Lecture 11
Image formation
Cameras and lenses

- Occlusion-based imaging
- Lens-based imaging
- Projection equations
The structure of ambient light
The structure of ambient light
The intensity $P$ can be parameterized as:

$$P (\theta, \phi, \lambda, t, X, Y, Z)$$

Adelson & Bergen, 91
Why don’t we generate an image when an object is in front of a white piece of paper?

Why is there no picture appearing on the paper?
Let’s check, do we get an image?
Let’s check, do we get an image? No
Measuring the light in the world

The camera obscura
The pinhole camera
Let’s try putting different occluders in between the scene and the sensor plane.
Let’s try putting different occluders in between
light on wall past pinhole
grocery bag pinhole camera
grocery bag pinhole camera

Put grocery bags over your head, outside in sun, with a grown up watching you.
grocery bag pinhole camera

view from outside the bag
http://www.youtube.com/watch?v=FZyCFxsyx8o

view from inside the bag
http://youtu.be/-rhZaAM3F44
Problem Set 6

http://www.foundphotography.com/PhotoThoughts/archives/2005/04/pinhole_camera_2.html
Problem Set 6
Effect of pinhole size

Wandell, Foundations of Vision, Sinauer, 1995
2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruerhardt, 1958.
Object size decreases with distance to the pinhole
There, given a single projection, if we know the size of the object we can know how far it is.
But for objects of unknown size, the 3D information seems to be lost.
Playing with pinholes
Two pinholes
What is the minimal distance between the two projected images?
Anaglyph pinhole camera
Anaglyph pinhole camera
Anaglyph pinhole camera
Synthesis of new views

Anaglyph
Problem set

• Build the device
• Take some pictures and put them in the report
• Take anaglyph images
• Work out the geometry
• Recover depth for some points in the image
Straw camera
Accidental pinhole and pinspeck cameras: Revealing the scene outside the picture

Antonio Torralba
William T. Freeman

See project page for videos:
http://people.csail.mit.edu/torralba/research/accidentalcameras/
Shadows?
Accidental pinhole camera
Window turned into a pinhole

View outside
Accidental pinhole camera

Outside scene

Anti-pinhole imaging

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Abstract. By complementing a pinhole to produce an isolated opaque spot, the light ordinarily blocked from the pinhole image is transmitted, and the light ordinarily transmitted is blocked. A negative geometrical image is formed, distinct from the familiar ‘bright-spot’ diffraction image. Anti-pinhole, or ‘pinspeck’ images are visible during a solar eclipse, when the shadows of objects appear crescent-shaped. Pinspecks demonstrate unlimited depth of field, freedom from distortion and large angular field. Images of different magnification may be formed simultaneously. Contrast is poor, but is improvable by averaging to remove noise and subtraction of a d.c. bias. Pinspecks may have application in X-ray space optics, and might be employed in the eyes of simple organisms.
Pinhole and Anti-pinhole cameras

Adam L. Cohen, 1982
To see the effect of pinspeck cameras, need to subtract the no-pinspeck image.
Shadows
Accidental anti-pinhole cameras
Background image

Input video

\[ \text{Negative of the shadow} \]
Background image - Input video = Negative of the shadow
Input video

Negative of the shadow
Inverted difference image

View behind the ball
The importance of the size of the occluder

Input video

Negative of the shadow
Size of the occluder

Antonio  Ball
Another occlusion-based camera: edge camera

show demo
Model of corner camera signal

Figure 3: Calculation of the contribution to the brightness of the target.
Corner Camera 1-D Image

Rectified Image

Kalman Gain Images
Experiment Proof of Concept
Experimental Proof of Concept
Experimental Proof of Concept
Experimental Proof of Concept
1-D Corner Camera Output

- How many people?
- Where slowed down, where moved quickly?
1-D Corner Camera Output

- How many people?
- How fast is each person moving?
Video Corresponding to 1-D Camera
More Corner Camera Videos

1 Person Walking in Circles

1 Person Randomly Walking

2 People Walking in Circles
Additional Results

Paper ID: 1983
A puzzle: light intensity goes down as $1/r^2$, where $r$ is the distance away. So then why does a long, uniform surface, like a road or a sidewalk, seem to have the same brightness, whether it’s nearby or far away? The difference in $1/r^2$ in this photo between the ground where the photographer is standing and the ground at the yellow sign is huge!
Influence of aperture size: with a small aperture, the image is sharp, but dim. A large aperture gives a bright, but blurry image.
A lens can focus light from one point in the world to one point on the sensor plane.
Images through large aperture, with and without lens present
Images through large aperture, with and without lens present
Derivation of Snell’s law

\[ \lambda_1 = \frac{c}{\omega n_1} \]  
\[ \lambda_1 = L \sin(\alpha_1) \]  
\[ n_1 \sin(\alpha_1) = \frac{c}{\omega L} = n_2 \sin(\alpha_2) \]

Rearranging the first two equations

\[ n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2) \]

Speed, and thus wavelength of light, scales inversely with \( n \). This requires that plane waves bend, according to the geometry at right.
Refraction: Snell’s law

\[ n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2) \]

For small angles, \( n_1 \alpha_1 \approx n_2 \alpha_2 \)
Spherical lens
For a spherical lens surface, we can define the relevant angles, apply Snell’s law, and find an expression telling how the lens focusses light.
That is easiest to do under the assumptions of “first order optics”: small bending angles, and a thin lens

\[ \sin(\theta) \approx \theta \]

\[ \theta \approx \frac{D/2}{f} \]
Paraxial refraction equation

\[ \alpha_1 = \gamma + \beta_1 \approx h \left( \frac{1}{R} + \frac{1}{d_1} \right) \]

Relates distance from sphere and sphere radius to bending angles of light ray, for lens with one spherical surface.

\[ n_1 \alpha_1 \approx n_2 \alpha_2 \]
Deriving the lensmaker’s formula

\[ \alpha_1 = h \left( \frac{1}{R} + \frac{1}{d_1} \right) \quad \text{small angle approx.} \]

\[ n_1 \alpha_1 \approx n_2 \alpha_2 \quad \text{Snell's law} \]

\[ \alpha_2 = 2 \gamma - \alpha_3 \quad \text{geometry} \]

\[ n_2 \alpha_3 \approx n_1 \alpha_4 \quad \text{Snell's law} \]

\[ \alpha_4 = h_1 \left( \frac{1}{R} + \frac{1}{d_2} \right) \quad \text{small angle} \]

\[ \gamma = \frac{h}{R} \]

\[ n_1 \alpha_1 = n_2 \left( \frac{2h}{R} \right) \left( \frac{n}{n_2} - \frac{1}{n} \alpha_4 \right) = h \left( \frac{1}{R} + \frac{1}{d_1} \right) \]

\[ \text{let } n_1 = 1, \ n_2 = n \]

\[ n \left( \frac{2}{R} - \frac{1}{n} \left( \frac{1}{R} + \frac{1}{d_2} \right) \right) = \frac{1}{R} + \frac{1}{d_1} \]

\[ \frac{2n}{R} - \frac{1}{R} - \frac{1}{d_2} = \frac{1}{R} + \frac{1}{d_1} \]

\[ 2 \frac{n}{R} - \frac{1}{R} = \frac{1}{d_1} + \frac{1}{d_2} \]

\[ 2(n-1) = \frac{1}{d_1} + \frac{1}{d_2} \]

“Lens maker’s formula”
The thin lens, first order optics

The lensmaker’s equation:

\[ \frac{1}{z'} + \frac{1}{z} = \frac{1}{f} \]

\[ f = \frac{R}{2(n-1)} \]

Forsyth & Ponce
How does the mapping from the 3-d world to the image plane compare for a lens and for a pinhole camera?

The perspective projection of a pinhole camera. But note that many more of the rays leaving from $P$ arrive at $P'$. 
Perspective projection

Cartesian coordinates:

We have, by similar triangles, that

\[(x, y, z) \rightarrow (dx/z, dy/z, df)\]

Ignore the third coordinate, and get

\[(x, y, z) \rightarrow (d \frac{x}{z}, d \frac{y}{z})\]
Lens demonstration

- Verify:
  - Focusing property
  - Lens maker’s equation
  - The relationship between distances in the world and distances in the sensor plane
Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image or half-planes.
- Polygons go to polygons

Degenerate cases
- line through focal point to point
- plane through focal point to line
Vanishing point for this 3-d line
Line in 3-space

\[ x(t) = x_0 + at \]
\[ y(t) = y_0 + bt \]
\[ z(t) = z_0 + ct \]

Perspective projection of that line

\[ x'(t) = \frac{fx}{z} = \frac{f(x_0 + at)}{z_0 + ct} \]
\[ y'(t) = \frac{fy}{z} = \frac{f(y_0 + bt)}{z_0 + ct} \]

In the limit as \( t \rightarrow \pm \infty \) we have (for \( c \neq 0 \)):

\[ x'(t) \rightarrow \frac{fa}{c} \]
\[ y'(t) \rightarrow \frac{fb}{c} \]

This tells us that any set of parallel lines (same \( a, b, c \) parameters) project to the same point (called the vanishing point).
Vanishing points

• Each set of parallel lines (=direction) meets at a different point
  – The *vanishing point* for this direction

• Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  – The line is called the *horizon* for that plane
What if you photograph a brick wall head-on?
Brick wall line in 3-space

\[ x(t) = x_0 + at \]
\[ y(t) = y_0 \]
\[ z(t) = z_0 \]

Perspective projection of that line

\[ x'(t) = \frac{f \cdot (x_0 + at)}{z_0} \]
\[ y'(t) = \frac{f \cdot y_0}{z_0} \]

All bricks have same \( z_0 \). Those in same row have same \( y_0 \)

Thus, a brick wall, photographed head-on, gets rendered as set of parallel lines in the image plane.
Other projection models: Orthographic projection

\[ (x, y, z) \rightarrow (x, y) \]
Other projection models: Weak perspective

- Issue
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: only approximate

\[(x, y, z) \rightarrow \left( \frac{fx}{z_0}, \frac{fy}{z_0} \right)\]
Three camera projections

(1) Perspective: \((x, y, z) \rightarrow \left( \frac{fx}{z}, \frac{fy}{z} \right)\)

(2) Weak perspective: \((x, y, z) \rightarrow \left( \frac{fx}{z_0}, \frac{fy}{z_0} \right)\)

(3) Orthographic: \((x, y, z) \rightarrow (x, y)\)
Two of those camera projections

Perspective projection

Parallel (orthographic) projection

Weak perspective?
More accurate models of real lenses

- Finite lens thickness
- Higher order approximation to $\sin(\theta)$
- Chromatic aberration
- Vignetting
Thick lens

Figure 1.11  A simple thick lens with two spherical surfaces.
Third order optics

\[ \sin(\theta) \approx \theta - \frac{\theta^3}{6} \]

\[ \theta \approx \frac{D/2}{f} - \frac{\left(\frac{D/2}{f}\right)^3}{6} \]
Paraxial refraction equation, 3rd order optics

\[
\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R} + h^2 \left[ \frac{n_1}{2d_1} \left( \frac{1}{R} + \frac{1}{d_1} \right)^2 + \frac{n_2}{2d_2} \left( \frac{1}{R} - \frac{1}{d_2} \right)^2 \right]
\]
Spherical aberration (from 3\textsuperscript{rd} order optics)
Other 3\textsuperscript{rd} order effects

- Coma, astigmatism, field curvature, distortion.

![Diagram of no distortion, pincushion distortion, and barrel distortion.](Forsyth&Ponce)
Chromatic aberration
(desirable for prisms, bad for lenses)
Other (possibly annoying) phenomena

- Chromatic aberration
  - Light at different wavelengths follows different paths; hence, some wavelengths are defocussed
  - Machines: coat the lens
  - Humans: live with it

- Scattering at the lens surface
  - Some light entering the lens system is reflected off each surface it encounters (Fresnel’s law gives details)
  - Machines: coat the lens, interior
  - Humans: live with it (various scattering phenomena are visible in the human eye)
Summary

- Want to make images
- Pinhole camera models the geometry of perspective projection
- Lenses make it work in practice
- Models for lenses
  - Thin lens, spherical surfaces, first order optics
  - Thick lens, higher-order optics, vignetting.