MIT CSAIL

6.869: Advances in Computer Vision

Bill Freeman and Antonio Torralba, 2017

MIT COMPUTER VISION

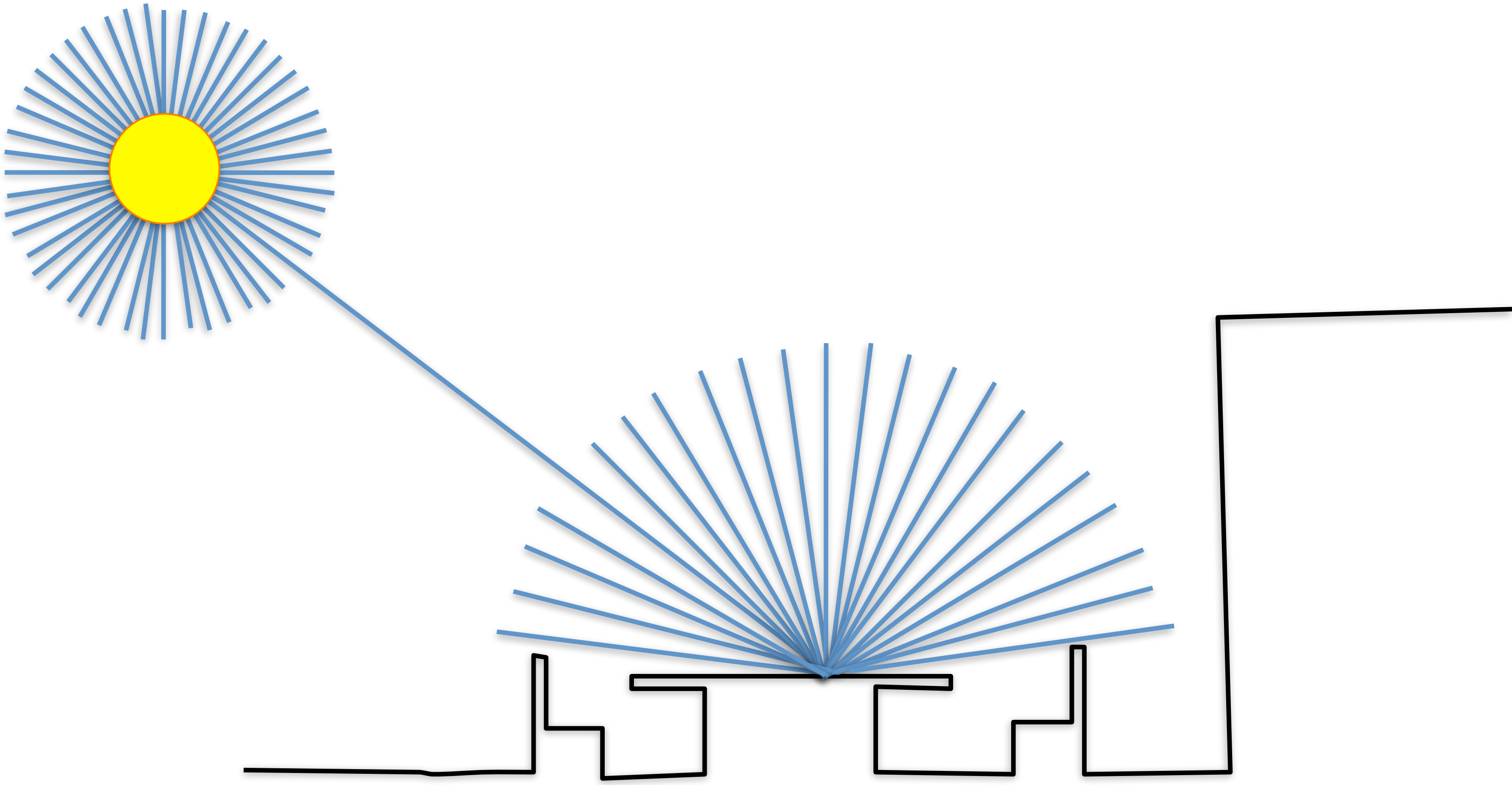
Lecture 11

Image formation

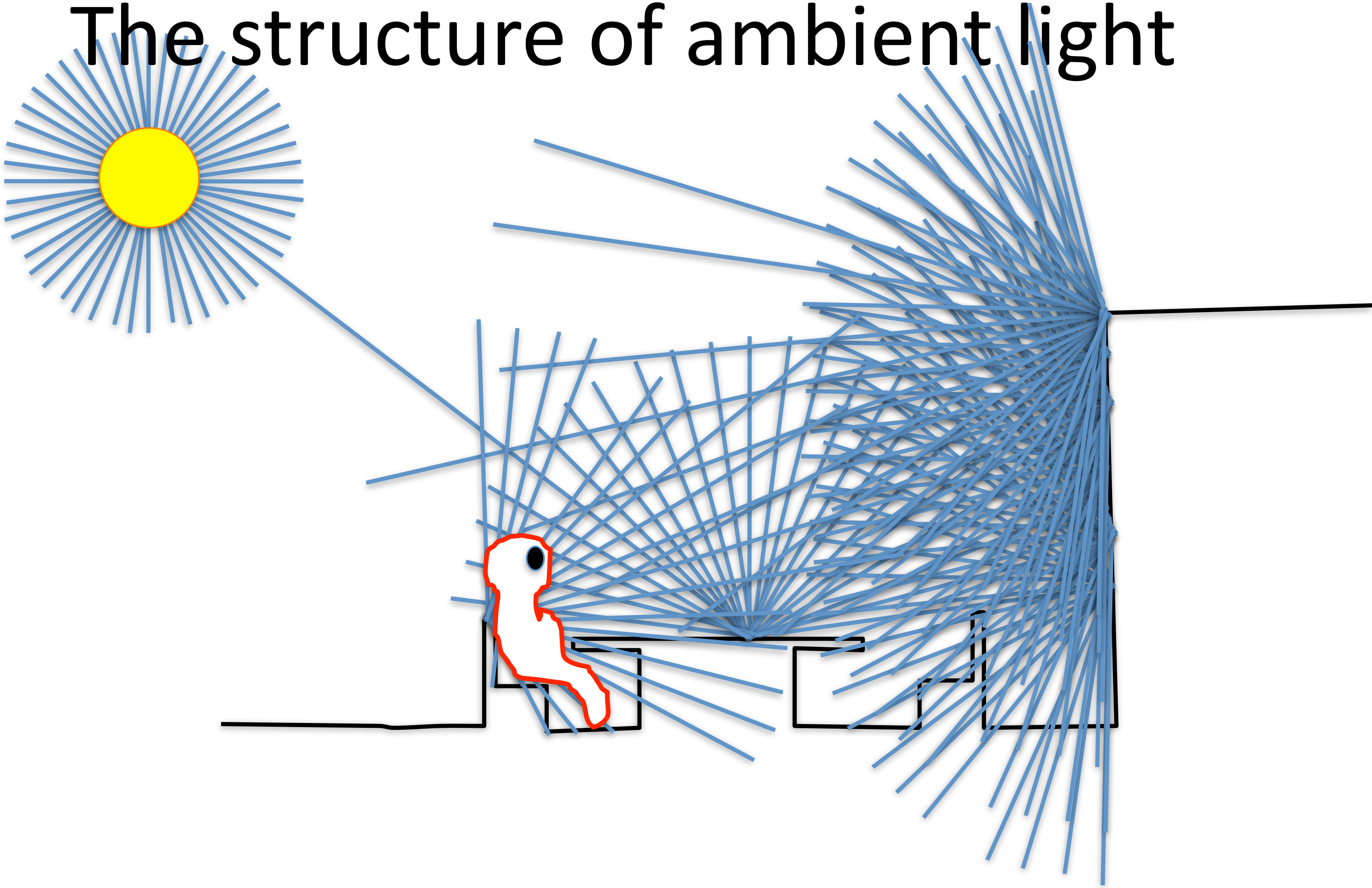
Cameras and lenses

- Occlusion-based imaging
- Lens-based imaging
- Projection equations

The structure of ambient light

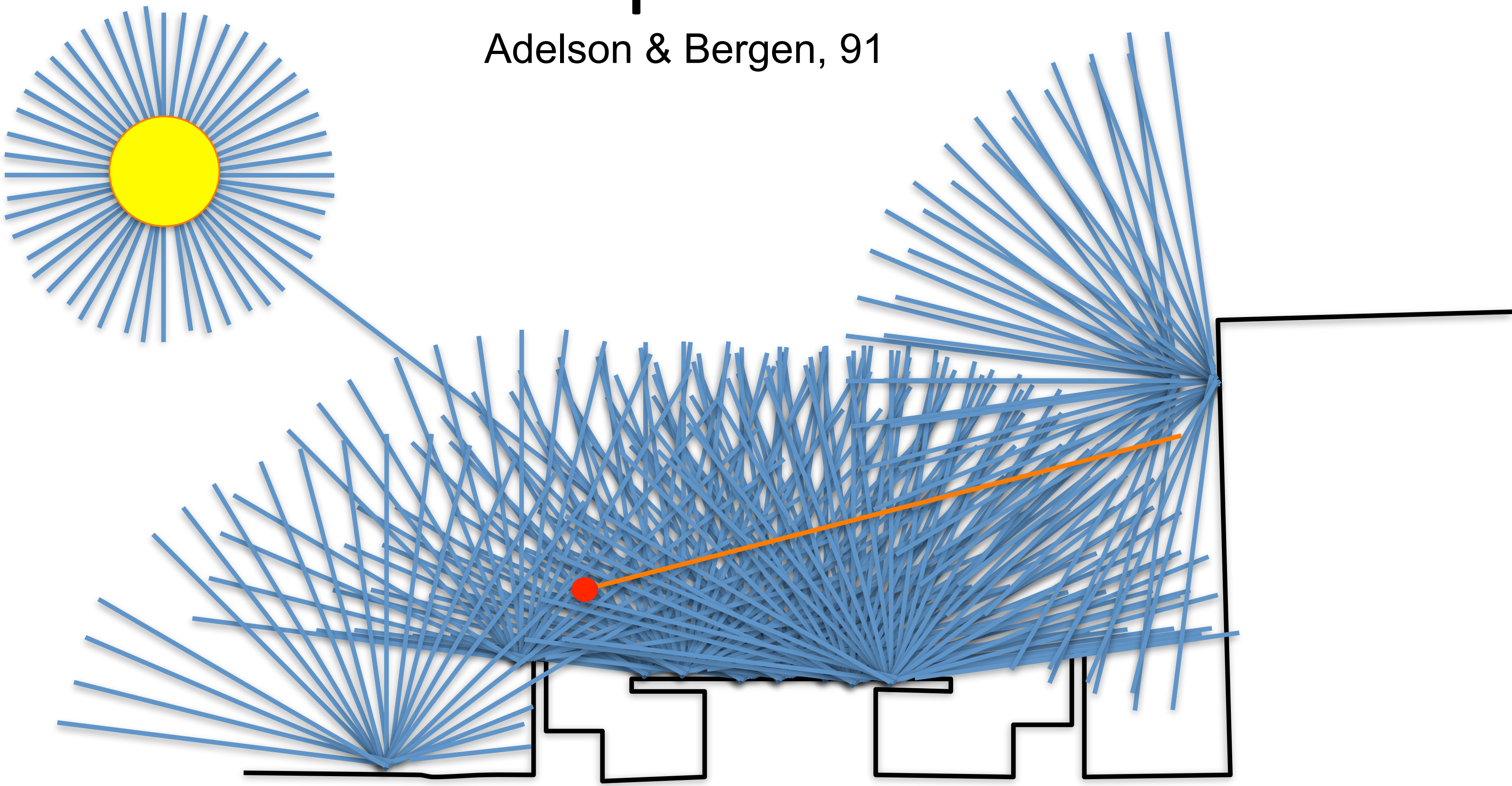


The structure of ambient light



The Plenoptic Function

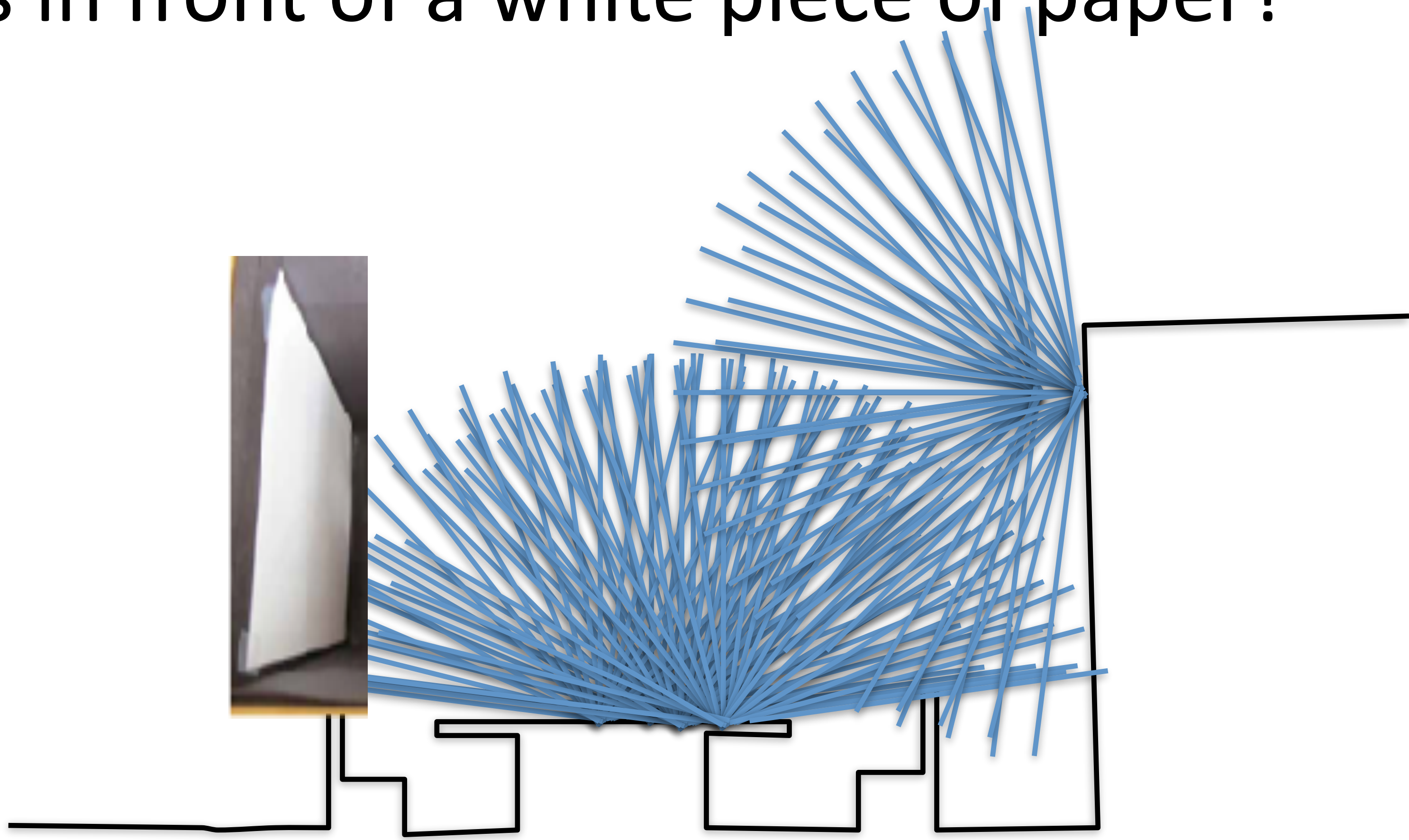
Adelson & Bergen, 91



The intensity P can be parameterized as:

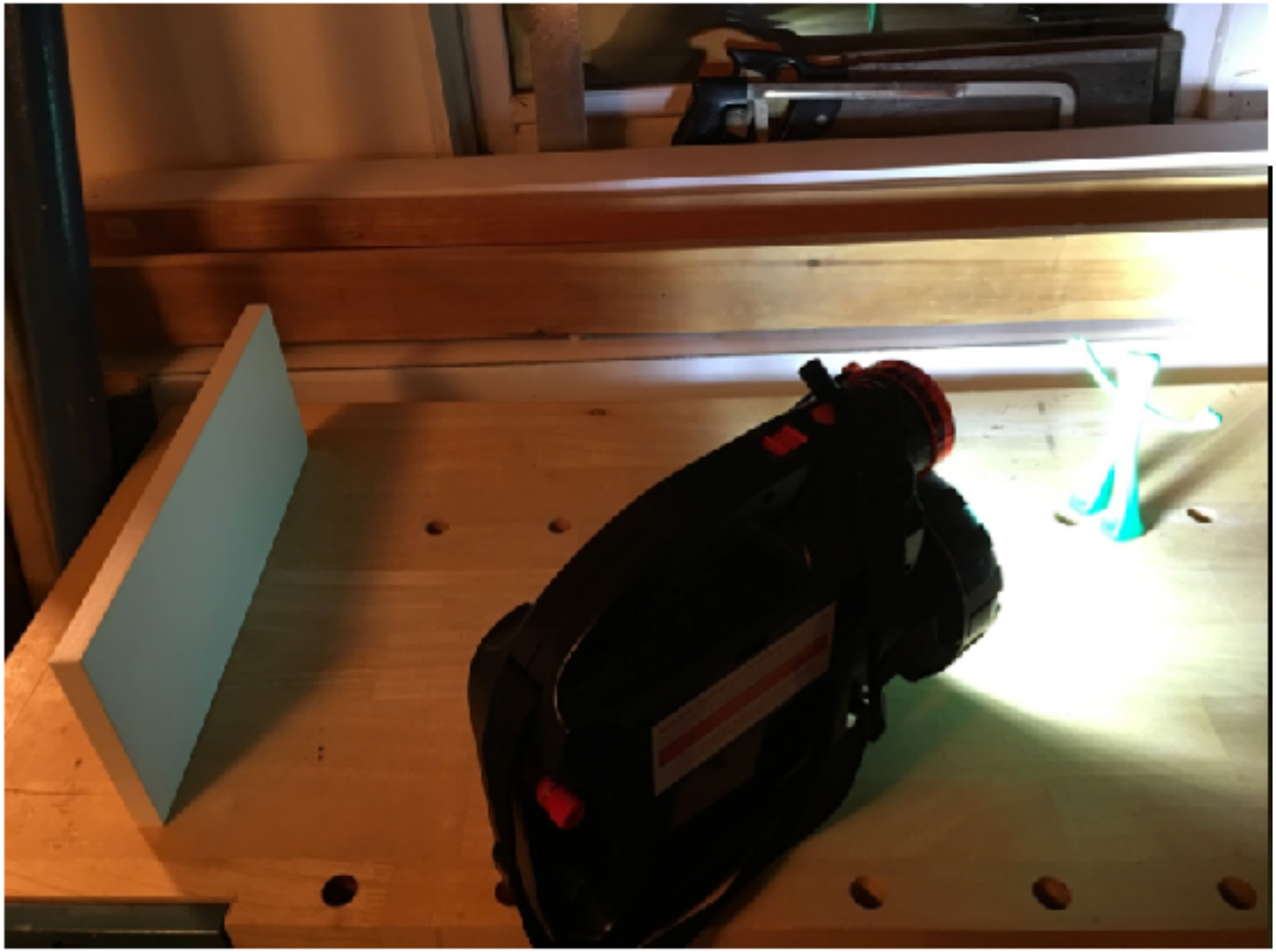
$$P(\theta, \phi, \lambda, t, X, Y, Z)$$

Why don't we generate an image when an object is in front of a white piece of paper?

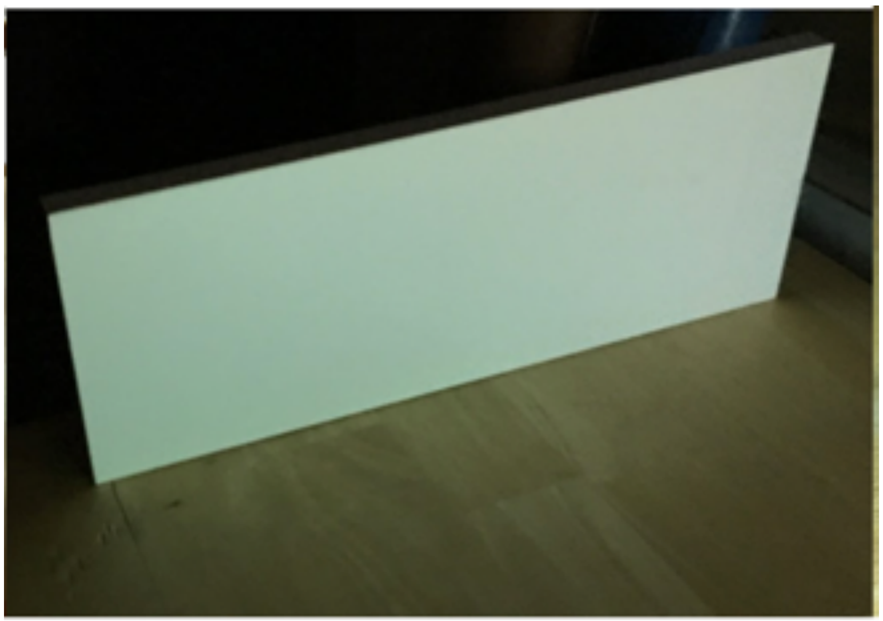


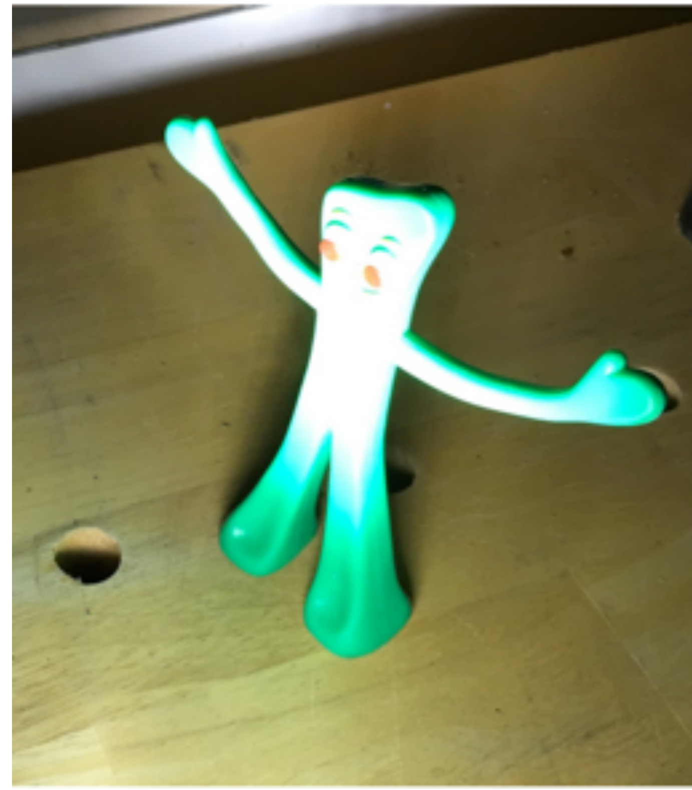
Why is there no picture appearing on the paper?

Let’s check, do we get an image?



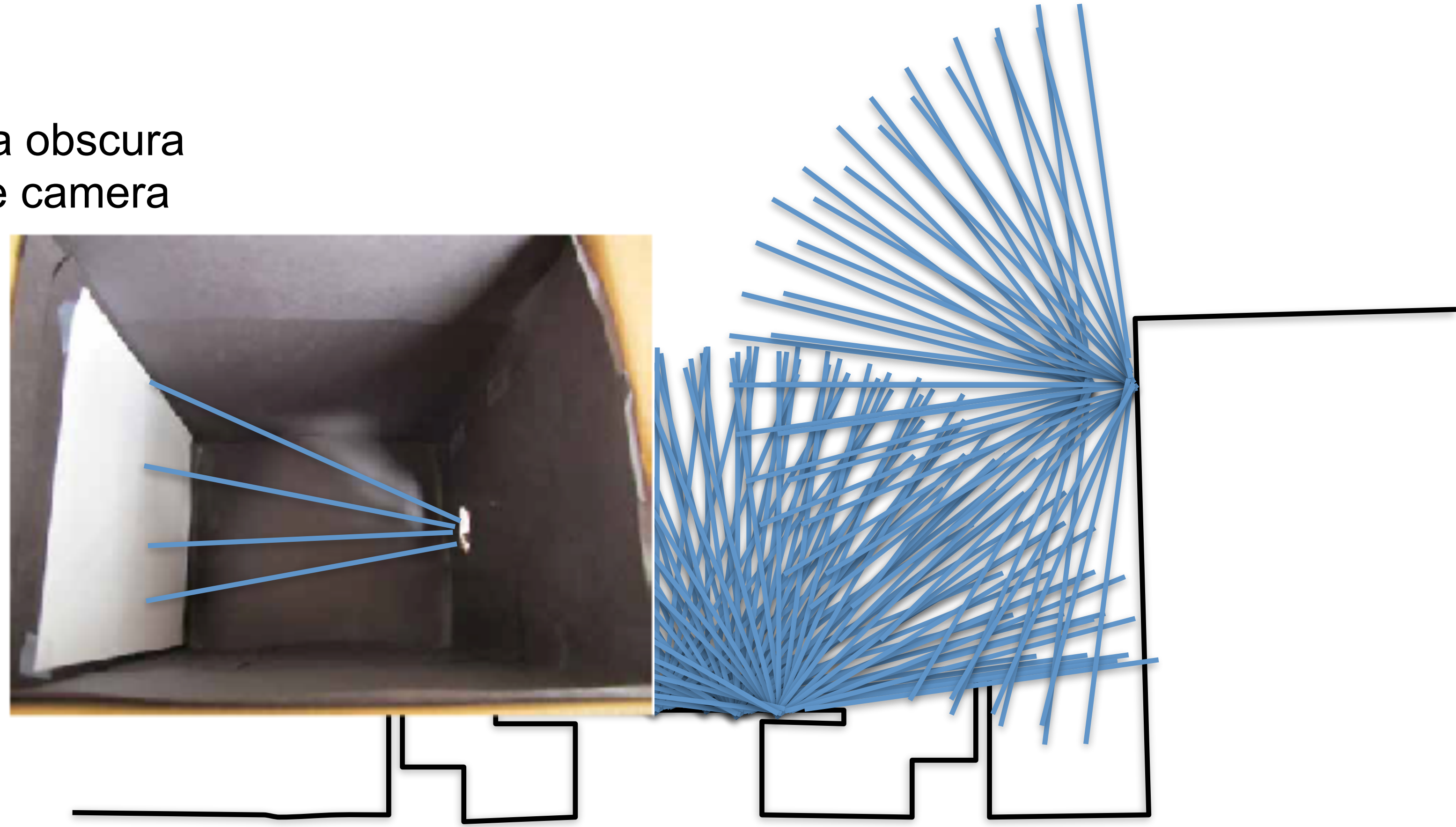
Let’s check, do we get an image? No



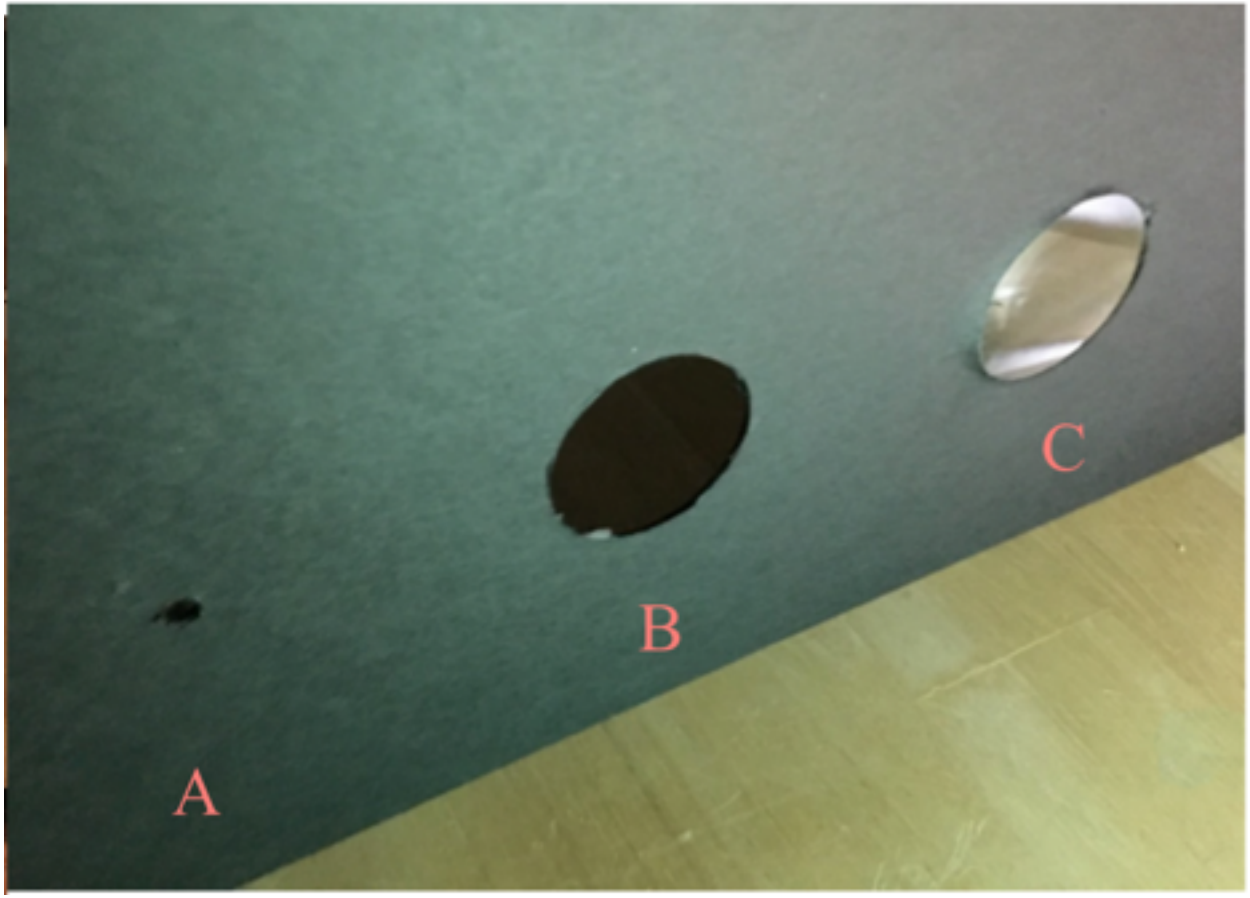


Measuring the light in the world

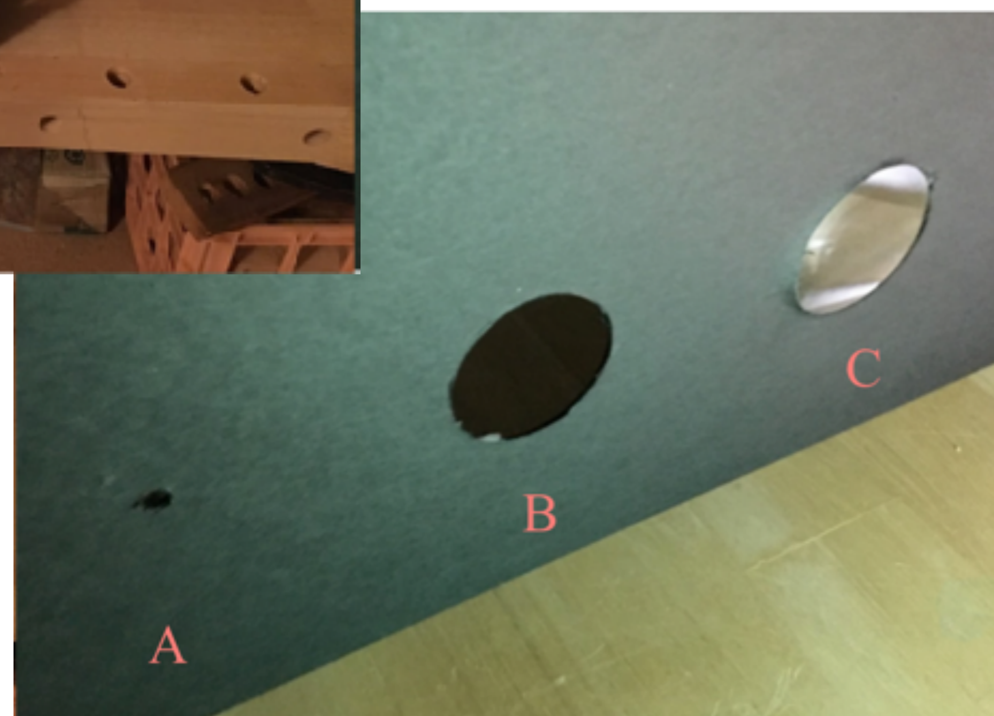
The camera obscura
The pinhole camera



Let’s try putting different occluders in between the scene and the sensor plane



Let’s try putting different occluders in between



light on wall past pinhole

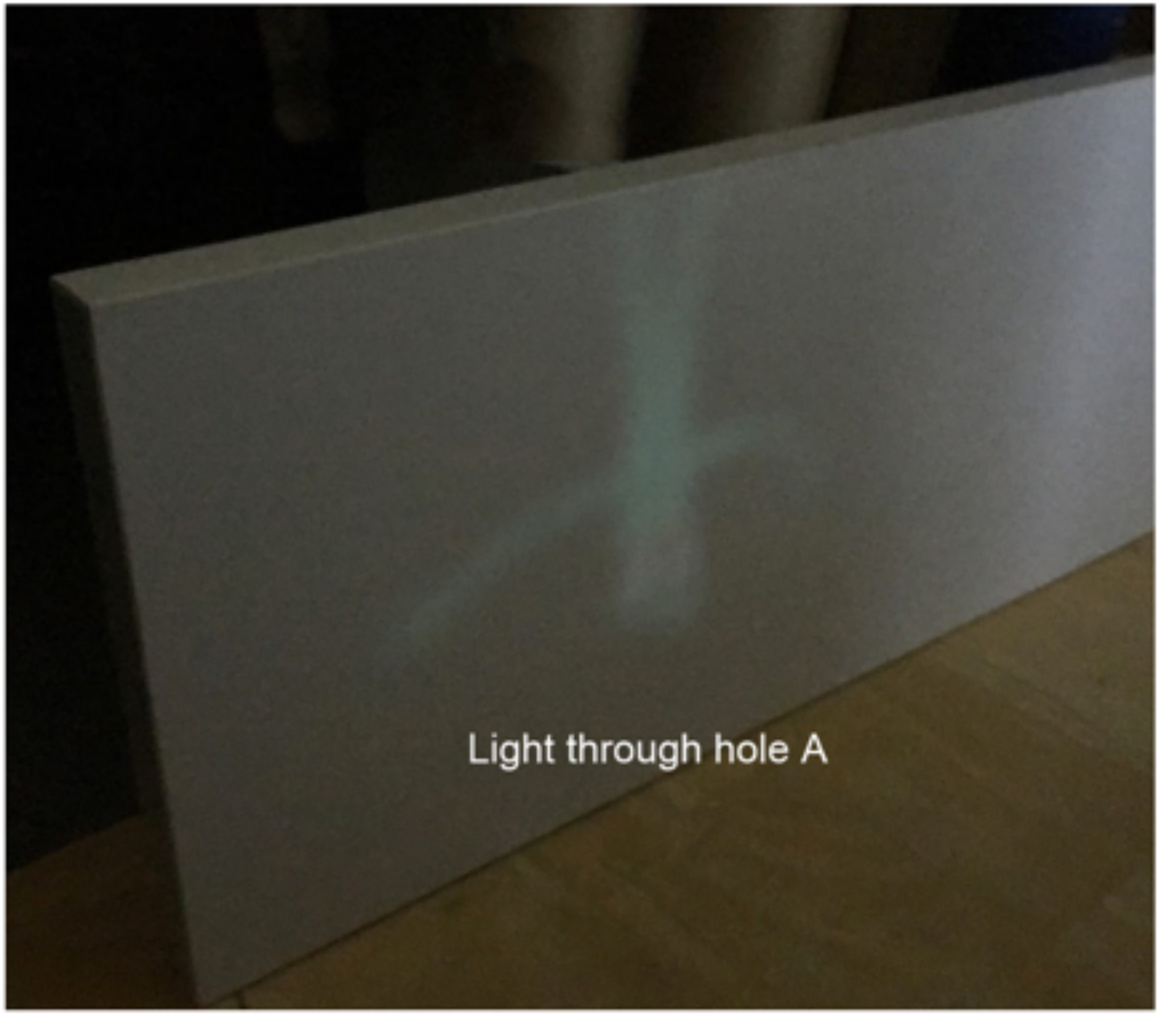
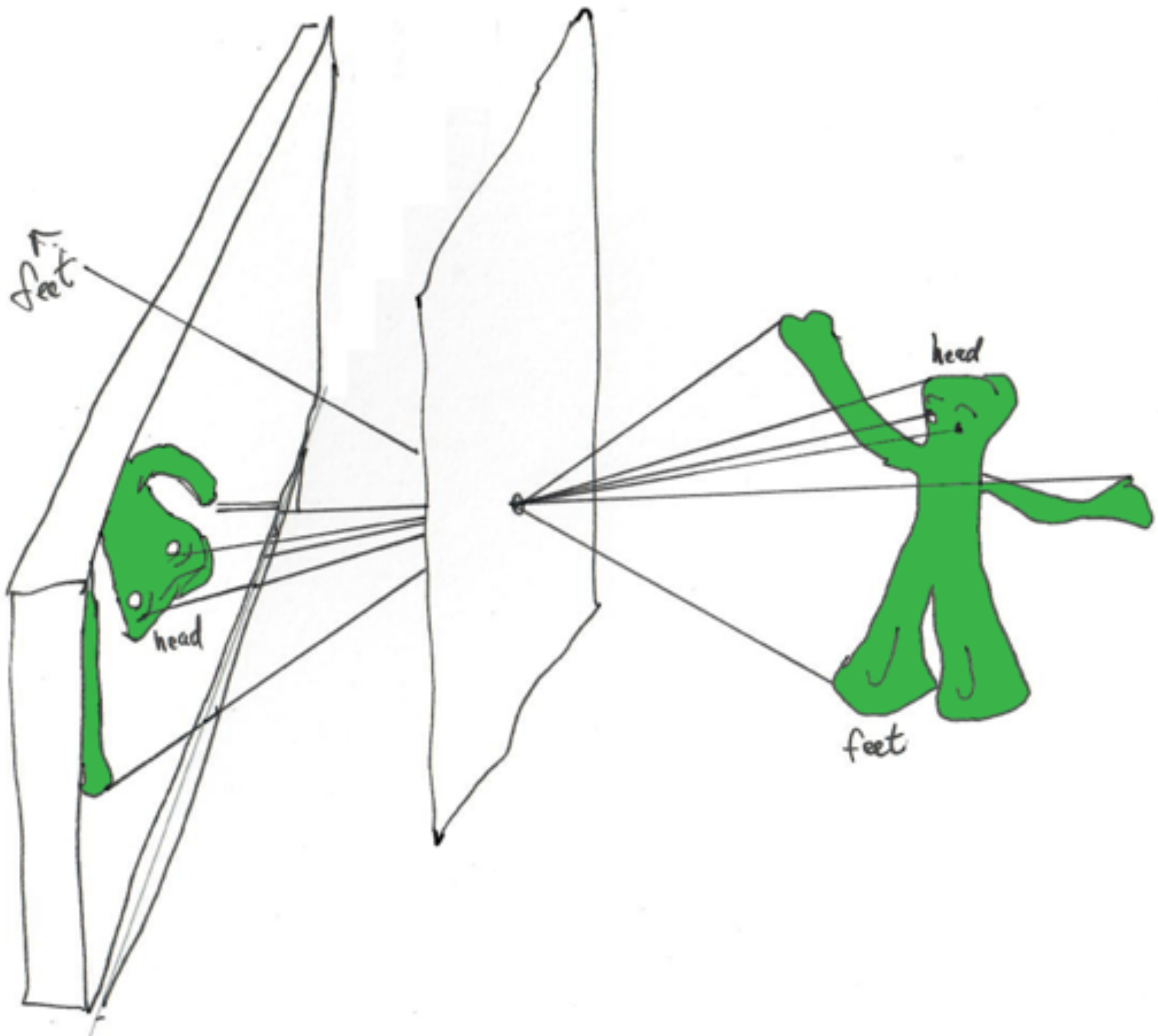


image is inverted



grocery bag pinhole camera

grocery bag pinhole camera

grocery bag pinhole camera

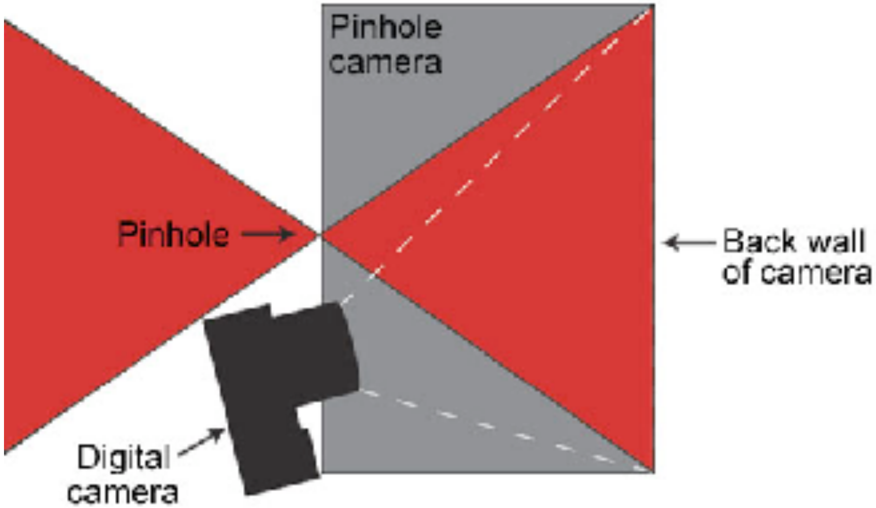
view from outside the bag

http://www.youtube.com/watch?v=FZyCFxsyx8o

view from inside the bag

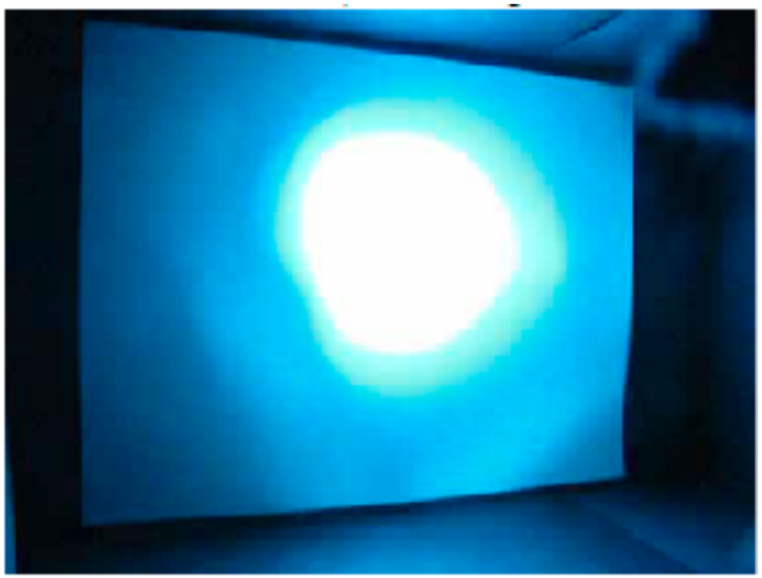
http://youtu.be/-rhZaAM3F44

Problem Set 6

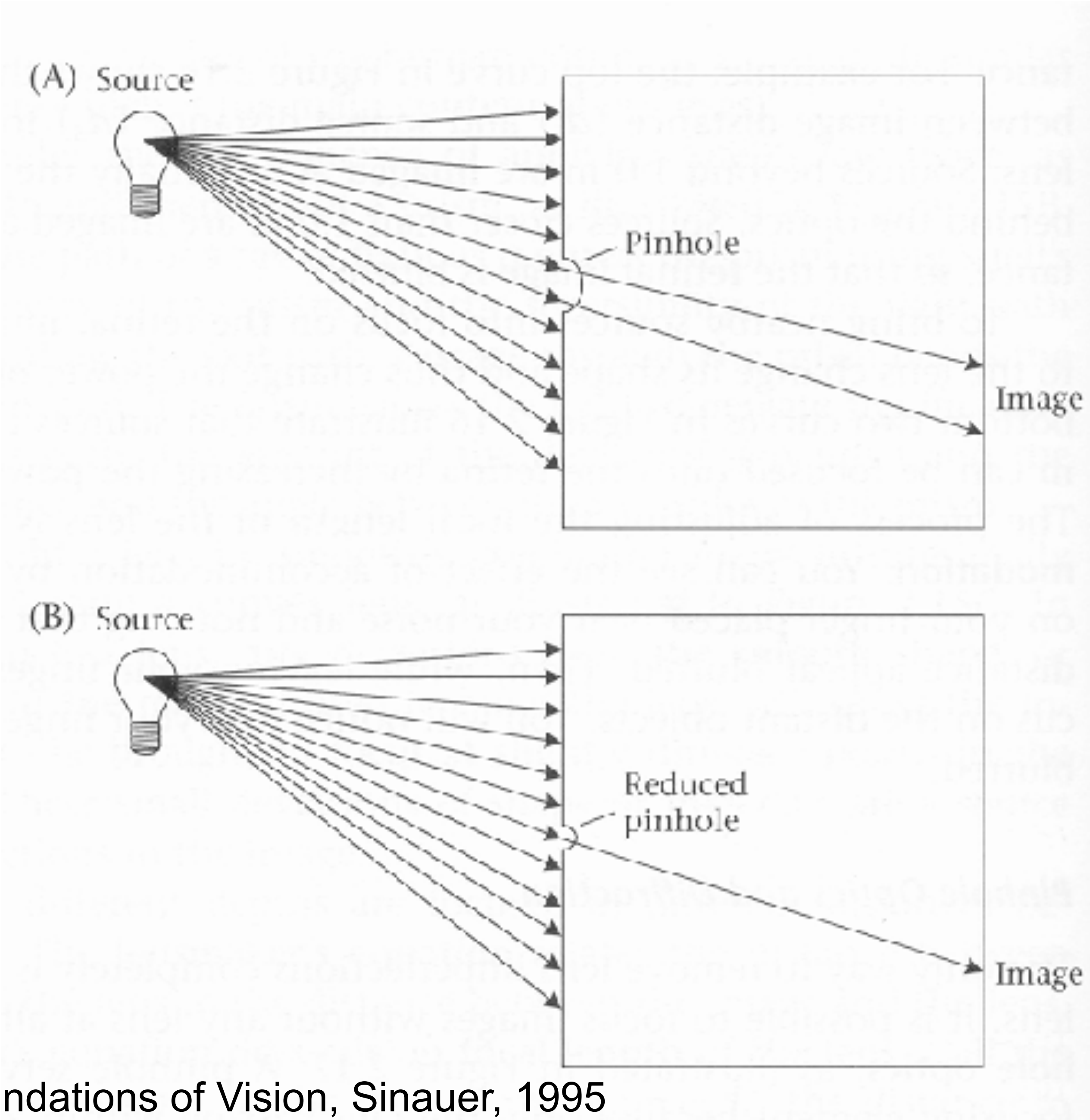


http://www.foundphotography.com/PhotoThoughts/archives/2005/04/pinhole_camera_2.html

Problem Set 6

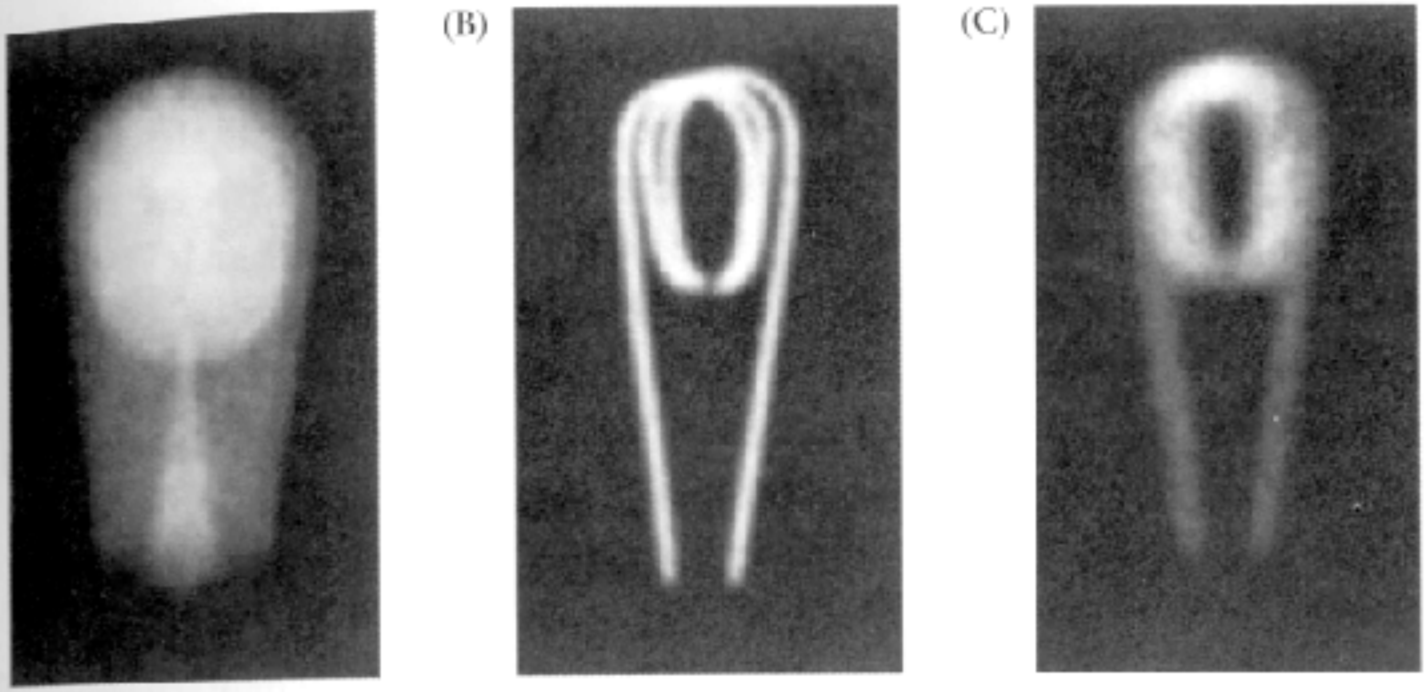


Effect of pinhole size



Wandell, Foundations of Vision, Sinauer, 1995

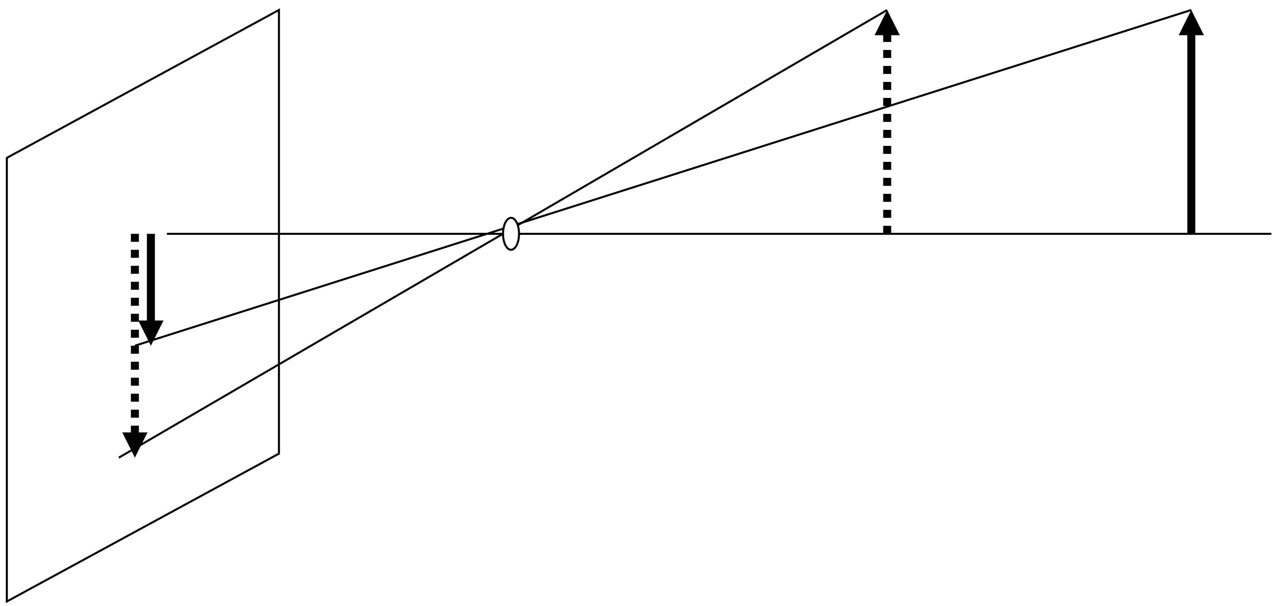
(A)



2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

Wandell, Foundations of Vision, Sinauer, 1995

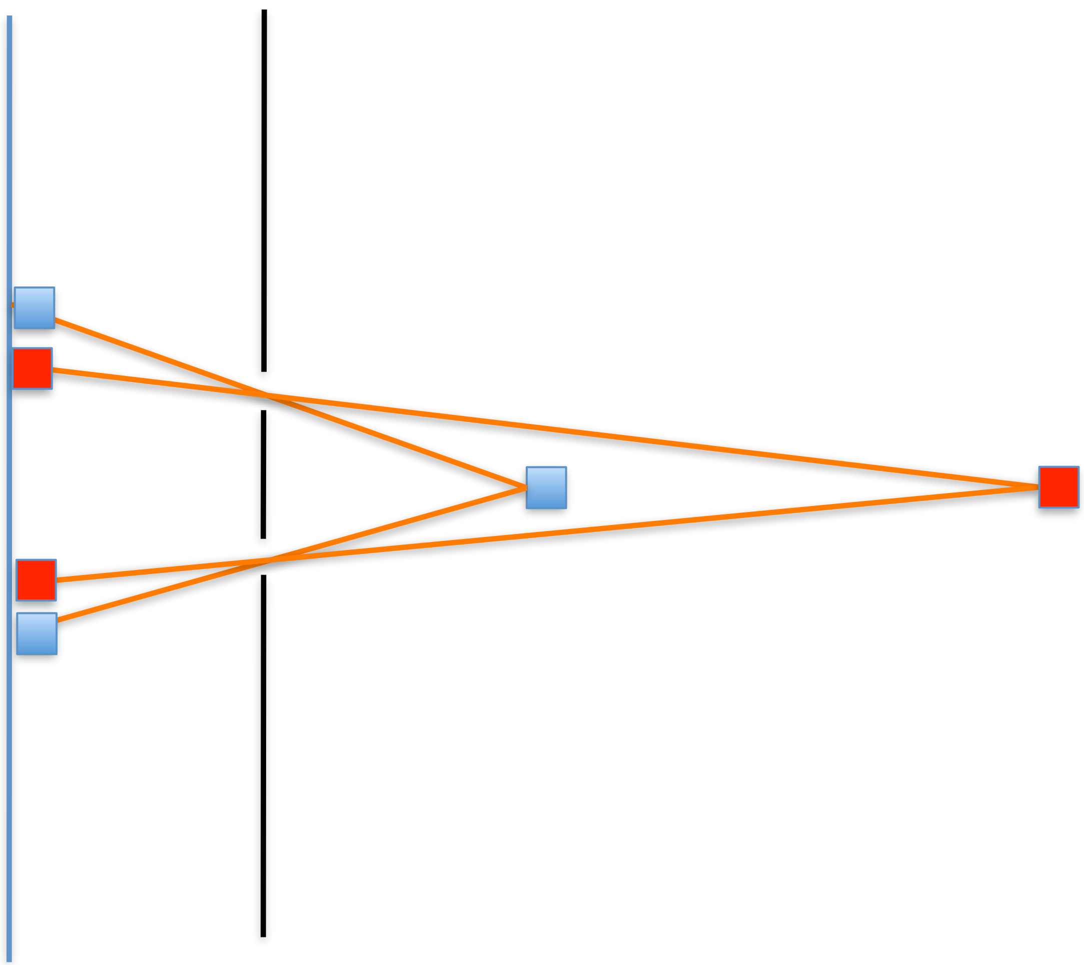
Measuring distance



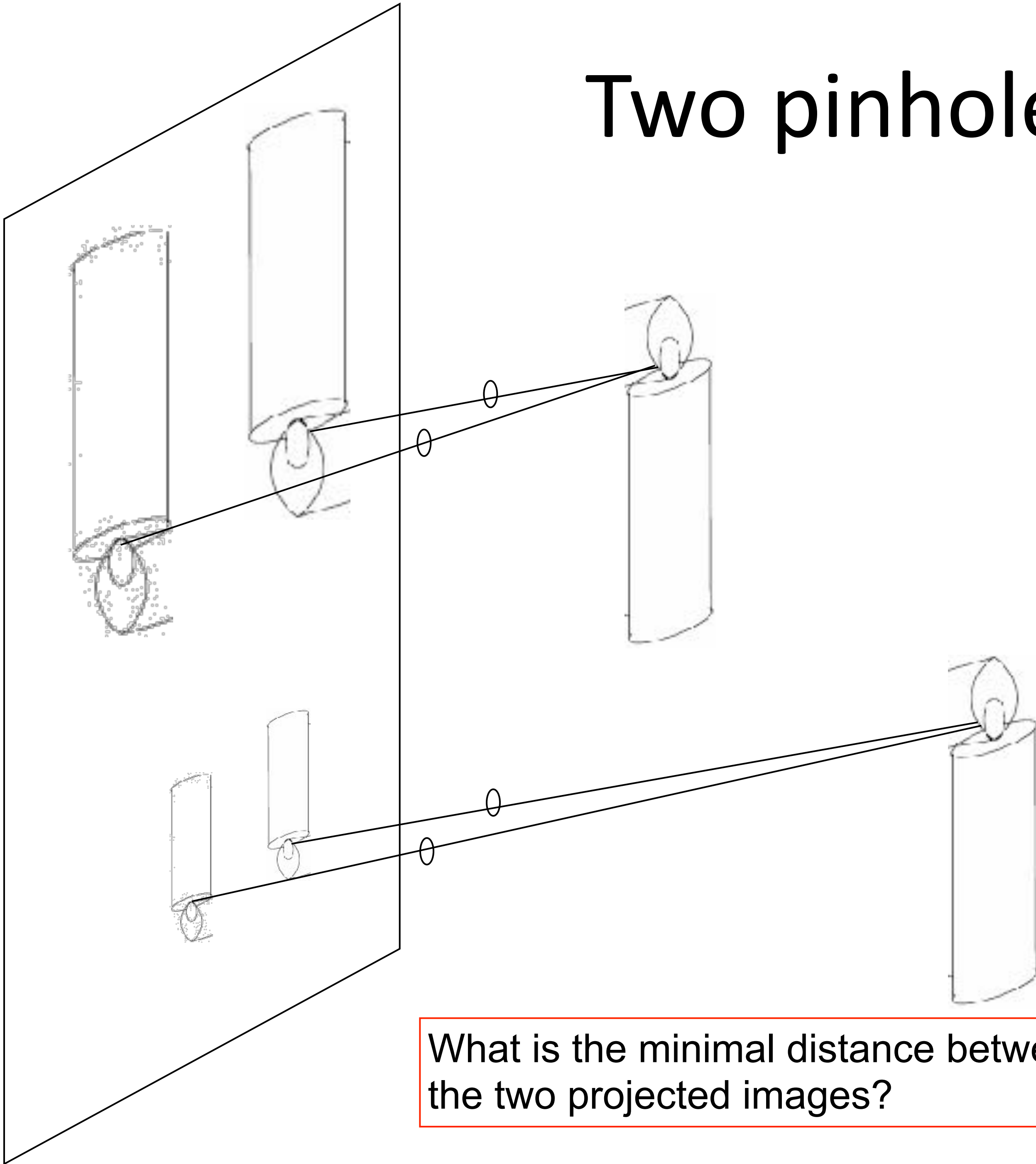
• Object size decreases with distance to the pinhole
• There, given a single projection, if we know the size of the object we can know how far it is.
• But for objects of unknown size, the 3D information seems to be lost.

Playing with pinholes

Two pinholes

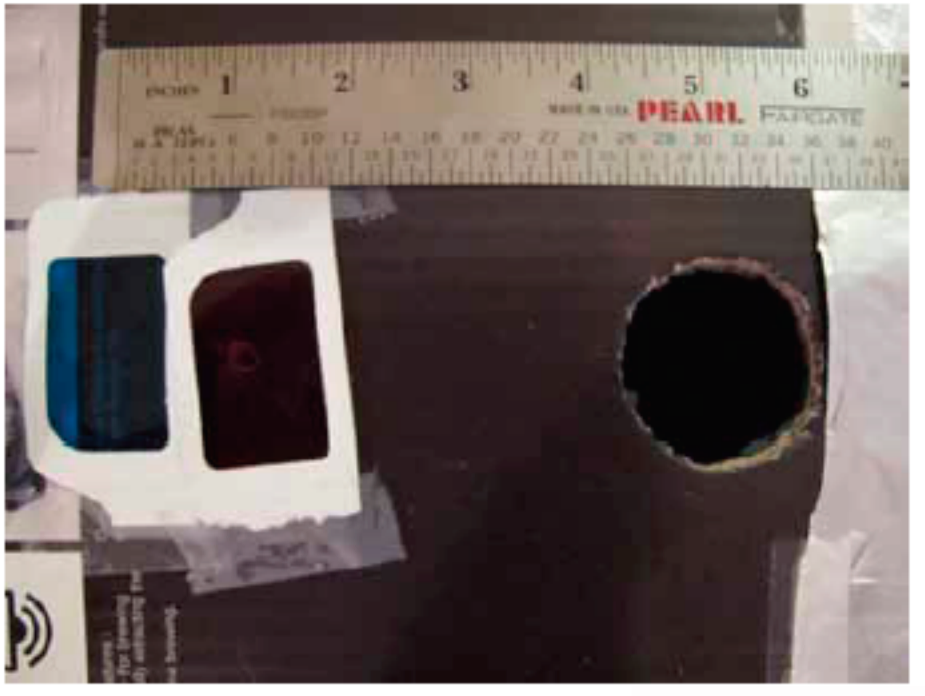


Two pinholes

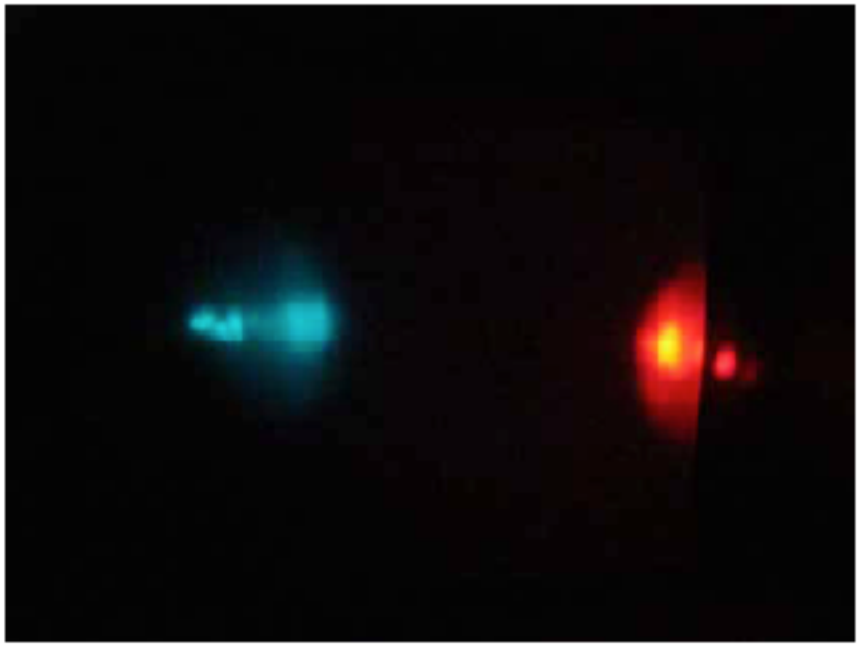


What is the minimal distance between the two projected images?

Anaglyph pinhole camera

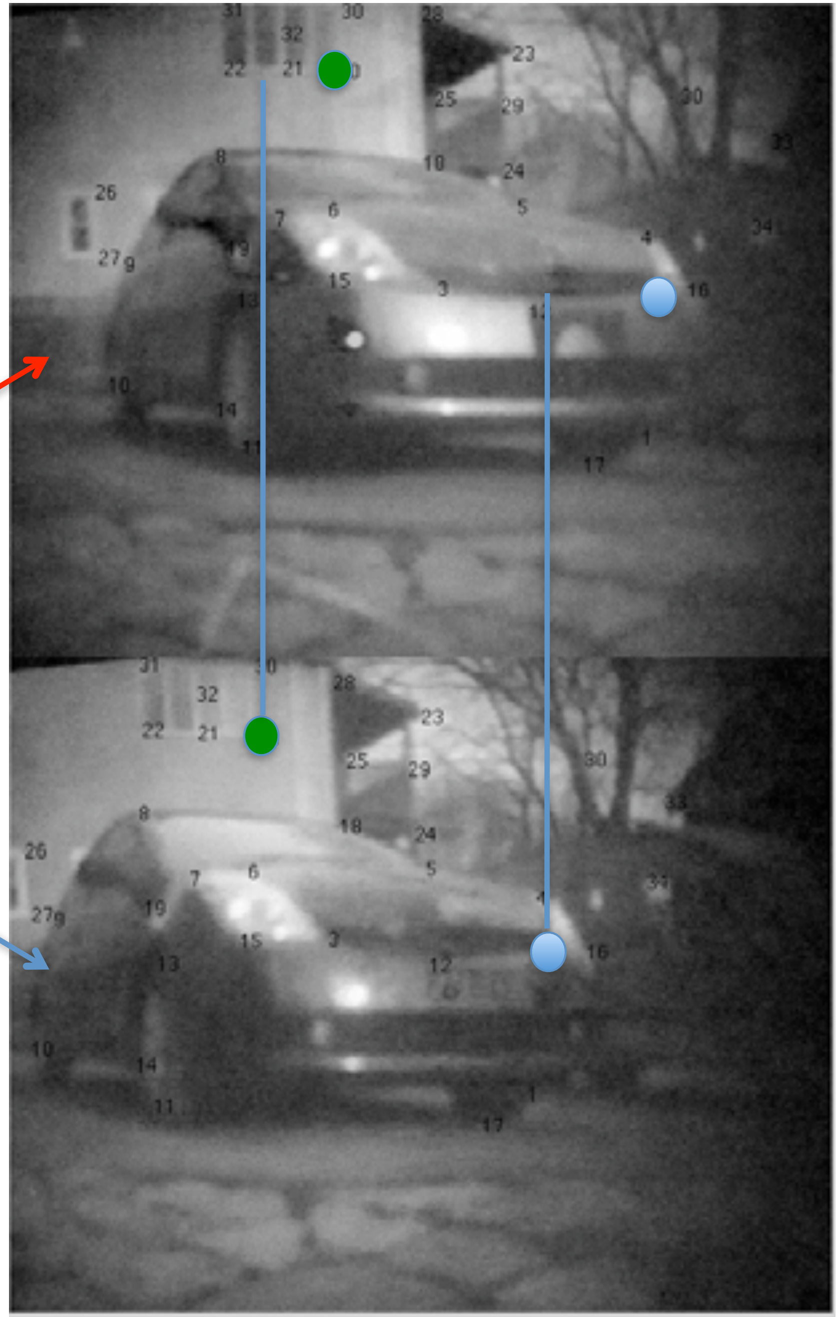


Anaglyph pinhole camera

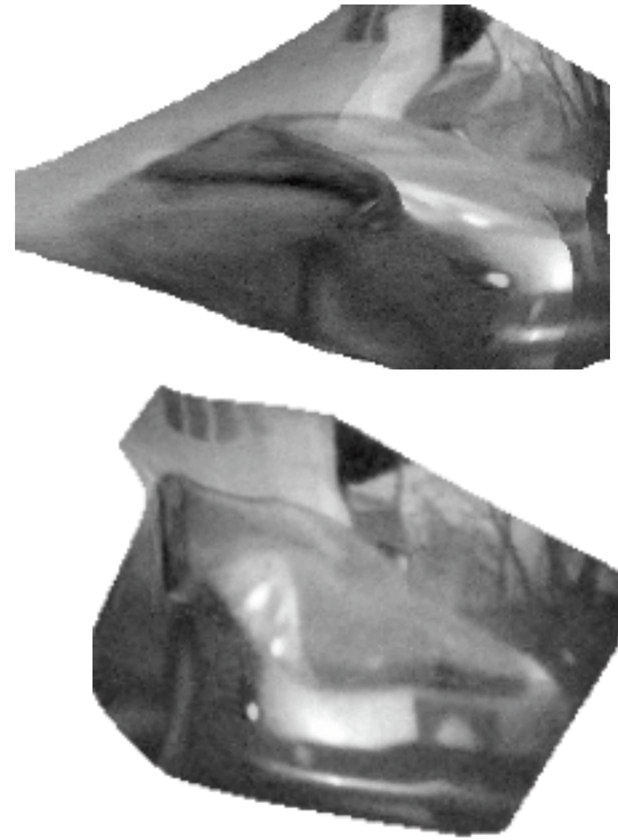


Anaglyph pinhole camera

Anaglyph



Synthesis of new views



Problem set

- Build the device
- Take some pictures and put them in the report
- Take anaglyph images
- Work out the geometry
- Recover depth for some points in the image

Straw camera

Accidental pinhole and pinspeck cameras: Revealing the scene outside the picture

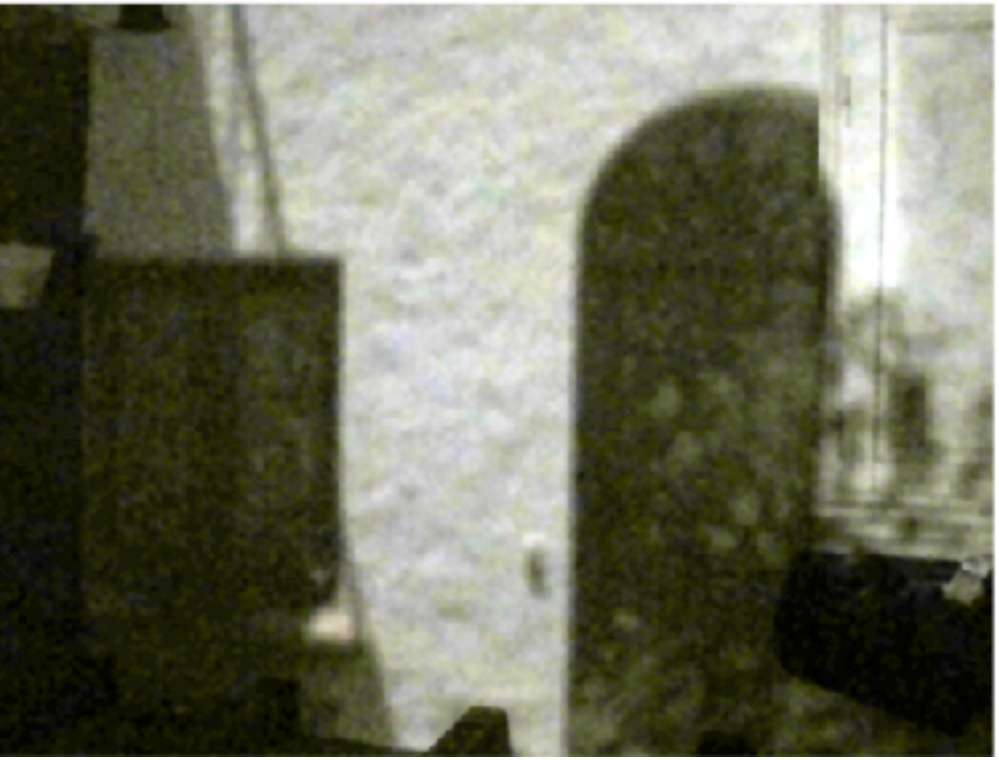
Antonio Torralba
William T. Freeman

See project page for videos:
http://people.csail.mit.edu/torralba/research/accidentalcameras/


Shadows?

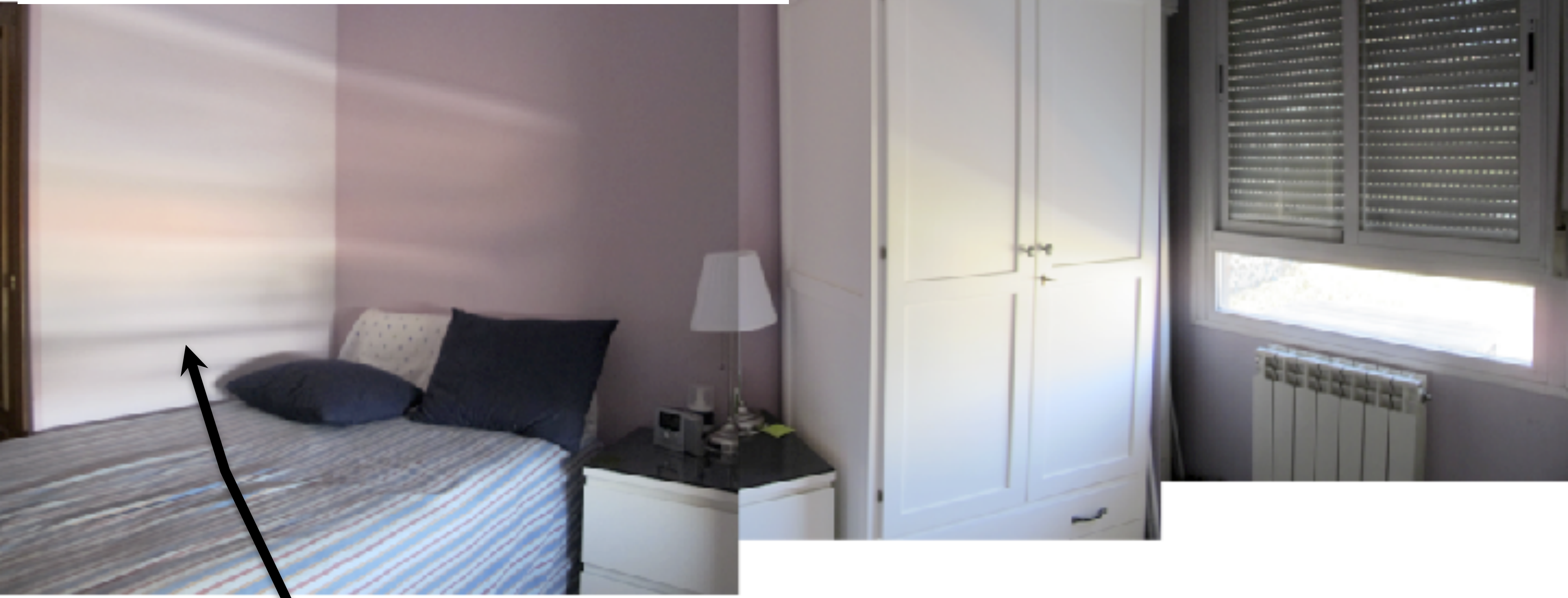
Accidental pinhole camera

Window turned into a pinhole



View outside

Accidental pinhole camera



Outside scene

*

Aperture

See Zomet, A.; Nayar, S.K. CVPR 2006 for a detailed analysis.

Anti-pinhole or Pinspeck cameras

Adam L. Cohen, 1982

OPTICA ACTA, 1982, VOL. 29, NO. 1, 63–67

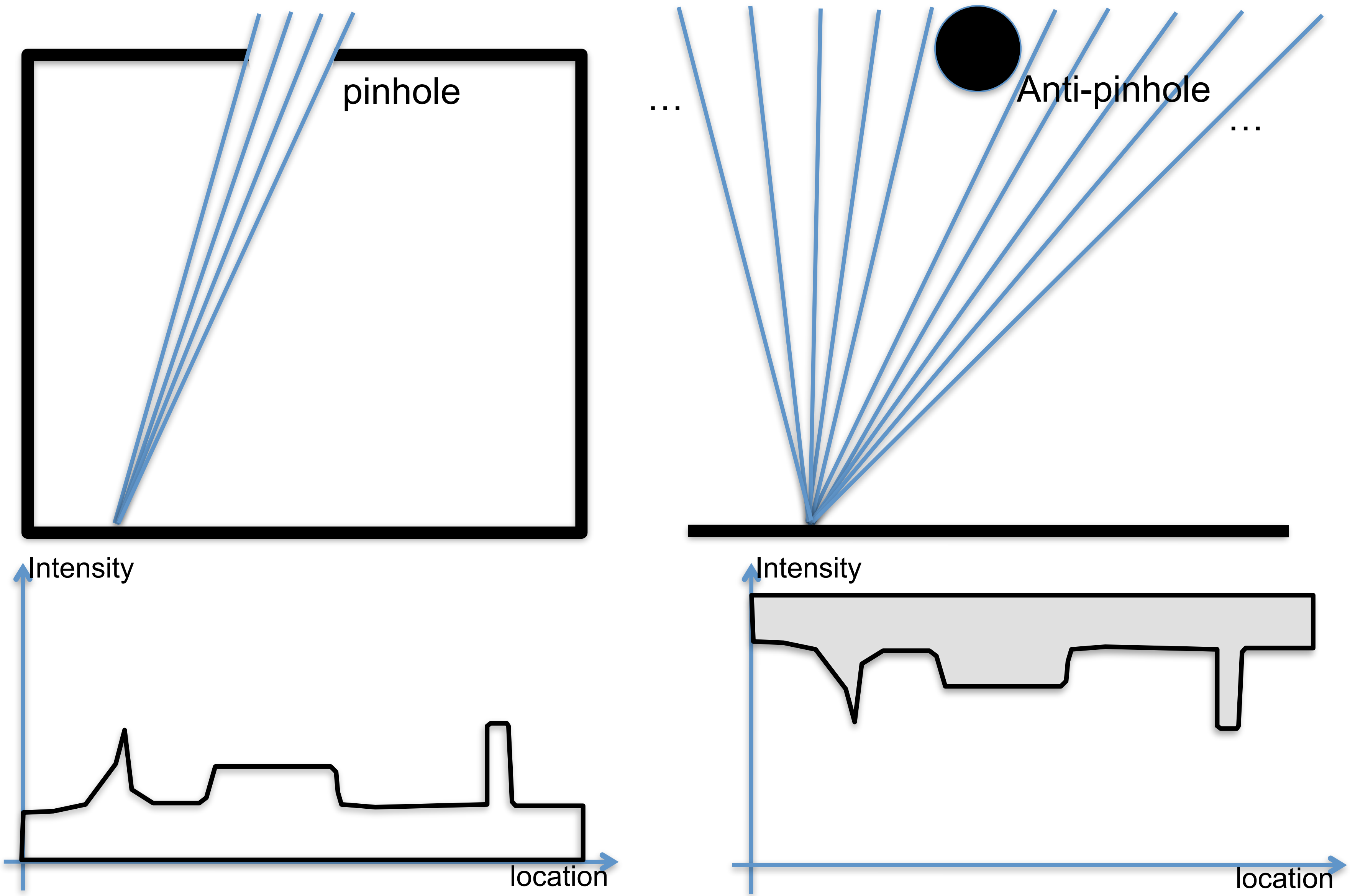
Anti-pinhole imaging

ADAM LLOYD COHEN
Parmly Research Institute, Loyola University of Chicago,
Chicago, Illinois 60626, U.S.A.

(*Received 16 April 1981; revision received 8 July 1981*)

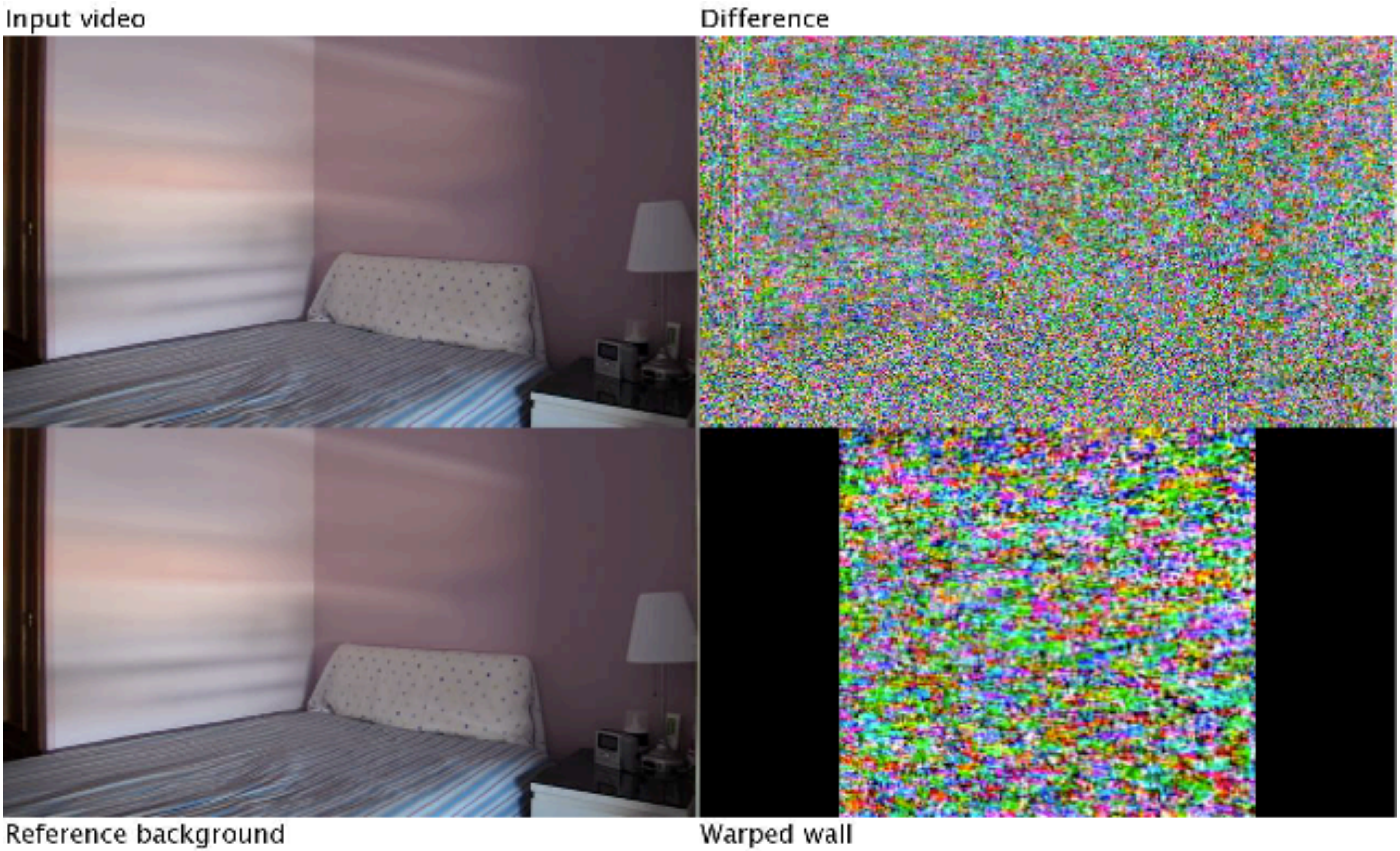
Abstract. By complementing a pinhole to produce an isolated opaque spot, the light ordinarily blocked from the pinhole image is transmitted, and the light ordinarily transmitted is blocked. A negative geometrical image is formed, distinct from the familiar 'bright-spot' diffraction image. Anti-pinhole, or 'pinspeck' images are visible during a solar eclipse, when the shadows of objects appear crescent-shaped. Pinspecks demonstrate unlimited depth of field, freedom from distortion and large angular field. Images of different magnification may be formed simultaneously. Contrast is poor, but is improvable by averaging to remove noise and subtraction of a d.c. bias. Pinspecks may have application in X-ray space optics, and might be employed in the eyes of simple organisms.

Pinhole and Anti-pinhole cameras

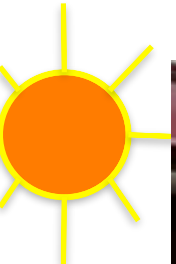


Adam L. Cohen, 1982

To see the effect of pinspeck cameras, need to subtract the no-pinspeck image



Shadows
Accidental anti-pinhole cameras



Background image

-

Input video

=

Negative of the shadow

Background image

-

Input video

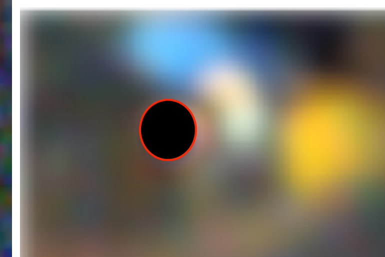
=

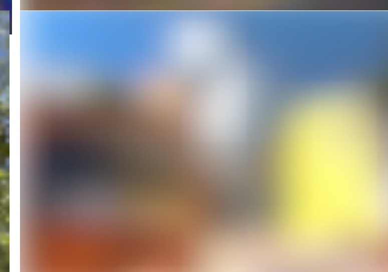
Negative of the shadow

Input video

Negative of the shadow

Inverted difference image





View behind the ball

The importance of the size of the occluder

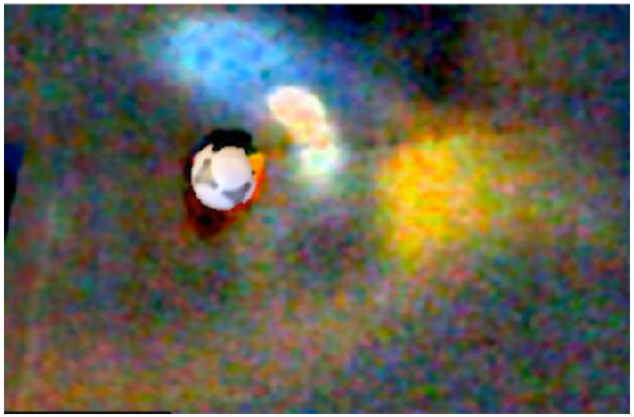
Input video

Negative of the shadow

Size of the occluder

Antonio

Ball



Another occlusion-based camera: edge camera

show demo

Model of corner camera signal

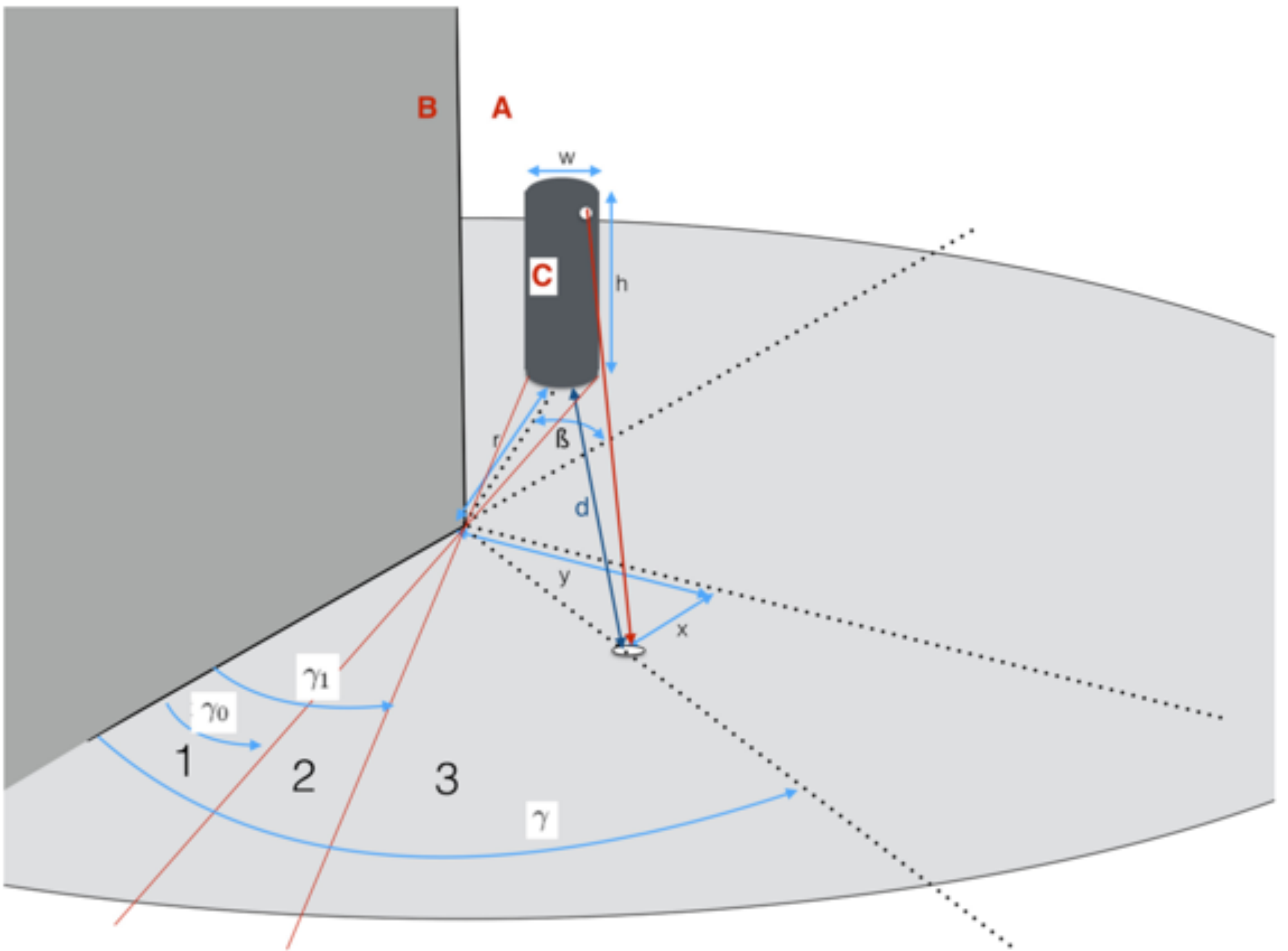
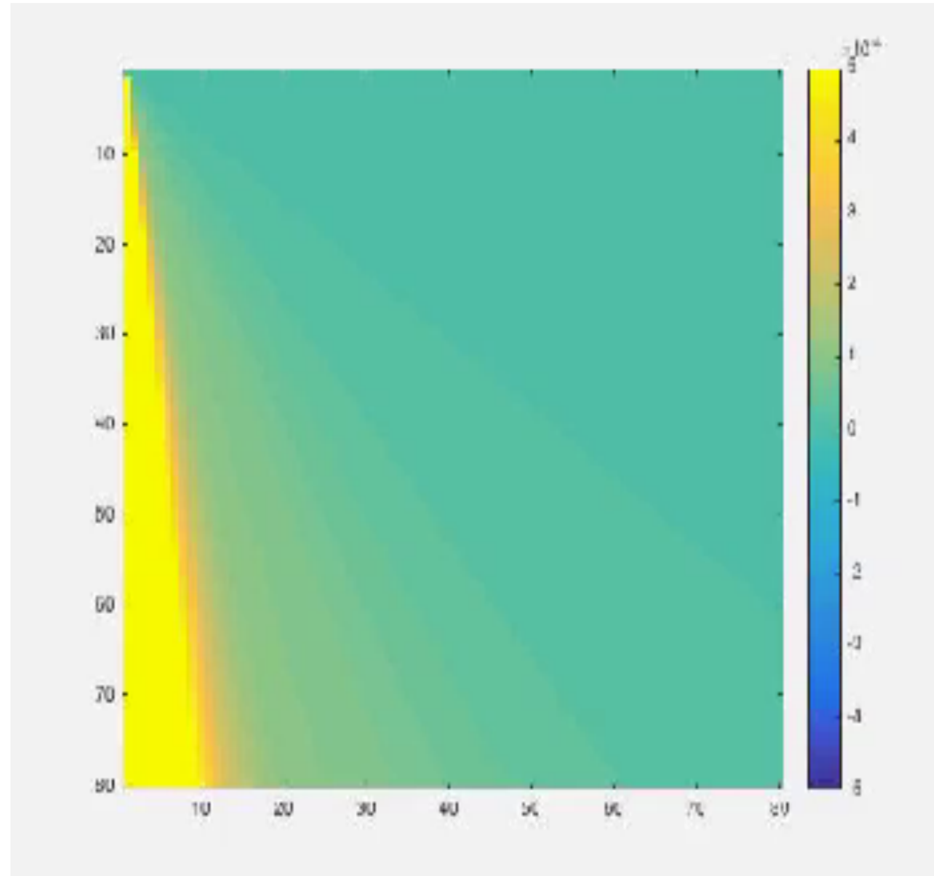


Figure 3: Calculation of the contribution to the brightness of the target.

Corner Camera 1-D Image

Rectified Image



Kalman Gain Images

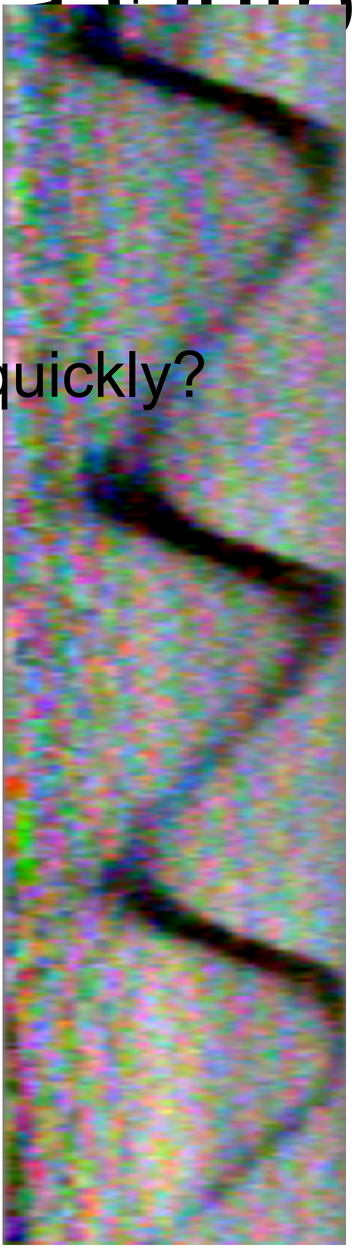
Experiment Proof of Concept

Experimental Proof of Concept

Experimental Proof of Concept

Experimental Proof of Concept

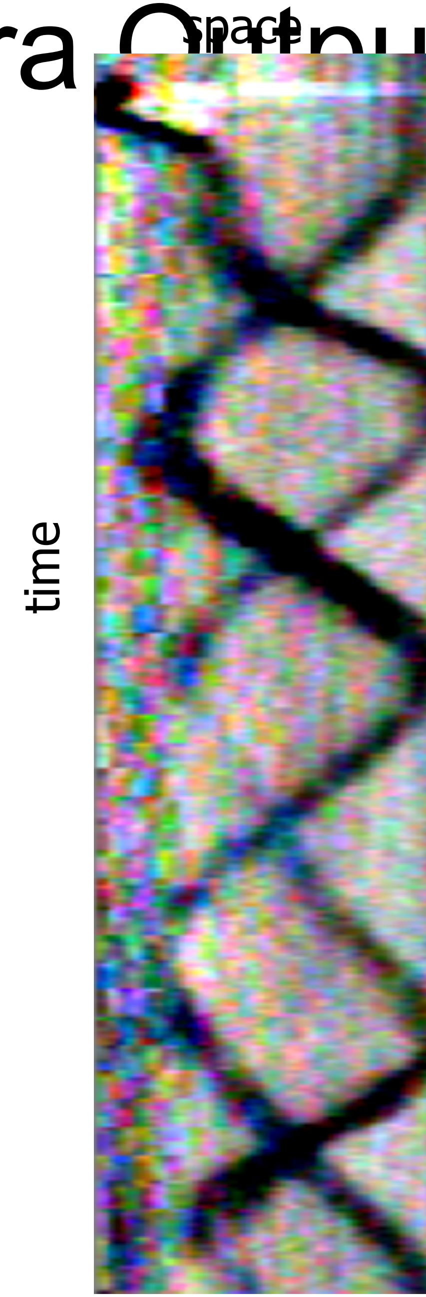
1-D Corner Camera Output



- How many people?
- Where slowed down, where moved quickly?

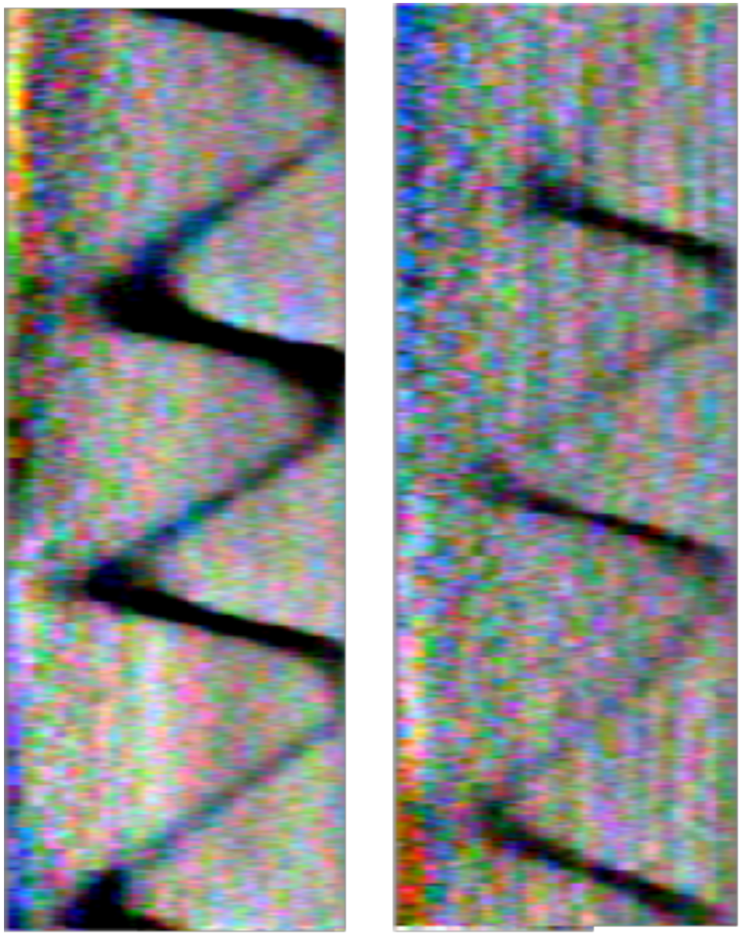
1-D Corner Camera Output

- How many people?
- How fast is each person moving?

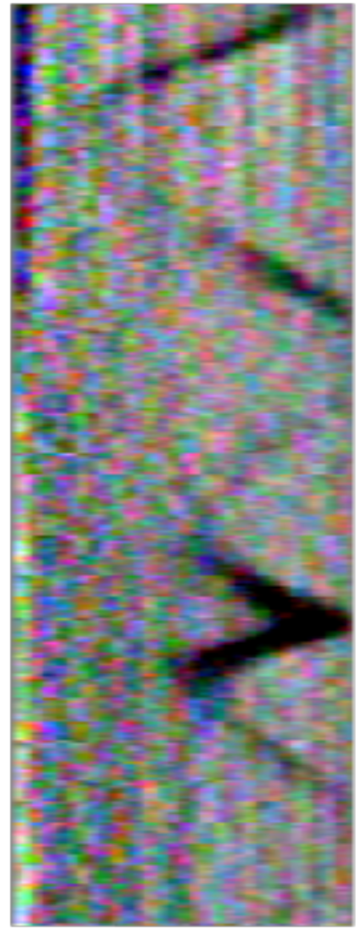


Video Corresponding to 1-D Camera

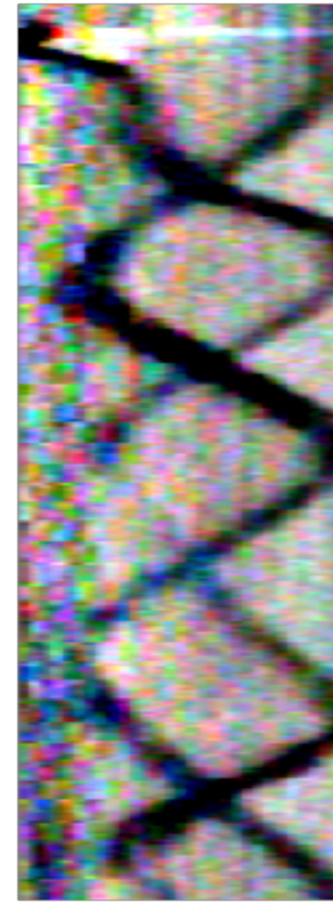
More Corner Camera Videos



1 Person Walking in Circles



1 Person
Randomly Walking



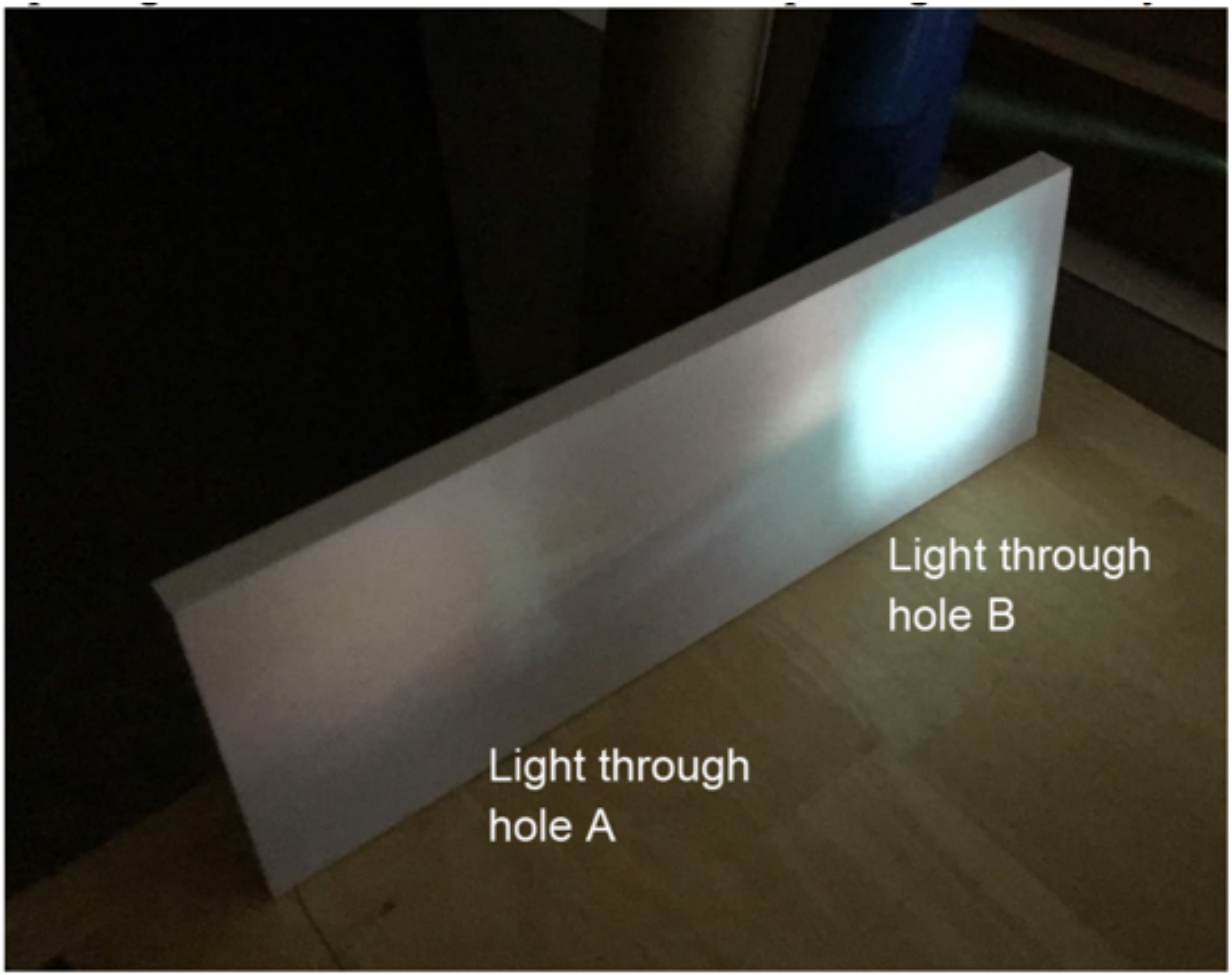
2 People Walking
in Circles

Additional Results

Paper ID: 1983

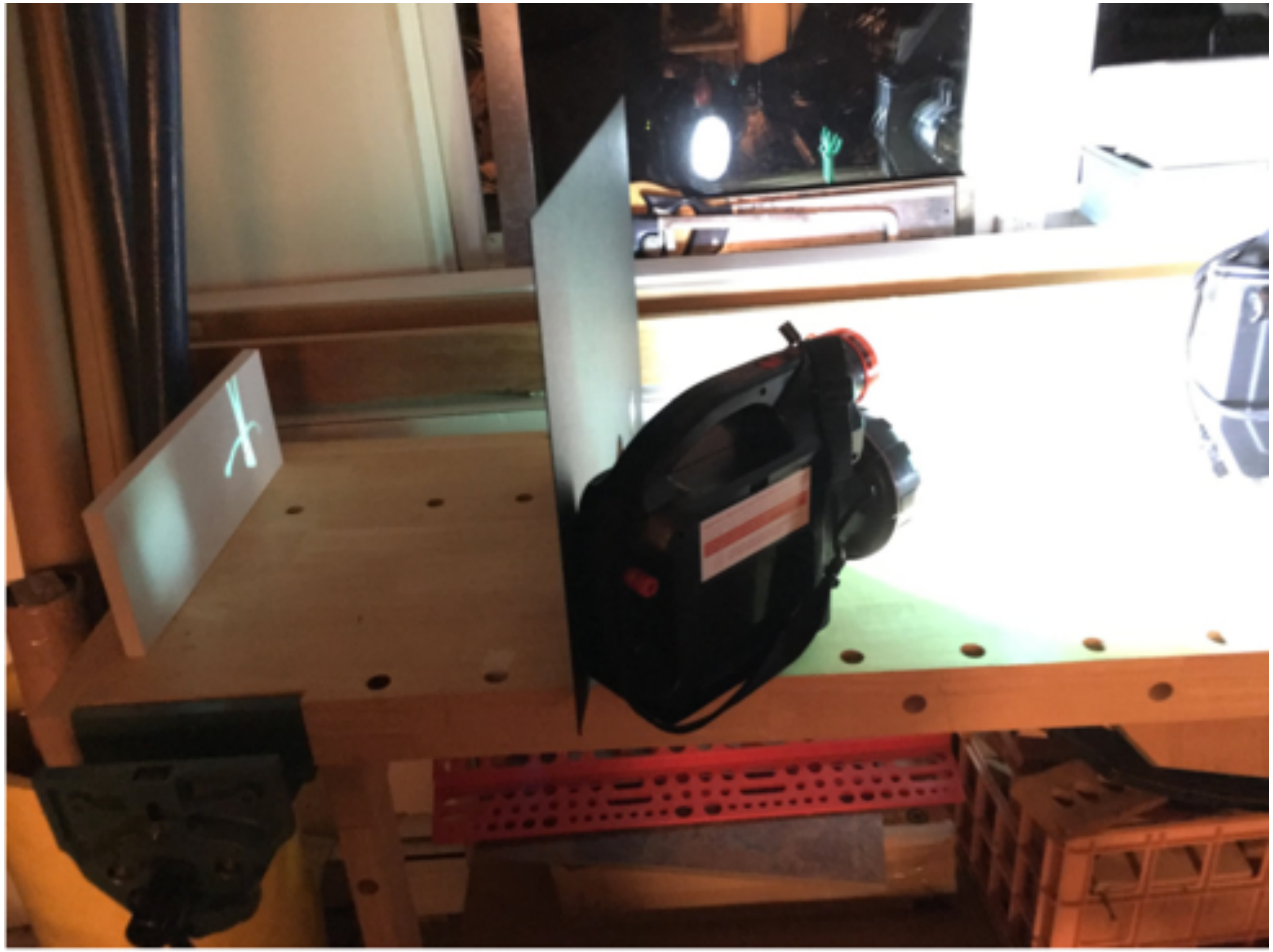
A puzzle: light intensity goes down as 1/r^2, where r is the distance away. So then why does a long, uniform surface, like a road or a sidewalk, seem to have the same brightness, whether it's nearby or far away? The difference in 1/r^2 in this photo between the ground where the photographer is standing and the ground at the yellow sign is huge!

Influence of aperture size: with a small aperture, the image is sharp, but dim. A large aperture gives a bright, but blurry image.

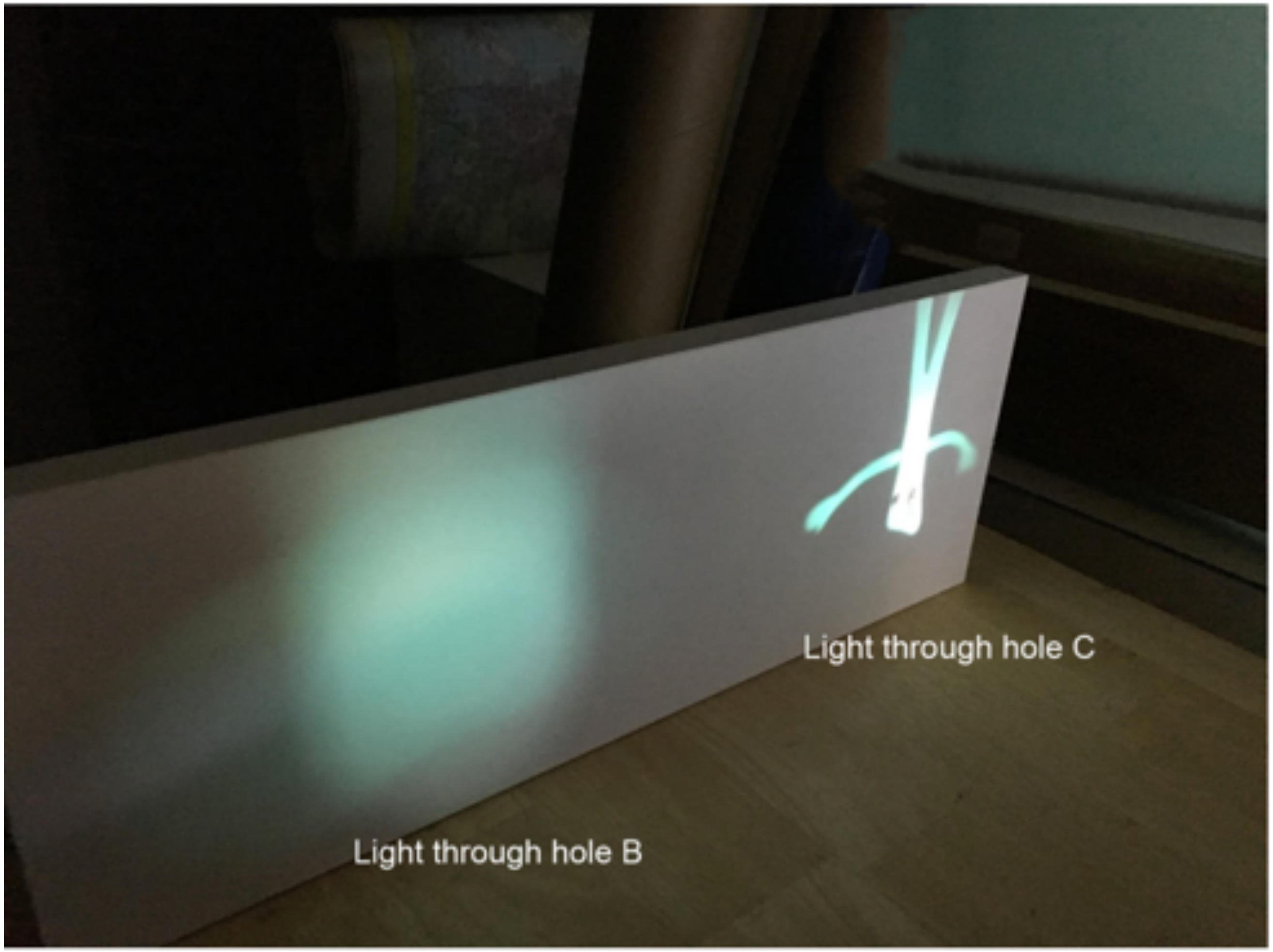


A lens can focus light from one point in the world to one point on the sensor plane.

Images through large aperture, with and without lens present



Images through large aperture, with and without lens present



Derivation of Snell's law

$$\lambda_1 = \frac{c}{\omega\, n_1}$$ wavelength is speed/ freq

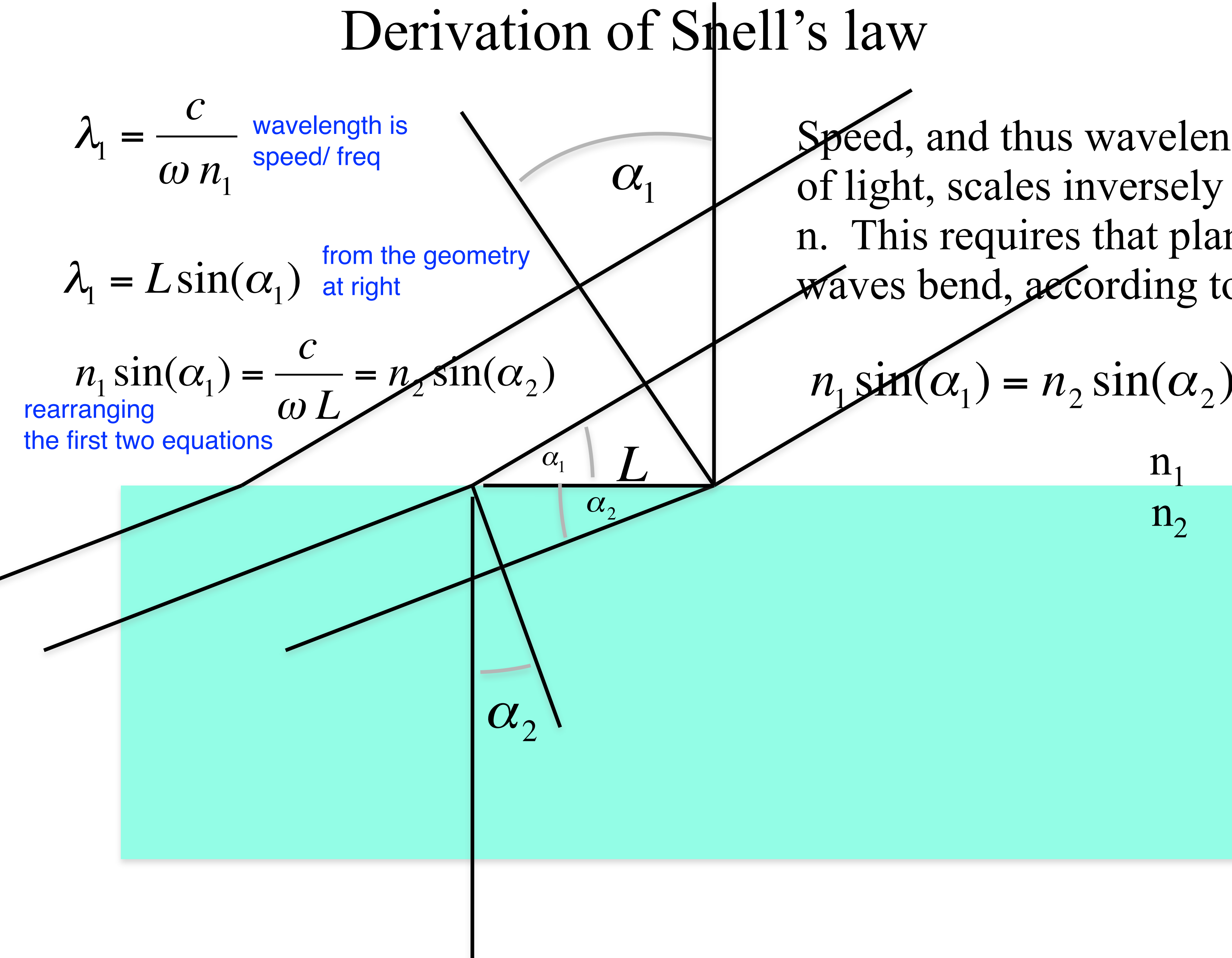
$$\lambda_1 = L \sin(\alpha_1)$$ from the geometry at right

$$n_1 \sin(\alpha_1) = \frac{c}{\omega\, L} = n_2 \sin(\alpha_2)$$

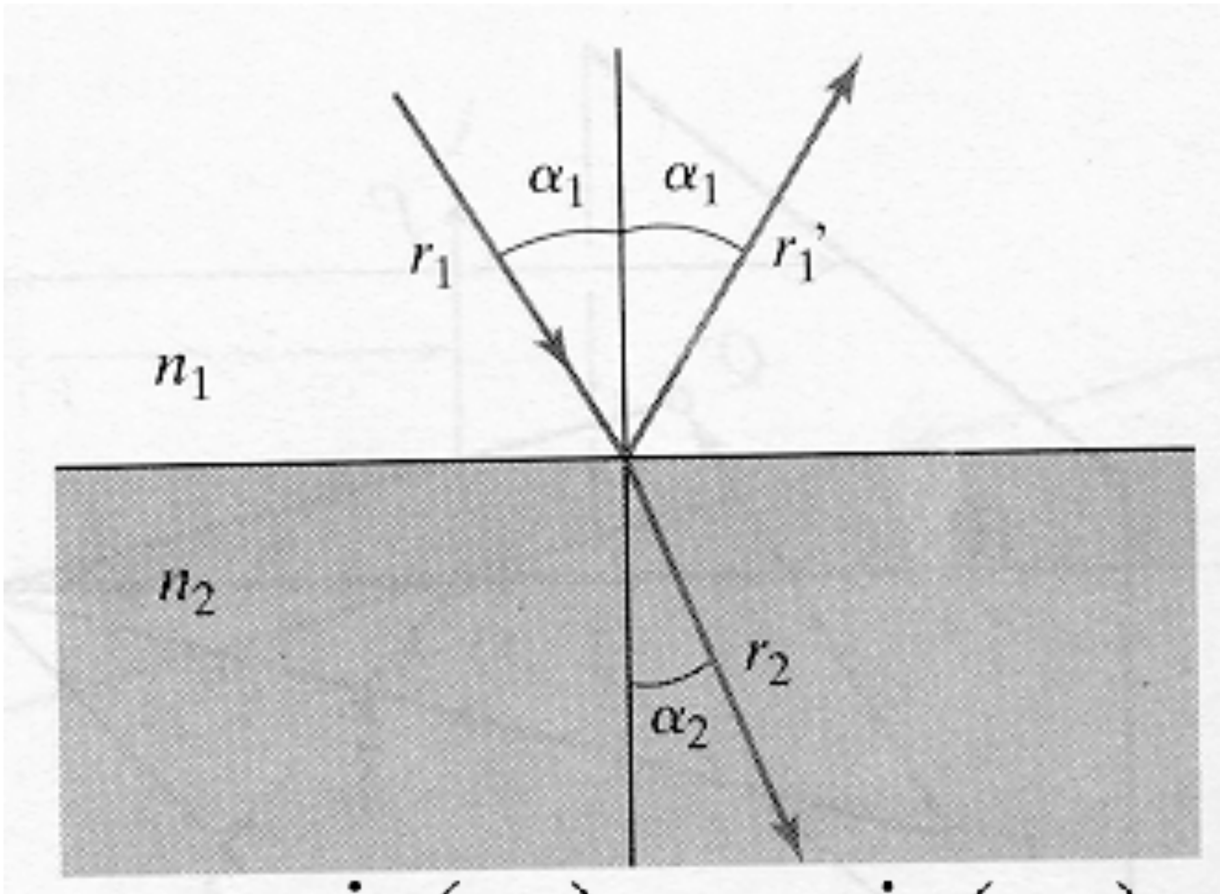
rearranging the first two equations

Speed, and thus wavelength of light, scales inversely with n. This requires that plane waves bend, according to

$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$$



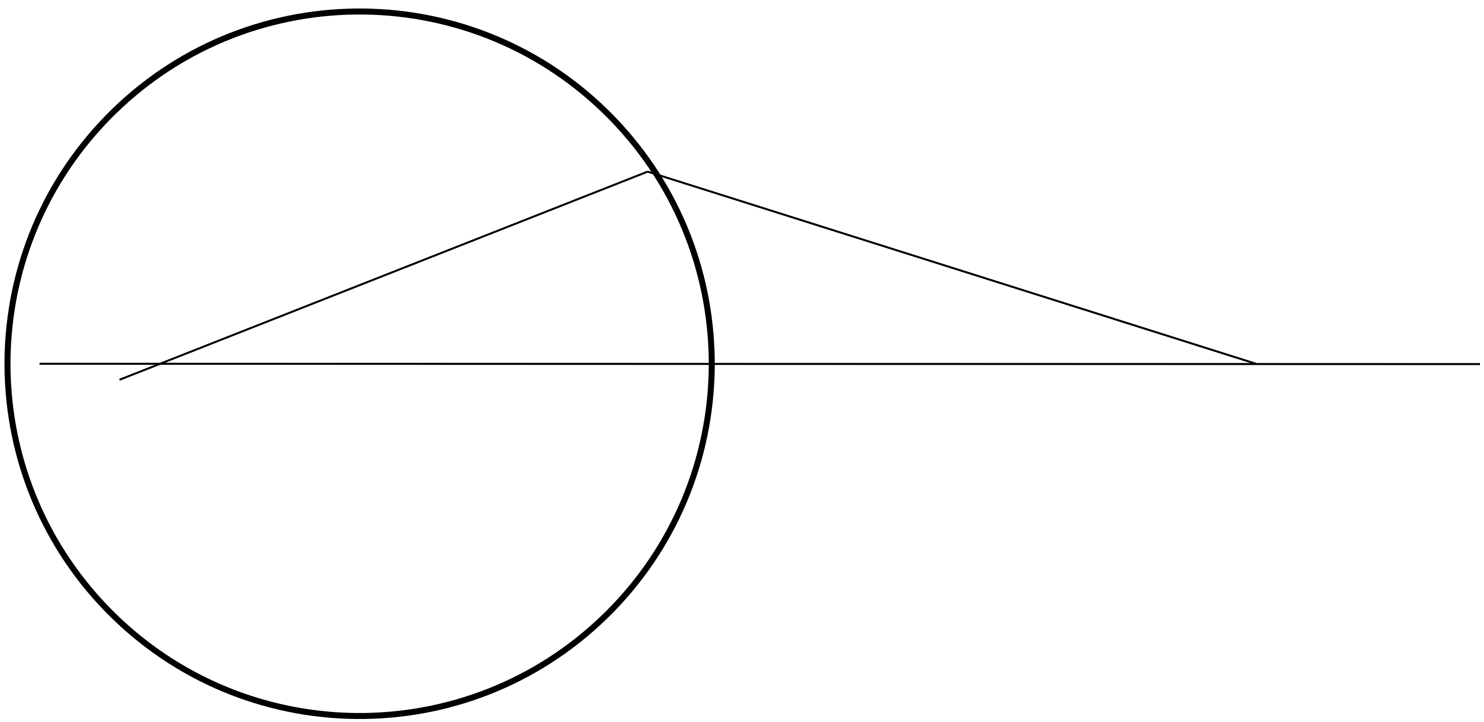
Refraction: Snell's law



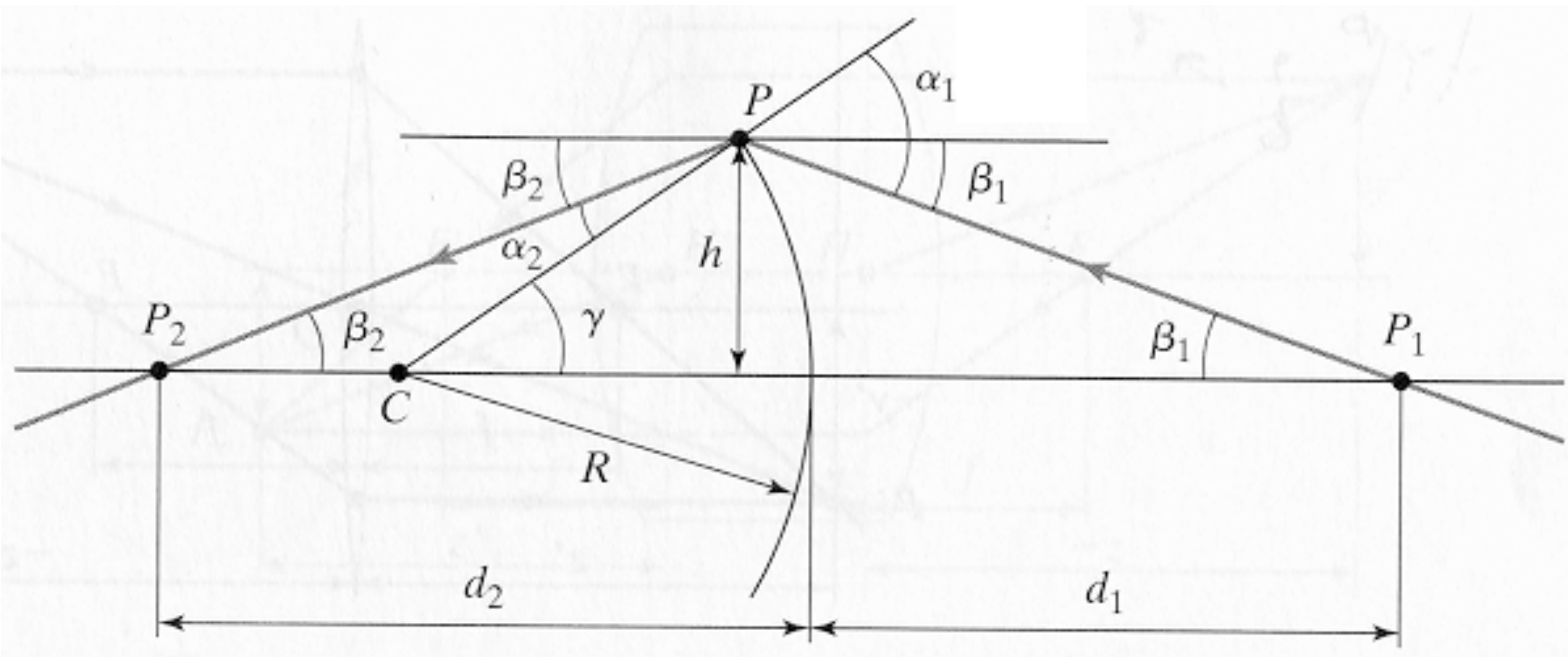
$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$$

For small angles, $n_1\alpha_1 \approx n_2\alpha_2$

Spherical lens



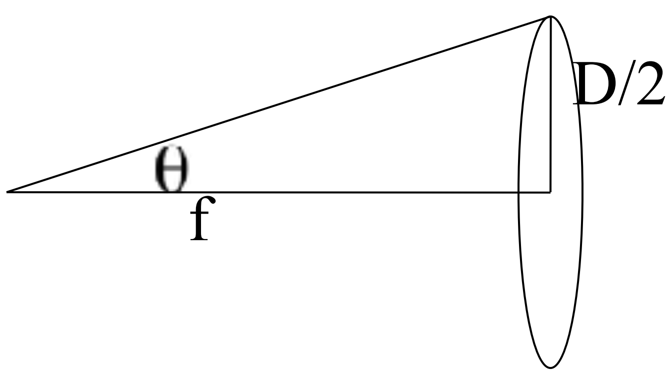
For a spherical lens surface, we can define the relevant angles, apply Snell's law, and find an expression telling how the lens focusses light



Forsyth and Ponce

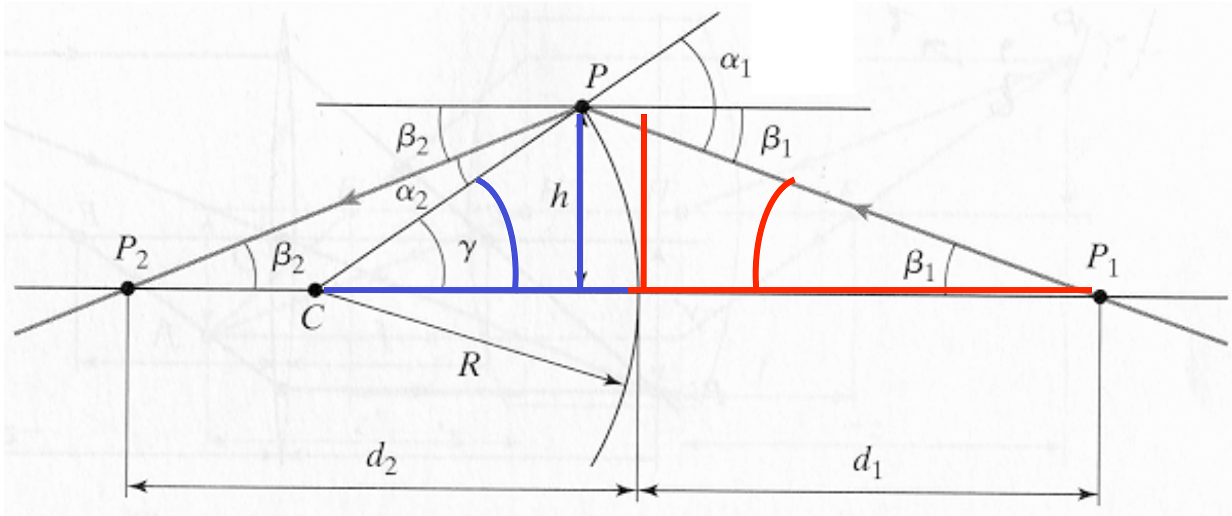
That is easiest to do under the assumptions of "first order optics": small bending angles, and a thin lens

$$\sin(\theta) \approx \theta$$



$$\theta \approx \frac{D/2}{f}$$

Paraxial refraction equation

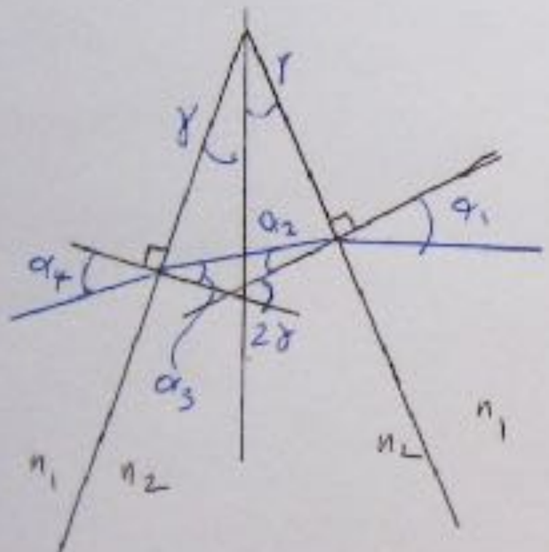


$$\alpha_1 = \boxed{\gamma} + \boxed{\beta_1} \approx h\left(\frac{1}{R} + \frac{1}{d_1}\right)$$

Relates distance from sphere and sphere radius to bending angles of light ray, for lens with one spherical surface.

$$n_1\alpha_1 \approx n_2\alpha_2$$

Deriving the lensmaker's formula



$$\alpha_1 = h\left(\frac{1}{R} + \frac{1}{d_1}\right)$$ small angle approx

$$n_1 \alpha_1 \cong n_2 \alpha_2$$ snell's law

$$\alpha_2 = 2\gamma - \alpha_3$$ geometry

$$n_2 \alpha_3 \cong n_1 \alpha_4$$ snell's law

$$\alpha_4 = h_1\left(\frac{1}{R} + \frac{1}{d_2}\right)$$ small angle

$$\gamma = \frac{h}{R}$$ small angle

$$n_1 \alpha_1 = n_2\left(\frac{2h}{R} - \frac{n_1}{n_2}\alpha_4\right) = h\left(\frac{1}{R} + \frac{1}{d_1}\right)$$

let $n_1 = 1$, $n_2 = n$

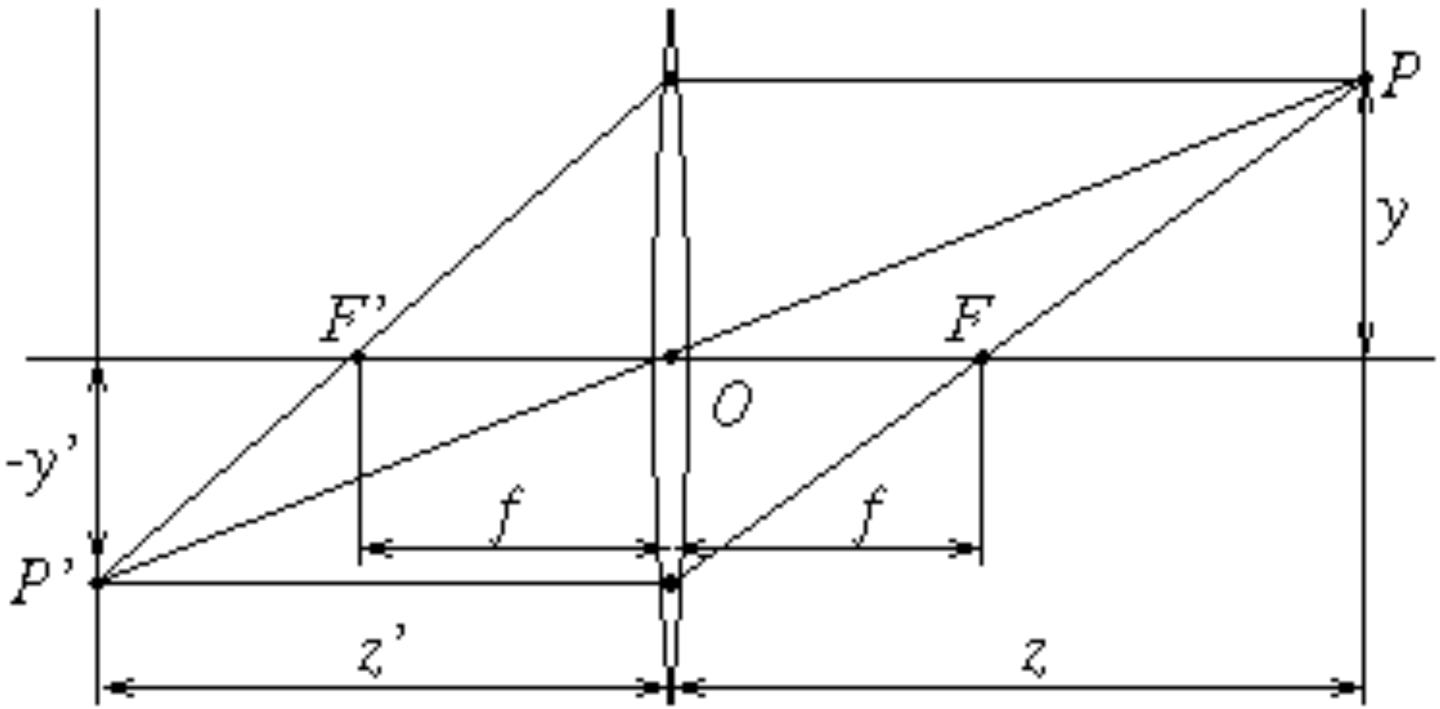
cancel h's

$$n\left(\frac{2}{R} - \frac{1}{n}\left(\frac{1}{R} + \frac{1}{d_2}\right)\right) = \frac{1}{R} + \frac{1}{d_1}$$

$$\frac{2n}{R} - \frac{1}{R} - \frac{1}{d_2} = \frac{1}{R} + \frac{1}{d_1}$$

$$\frac{2(n-1)}{R} = \frac{1}{d_1} + \frac{1}{d_2}$$ "Lens maker's formula"

The thin lens, first order optics



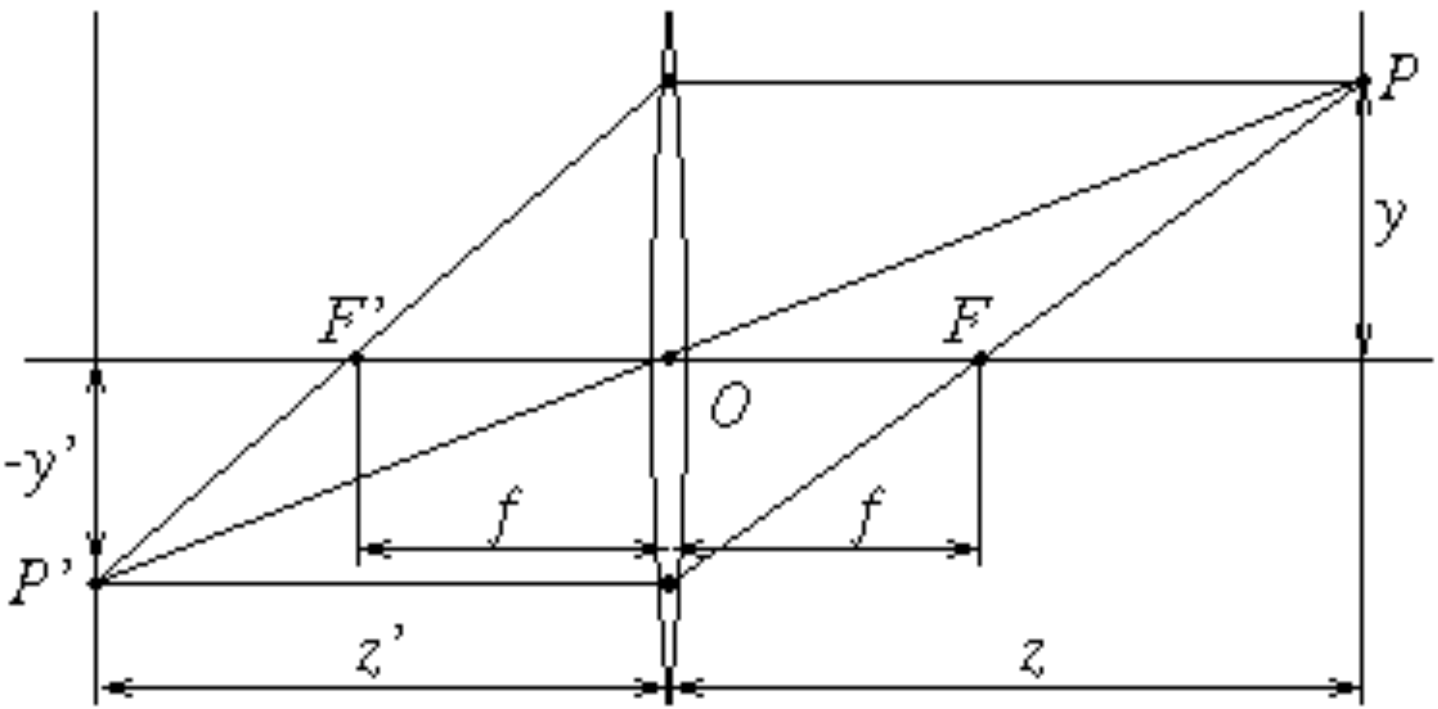
The lensmaker's equation:

$$\frac{1}{z'} + \frac{1}{z} = \frac{1}{f} \qquad f = \frac{R}{2(n-1)}$$

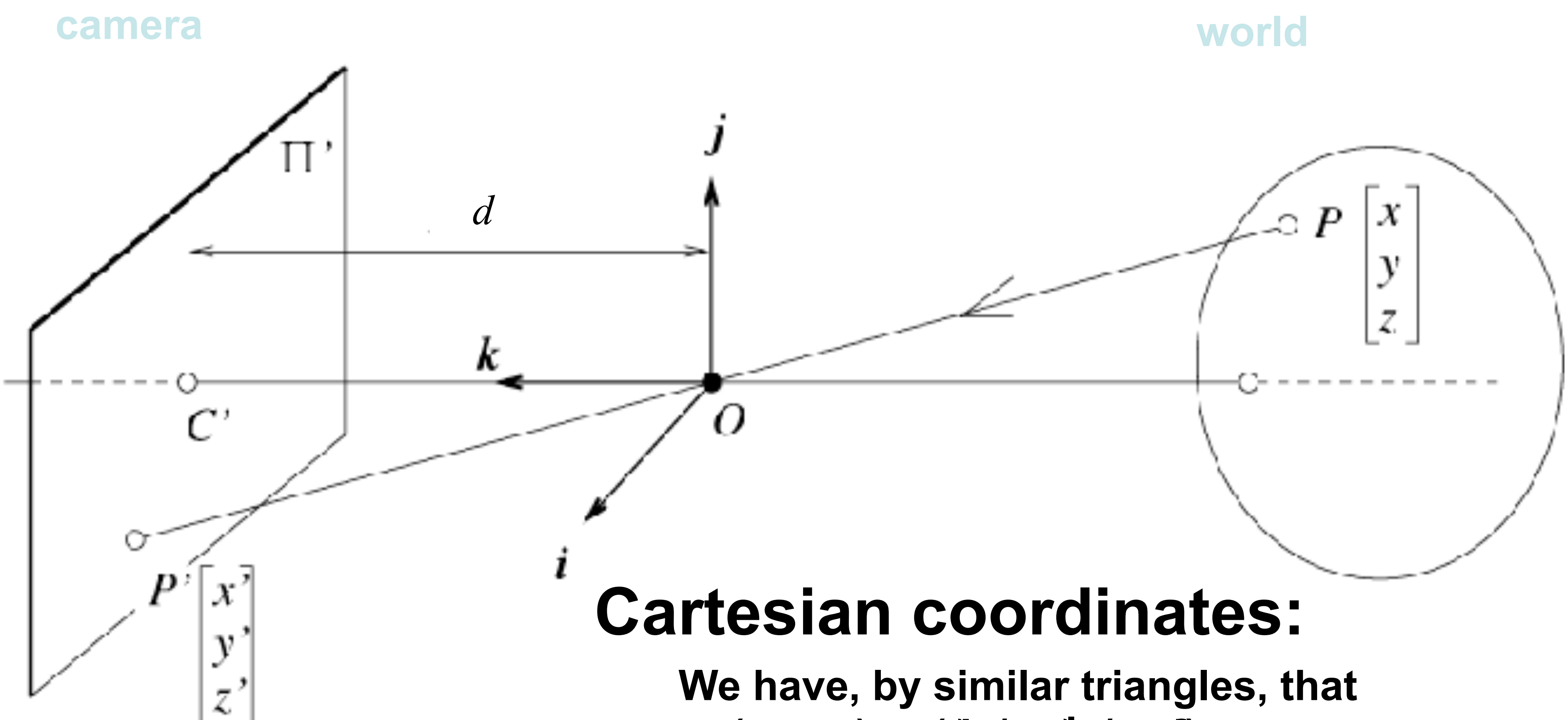
Forsyth&Ponce

How does the mapping from the 3-d world to the image plane compare for a lens and for a pinhole camera?

The perspective projection of a pinhole camera. But note that many more of the rays leaving from P arrive at P'



Perspective projection



Cartesian coordinates:

We have, by similar triangles, that

(x, y, z) -> (d'x/z, d'y/z, d')

Ignore the third coordinate, and get

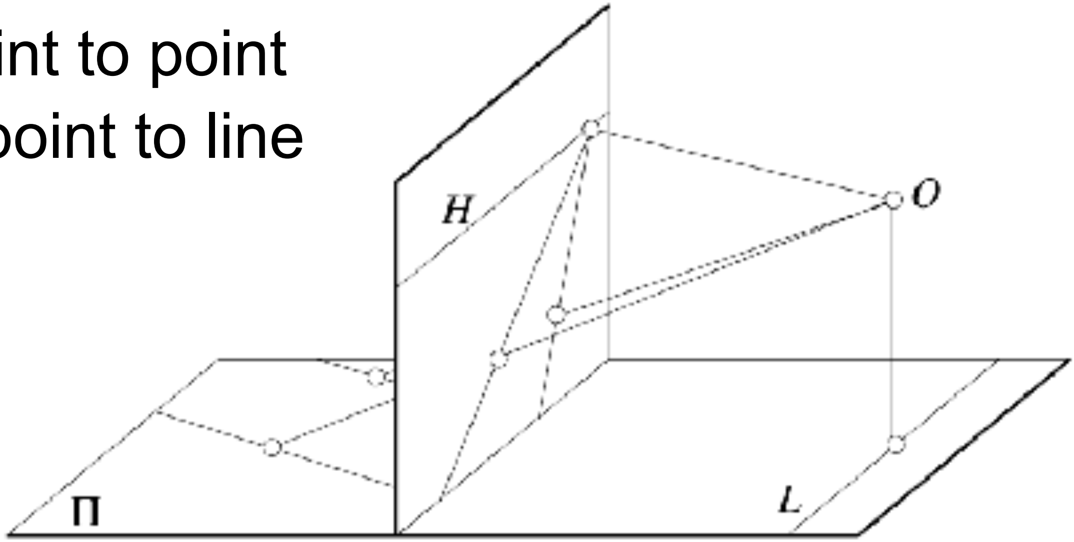
$$(x,y,z) \rightarrow (d\frac{x}{z}, d\frac{y}{z})$$

Lens demonstration

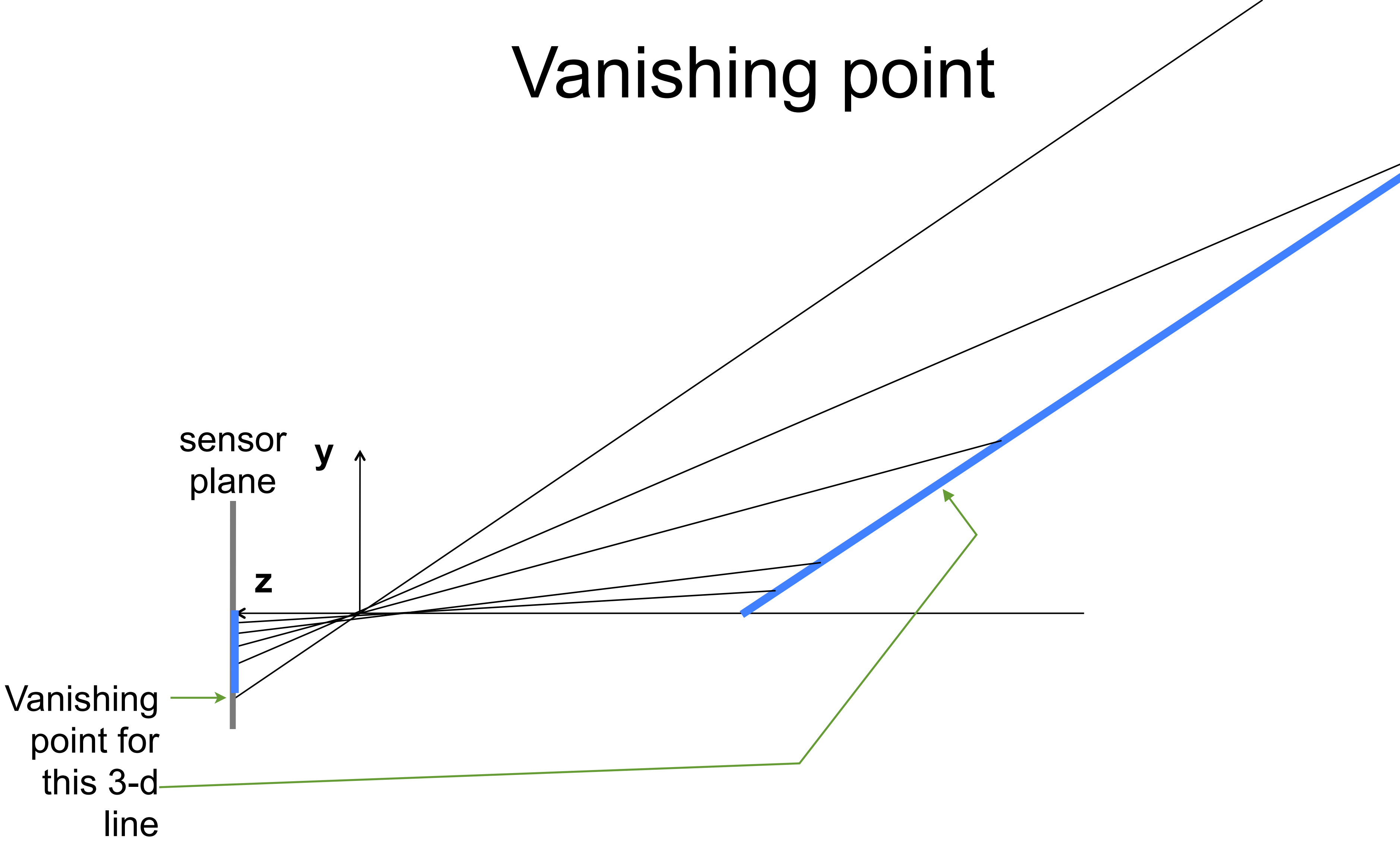
- Verify:
  - Focusing property
  - Lens maker’s equation
  - The relationship between distances in the world and distances in the sensor plane

Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image or half-planes.
- Polygons go to polygons
- Degenerate cases
  - line through focal point to point
  - plane through focal point to line



Vanishing point



Line in 3-space

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

Perspective projection of that line

$$x'(t) = \frac{fx}{z} = \frac{f(x_0 + at)}{z_0 + ct}$$

$$y'(t) = \frac{fy}{z} = \frac{f(y_0 + bt)}{z_0 + ct}$$

In the limit as $t \rightarrow \pm\infty$ **we have (for** $c \neq 0$**):**

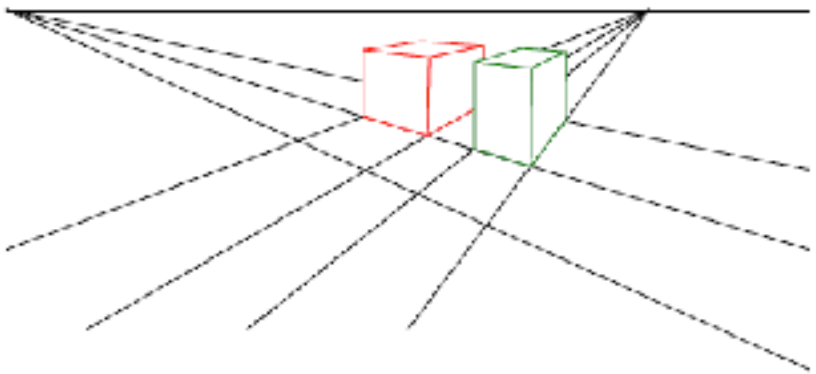
$$x'(t) \longrightarrow \frac{fa}{c}$$

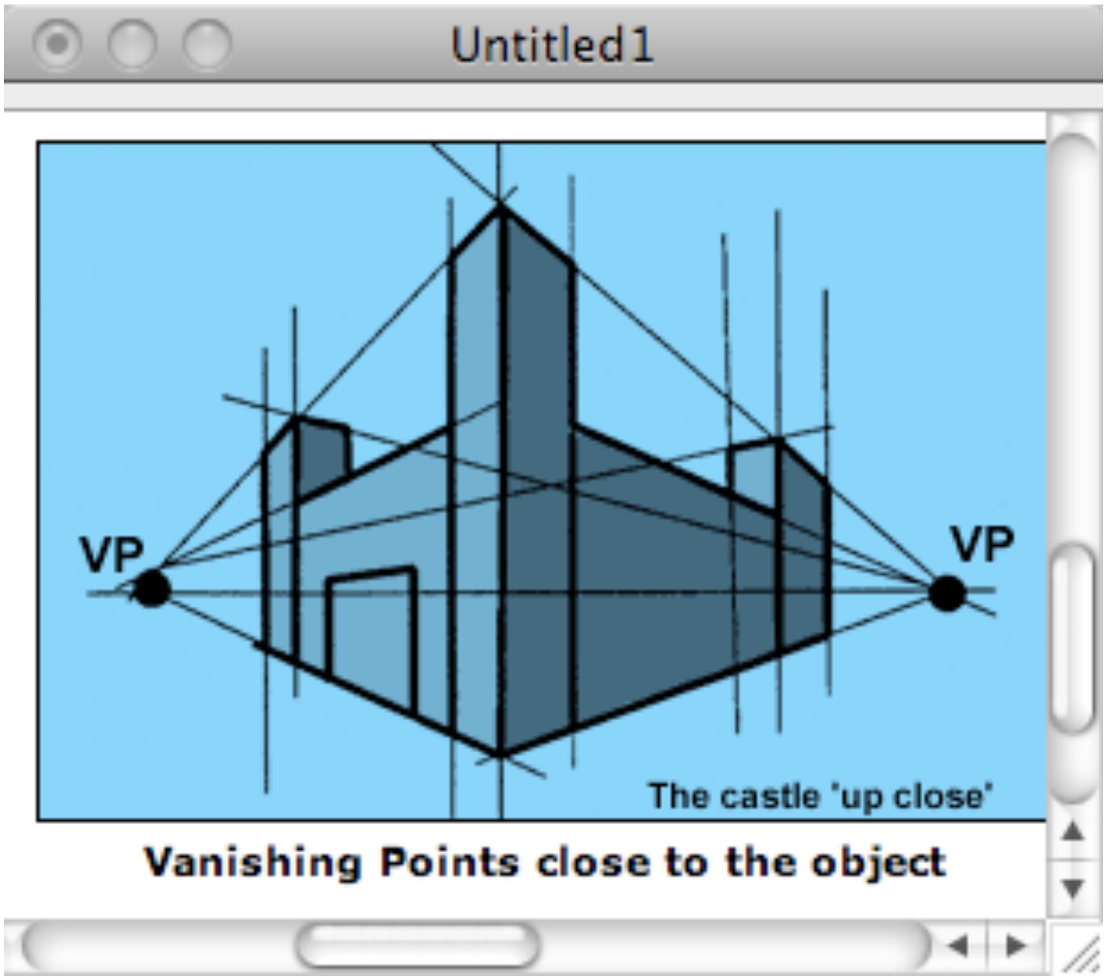
$$y'(t) \longrightarrow \frac{fb}{c}$$

This tells us that any set of parallel lines (same a, b, c parameters) project to the same point (called the vanishing point).

Vanishing points

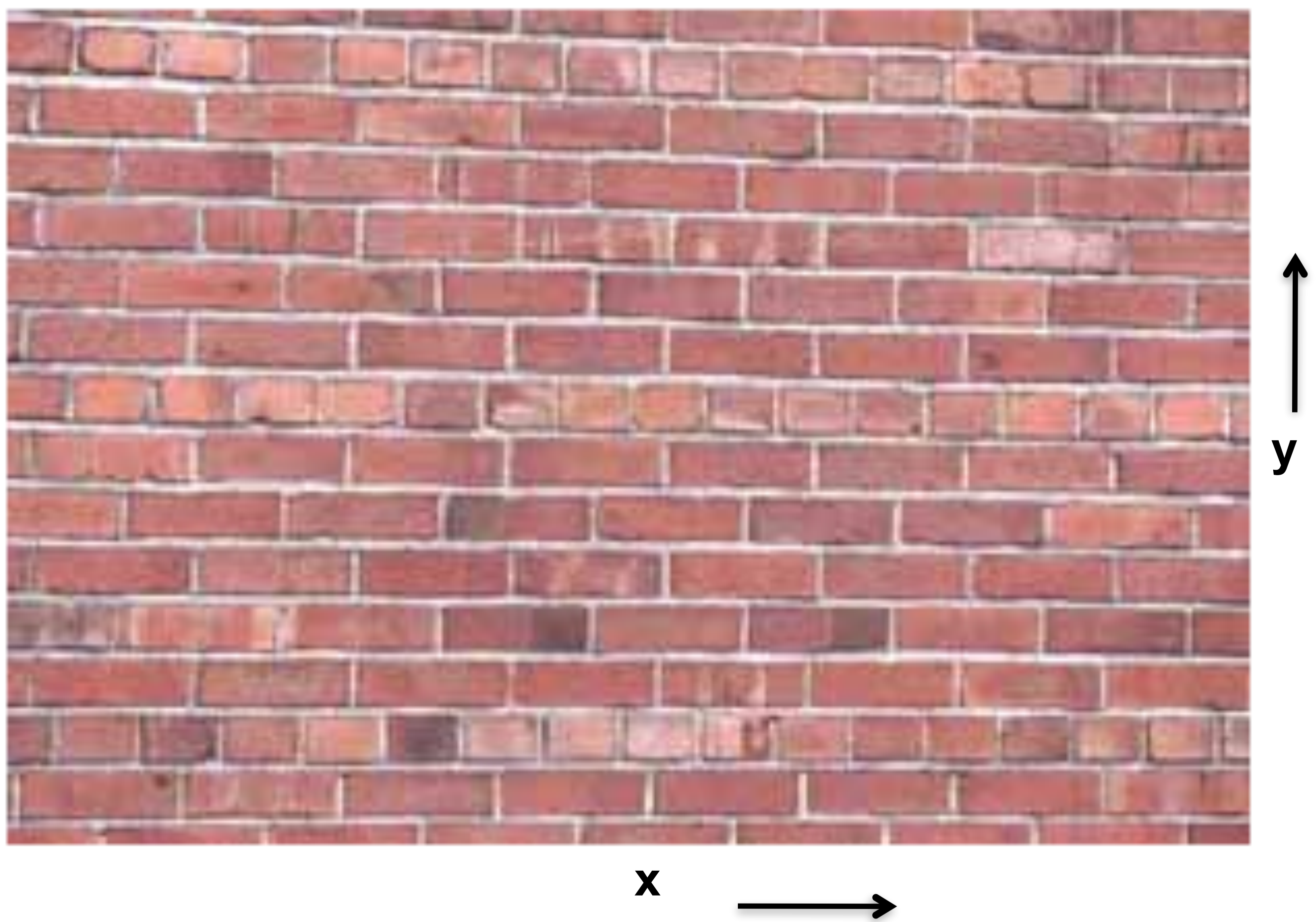
- Each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane





http://www.ider.herts.ac.uk/school/courseware/graphics/two_point_perspective.html

What if you photograph a brick wall head-on?



Brick wall line in 3-space

$$x(t) = x_0 + at$$

$$y(t) = y_0$$

$$z(t) = z_0$$

Perspective projection of that line

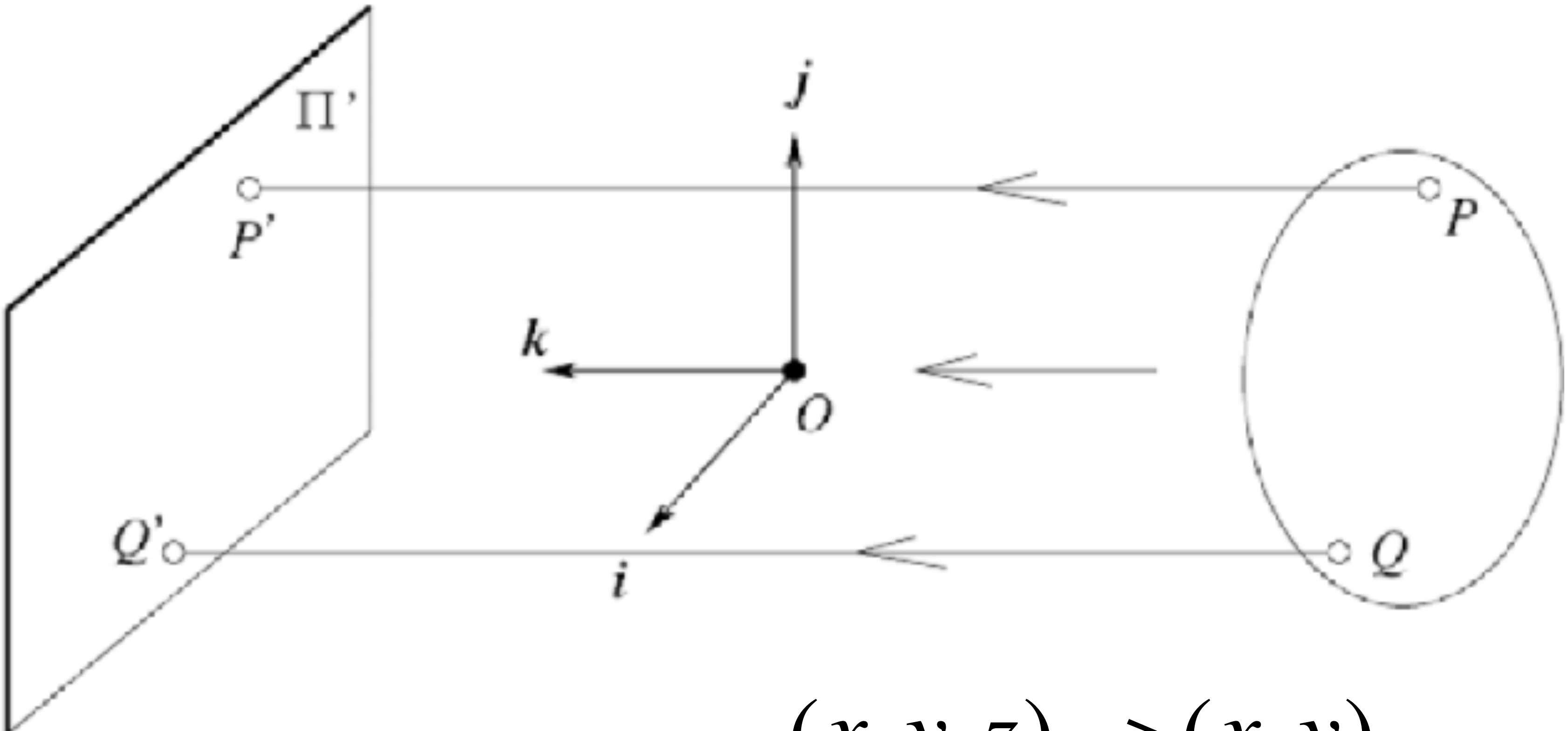
$$x'(t) = \frac{f \cdot (x_0 + at)}{z_0}$$

$$y'(t) = \frac{f \cdot y_0}{z_0}$$

All bricks have same z_0. Those in same row have same y_0

Thus, a brick wall, photographed head-on, gets rendered as set of parallel lines in the image plane.

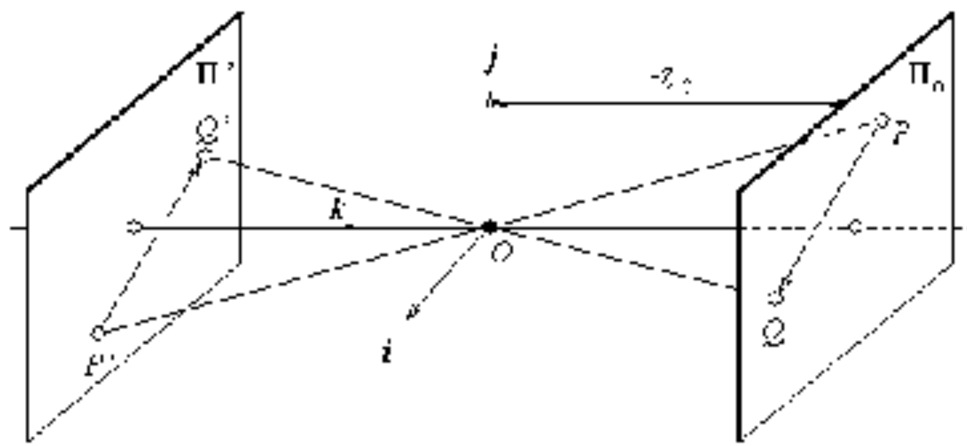
Other projection models: Orthographic projection



$$(x, y, z) \rightarrow (x, y)$$

Other projection models: Weak perspective

- Issue
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: only approximate



$$(x, y, z) \rightarrow \left(\frac{fx}{z_0}, \frac{fy}{z_0} \right)$$

Three camera projections

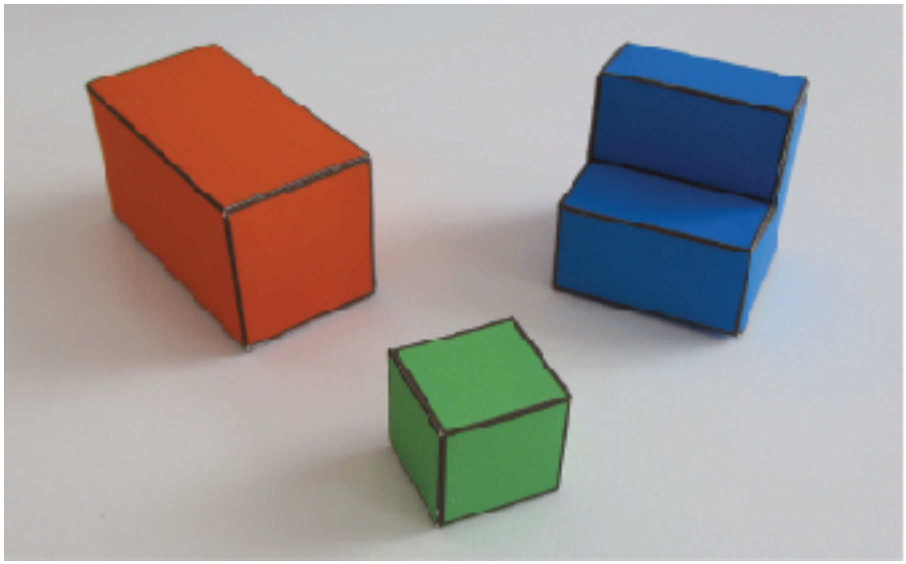
3-d point **2-d image position**

(1) Perspective: $(x, y, z) \rightarrow \left(\frac{fx}{z}, \frac{fy}{z} \right)$

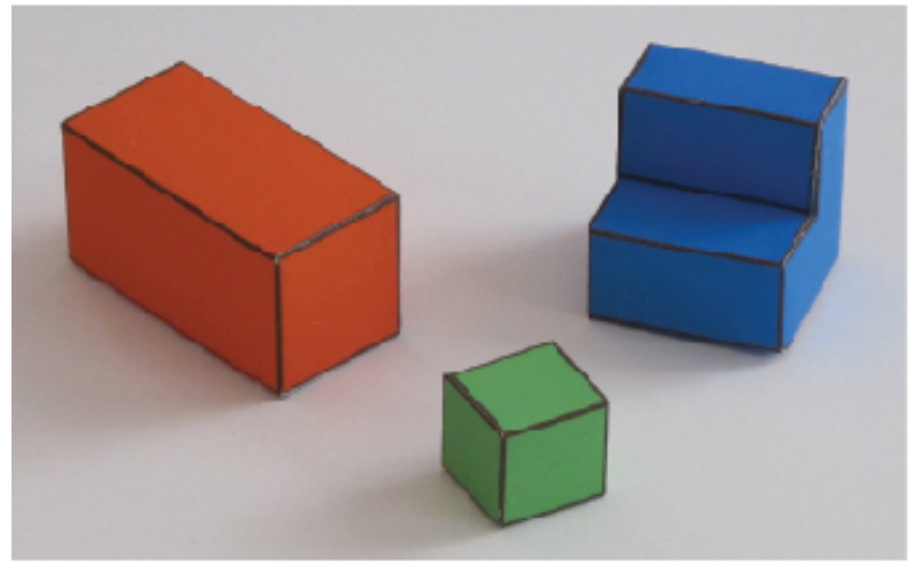
(2) Weak perspective: $(x, y, z) \rightarrow \left(\frac{fx}{z_0}, \frac{fy}{z_0} \right)$

(3) Orthographic: $(x, y, z) \rightarrow (x, y)$

Two of those camera projections



Perspective projection



Parallel (orthographic) projection

Weak perspective?

More accurate models of real lenses

- Finite lens thickness
- Higher order approximation to $\sin(\theta)$
- Chromatic aberration
- Vignetting

Thick lens

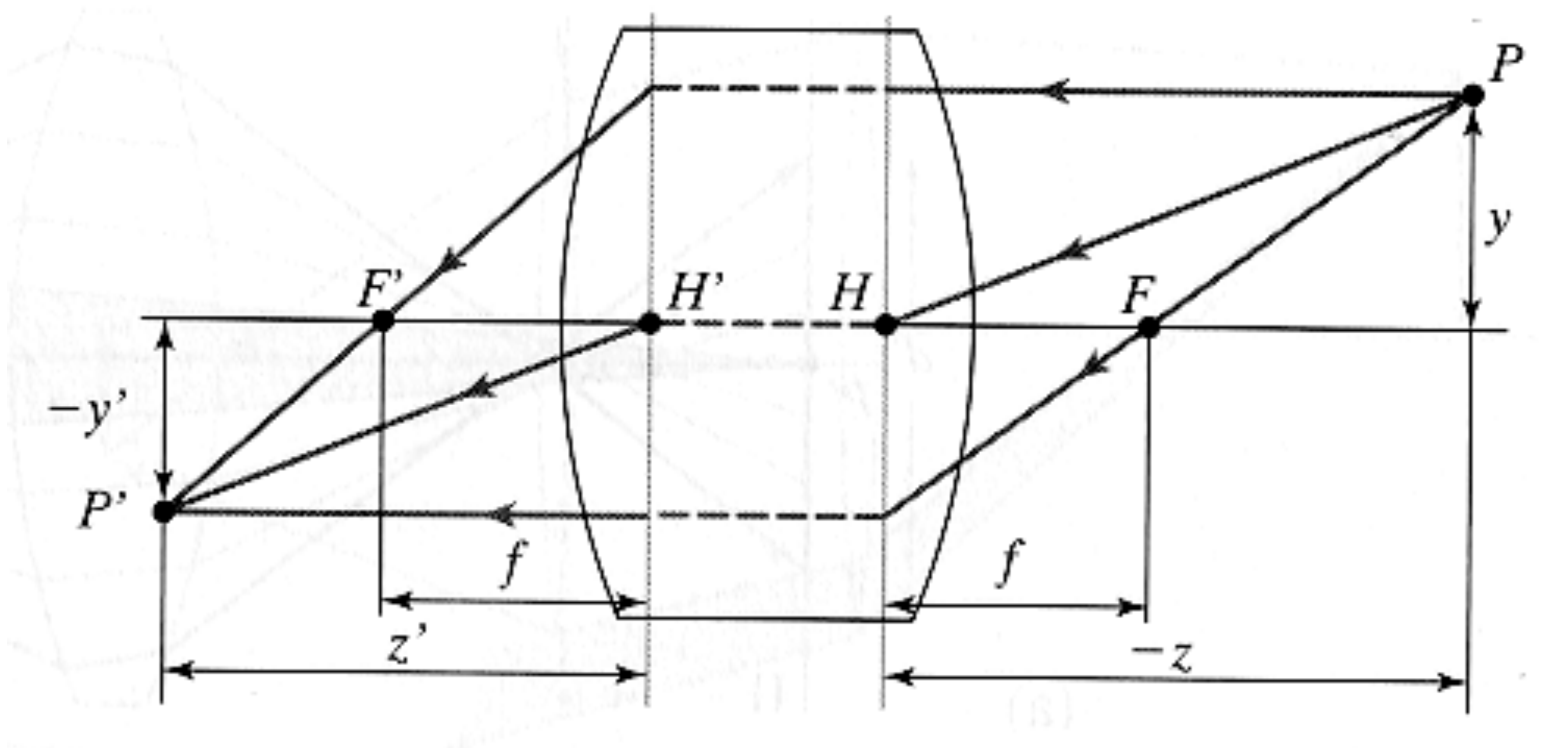
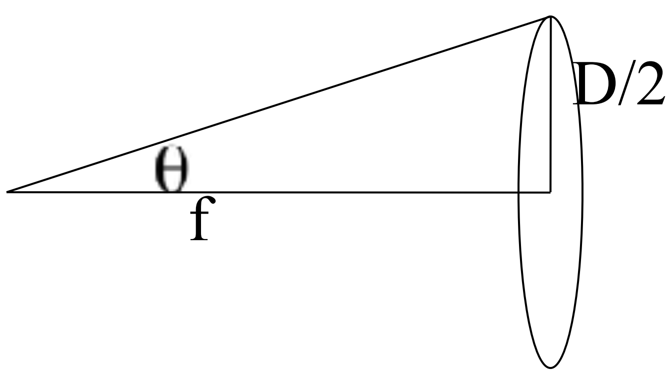


Figure 1.11 A simple thick lens with two spherical surfaces.

Forsyth&Ponce

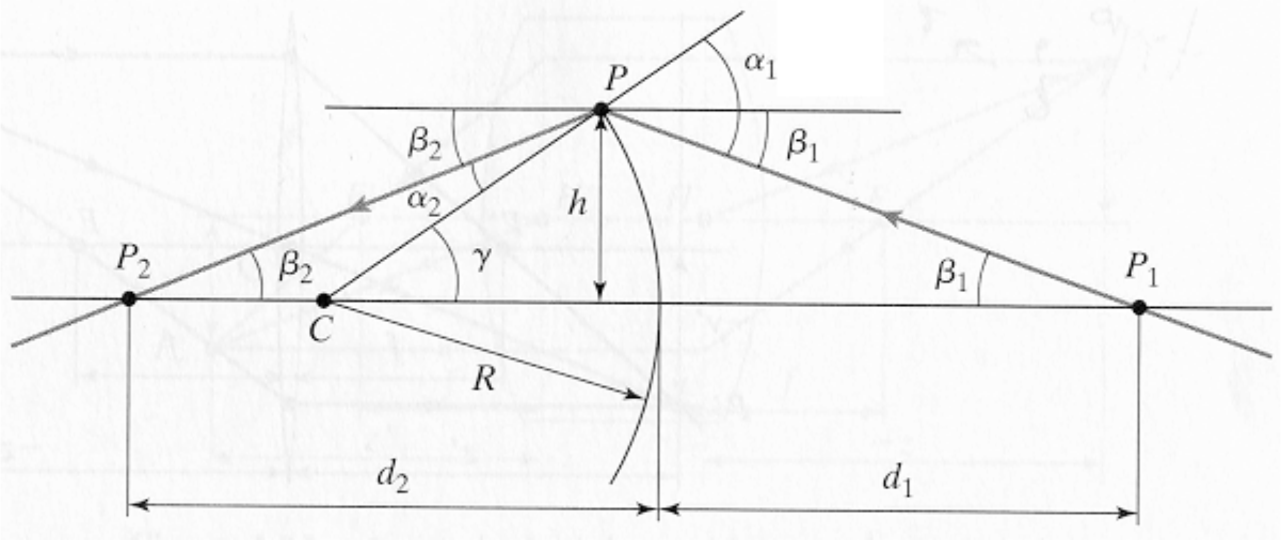
Third order optics

$$\sin(\theta) \approx \theta - \frac{\theta^3}{6}$$



$$\theta \approx \frac{D/2}{f} - \frac{\left(\frac{D/2}{f}\right)^3}{6}$$

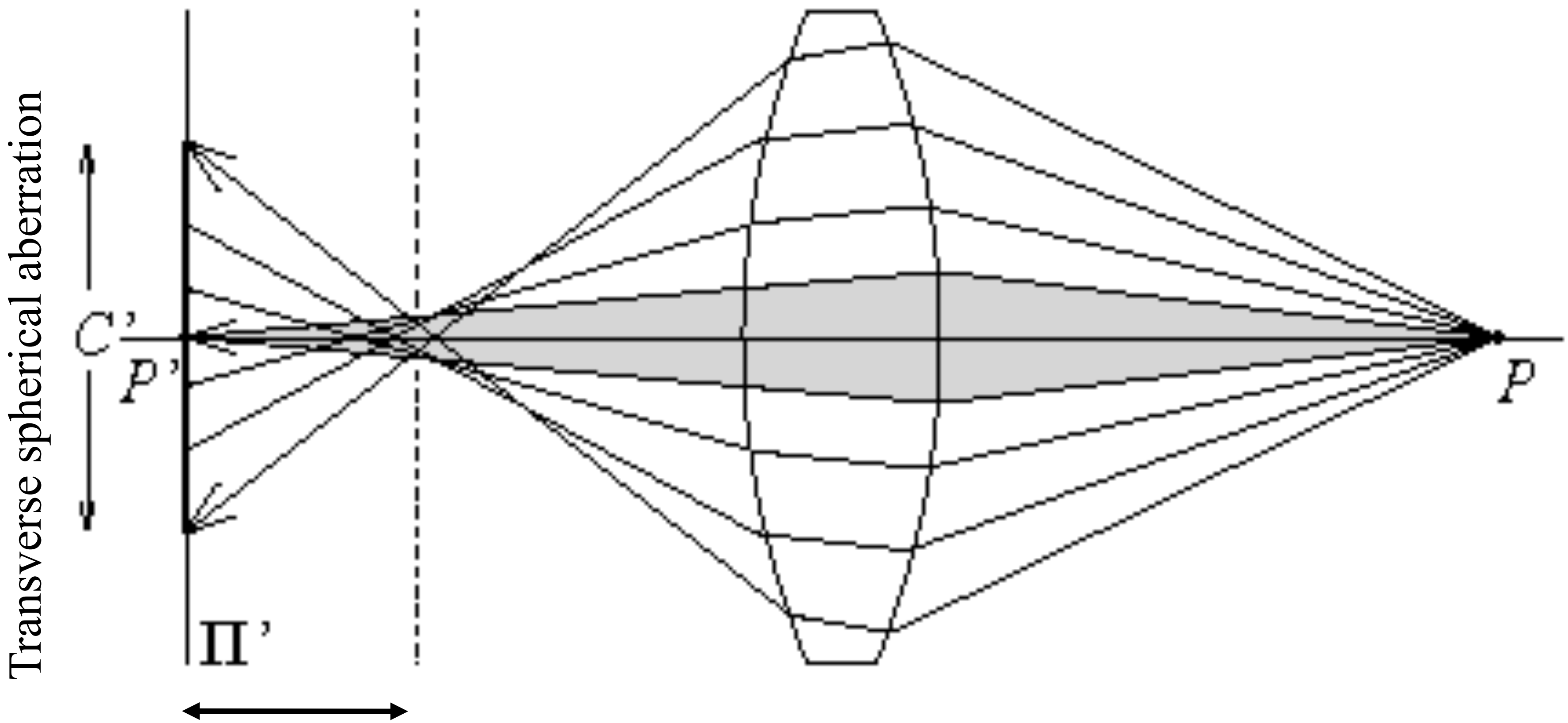
Paraxial refraction equation, 3rd order optics



$$\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R} + h^2 \left[\frac{n_1}{2d_1}\left(\frac{1}{R} + \frac{1}{d_1}\right)^2 + \frac{n_2}{2d_2}\left(\frac{1}{R} - \frac{1}{d_2}\right)^2 \right]$$

Forsyth&Ponce

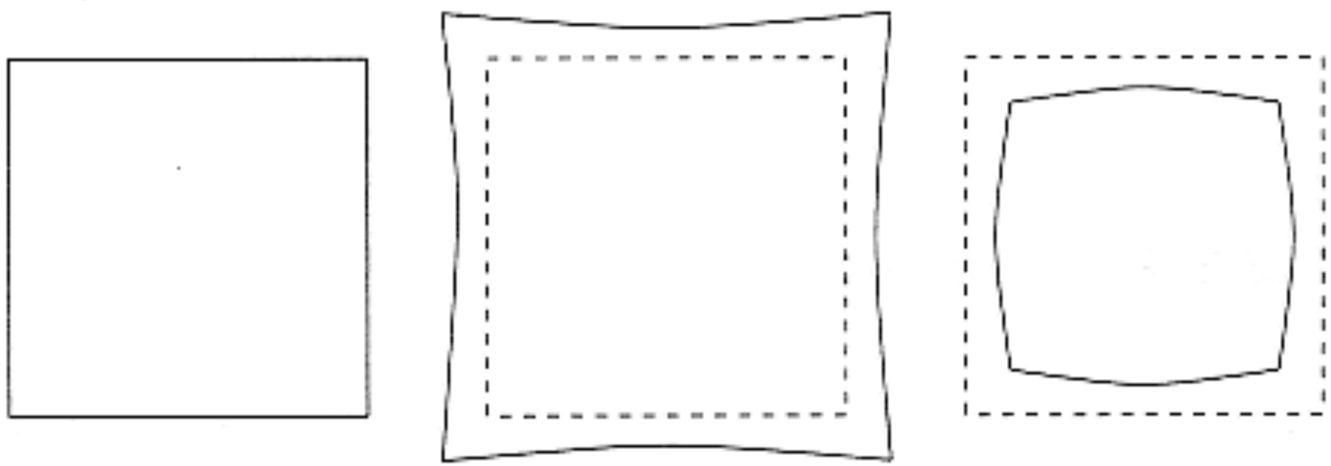
Spherical aberration (from 3rd order optics



Forsyth&Ponce

Other 3rd order effects

- Coma, astigmatism, field curvature, distortion.



Forsyth&Ponce

Chromatic aberration

(desirable for prisms, bad for lenses)

Other (possibly annoying) phenomena

- Chromatic aberration
  - Light at different wavelengths follows different paths; hence, some wavelengths are defocussed
  - Machines: coat the lens
  - Humans: live with it
- Scattering at the lens surface
  - Some light entering the lens system is reflected off each surface it encounters (Fresnel’s law gives details)
  - Machines: coat the lens, interior
  - Humans: live with it (various scattering phenomena are visible in the human eye)

Summary

- Want to make images
- Pinhole camera models the geometry of perspective projection
- Lenses make it work in practice
- Models for lenses
  - Thin lens, spherical surfaces, first order optics
  - Thick lens, higher-order optics, vignetting.