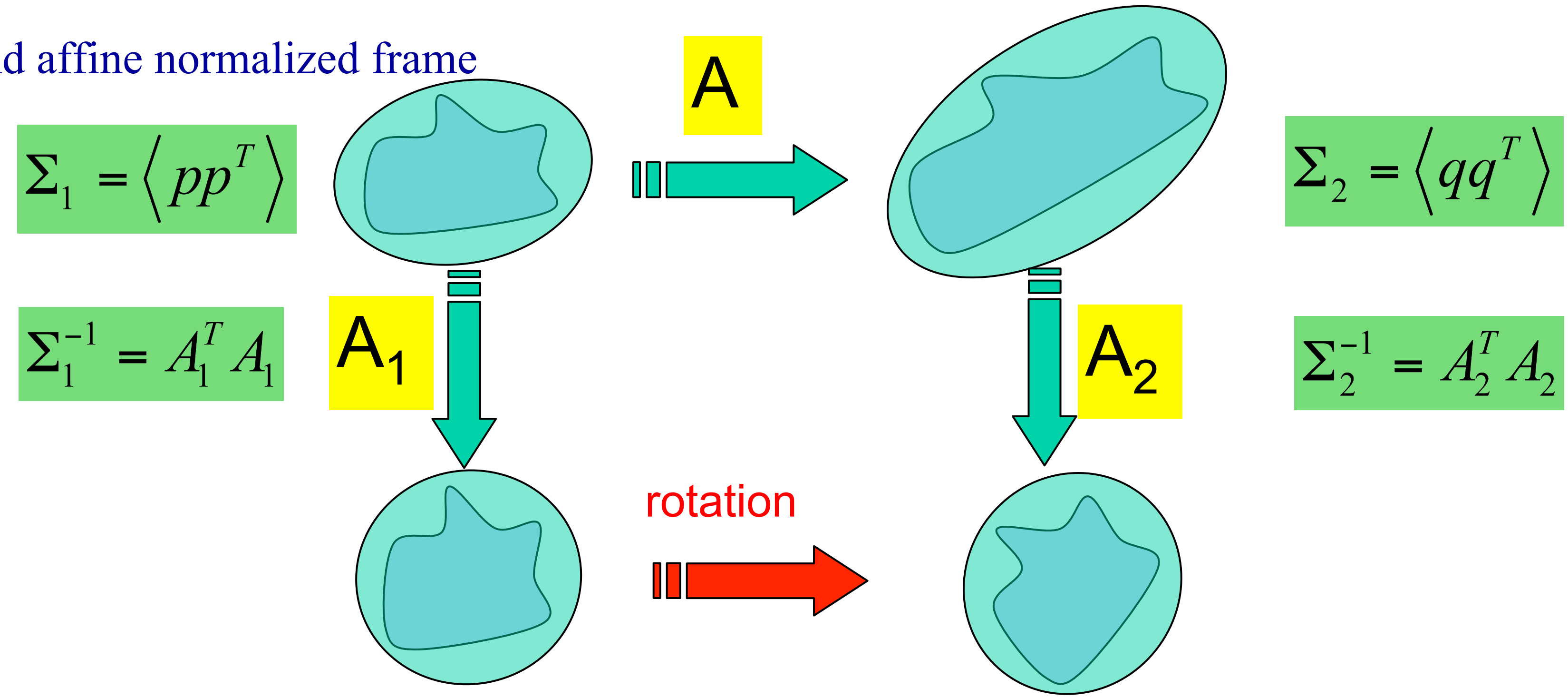
- Match features after random change in image scale & orientation, with 2% image noise, and affine distortion
- Find nearest neighbor in database of 30,000 features



Affine Invariant Descriptors

If a wide range of affine invariance is desired, such as for a surface that is known to be planar, then a simple solution is to adopt the approach of Pritchard and Heidrich (2003) in which additional SIFT features are generated from 4 affine-transformed versions of the training image corresponding to 60 degree viewpoint changes. This allows for the use of standard SIFT features with no additional cost when processing the image to be recognized, but results in an increase in the size of the feature database by a factor of 3.

Find affine normalized frame

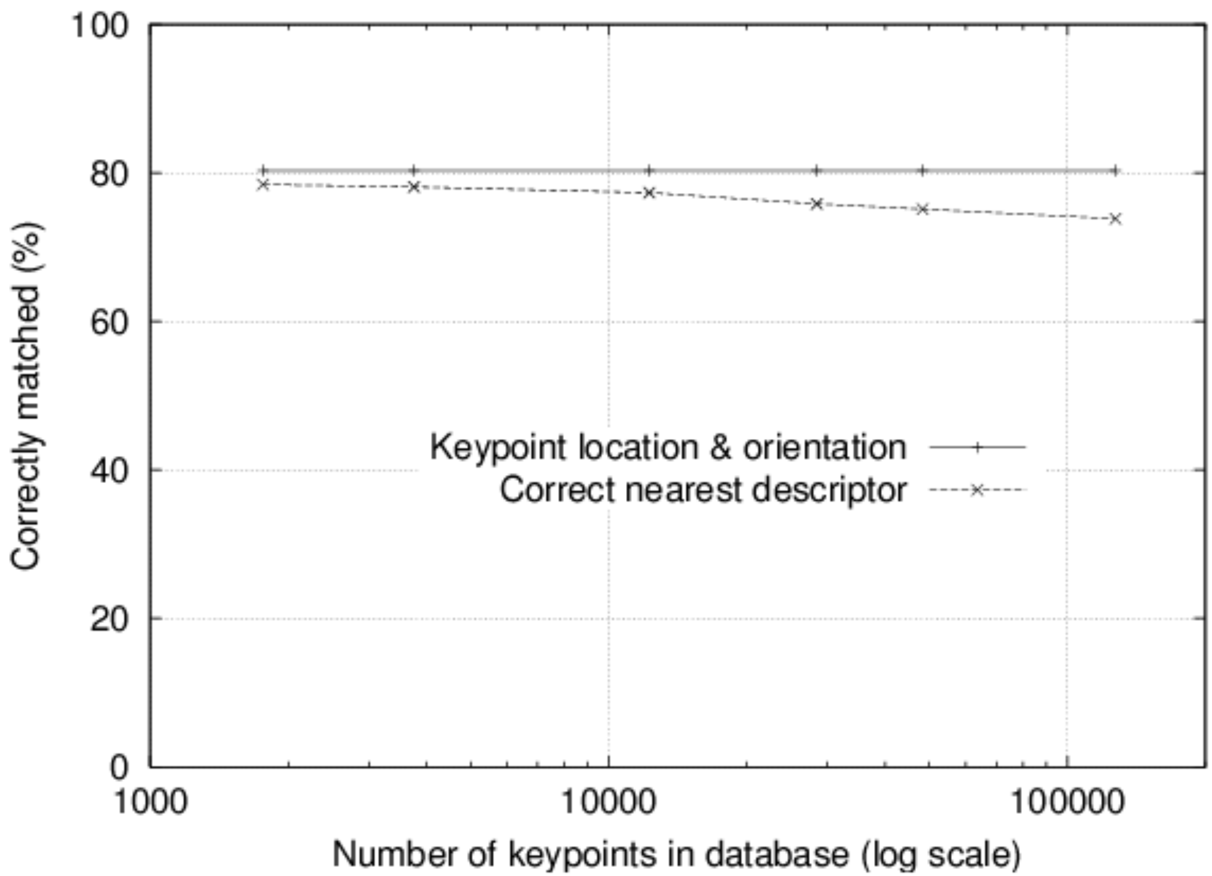


Compute rotational invariant descriptor in this normalized frame

J.Matas et.al. “Rotational Invariants for Wide-baseline Stereo”. Research Report of CMP, 2003

Distinctiveness of features

- Vary size of database of features, with 30 degree affine change, 2% image noise
- Measure % correct for single nearest neighbor match



Application of invariant local features to object (instance) recognition.

Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters

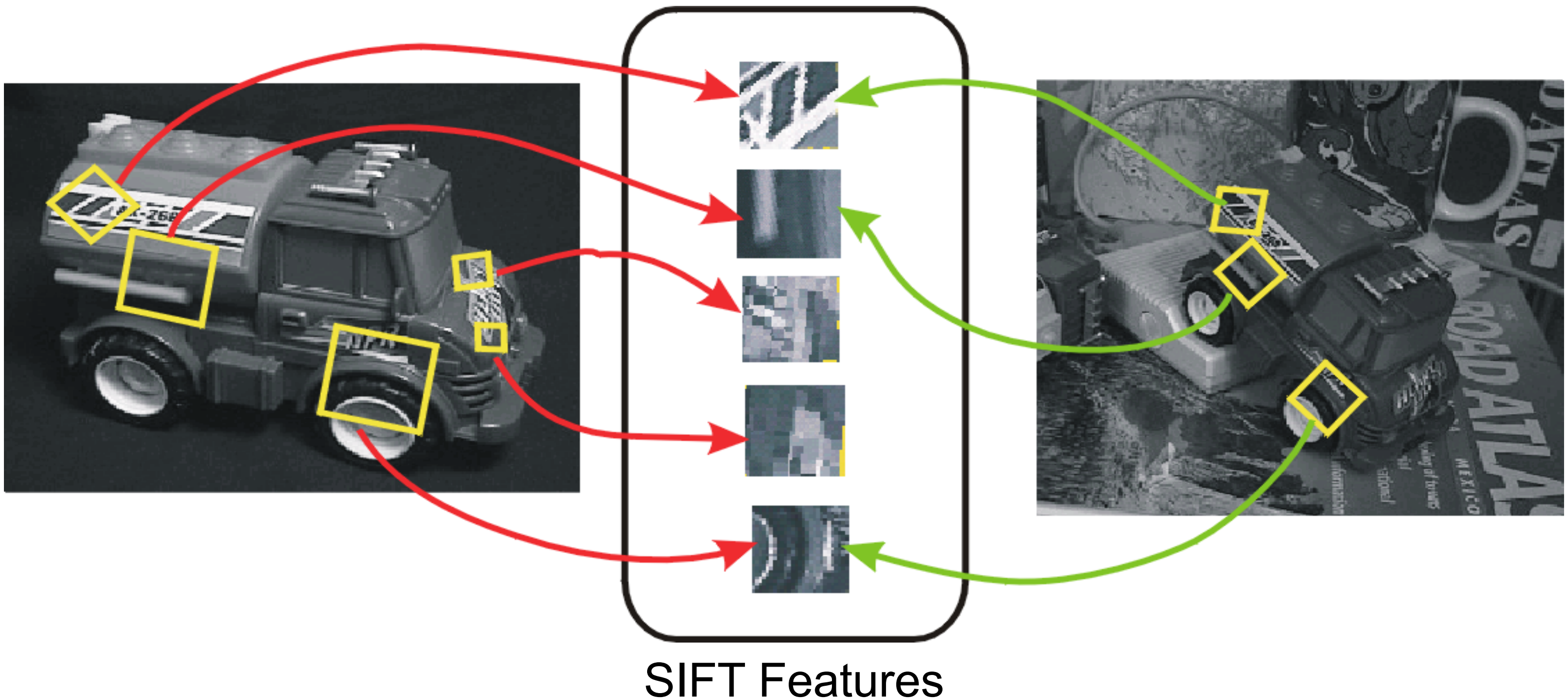


Figure 12: The training images for two objects are shown on the left. These can be recognized in a cluttered image with extensive occlusion, shown in the middle. The results of recognition are shown on the right. A parallelogram is drawn around each recognized object showing the boundaries of the original training image under the affi ne transformation solved for during recognition. Smaller squares indicate the keypoints that were used for recognition.

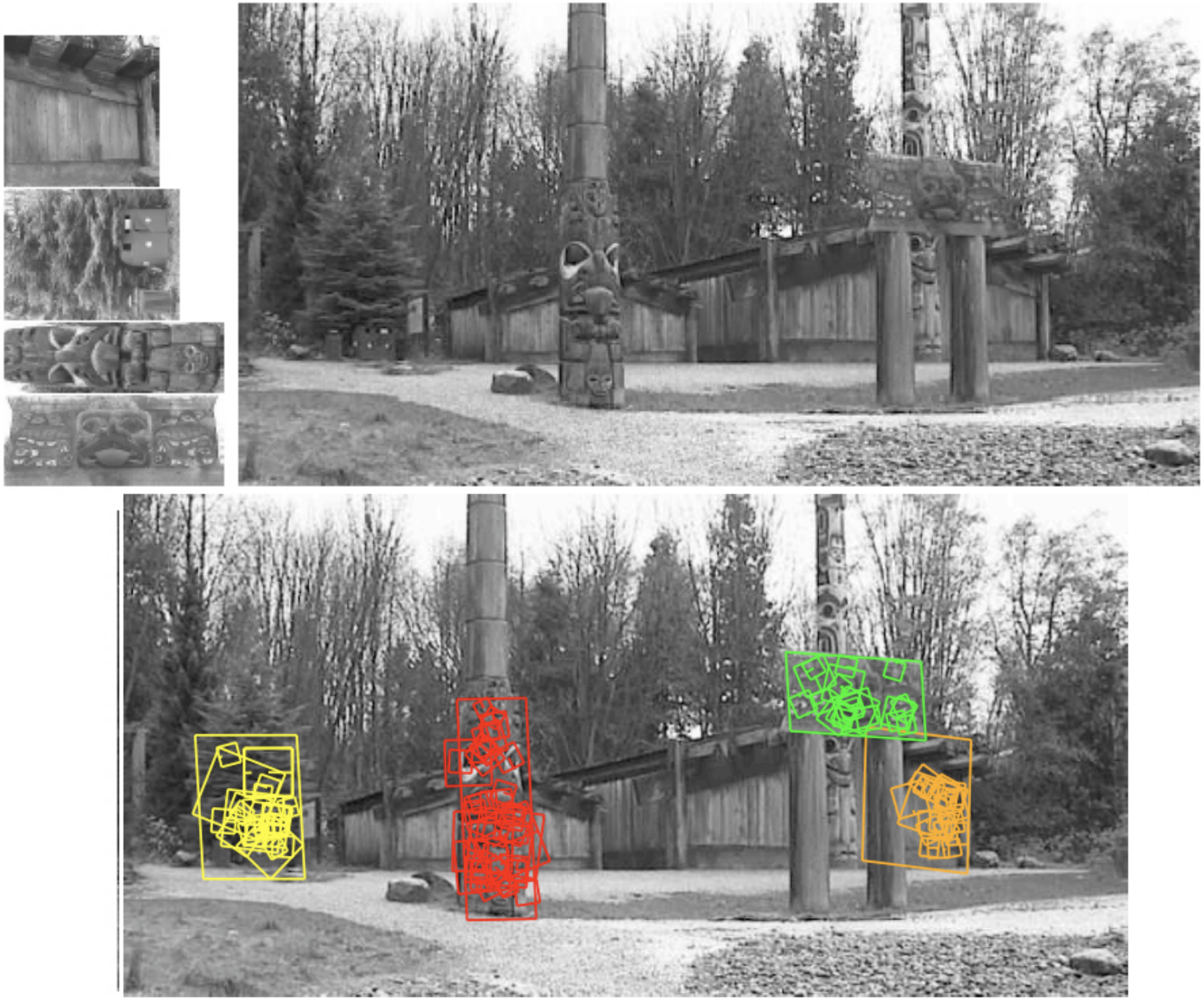


Figure 13: This example shows location recognition within a complex scene. The training images for locations are shown at the upper left and the 640x315 pixel test image taken from a different viewpoint is on the upper right. The recognized regions are shown on the lower image, with keypoints shown as squares and an outer parallelogram showing the boundaries of the training images under the affi ne transform used for recognition.

SIFT features impact

SIFT feature paper citations:

Distinctive image features from scale-invariant keypointsDG Lowe - International journal of computer vision, 2004 - Springer International Journal of Computer Vision 60(2), 91–110, 2004 cc 2004 Kluwer Academic Publishers. Computer Science Department, University of British Columbia ...Cited by 16232 (google)

A good SIFT features tutorial:

http://www.cs.toronto.edu/~jepson/csc2503/tutSIFT04.pdf

By Estrada, Jepson, and Fleet.

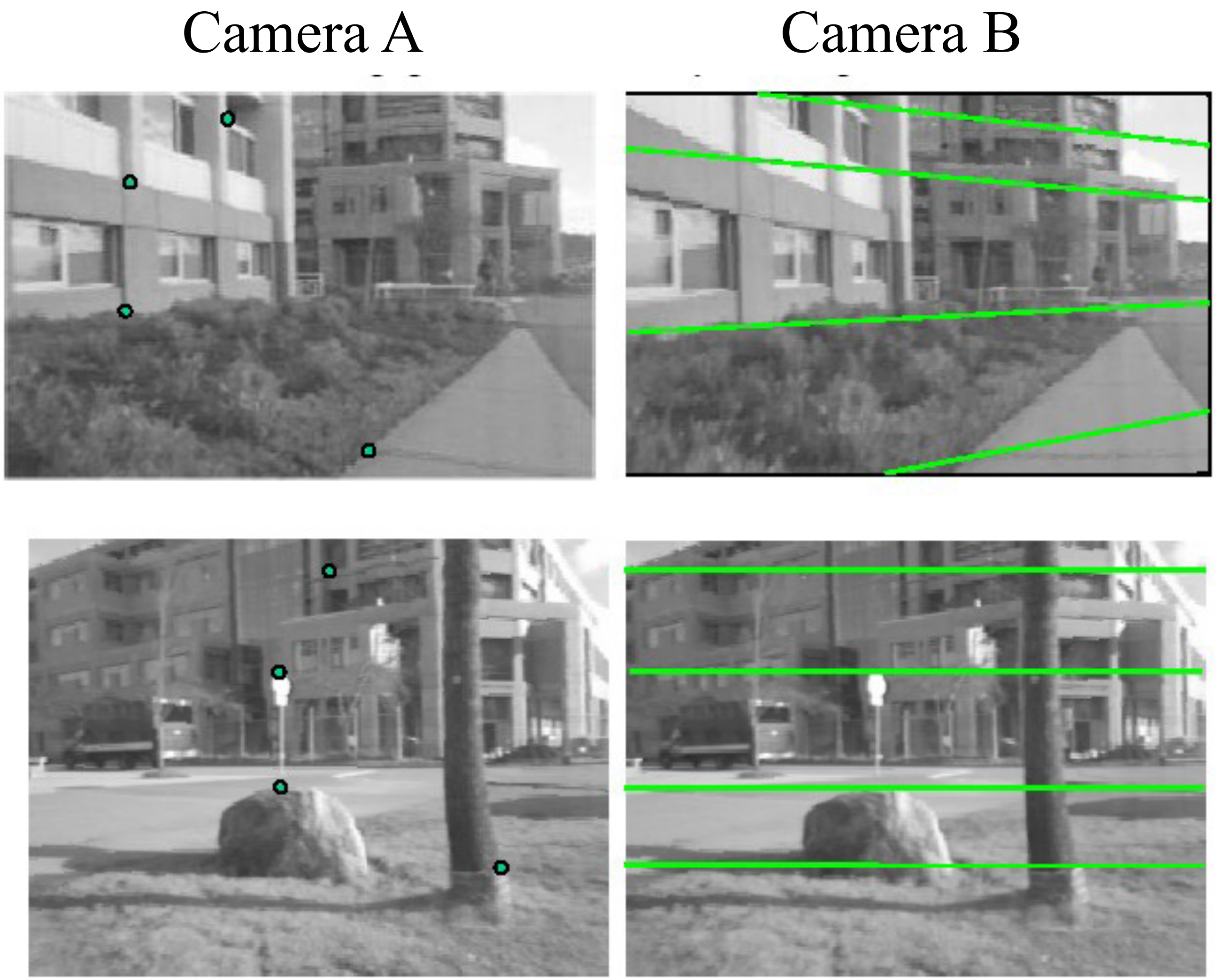
The original SIFT paper:
http://www.cs.ubc.ca/~lowe/papers/ijcv04.pdf

Now we have

- Well-localized feature points
- Distinctive descriptor

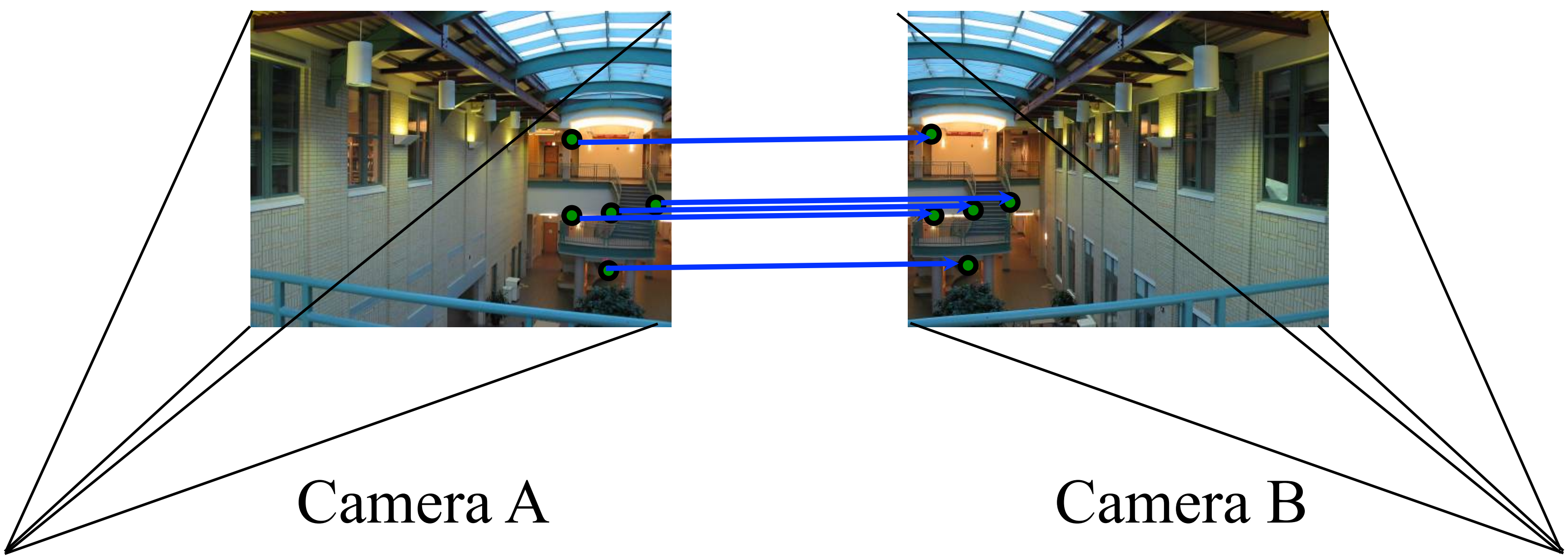
- Now we need to
  - match pairs of feature points in different images
  - Robustly compute homographies (in the presence of errors/outliers)

Depth-based ambiguity of position

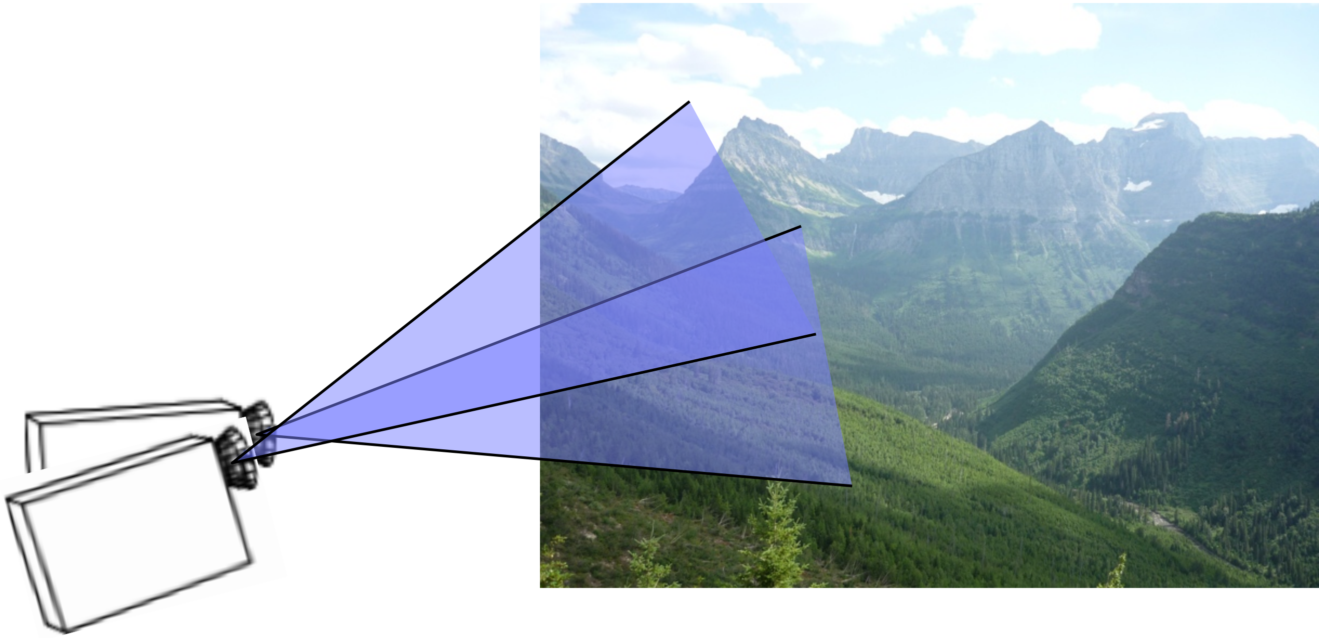


In general, matches are constrained to lie on the epipolar lines, but… that's it?, there are no more constraints?

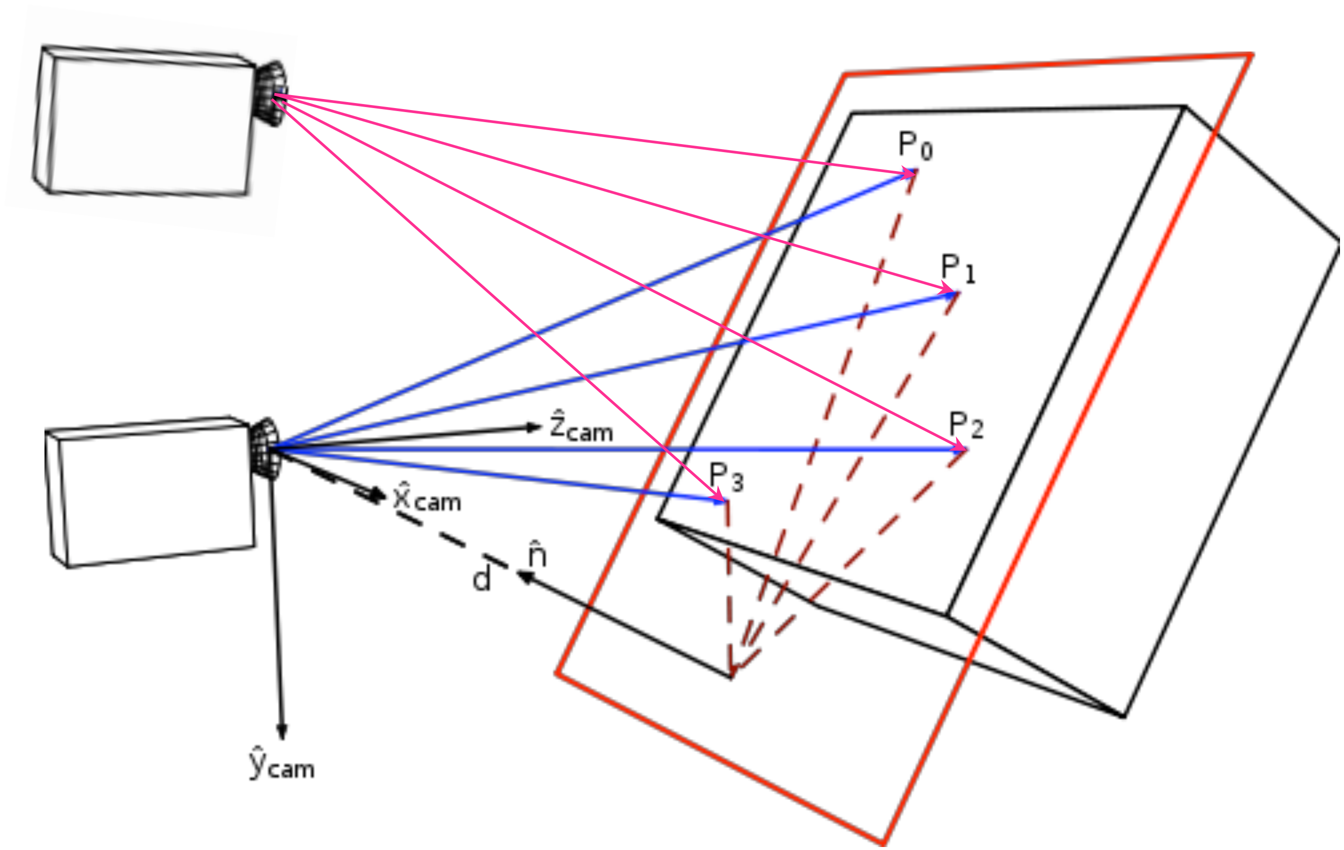
Under what conditions can you know where to translate each point of image A to where it would appear in camera B (with calibrated cameras), knowing nothing about image depths?



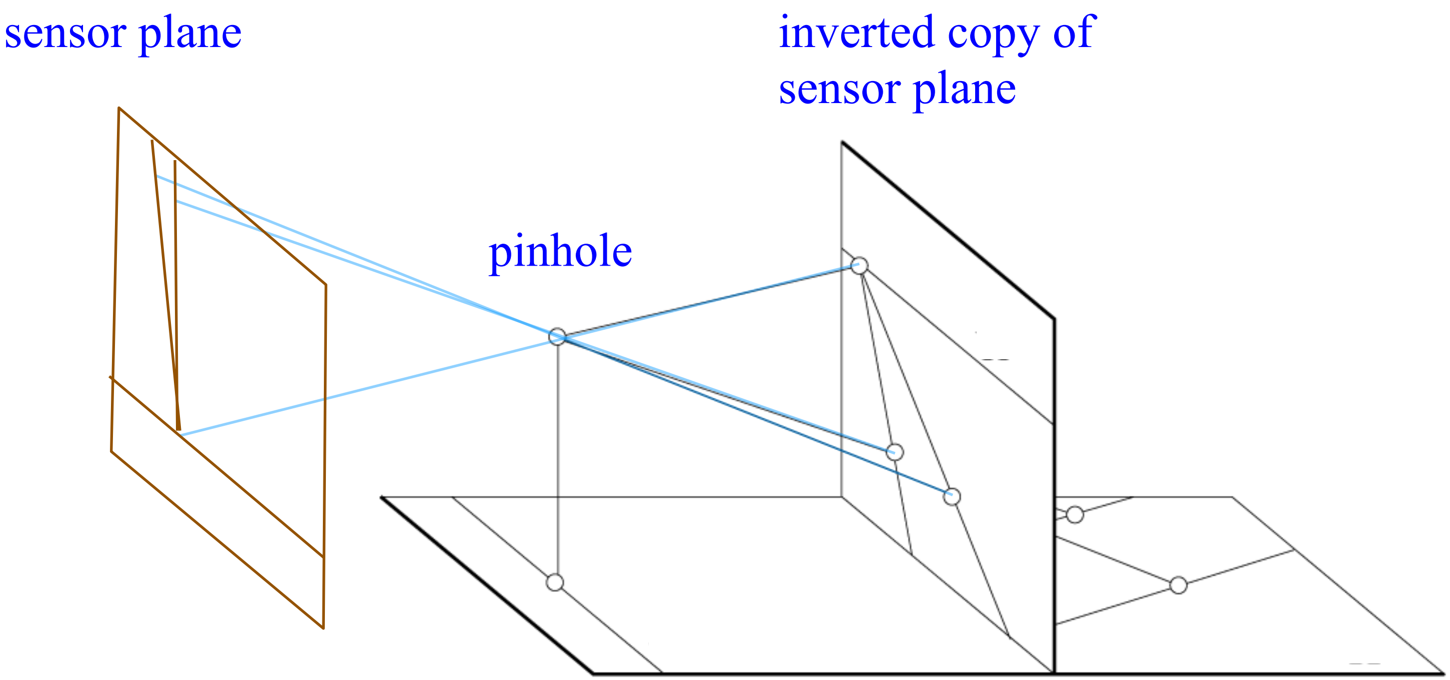
(a) camera rotation



and (b) imaging a planar surface

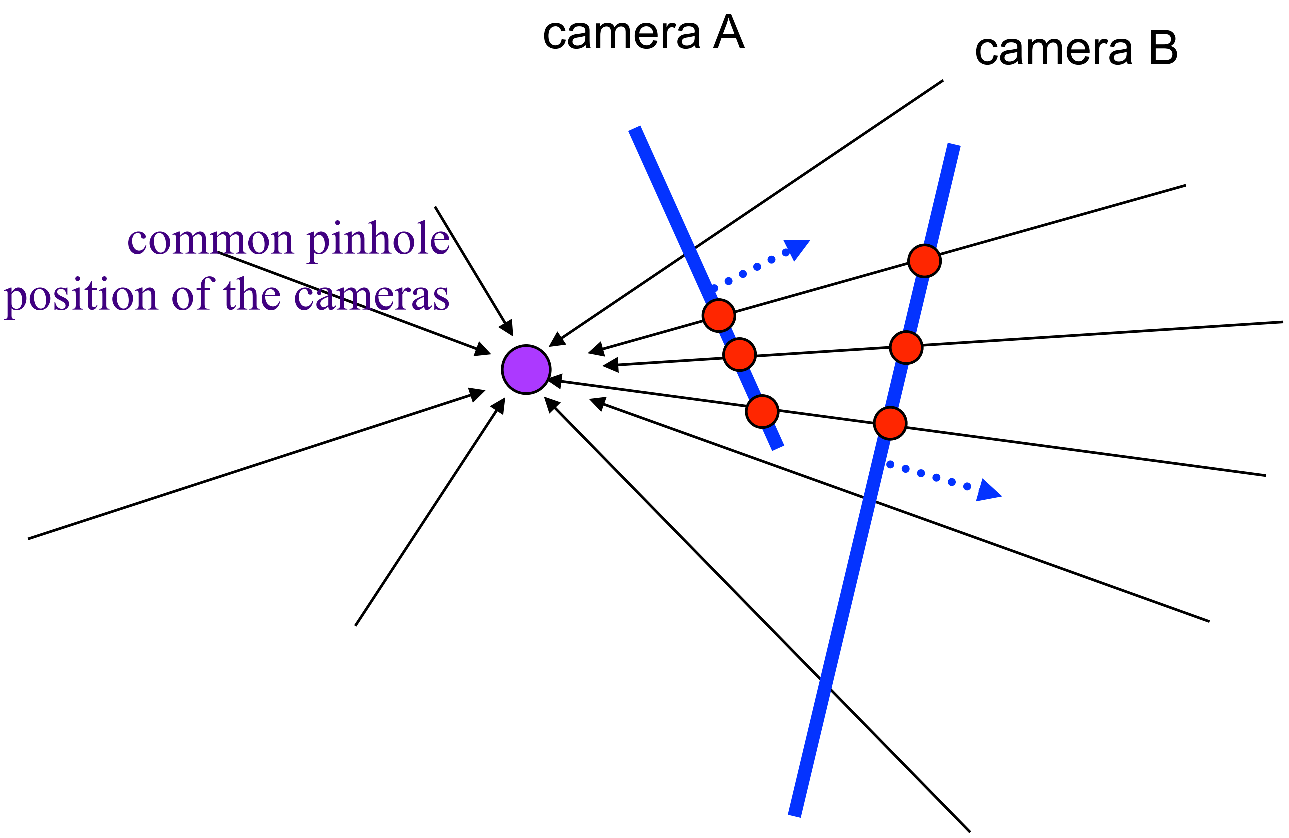


Geometry of perspective projection



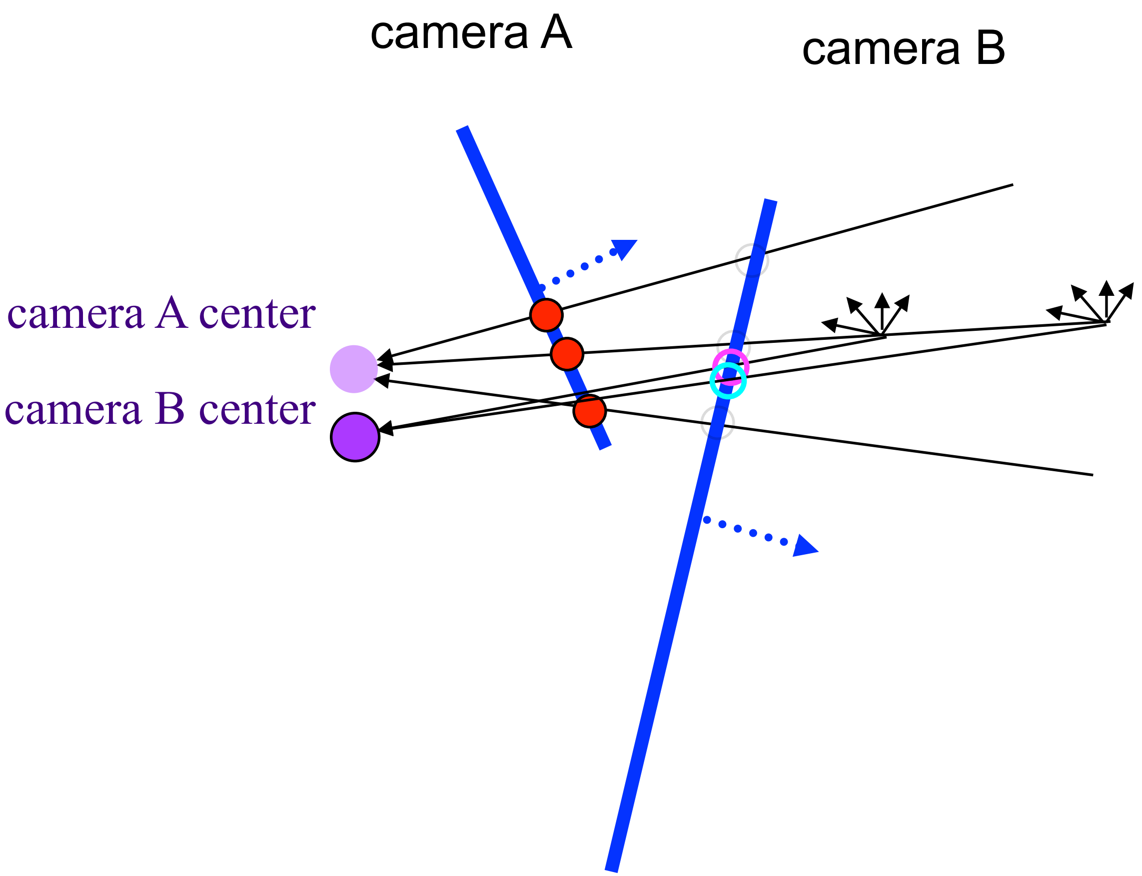
Let's look at this scene from above...

Two cameras with same center of projection



Can generate any synthetic camera view
as long as it has the same center of projection!

Two cameras with offset centers of projection

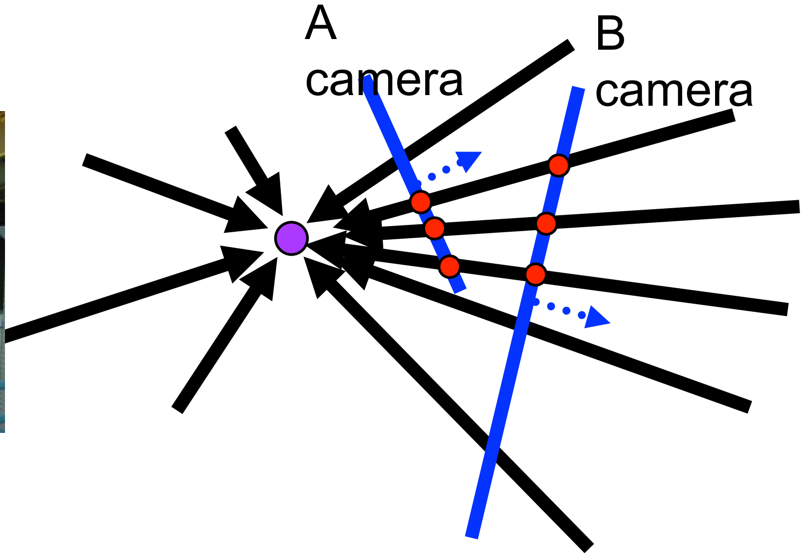


Entrance pupil

- Often wrongly called nodal point
- When camera is rotated around entrance pupil, there is no parallax
  - That is, if two 3D points are superimposed for one orientation, they remain superimposed after rotation
- Finding the entrance pupil is painful
  - http://www.reallyrightstuff.com/pano/index.html
  - http://www.path.unimelb.edu.au/~bernardk/tutorials/360/photo/nodal.html

Recap

- When we only rotate the camera (around nodal point) depth does not matter
- It only performs a 2D warp
  - one-to-one mapping of the 2D plane
  - plus of course reveals stuff that was outside the field of view



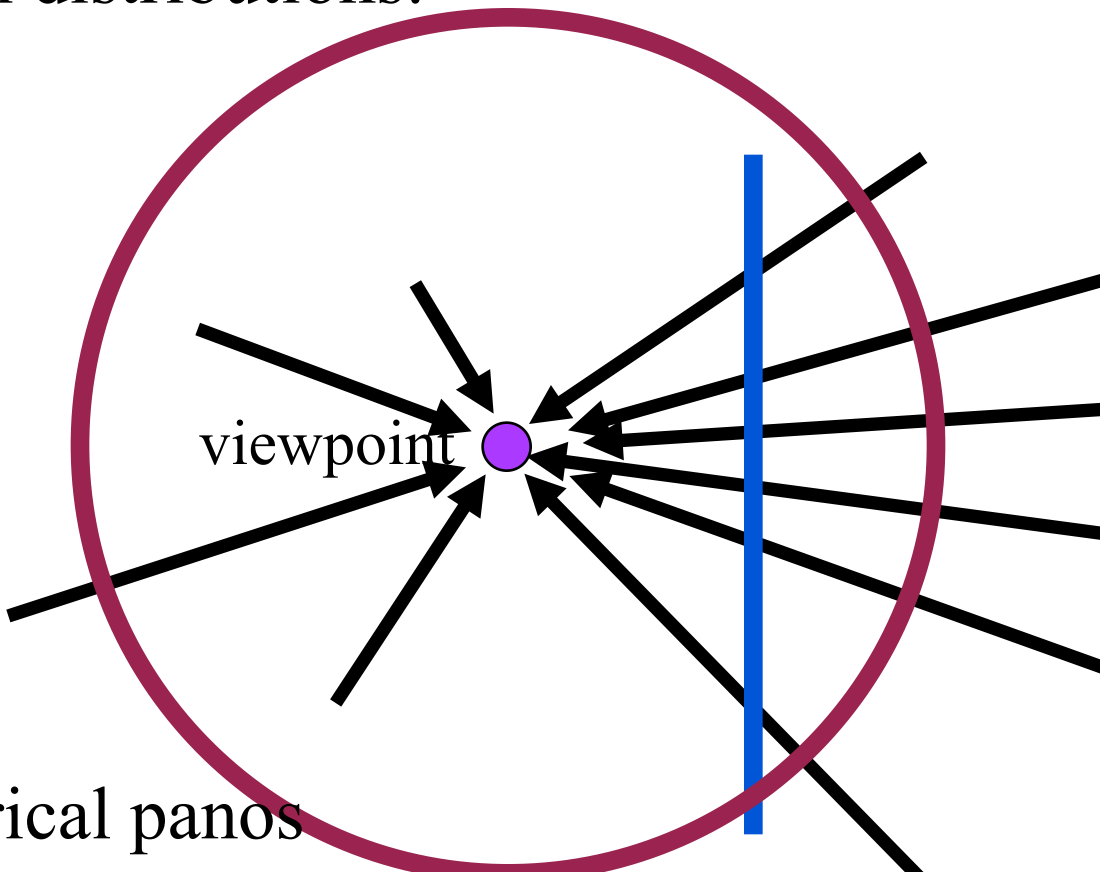
- Now we just need to figure out this mapping

Other interpretation

- Depth does not matter
- We can pretend that each pixel is at a convenient depth

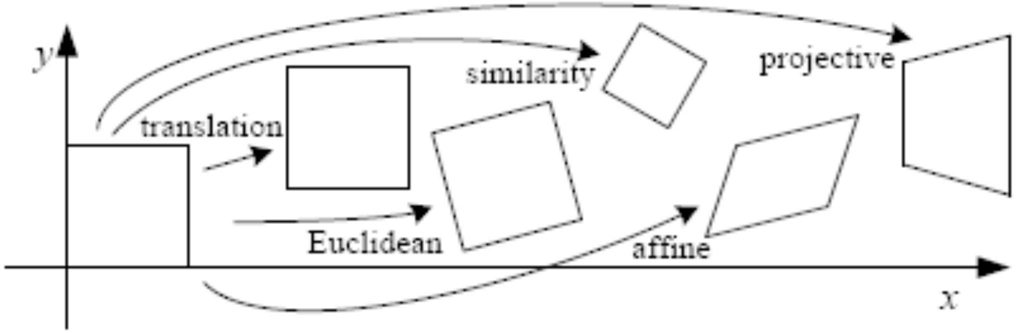
- Three convenient depth distributions:
  - – spherical
  - – planar
  - – cylindrical

- We focus on planar
  - it makes life more linear
  - Still useful for spherical panos



Aligning images

- We have established that pairs of images from the same viewpoint can be aligned through a simple 2D spatial transformation (warp).
- What kind of transformation?



Aligning images: translation?


left on top

right on top


Translations are not enough to align the images

Image Warping

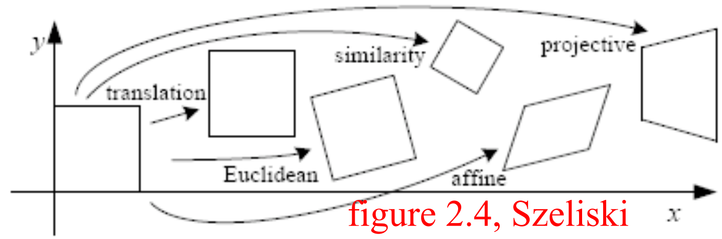
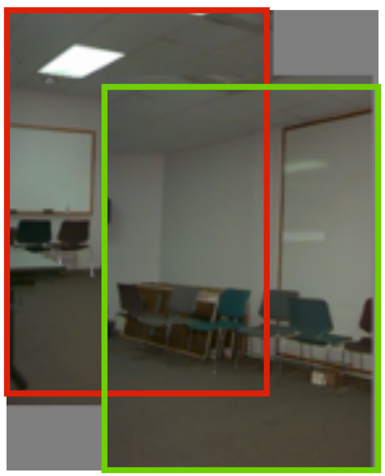
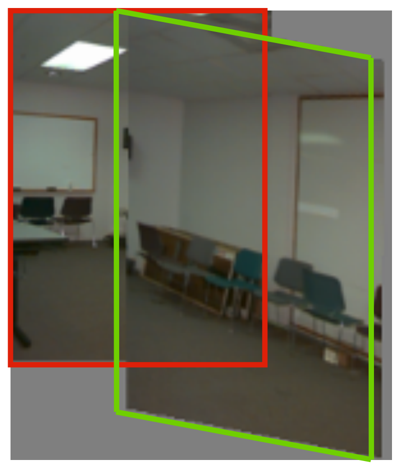
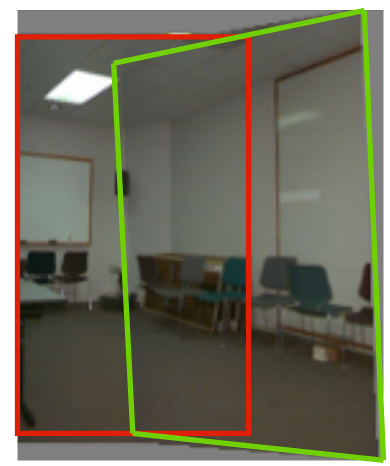


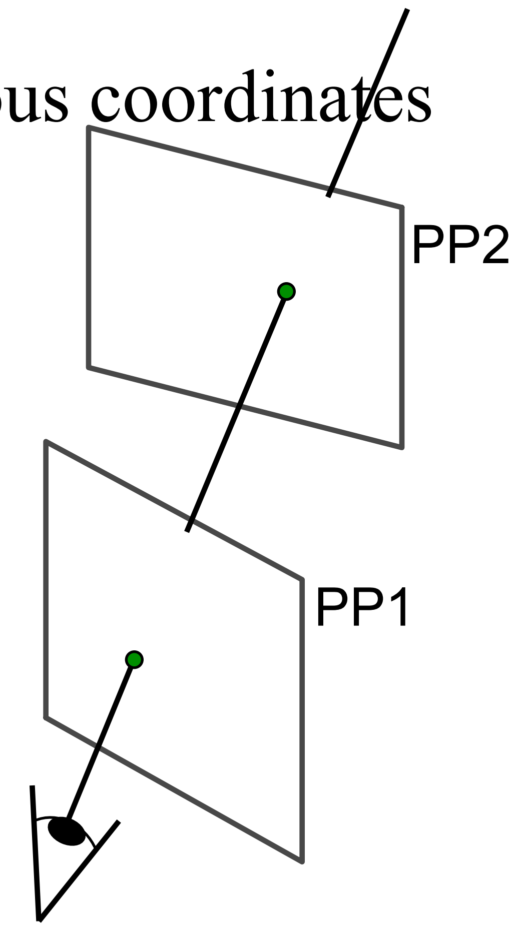
figure 2.4, Szeliski

Translation	Affine	Projective
		
2 unknowns	6 unknowns	8 unknowns

Homography

- Projective – mapping between any two projection planes with the same center of projection
- called Homography
- represented as 3x3 matrix in homogenous coordinates

$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{p'} = \underbrace{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{p}$$

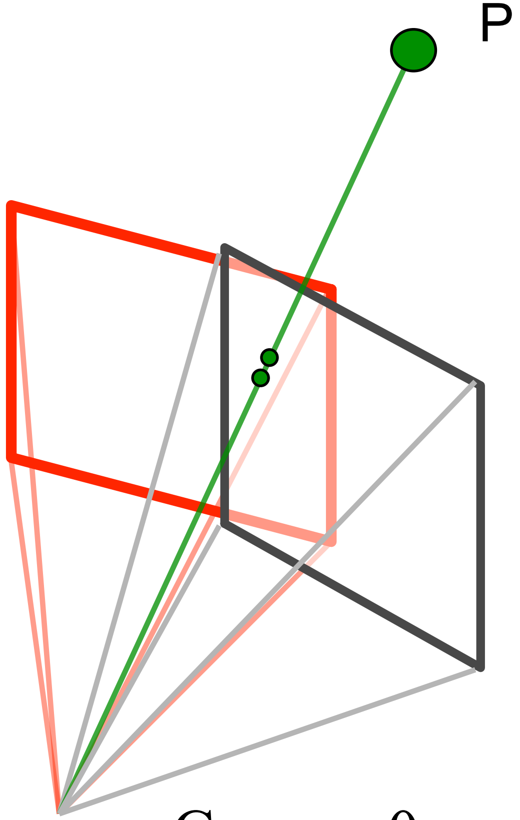


To apply a homography H

- Compute p' = Hp (regular matrix multiply)
- Convert p' from homogeneous to image coordinates (divide by w)

See Szeliski Sect 2.1.5, Mapping from one camera to another.

homography



$x_0 = A_0 P$

$x_1 = A_1 P$

$$A_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{pmatrix}$$

$$A_1 = A_0 \begin{pmatrix} \begin{pmatrix} \text{3x3} \\ \text{rotation} \\ \text{matrix} \end{pmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix} = A_0 R$$

Camera 1
parameters: A_1

Camera 0
parameters: A_0

homography

we seek M_{10} such that

$$\underline{x}_0 = M_{10}\,\underline{x}_1 \qquad \text{for all } \underline{x}_0, \underline{x}_1$$

$$\underbrace{A_0 \underline{P}}_{x_0} = M_{10}\, \underbrace{A_0 R \underline{P}}_{x_1} \qquad \text{for all } \underline{P}, \text{ so}$$

$$A_0 = M_{10} A_0 R \qquad \text{mult by } R^{-1} = \begin{pmatrix} \begin{pmatrix}\text{inverse} \\ \text{rotation}\end{pmatrix} & \begin{matrix}0\\0\\0\end{matrix} \\ 0\;0\;0 & 1 \end{pmatrix}$$

$$A_0 R^{-1} = M_{10} A_0$$

$$A_0 R^{-1} B = M_{10}$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & f & 0 \end{pmatrix}}_{A_0} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & f \\ 0 & 0 & 0 \end{pmatrix}}_{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

homography

$$M_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{pmatrix} R^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & f \\ 0 & 0 & 0 \end{pmatrix}$$

$$\underline{x}_0 = M_{10}\, \underline{x}_1 \qquad \text{for all } \underline{x}_0, \underline{x}_1$$

How many pairs of points does it take to specify M_10?

Planar objects

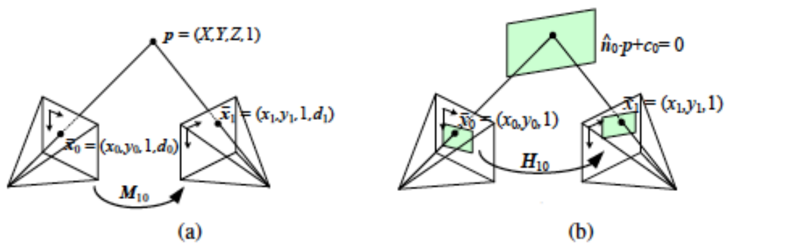


Figure 2.12 A point is projected into two images: (a) relationship between the 3D point coordinate $(X, Y, Z, 1)$ and the 2D projected point $(x, y, 1, d)$; (b) planar homography induced by points all lying on a common plane $\hat{n}_0 \cdot p + c_0 = 0$.

Mapping from one camera to another

What happens when we take two images of a 3D scene from different camera positions or orientations (Figure 2.12a)? Using the full rank 4×4 camera matrix $\tilde{P} = \tilde{K}E$ from (2.64), we can write the projection from world to screen coordinates as

$$\bar{x}_0 \sim \tilde{K}_0 E_0 p = \tilde{P}_0 p. \quad (2.68)$$

Assuming that we know the z-buffer or disparity value d_0 for a pixel in one image, we can compute the 3D point location p using

$$p \sim E_0^{-1} \tilde{K}_0^{-1} \bar{x}_0 \quad (2.69)$$

and then project it into another image yielding

$$\bar{x}_1 \sim \tilde{K}_1 E_1 p = \tilde{K}_1 E_1 E_0^{-1} \tilde{K}_0^{-1} \bar{x}_0 = \tilde{P}_1 \tilde{P}_0^{-1} \bar{x}_0 = M_{10} \bar{x}_0. \quad (2.70)$$

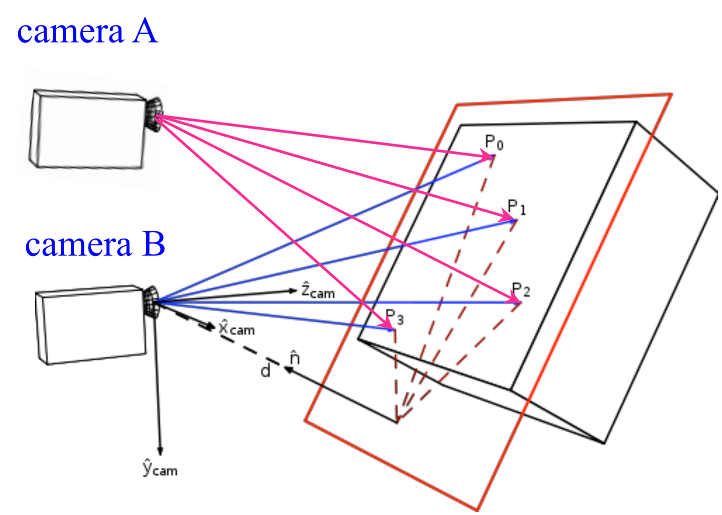
Unfortunately, we do not usually have access to the depth coordinates of pixels in a regular photographic image. However, for a *planar scene*, as discussed above in (2.66), we can replace the last row of P_0 in (2.64) with a general *plane equation*, $\hat{n}_0 \cdot p + c_0$ that maps points on the plane to $d_0 = 0$ values (Figure 2.12b). Thus, if we set $d_0 = 0$, we can ignore the last column of M_{10} in (2.70) and also its last row, since we do not care about the final z-buffer depth. The mapping equation (2.70) thus reduces to

$$\tilde{x}_1 \sim \tilde{H}_{10} \tilde{x}_0, \quad (2.71)$$

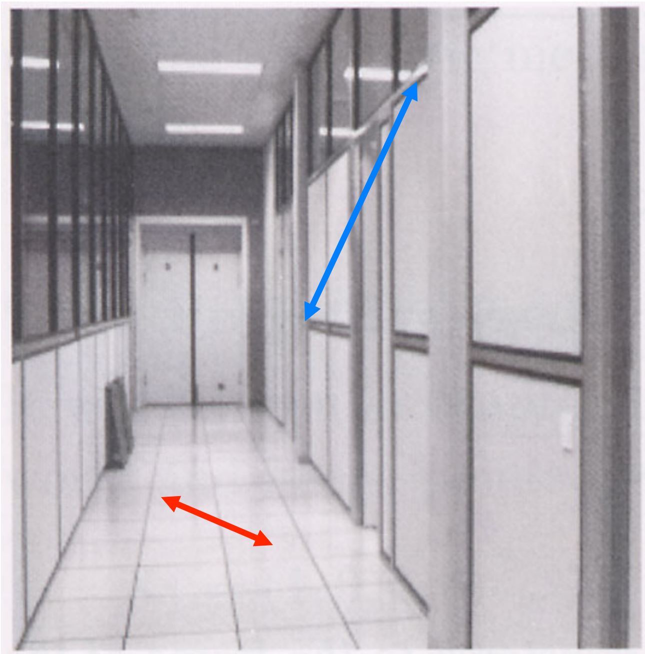
where $\tilde{H}_{10}$ is a general 3×3 homography matrix and $\tilde{x}_1$ and $\tilde{x}_0$ are now 2D homogeneous coordinates (i.e., 3-vectors) (Szeliski 1996).This justifies the use of the 8-parameter homography as a general alignment model for mosaics of planar scenes (Mann and Picard 1994; Szeliski 1996).

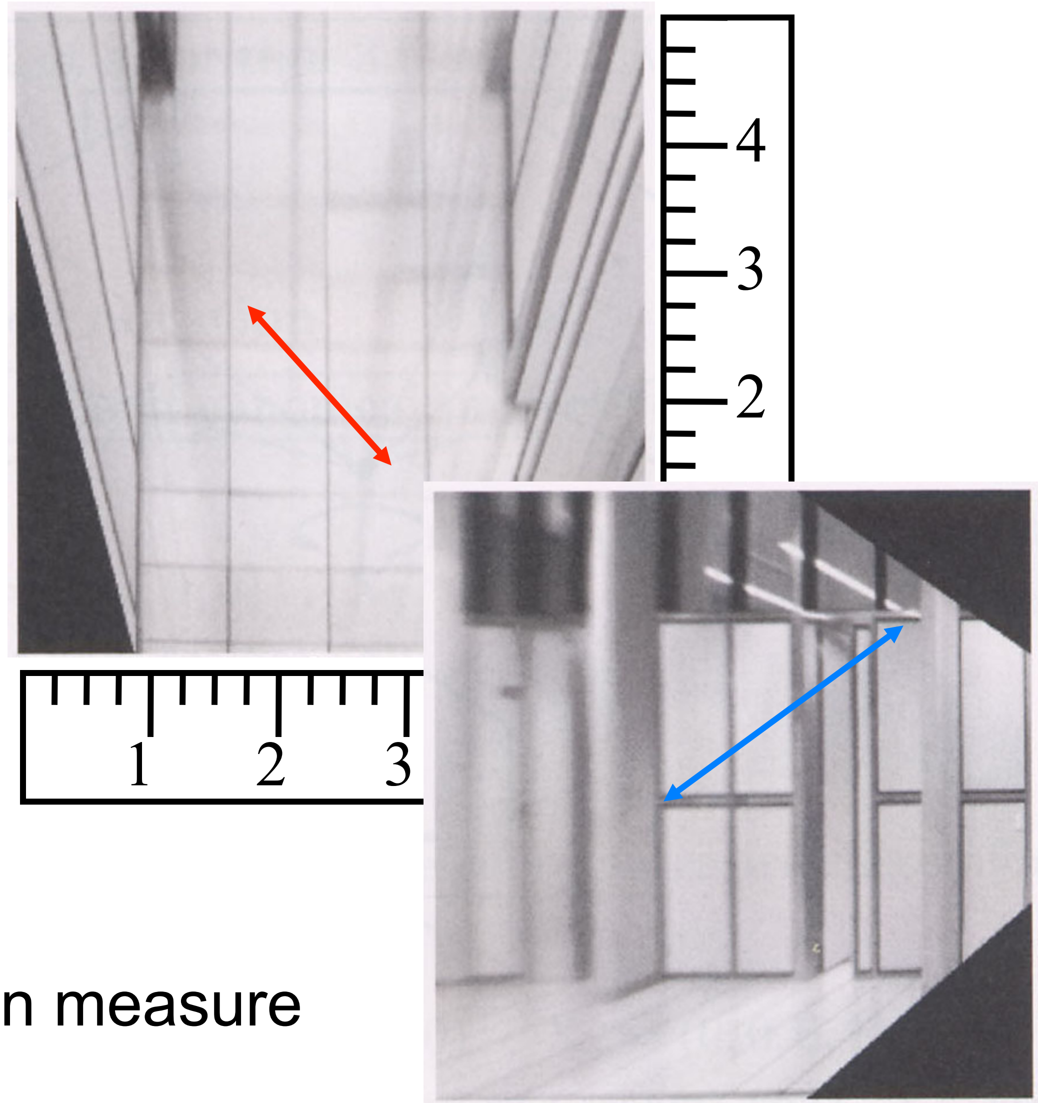
From Szeliski book

Images of planar objects, taken by generically offset cameras, are also related by a homography.



Measurements on planes

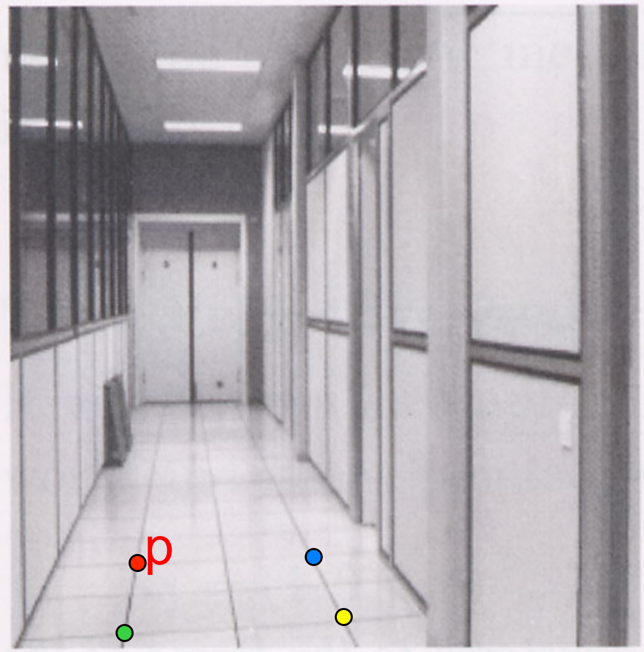


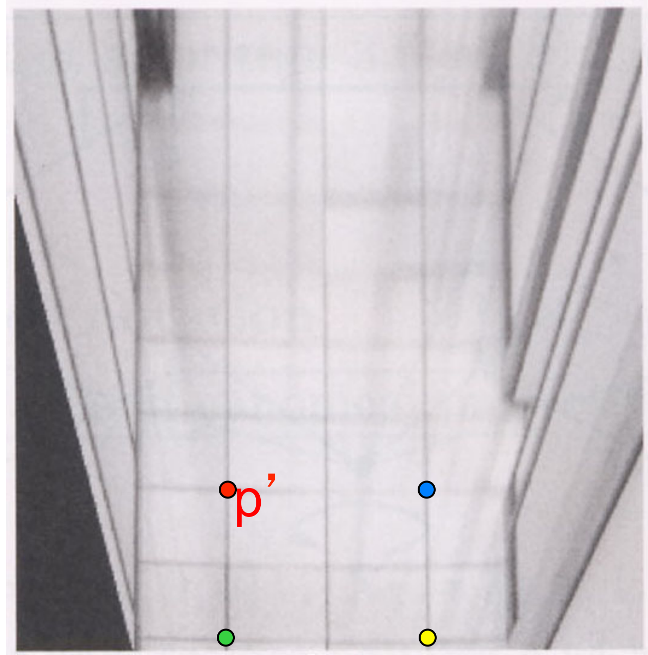


Approach: unwarp then measure

How to unwarp?

Image rectification





To unwarp (rectify) an image

- solve for homography H given p and p'
- solve equations of the form: wp' = Hp
  - linear in unknowns: w and coefficients of H
  - H is defined up to an arbitrary scale factor
  - how many points are necessary to solve for H?

Solving for homographies

$$\begin{bmatrix} w x_i' \\ w y_i' \\ w \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$x_i' = \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$y_i' = \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$x_i'(h_{20}x_i + h_{21}y_i + h_{22}) = h_{00}x_i + h_{01}y_i + h_{02}$$

$$y_i'(h_{20}x_i + h_{21}y_i + h_{22}) = h_{10}x_i + h_{11}y_i + h_{12}$$

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x_i'x_i & -x_i'y_i & -x_i' \\ 0 & 0 & 0 & x_i & y_i & 1 & -y_i'x_i & -y_i'y_i & -y_i' \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solving for homographies

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ & & & & \vdots & & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

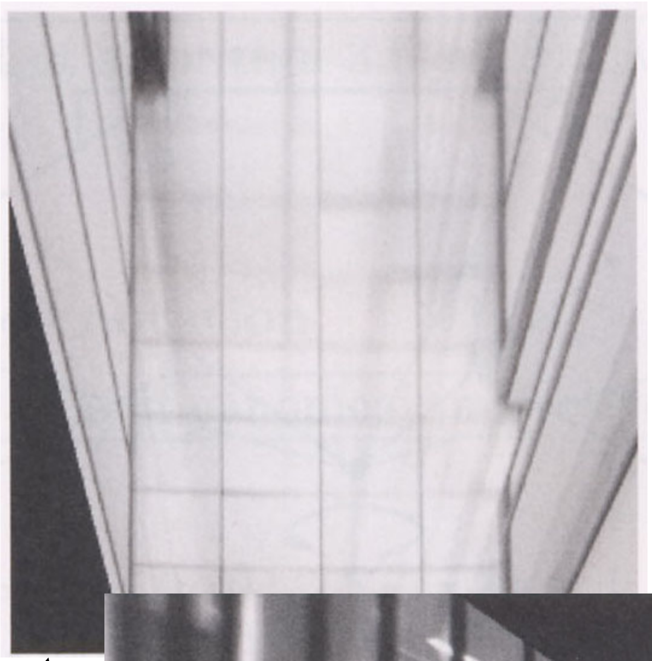
A ($2n \times 9$)　　h (9)　　0 ($2n$)

Defines a least squares problem: $\text{minimize } \|\mathbf{Ah} - \mathbf{0}\|^2$

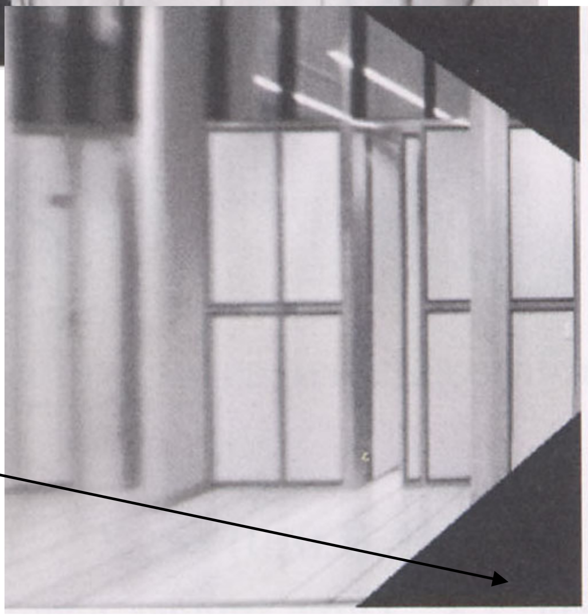
- Since h is only defined up to scale, solve for unit vector $\hat{h}$
- Solution: $\hat{h}$ = eigenvector of A^TA with smallest eigenvalue
- Works with 4 or more points

Image warping with homographies

homography so that image is parallel to floor



homography so that image is parallel to right wall

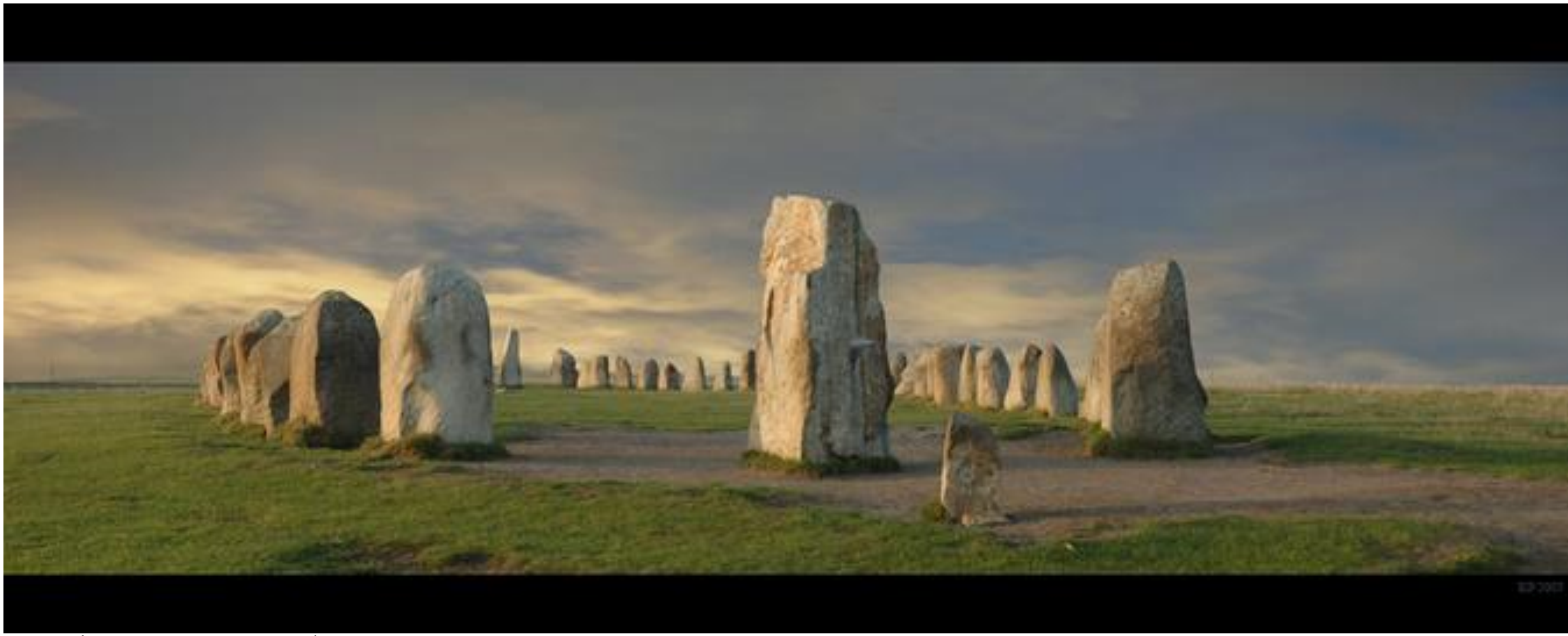


black area where no pixel maps to

automatic image mosaicing

- Basic Procedure
  - Take a sequence of images from the same position.
    - Rotate the camera about its optical center (entrance pupil).
  - Robustly compute the homography transformation between second image and first.
  - Transform (warp) the second image to overlap with first.
  - Blend the two together to create a mosaic.
  - If there are more images, repeat.

Robust feature matching through RANSAC



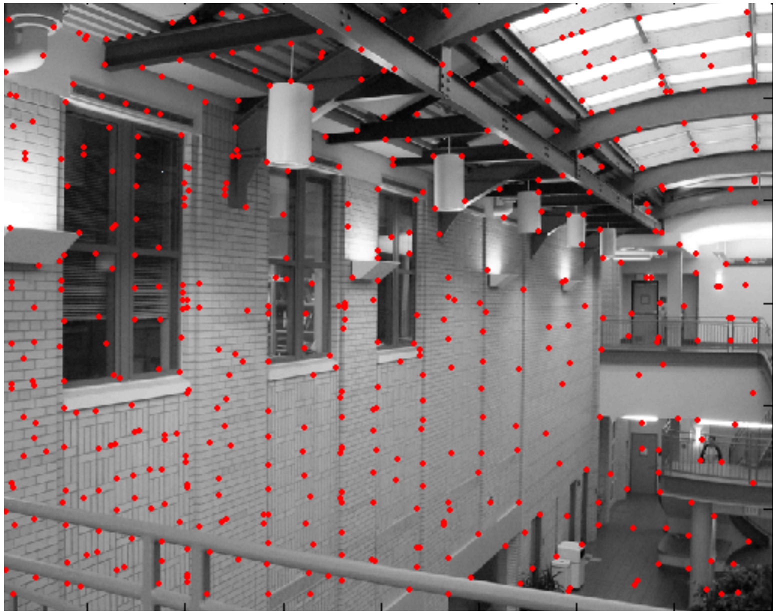
© Krister Parmstrand

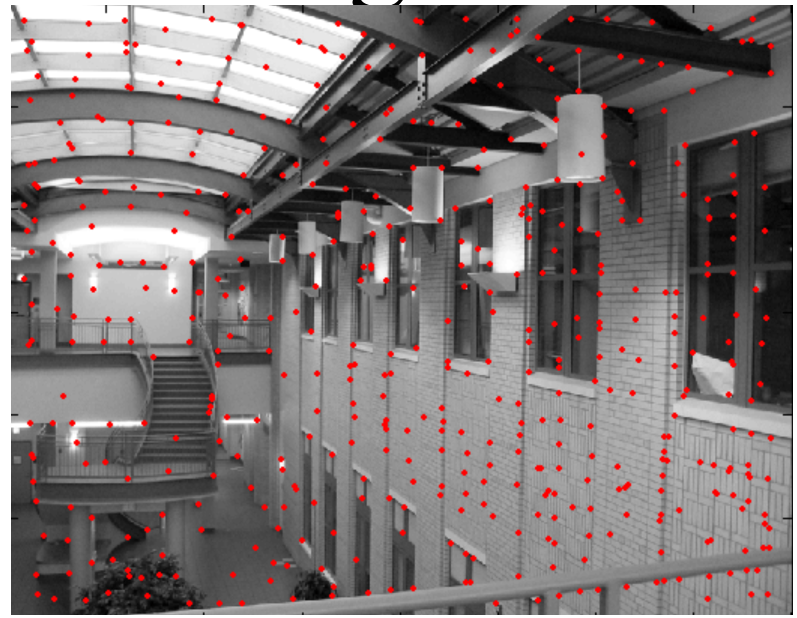
Nikon D70. Stitched Panorama. The sky has been retouched. No other image manipulation.

with a lot of slides stolen from
Steve Seitz and Rick Szeliski

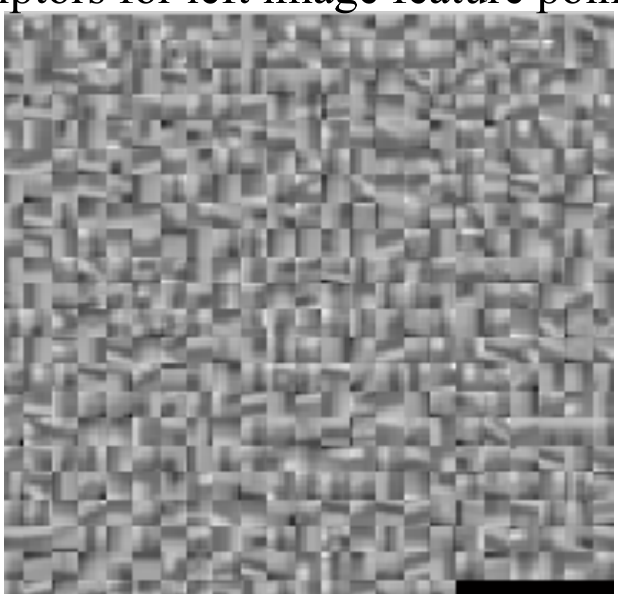
15-463: Computational Photography
Alexei Efros, CMU, Fall 2005

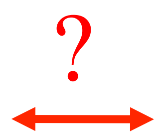
Feature matching



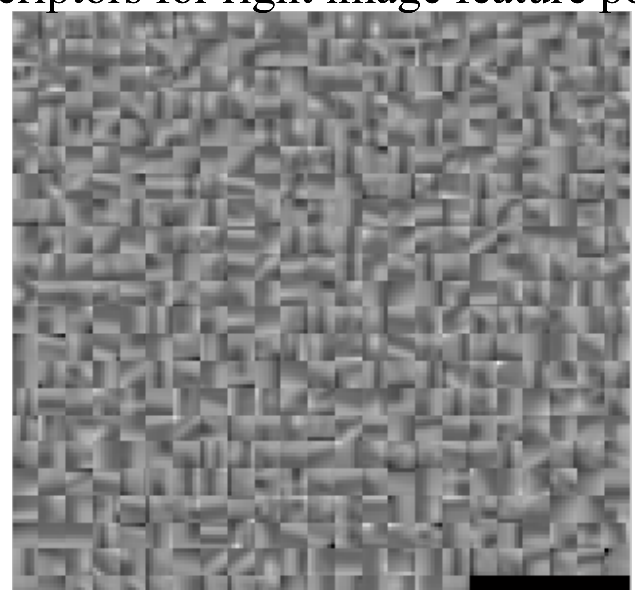


descriptors for left image feature points





descriptors for right image feature points



Strategies to match images robustly

(a) Working with individual features: For each feature point, find most similar point in other image (SIFT distance)

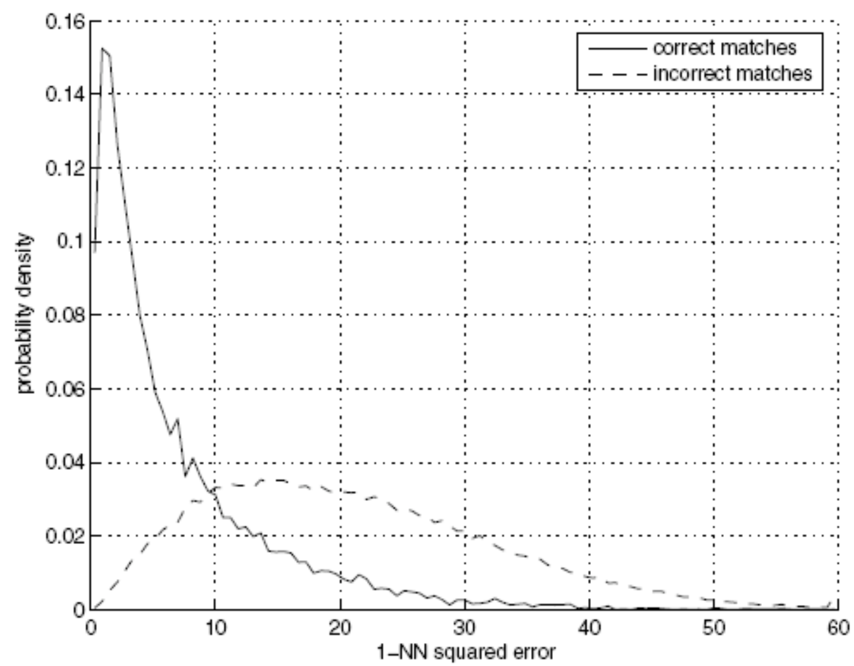
Reject ambiguous matches where there are too many similar points

(b) Working with all the features: Given some good feature matches, look for possible homographies relating the two images

Reject homographies that don't have many feature matches.

(a) Feature-space outlier rejection

- Let's not match all features, but only these that have "similar enough" matches?
- How can we do it?
  - SSD (patch1,patch2) < threshold
  - How to set threshold? Not so easy.

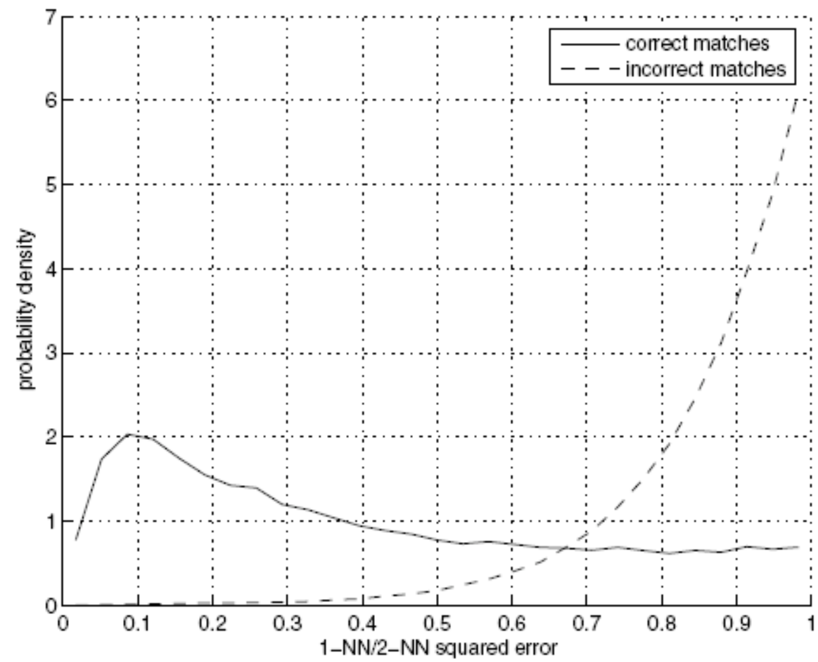


Feature matching

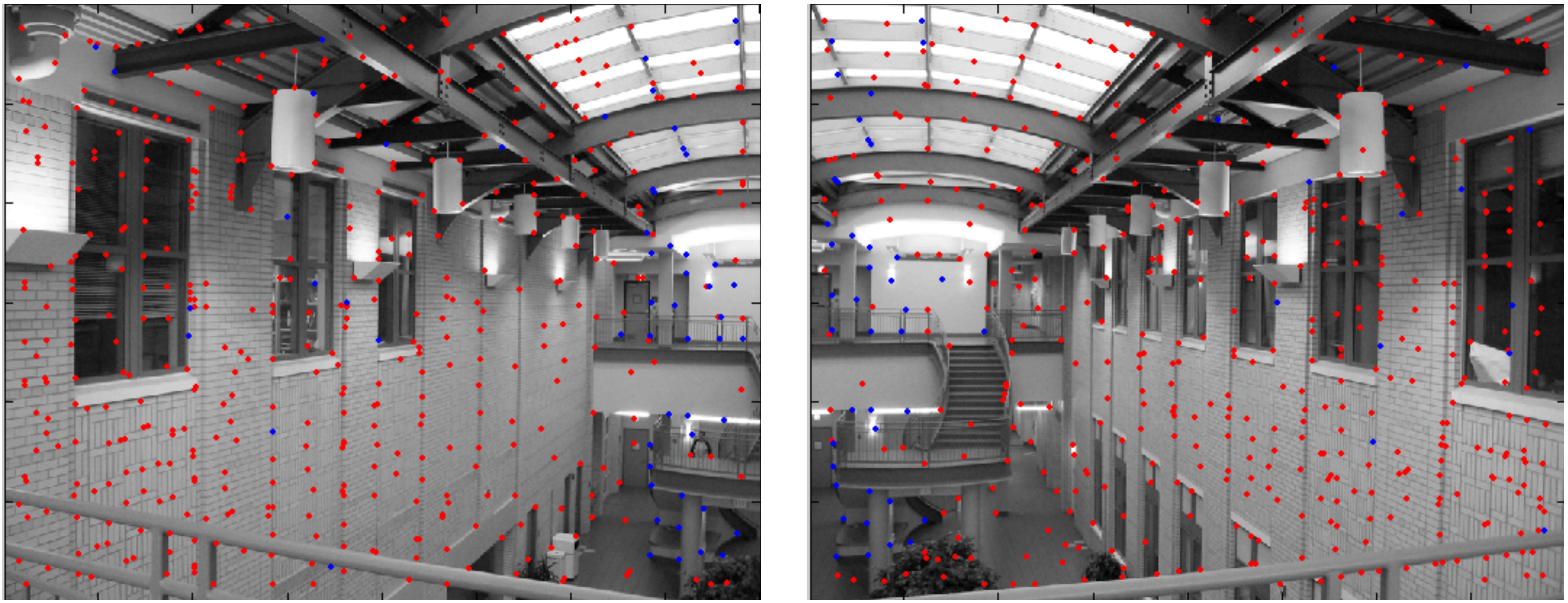
- Exhaustive search
  - for each feature in one image, look at all the other features in the other image(s)
  - Usually not so bad
- Hashing
  - compute a short descriptor from each feature vector, or hash longer descriptors (randomly)
- Nearest neighbor techniques
  - k-trees and their variants (Best Bin First)

Feature-space outlier rejection

- A better way [Lowe, 1999]:
  - 1-NN: SSD of the closest match
  - 2-NN: SSD of the <u>second-closest</u> match
  - Look at how much better 1-NN is than 2-NN, e.g. 1-NN/2-NN
  - That is, is our best match so much better than the rest?



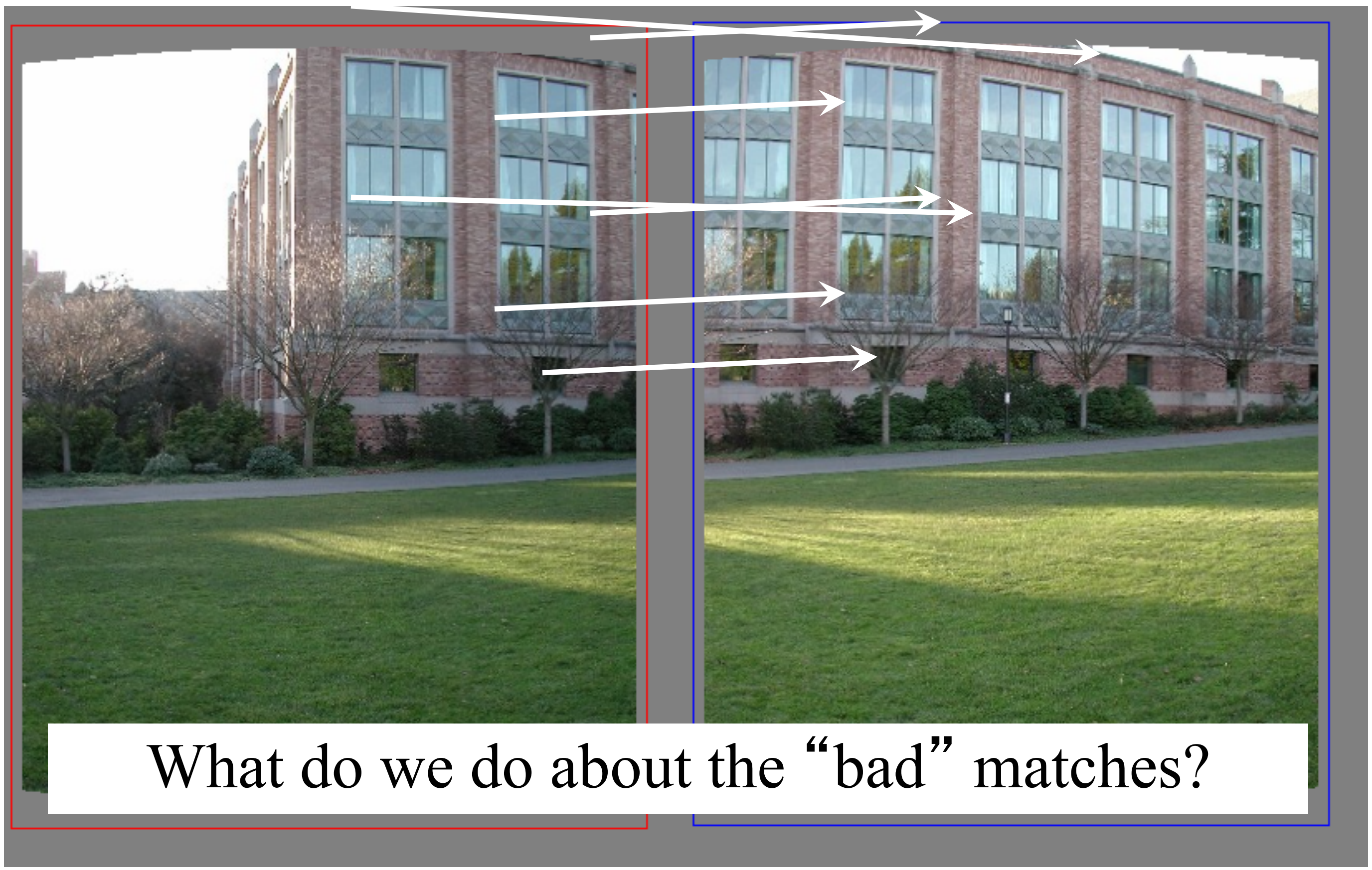
Feature-space outlier rejection



- Can we now compute H from the blue points?
  - No! Still too many outliers…
  - What can we do?

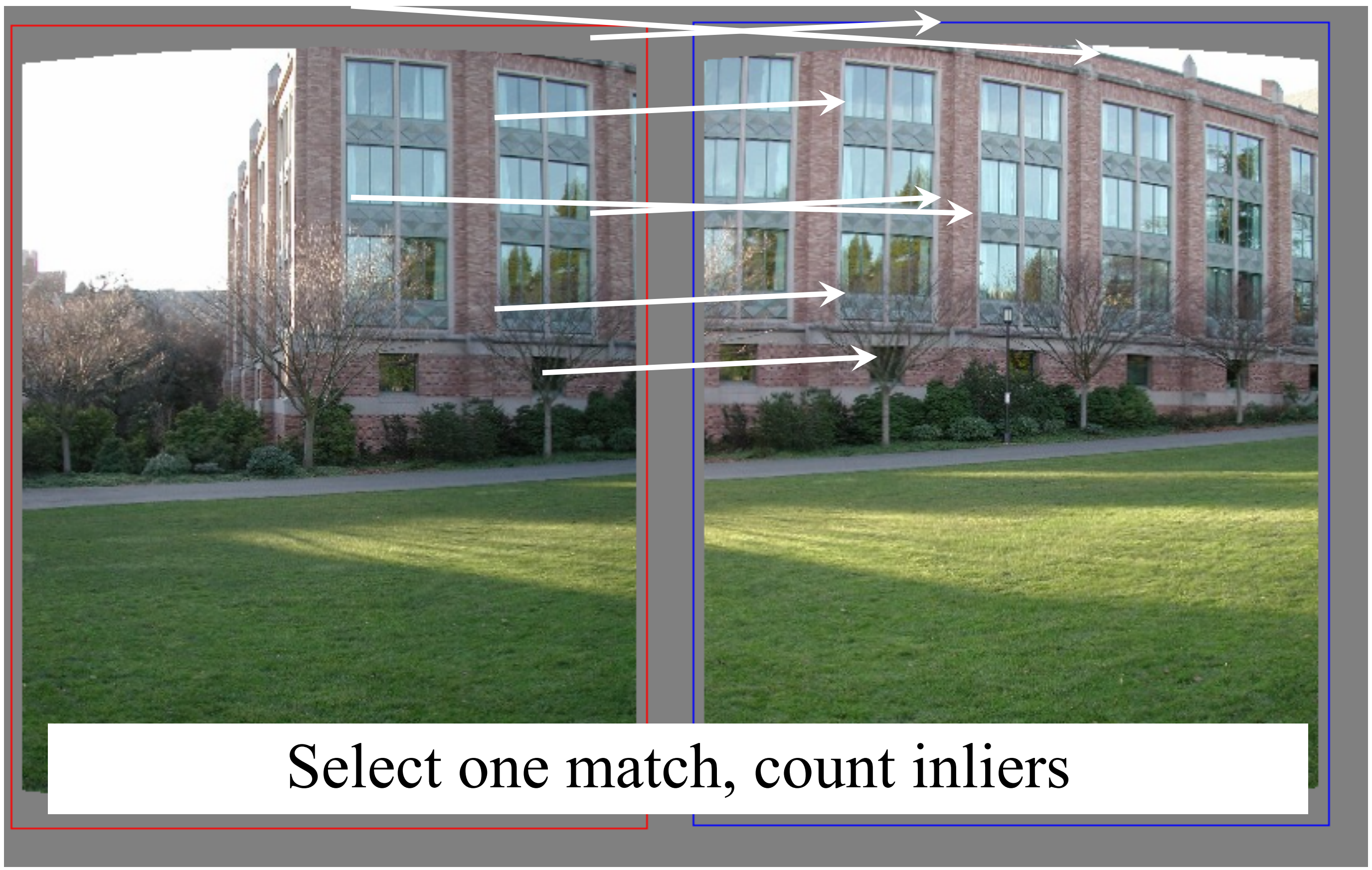
(b) Matching many features--looking for a good homography

Simplified illustration with translation instead of homography



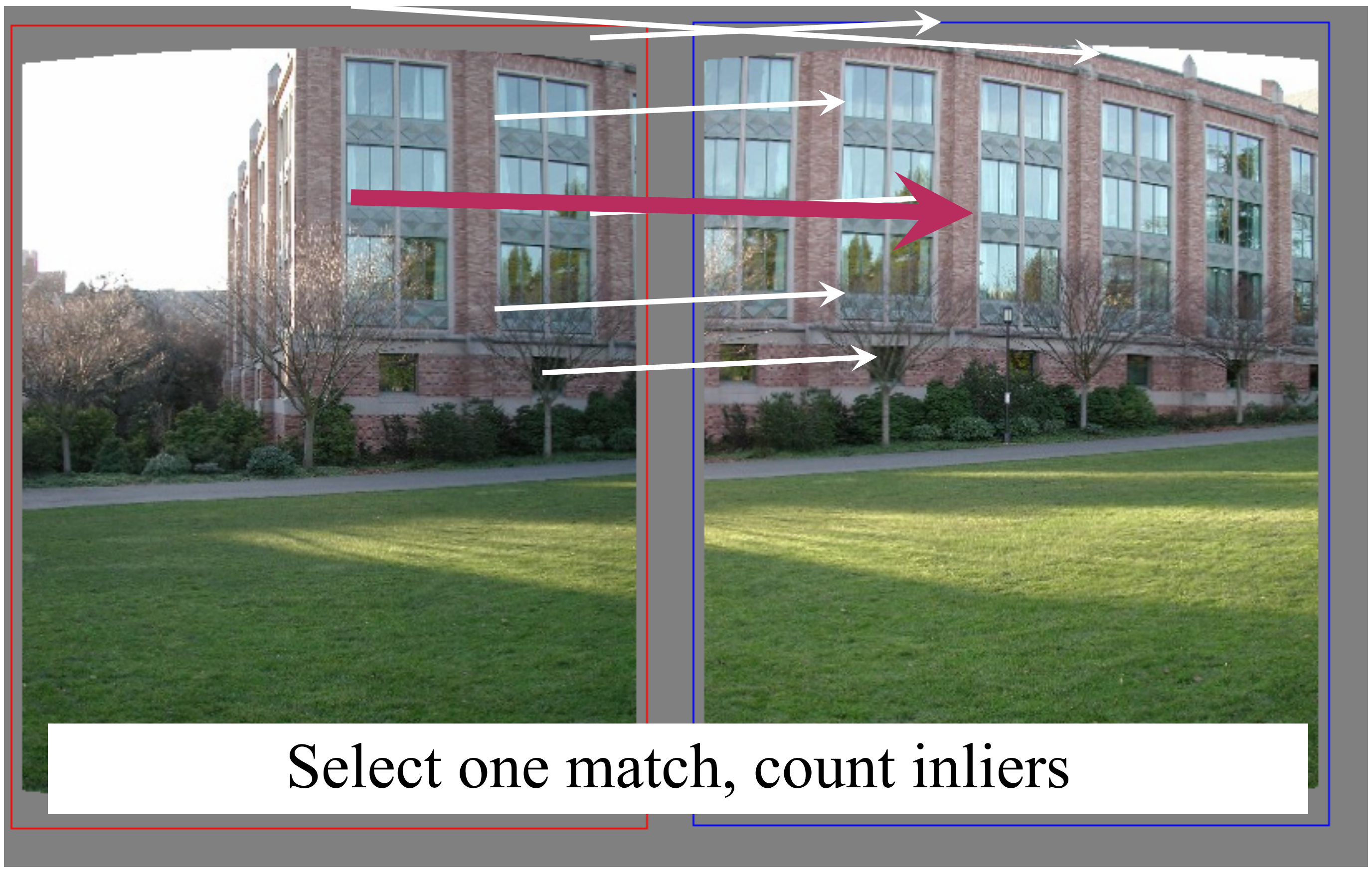
Note: at this point we don't know which ones are good/bad

RAndom SAmple Consensus



Select one match, count inliers

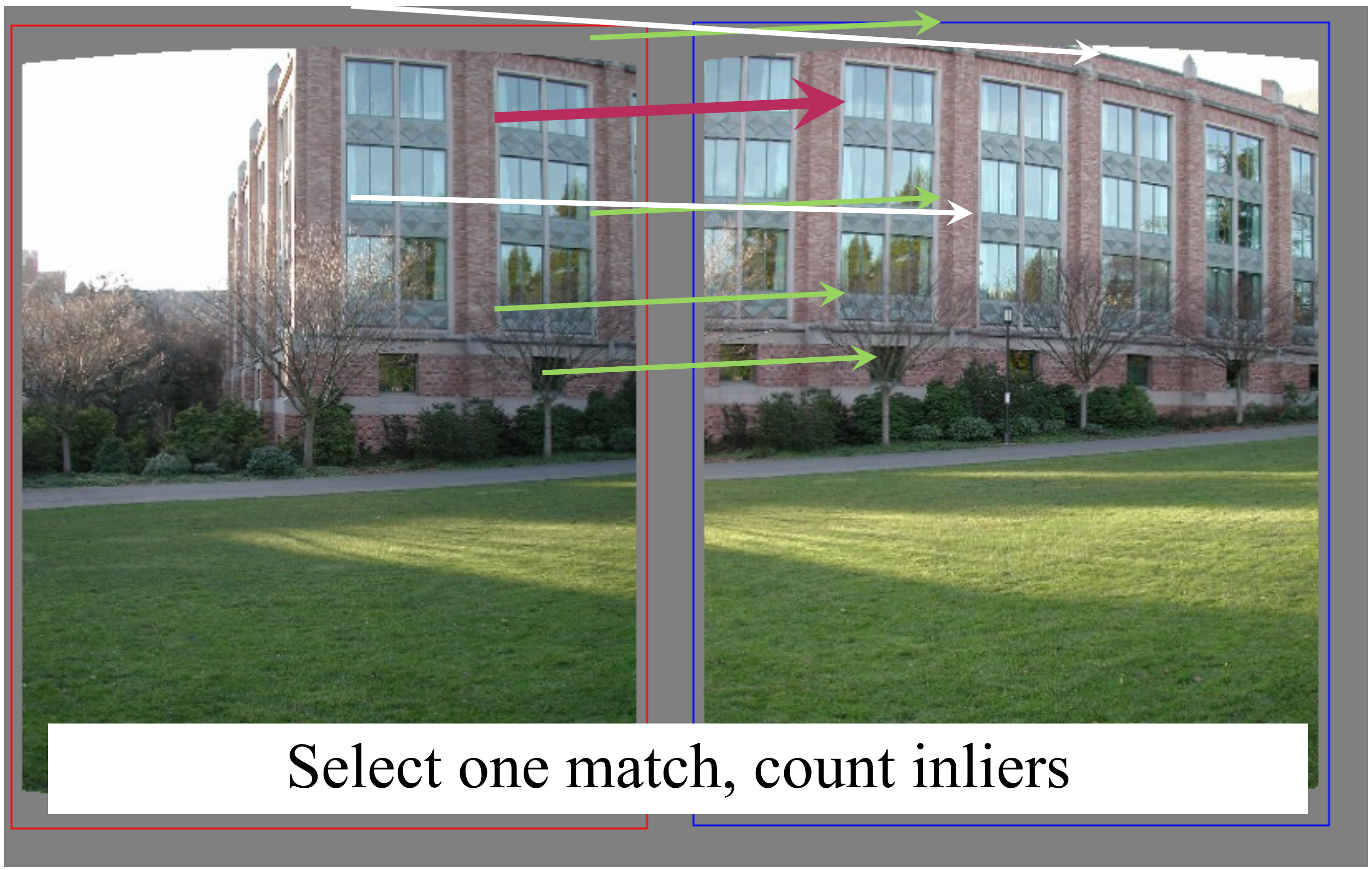
RAndom SAmple Consensus



Select one match, count inliers

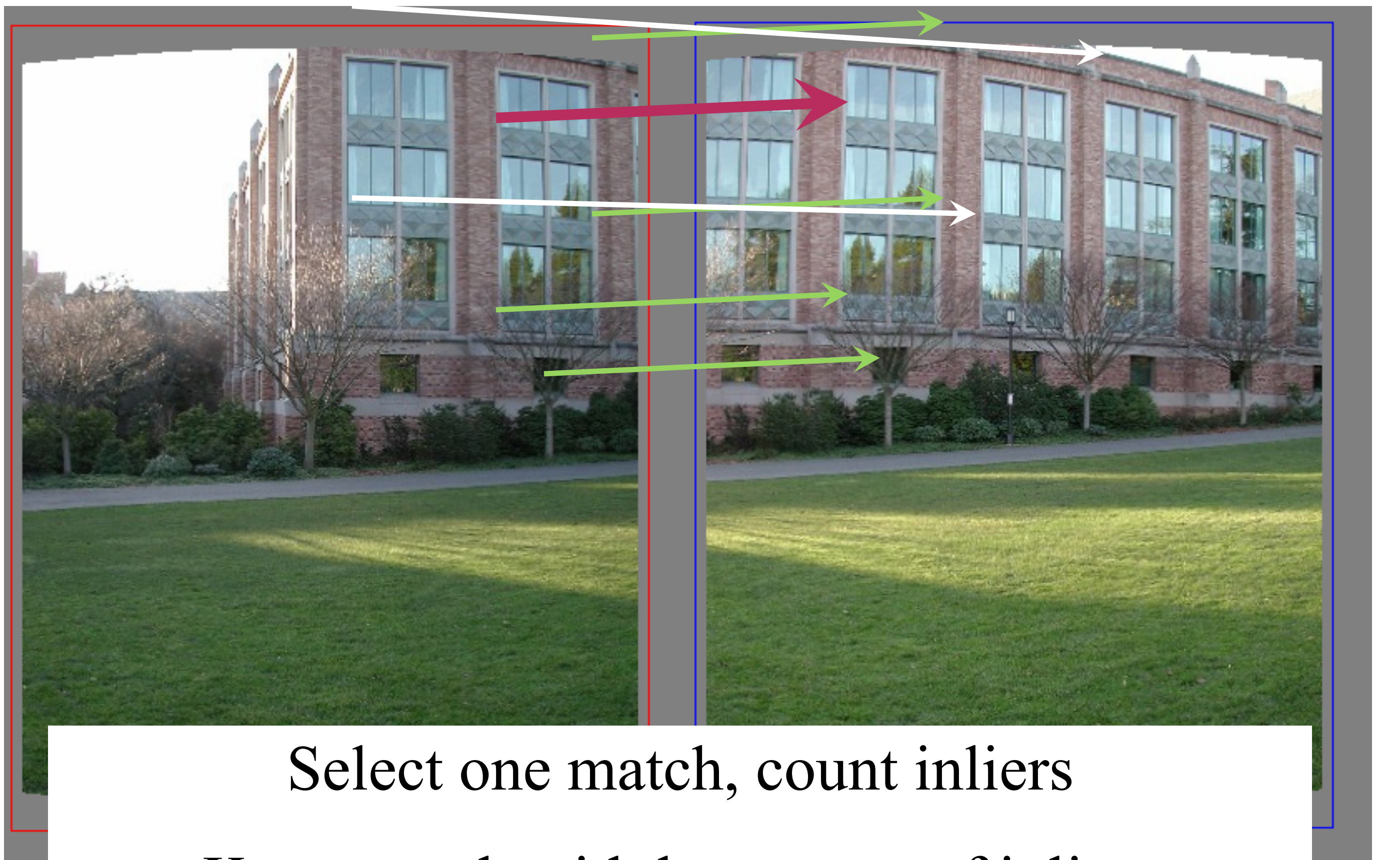
0 inliers

RAndom SAmple Consensus



4 inliers

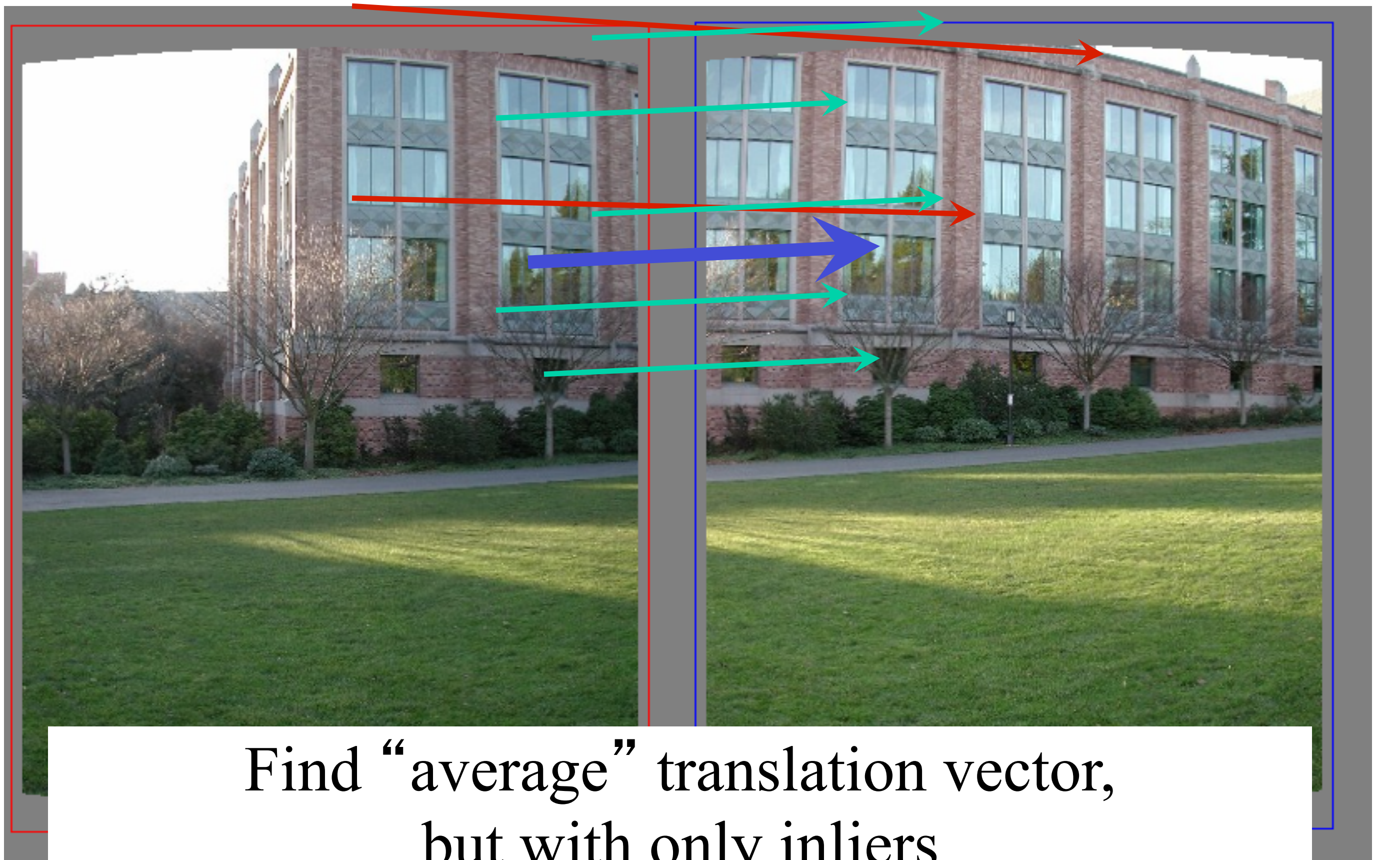
RAndom SAmple Consensus



Select one match, count inliers

Keep match with largest set of inliers

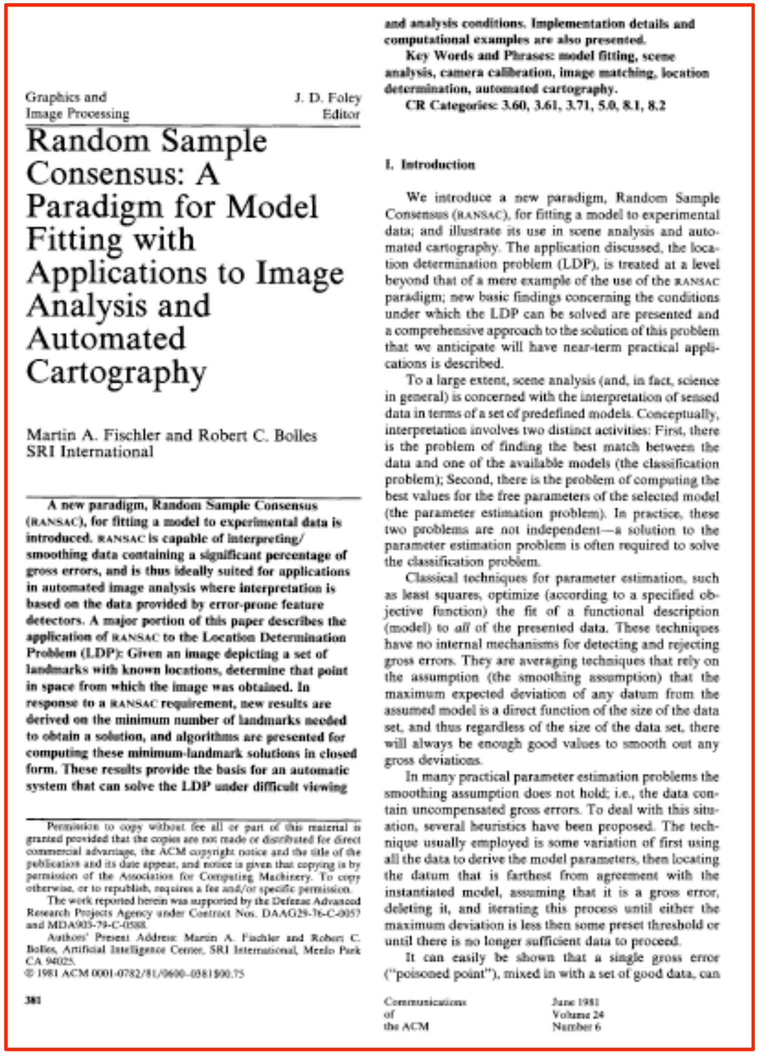
At the end: Least squares fit



Find “average” translation vector, but with only inliers

Reference

- M. A. Fischler, R. C. Bolles. Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography. Comm. of the ACM, Vol 24, pp 381-395, 1981.
- http://portal.acm.org/citation.cfm?id=358692



Graphics and Image Processing — J. D. Foley, Editor

Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography

Martin A. Fischler and Robert C. Bolles
SRI International

A new paradigm, Random Sample Consensus (RANSAC), for fitting a model to experimental data is introduced. RANSAC is capable of interpreting/smoothing data containing a significant percentage of gross errors, and is thus ideally suited for applications in automated image analysis where interpretation is based on the data provided by error-prone feature detectors. A major portion of this paper describes the application of RANSAC to the Location Determination Problem (LDP): Given an image depicting a set of landmarks with known locations, determine that point in space from which the image was obtained. In response to a RANSAC requirement, new results are derived on the minimum number of landmarks needed to obtain a solution, and algorithms are presented for computing these minimum-landmark solutions in closed form. These results provide the basis for an automatic system that can solve the LDP under difficult viewing and analysis conditions. Implementation details and computational examples are also presented.

Key Words and Phrases: model fitting, scene analysis, camera calibration, image matching, location determination, automated cartography.

CR Categories: 3.60, 3.61, 3.71, 5.0, 8.1, 8.2

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I. Introduction

We introduce a new paradigm, Random Sample Consensus (RANSAC), for fitting a model to experimental data; and illustrate its use in scene analysis and automated cartography. The application discussed, the location determination problem (LDP), is treated at a level beyond that of a mere example of the use of the RANSAC paradigm; new basic findings concerning the conditions under which the LDP can be solved are presented and a comprehensive approach to the solution of this problem that we anticipate will have near-term practical applications is described.

To a large extent, scene analysis (and, in fact, science in general) is concerned with the interpretation of sensed data in terms of a set of predefined models. Conceptually, interpretation involves two distinct activities: First, there is the problem of finding the best match between the data and one of the available models (the classification problem); Second, there is the problem of computing the best values for the free parameters of the selected model (the parameter estimation problem). In practice, these two problems are not independent—a solution to the parameter estimation problem is often required to solve the classification problem.

Classical techniques for parameter estimation, such as least squares, optimize (according to a specified objective function) the fit of a functional description (model) to *all* of the presented data. These techniques have no internal mechanisms for detecting and rejecting gross errors. They are averaging techniques that rely on the assumption (the smoothing assumption) that the maximum expected deviation of any datum from the assumed model is a direct function of the size of the data set, and thus regardless of the size of the data set, there will always be enough good values to smooth out any gross deviations.

In many practical parameter estimation problems the smoothing assumption does not hold; i.e., the data contain uncompensated gross errors. To deal with this situation, several heuristics have been proposed. The technique usually employed is some variation of first using all the data to derive the model parameters, then locating the datum that is farthest from agreement with the instantiated model, assuming that it is a gross error, deleting it, and iterating this process until either the maximum deviation is less then some preset threshold or until there is no longer sufficient data to proceed.

It can easily be shown that a single gross error ("poisoned point"), mixed in with a set of good data, can

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RANSAC for estimating homography

RANSAC loop:

Select four feature pairs (at random)

Compute homography H (exact)

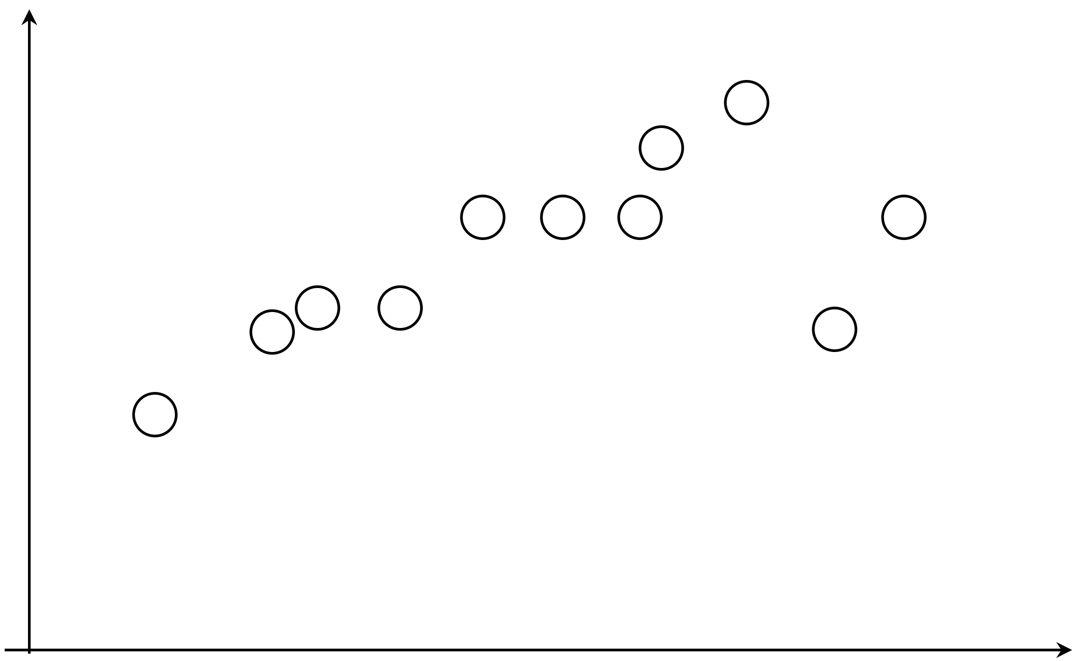
Compute inliers where $||p_i', H p_i|| < \varepsilon$

Keep largest set of inliers

Re-compute least-squares H estimate using all of the inliers

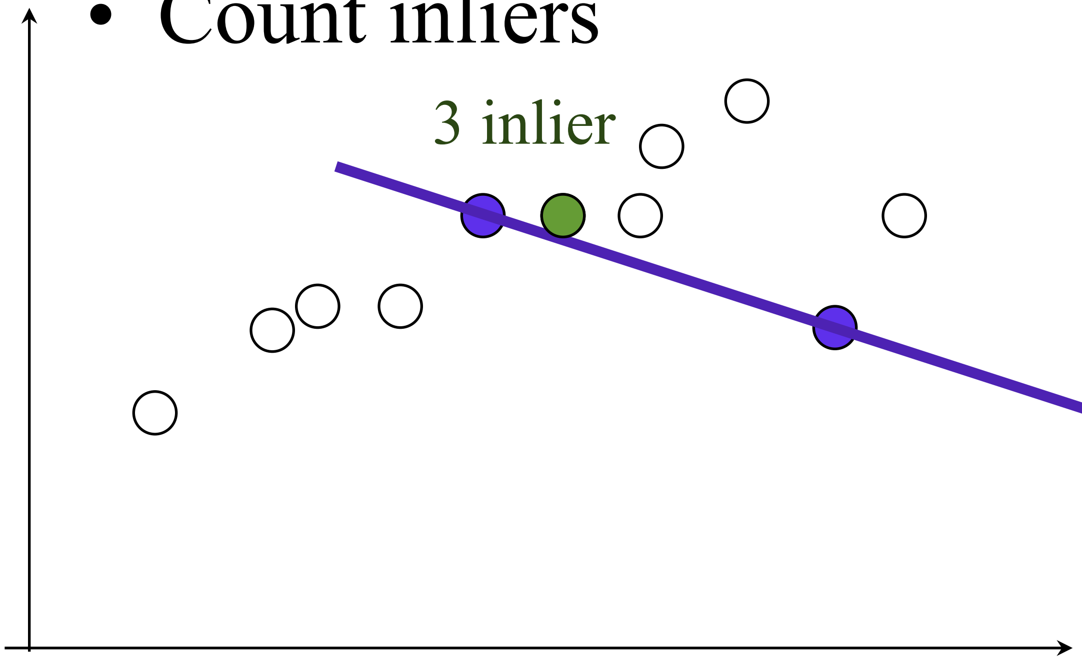
Simple example: fit a line

- Rather than homography H (8 numbers) fit y=ax+b (2 numbers a, b) to 2D pairs



Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers



Simple example: fit a line

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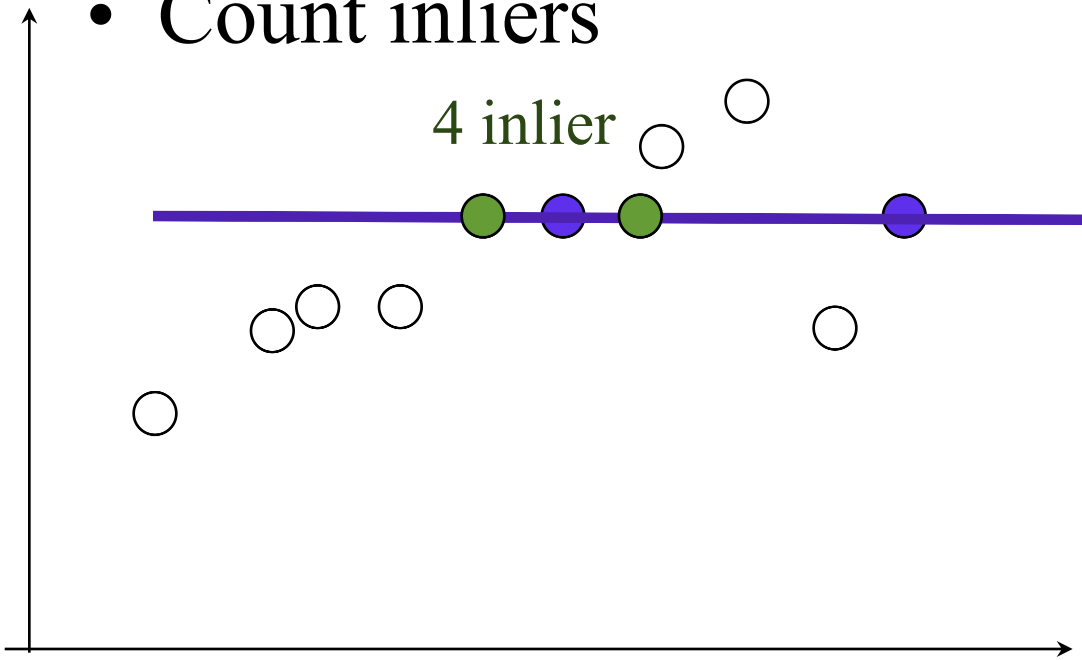
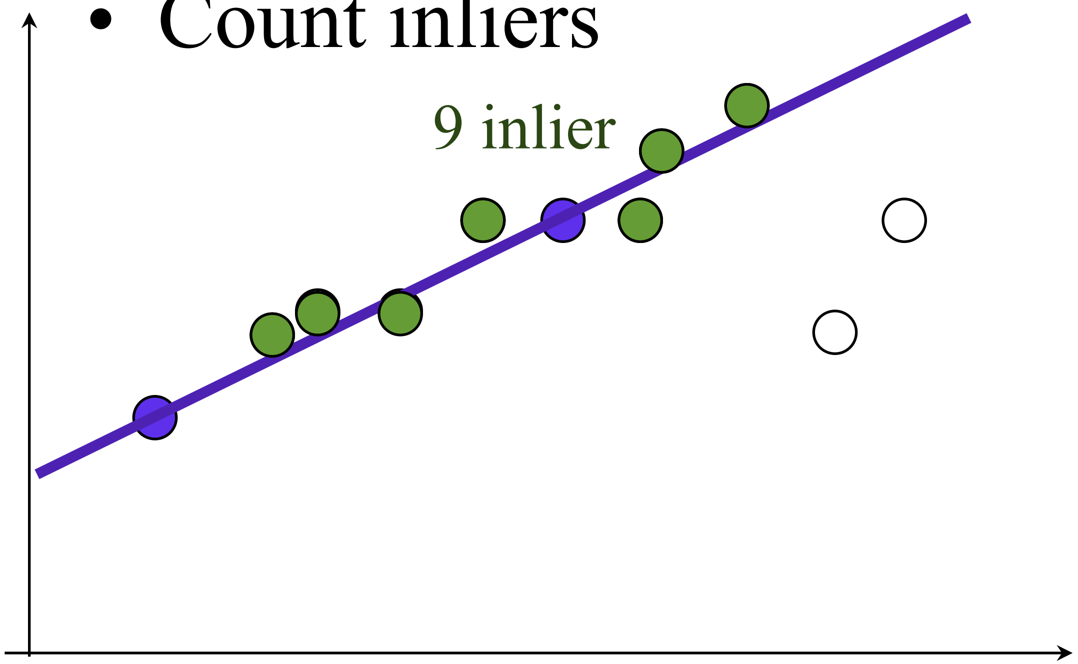
- Fit line
- Count inliers



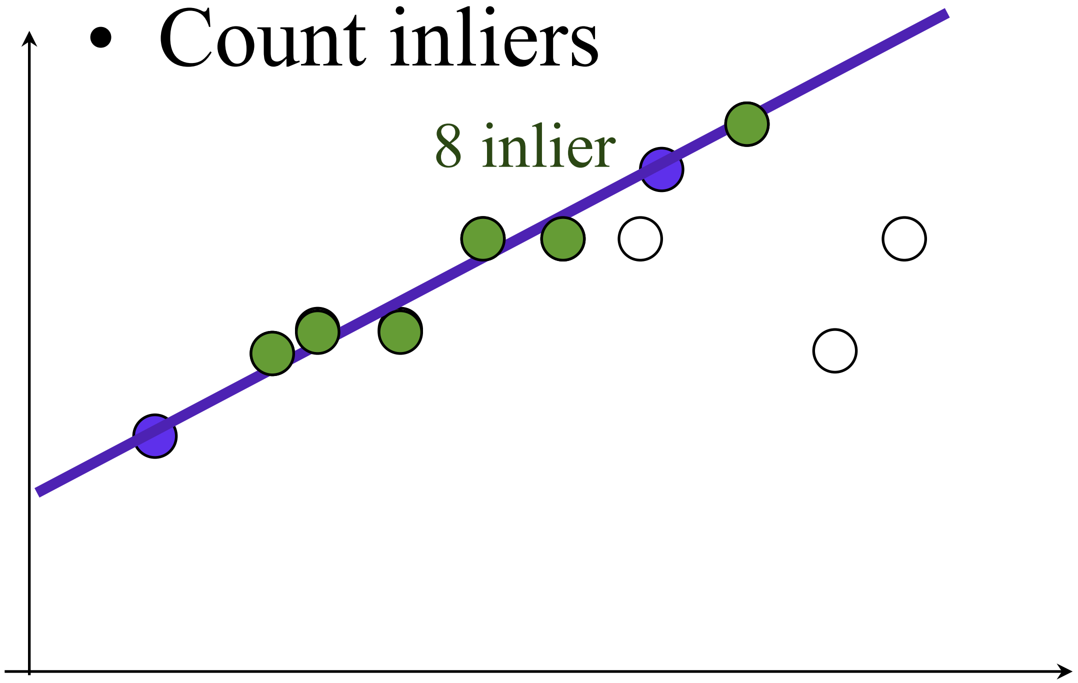
Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers



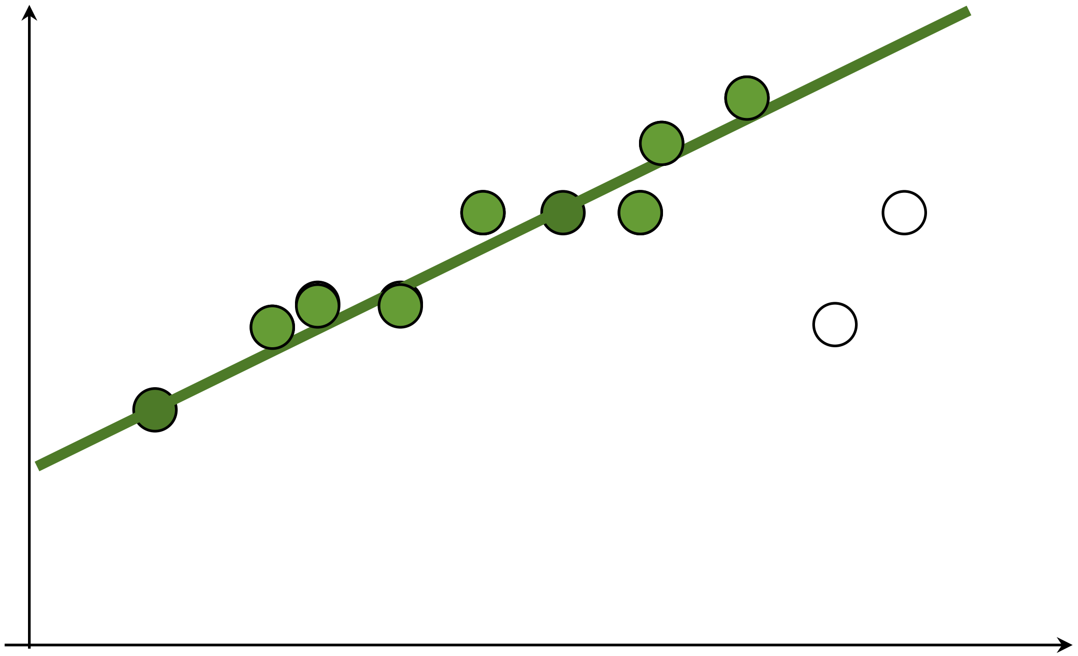
Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers

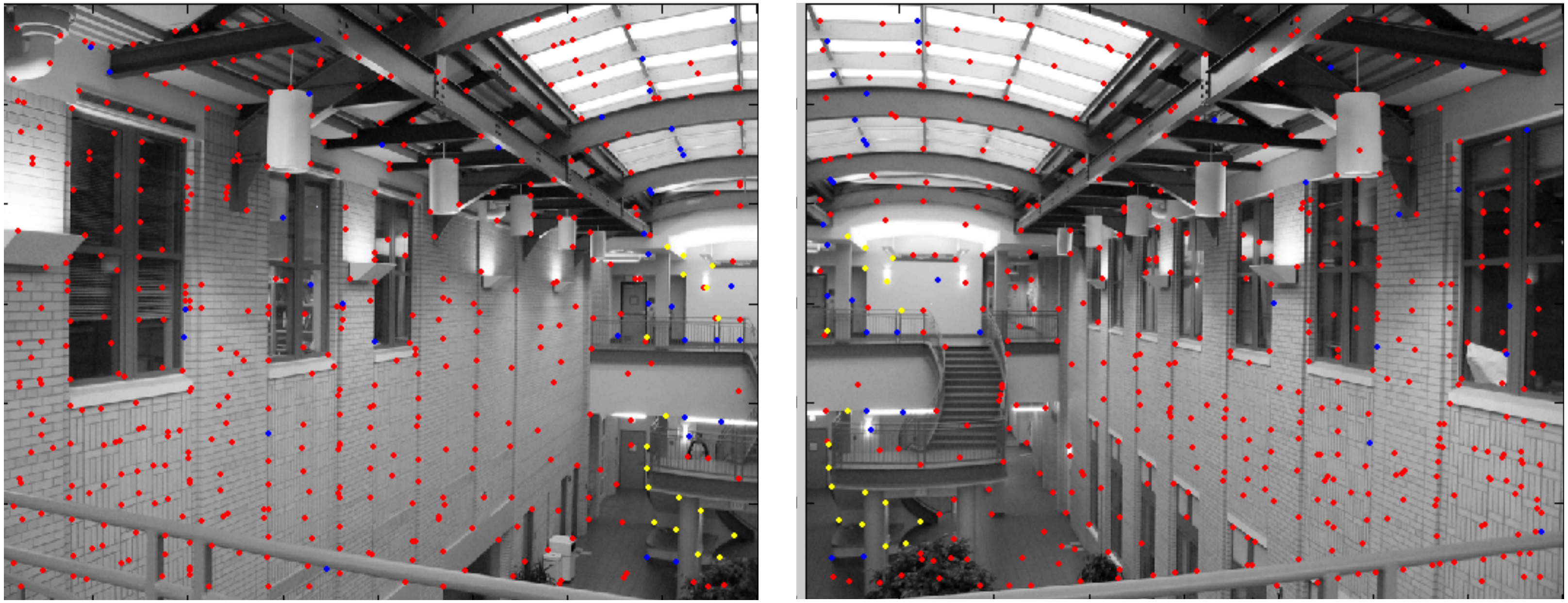


Simple example: fit a line

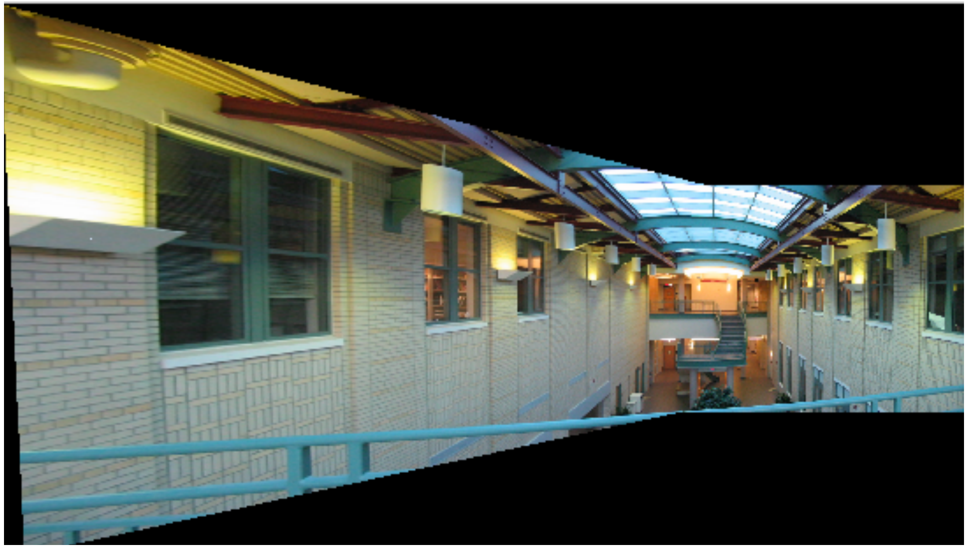
- Use biggest set of inliers
- Do least-square fit



RANSAC



red:
rejected by 2nd nearest neighbor criterion
blue:
Ransac outliers
yellow:
inliers



Robustness

- Proportion of inliers in our pairs is G (for "good")
- Our model needs P pairs

  P=4 for homography

- Probability that we pick P inliers?

  $$G^P$$

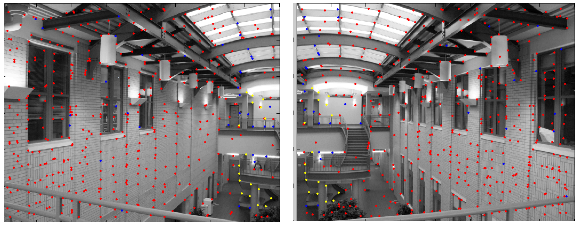
- Probability that after N RANSAC iterations we have not picked a set of inliers?

  $$(1-G^P)^N$$

Robustness: example

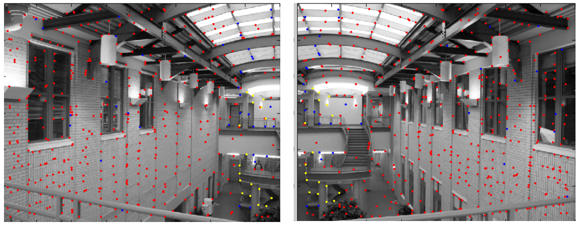
- Proportion of inliers G=0.5
- Probability that we pick P=4 inliers?
  - $0.5^4=0.0625$ (6% chance)
- Probability that we have not picked a set of inliers?
  - N=100 iterations:
    $(1-0.5^4)^{100}=0.00157$ (1 chance in 600)
  - N=1000 iterations:
    1 chance in 1e28

Robustness: example



- Proportion of inliers G=0.3
- Probability that we pick P=4 inliers?
  - $0.3^4=0.0081$ (0.8% chance)
- Probability that we have not picked a set of inliers?
  - N=100 iterations:
    $(1-0.3^4)^{100}=0.44$ (1 chance in 2)
  - N=1000 iterations:
    1 chance in 3400

Robustness: example



- Proportion of inliers G=0.1
- Probability that we pick P=4 inliers?
  - $0.1^4=0.0001$ (0.01% chances, 1 in 10,000)
- Probability that we have not picked a set of inliers?
  - N=100 iterations: $(1\text{-}0.1^4)^{100}=0.99$
  - N=1000 iterations: 90%
  - N=10,000: 36%
  - N=100,000: 1 in 22,000

Robustness: conclusions

- Effect of number of parameters of model/ number of necessary pairs
  - Bad exponential
- Effect of percentage of inliers
  - Base of the exponential
- Effect of number of iterations
  - Good exponential

RANSAC recap

- For fitting a model with low number P of parameters (8 for homographies)
- Loop
  - Select P random data points
  - Fit model
  - Count inliers
    (other data points well fit by this model)
- Keep model with largest number of inliers

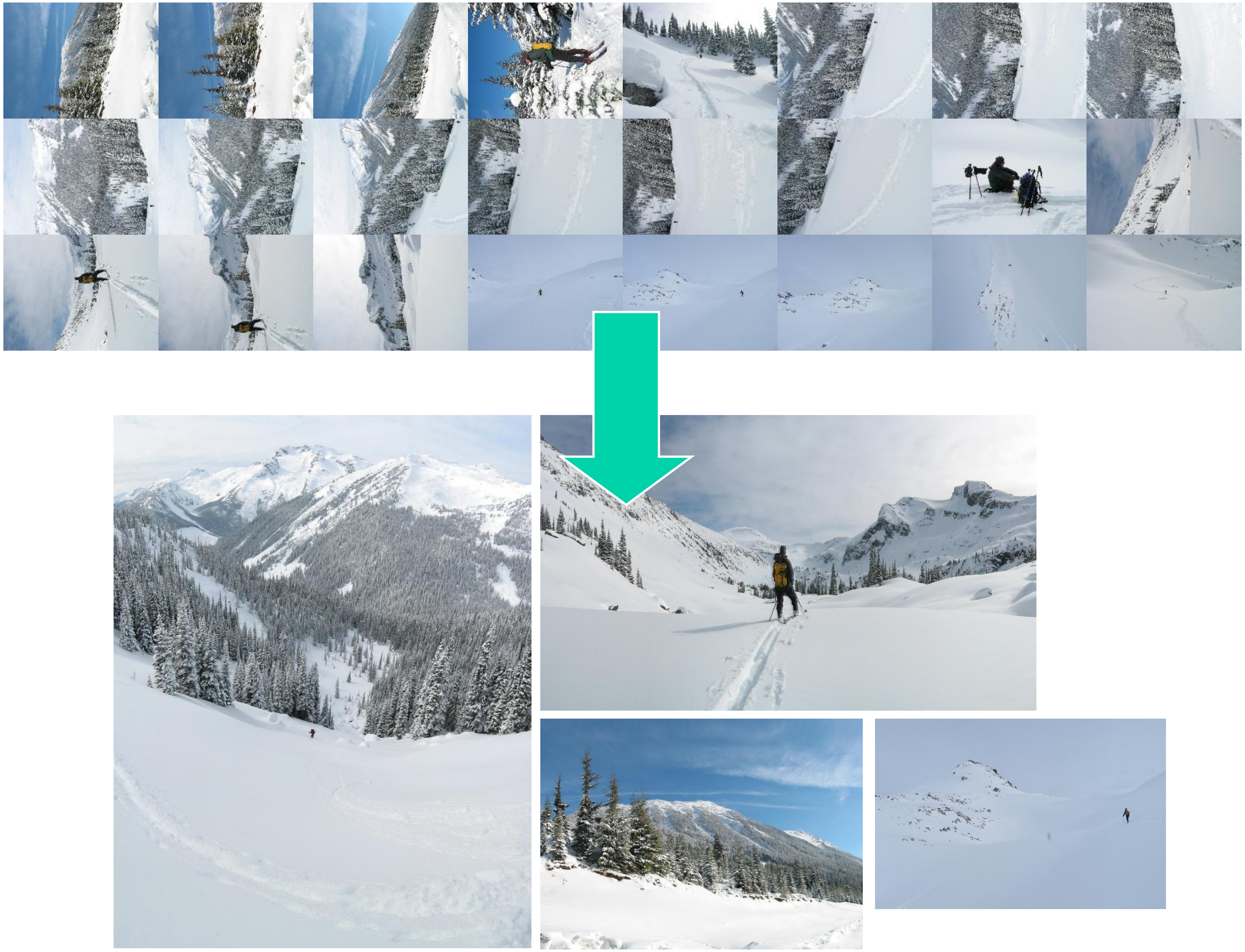
Example: Recognising Panoramas

M. Brown and D. Lowe,

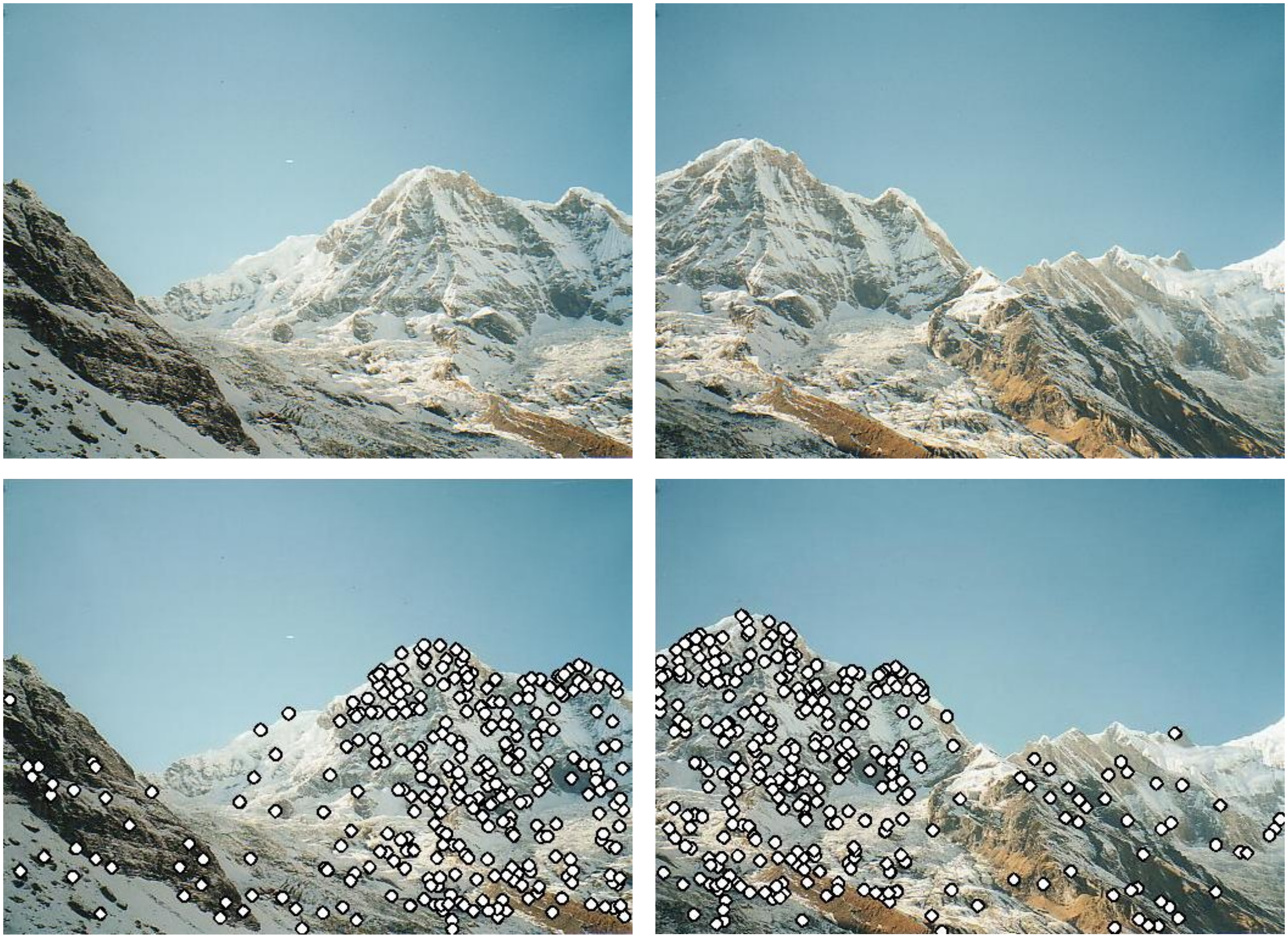
University of British Columbia

* M. Brown and D. Lowe. Automatic Panoramic Image Stitching using Invariant Features. International Journal of Computer Vision, 74(1), pages 59-73, 2007 (pdf 3.5Mb | bib) * M. Brown and D. G. Lowe. Recognising Panoramas. In Proceedings of the 9th International Conference on Computer Vision (ICCV2003), pages 1218-1225, Nice, France, 2003 (pdf 820kb | ppt | bib)

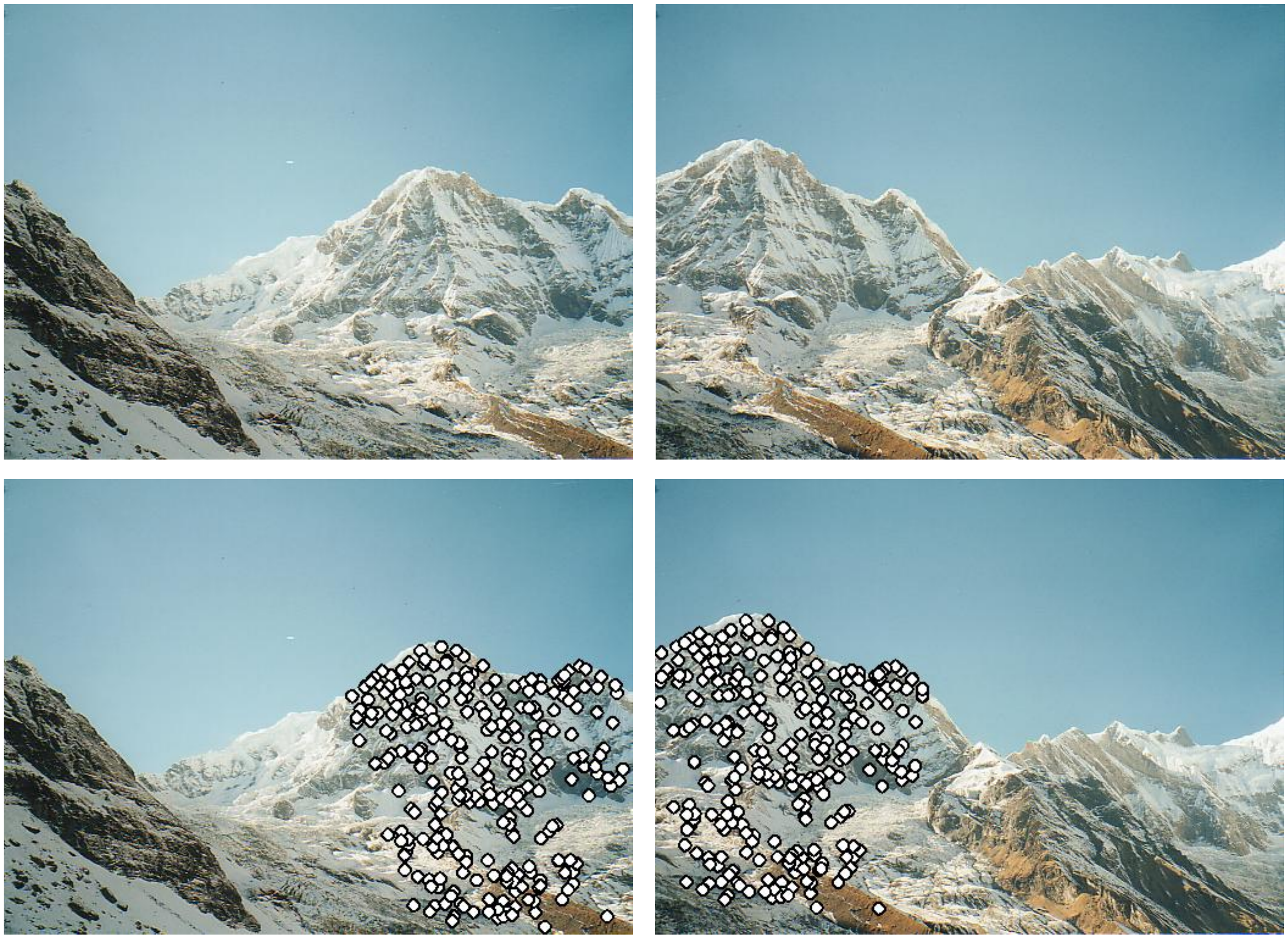
“Recognising Panoramas”?



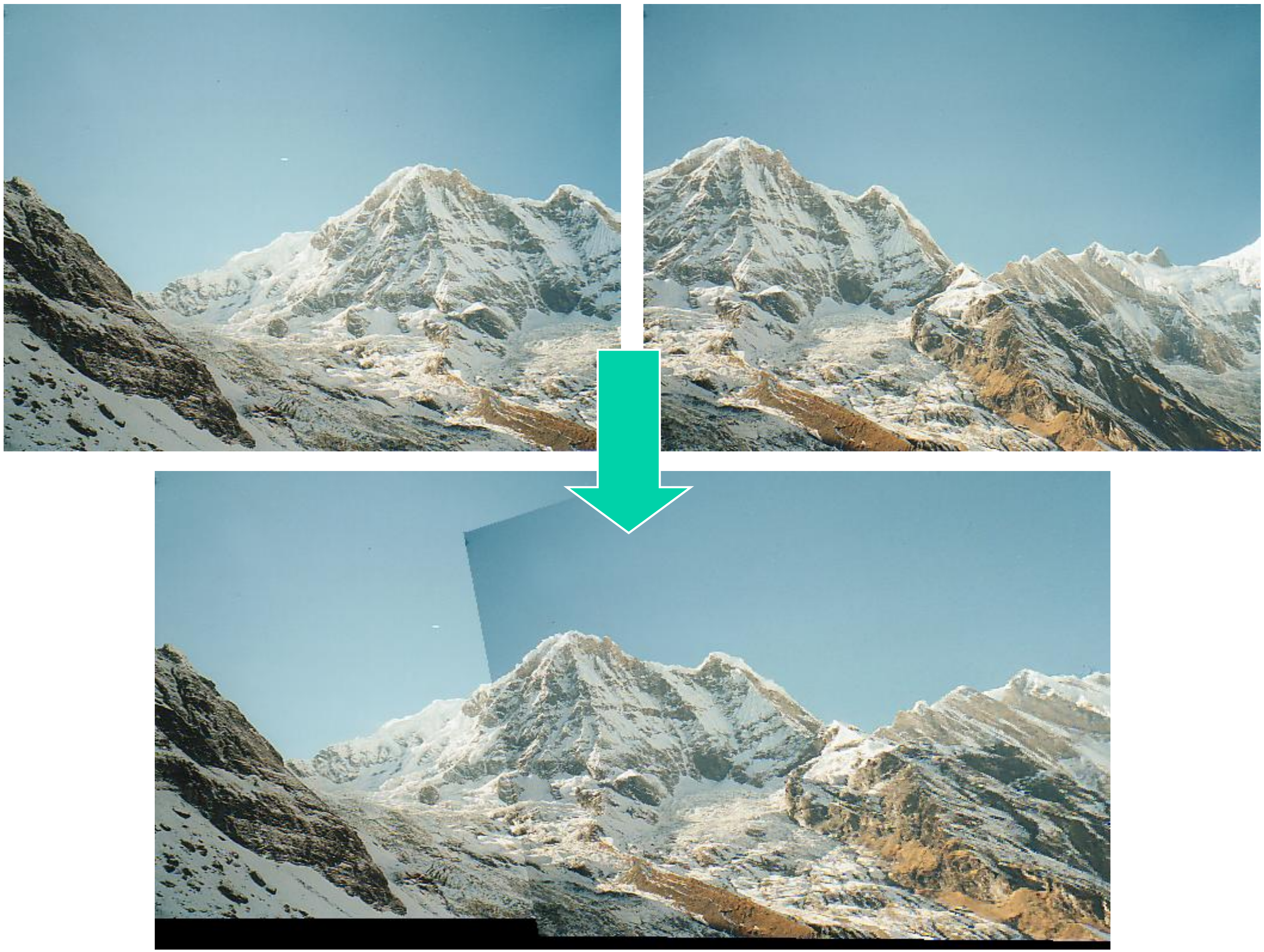
RANSAC for Homography



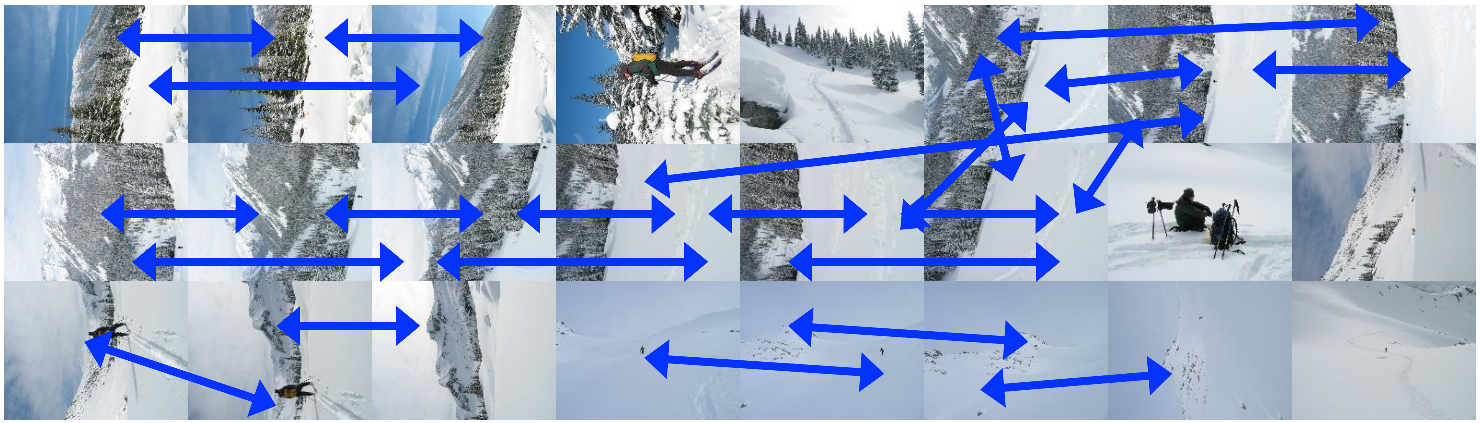
RANSAC for Homography



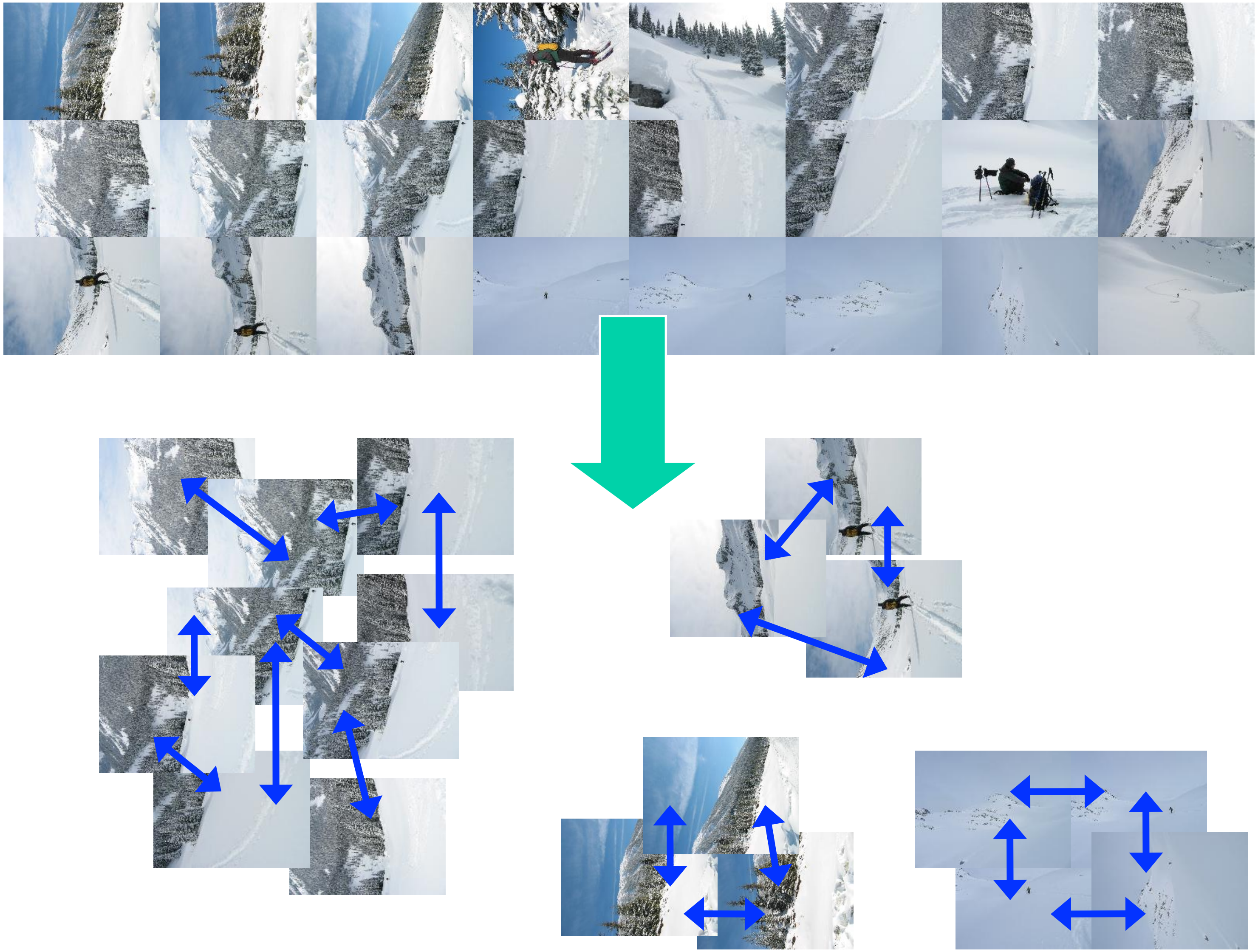
RANSAC for Homography



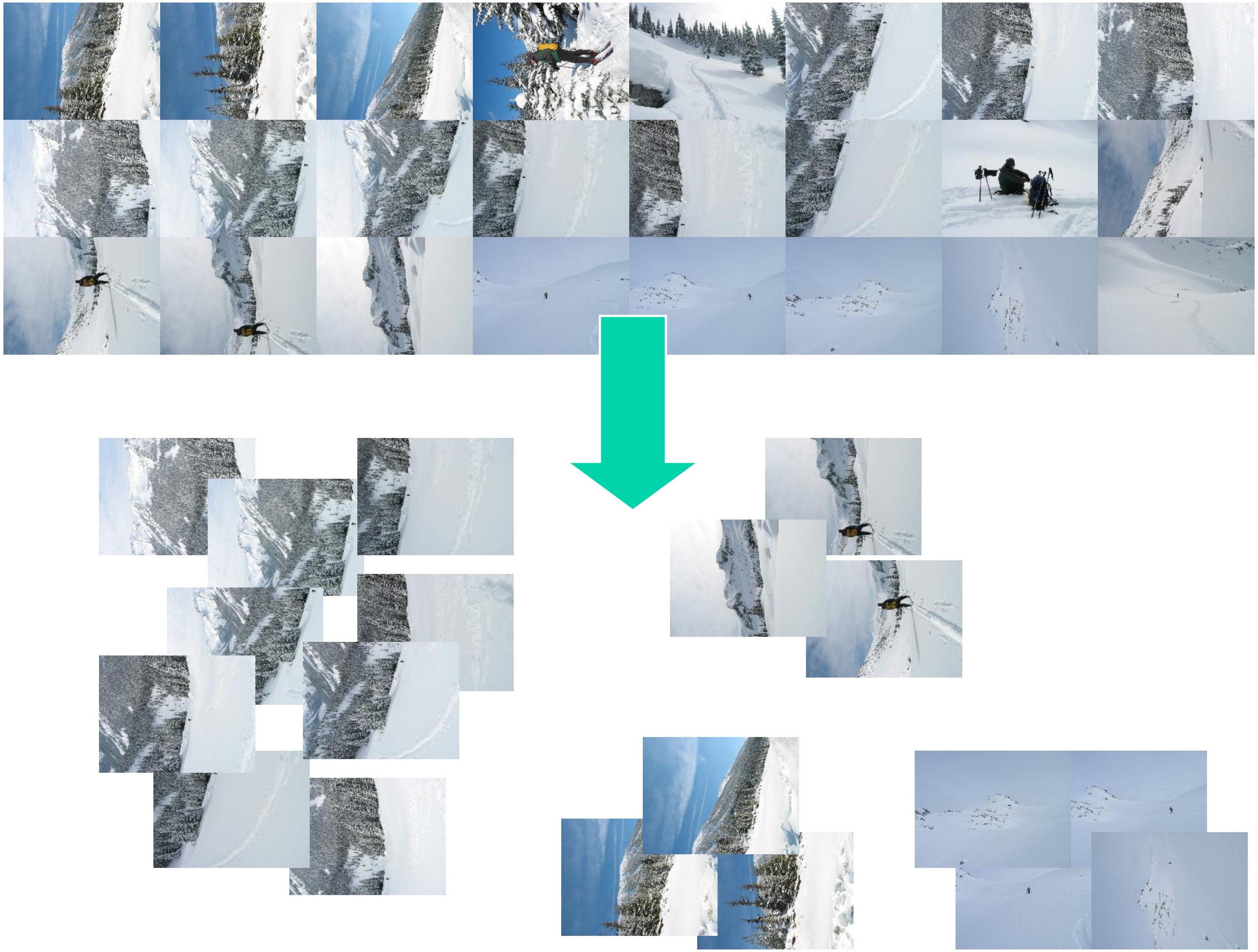
Finding the panoramas



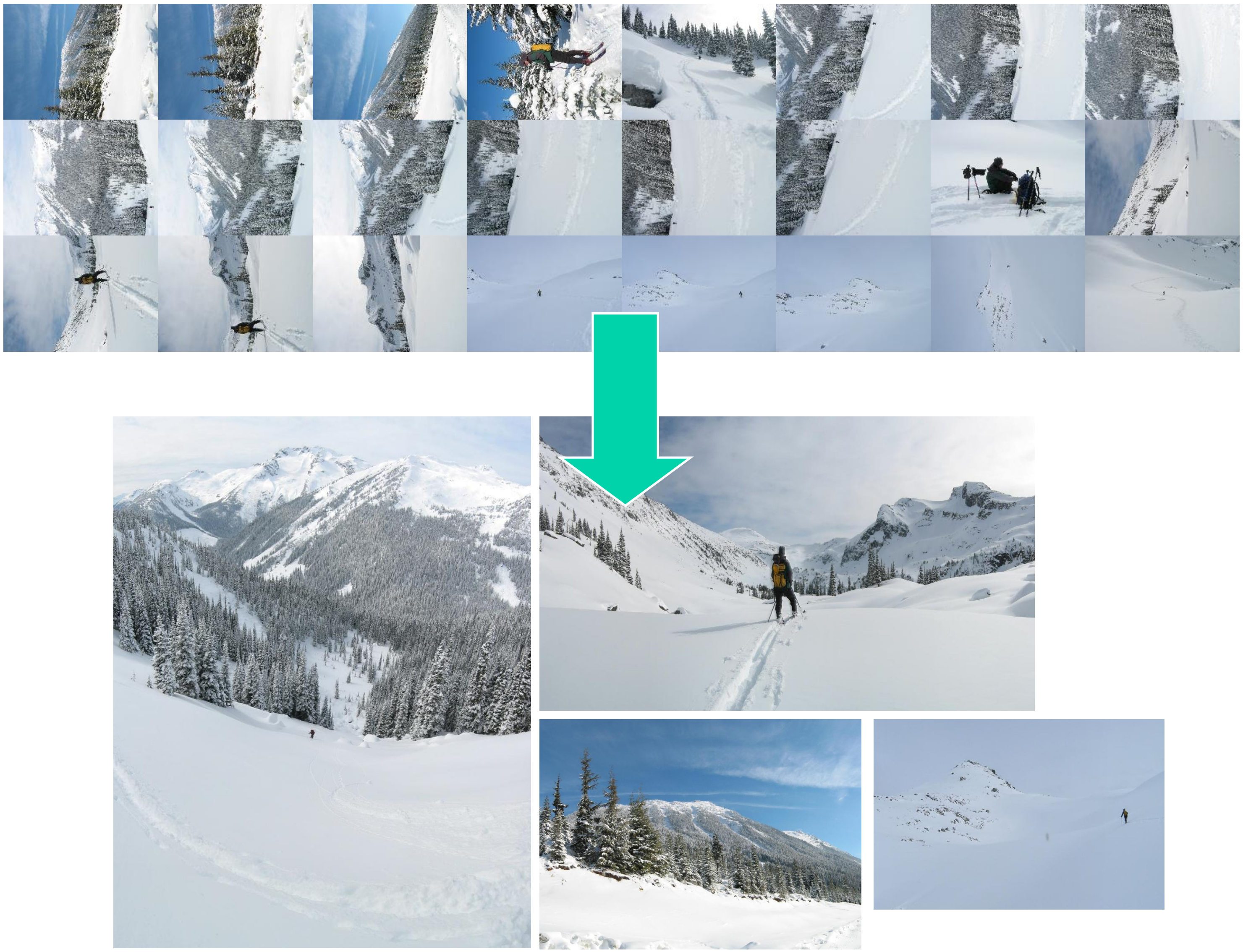
Finding the panoramas



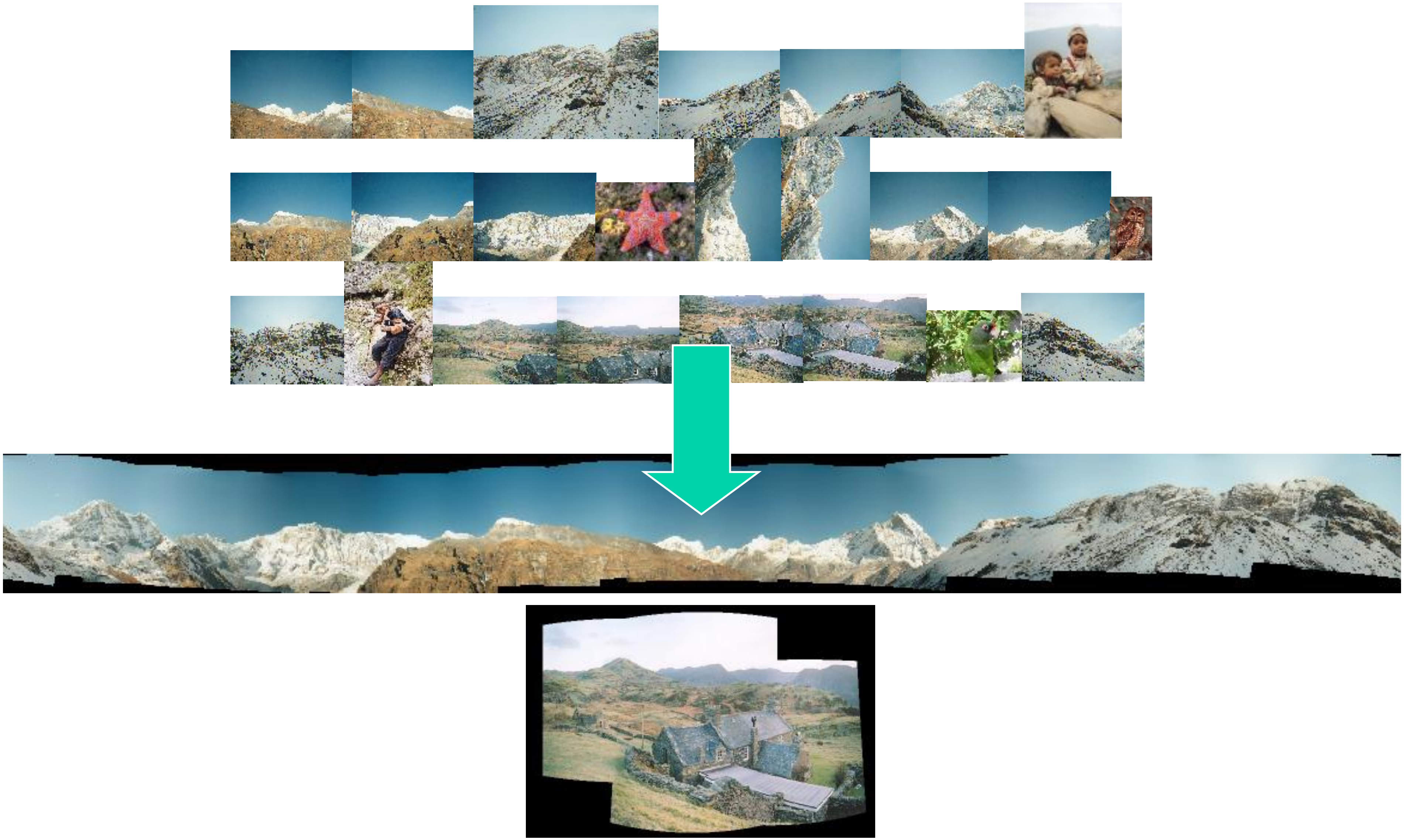
Finding the panoramas



Finding the panoramas



Results



AUTOSTITCH

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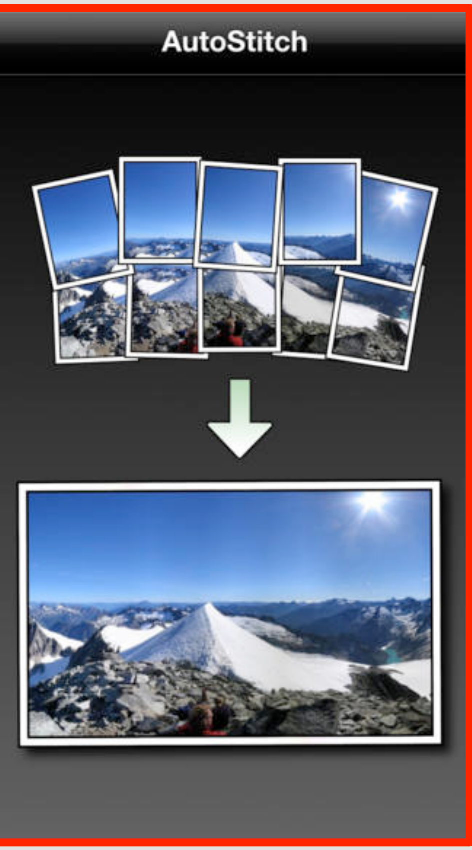
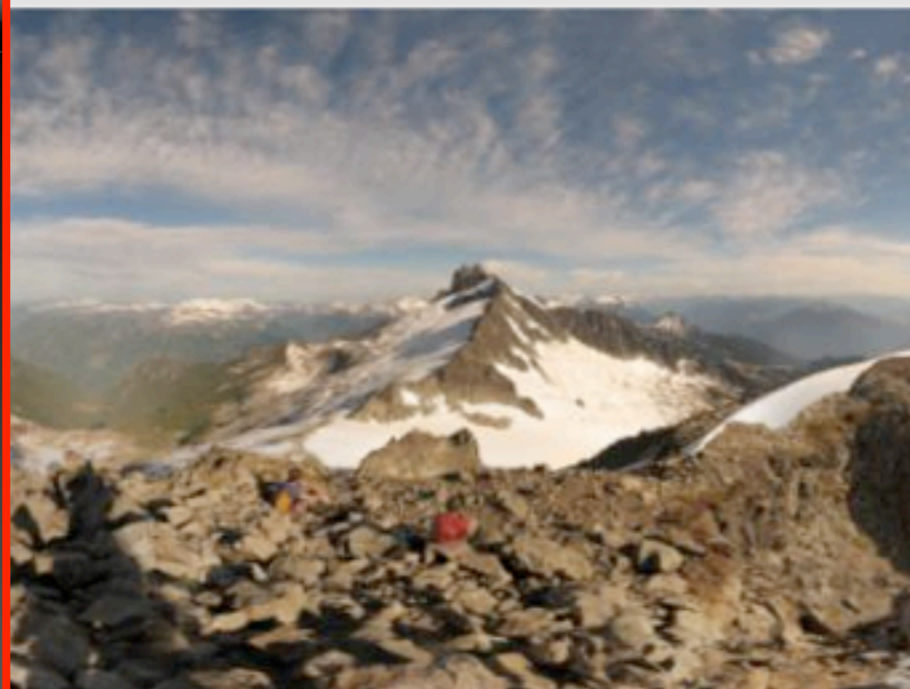
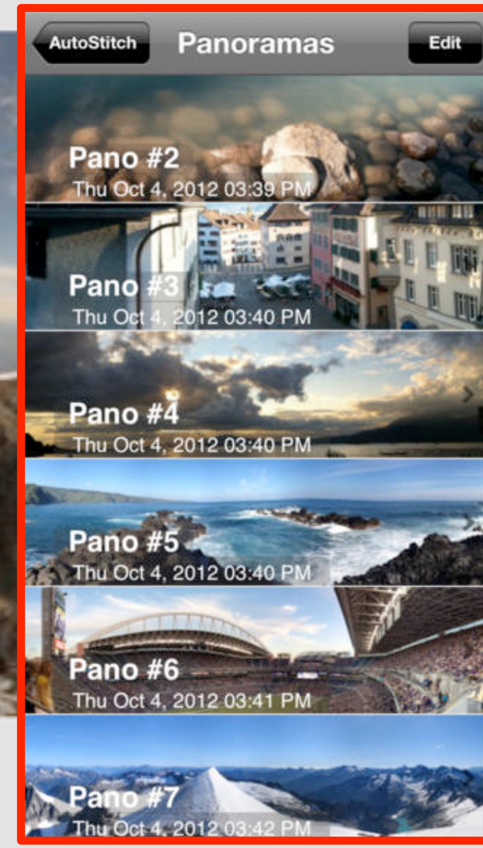
AutoStitch :: a new dimension in automatic image stitching





Serratus



Welcome to AutoStitch. If you have an iPhone, please check out our new iPhone version of AutoStitch below! If you're looking for the Windows demo version, you can download it using the link above, or read on to find out more about AutoStitch. Thanks for visiting!

Benefits of Laplacian image compositing

(a) Linear blending

(b) Multi-band blending

Figure 7. Comparison of linear and multi-band blending. The image on the right was blended using multi-band blending using 5 bands and $\sigma = 5$ pixels. The image on the left was linearly blended. In this case matches on the moving person have caused small misregistrations between the images, which cause blurring in the linearly blended result, but the multi-band blended image is clear.

M. Brown and D. Lowe. Automatic Panoramic Image Stitching using Invariant Features. International Journal of Computer Vision, 74(1), pages 59-73, 2007

Photo Tourism:
Exploring Photo Collections in 3D

Noah Snavely

Steven M. Seitz

University of Washington

Richard Szeliski

Microsoft Research

15,464

37,383

76,389

Photo Tourism

Exploring photo collections in 3D

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University of Washington *Microsoft Research*

SIGGRAPH 2006

Rendering

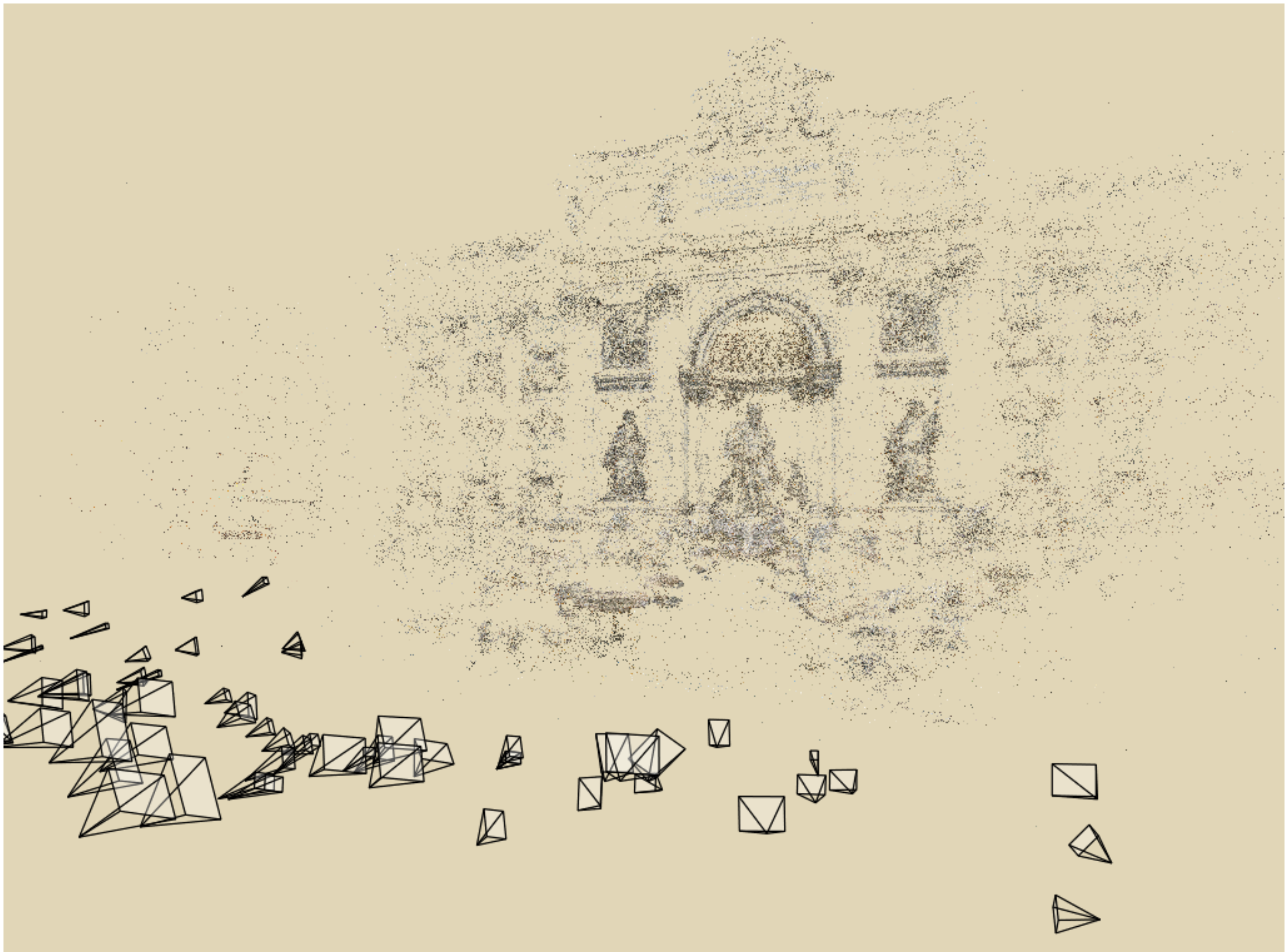
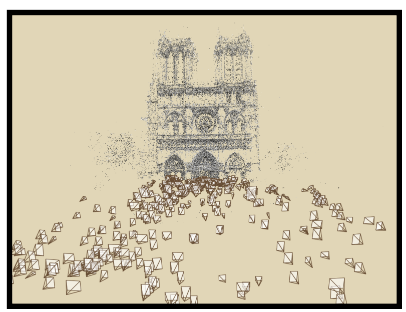


Photo Tourism overview

Input photographs

Scene reconstruction



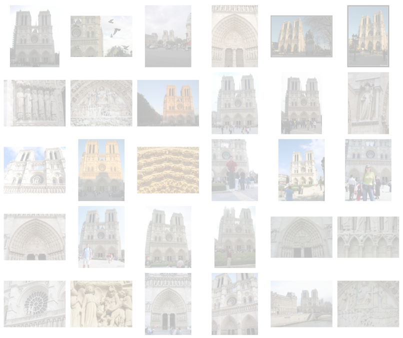
Relative camera positions and orientations

Point cloud

Sparse correspondence

Photo Explorer

Photo Tourism overview



Input photographs

Scene reconstruction

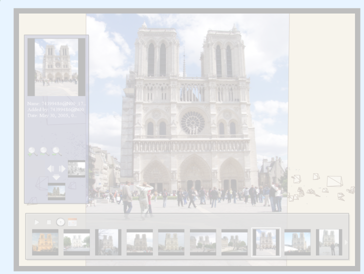
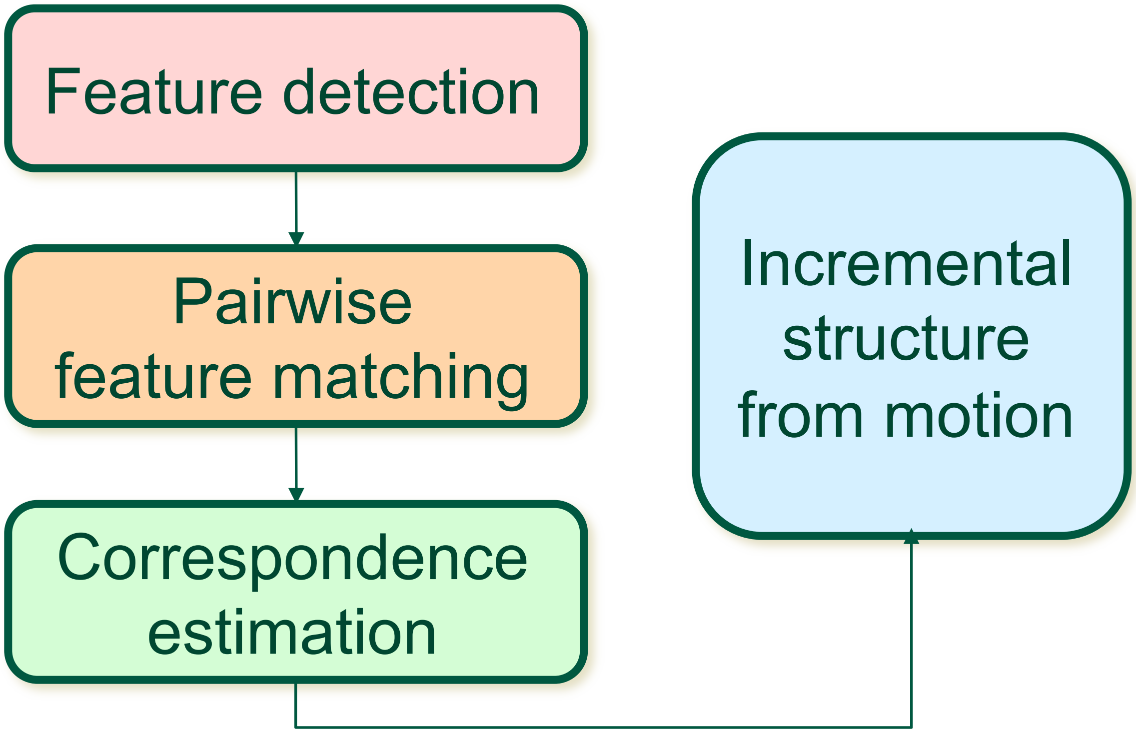


Photo Explorer

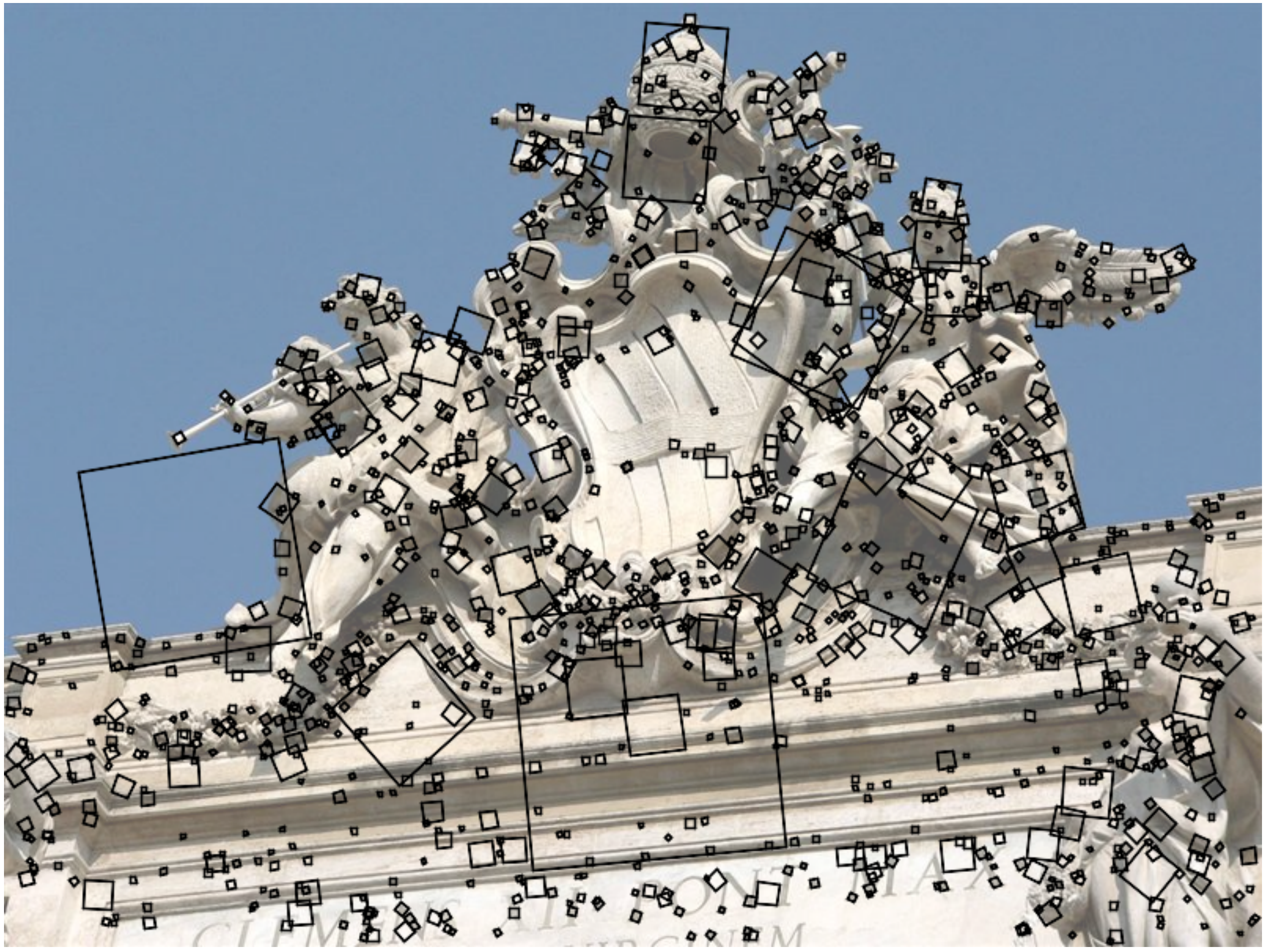
Scene reconstruction

- Automatically estimate
  - position, orientation, and focal length of cameras
  - 3D positions of feature points



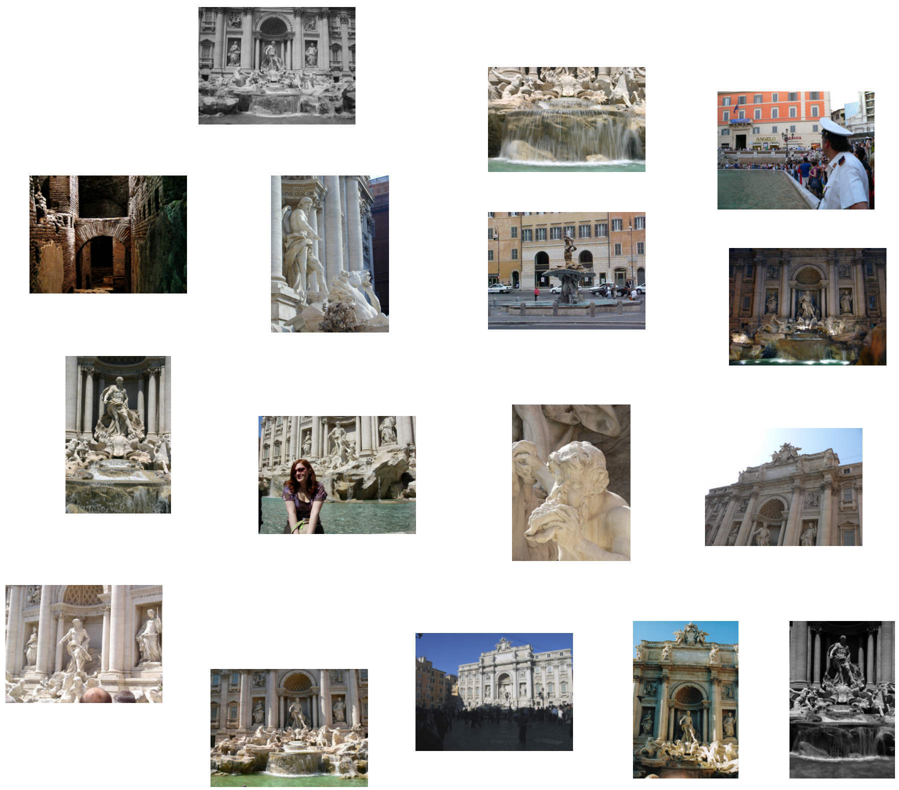
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



Feature detection

Detect features using SIFT [Lowe, IJCV 2004]

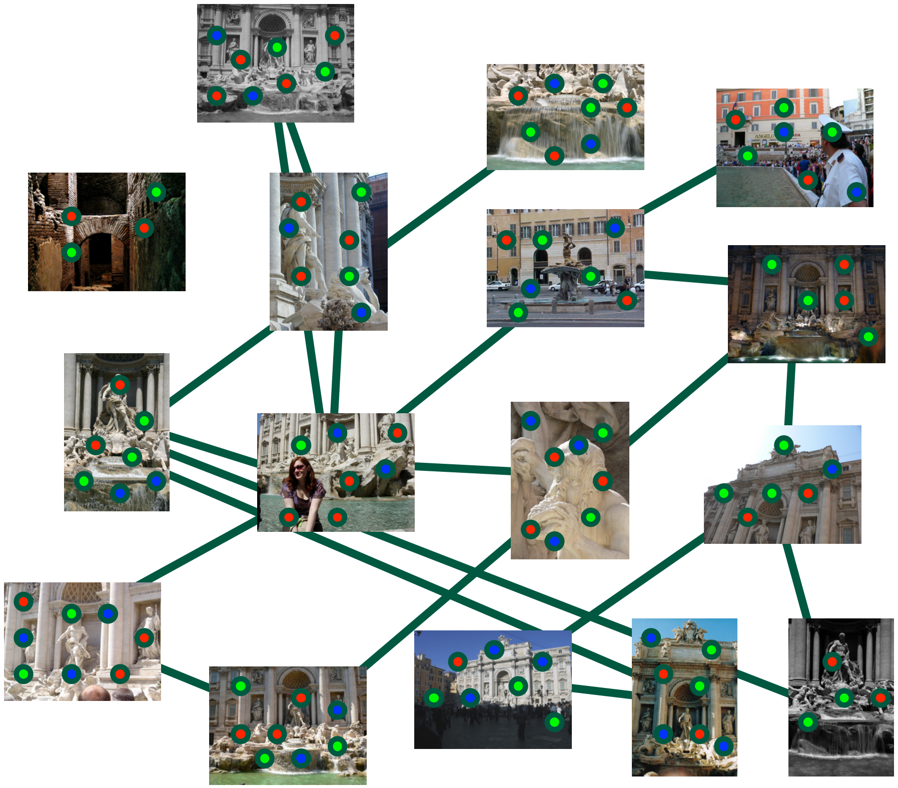


Feature detection

Detect features using SIFT [Lowe, IJCV 2004]

Feature matching

Match features between each pair of images

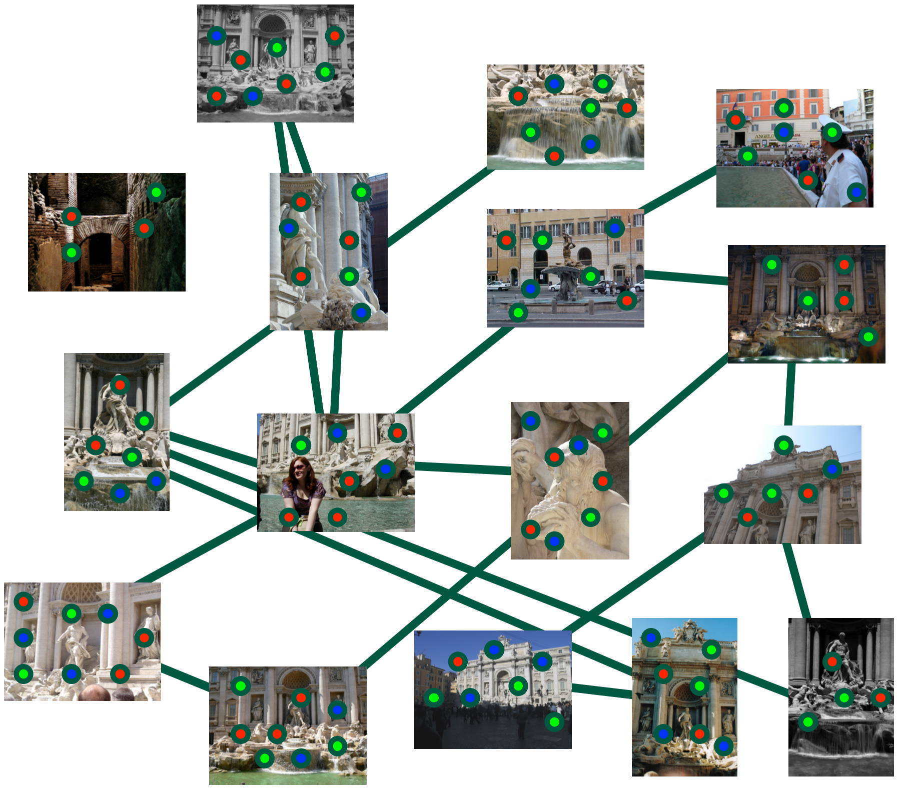


Feature matching

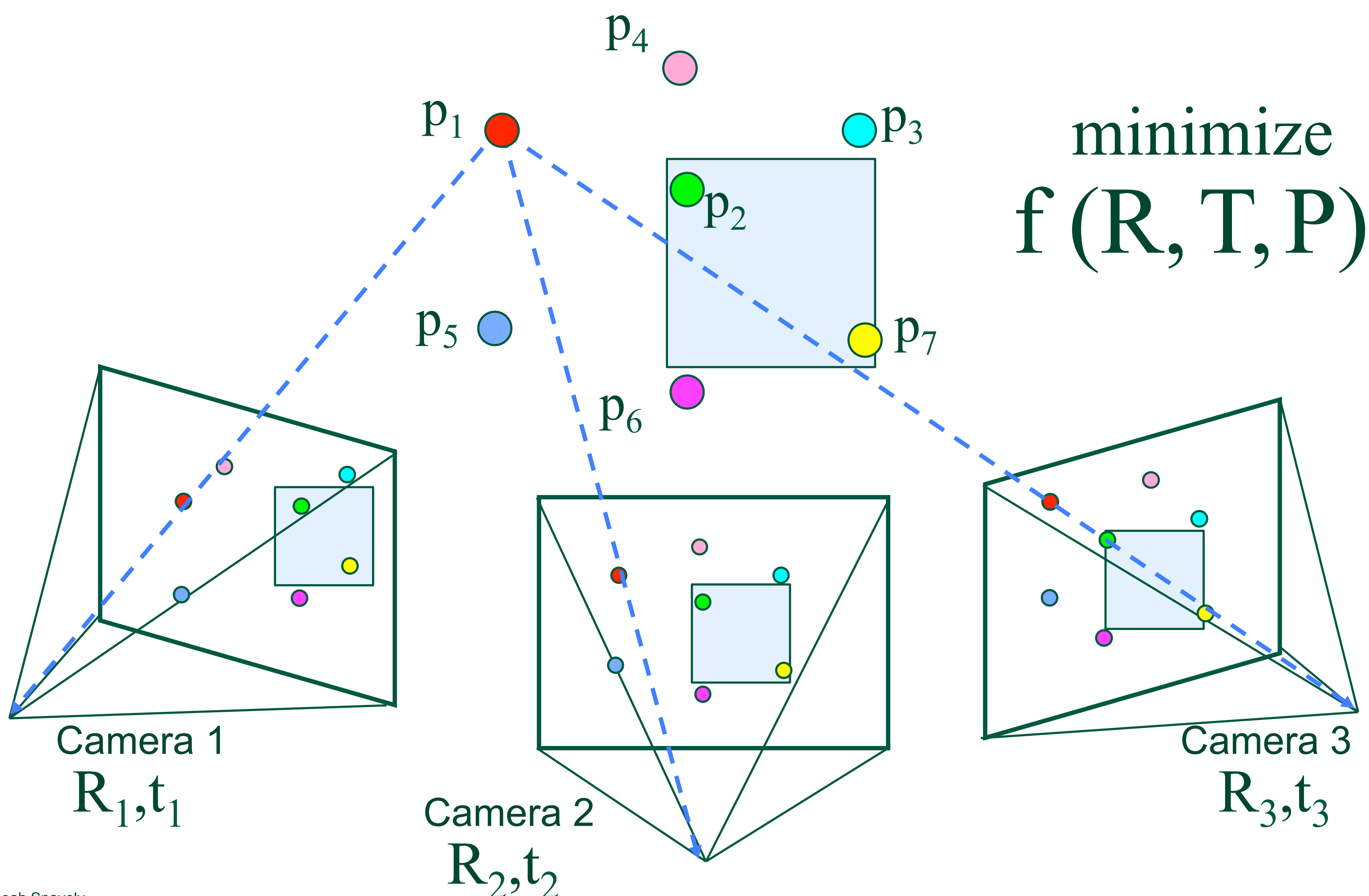
Refine matching using RANSAC [Fischler & Bolles 1987] to estimate fundamental matrices between pairs

(See 6.801/6.866 for fundamental matrix, or Hartley and Zisserman, Multi-View Geometry.

See also the fundamental matrix song: http://danielwedge.com/fmatrix/)



Structure from motion



minimize

$$f(R, T, P)$$

Links

- Code available: http://phototour.cs.washington.edu/bundler/
- http://phototour.cs.washington.edu/
- http://livelabs.com/photosynth/
- http://www.cs.cornell.edu/~snavely/