MIT CSAIL

6.869: Advances in Computer Vision

Antonio Torralba & William T. Freeman

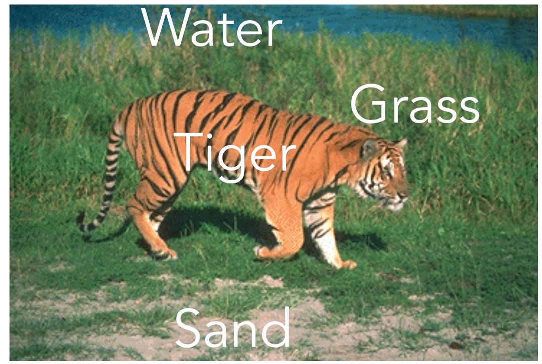
MIT COMPUTER VISION

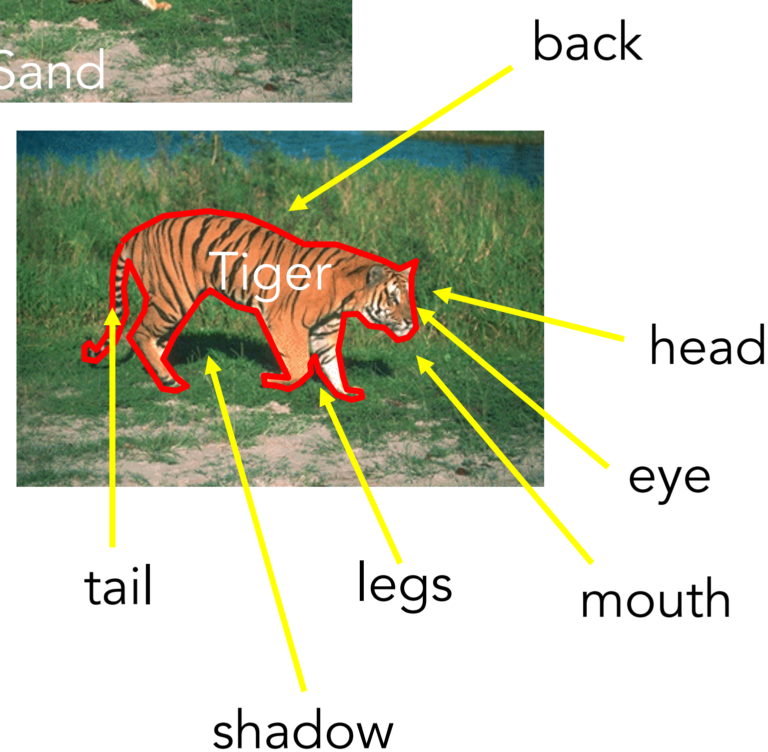
Lecture 15

Edges and segmentation

From Pixels to Perception:

Mid-level operations of Segmentation and Grouping





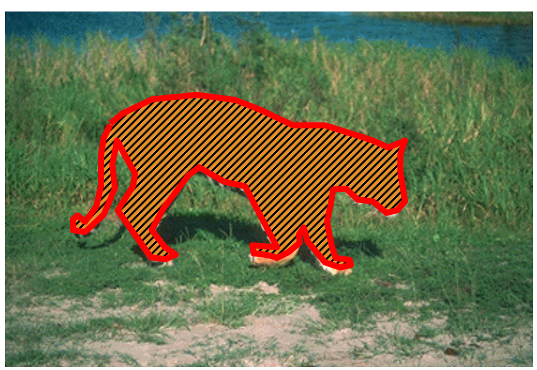


Figure / Ground

Finding groups of pixels that go together (parts, objects, textures, holes)

Predicted scene categories:

forest - broadleaf (0.498), swimming hole (0.402), bayou (0.062)

Predicted scene categories:

forest - broadleaf (0.979)

Figure / Ground

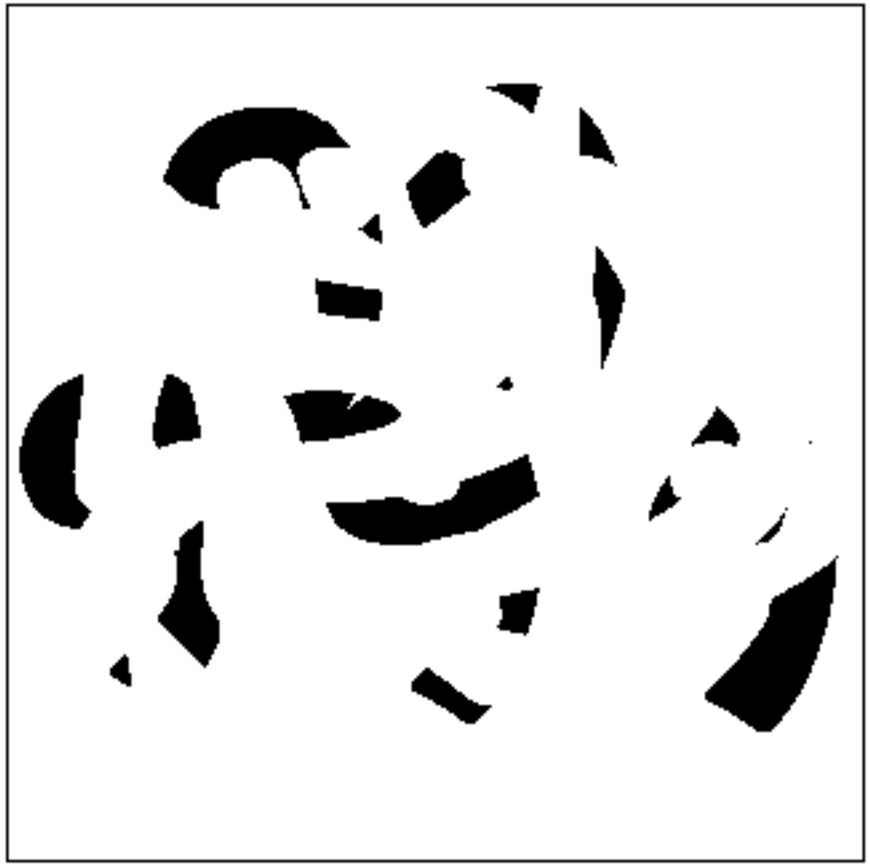
http://twistedsifter.com/2015/03/mind-bending-optical-illusion-paintings-by-rob-gonsalves/

A “simple” segmentation problem

It can get a lot harder

Brady, M. J., & Kersten, D. (2003). Bootstrapped learning of novel objects. J Vis, 3(6), 413-422

Segmentation is a global process



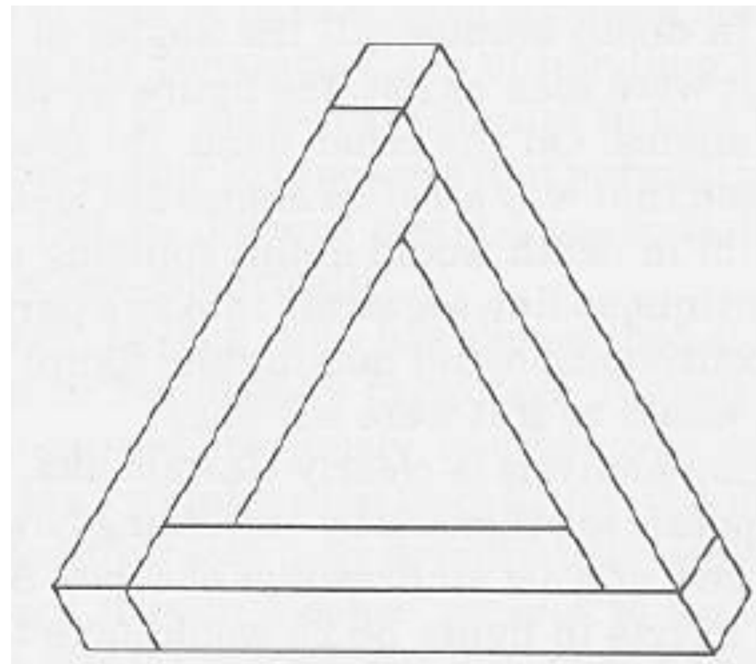
What are the occluded numbers?

Segmentation is a global process

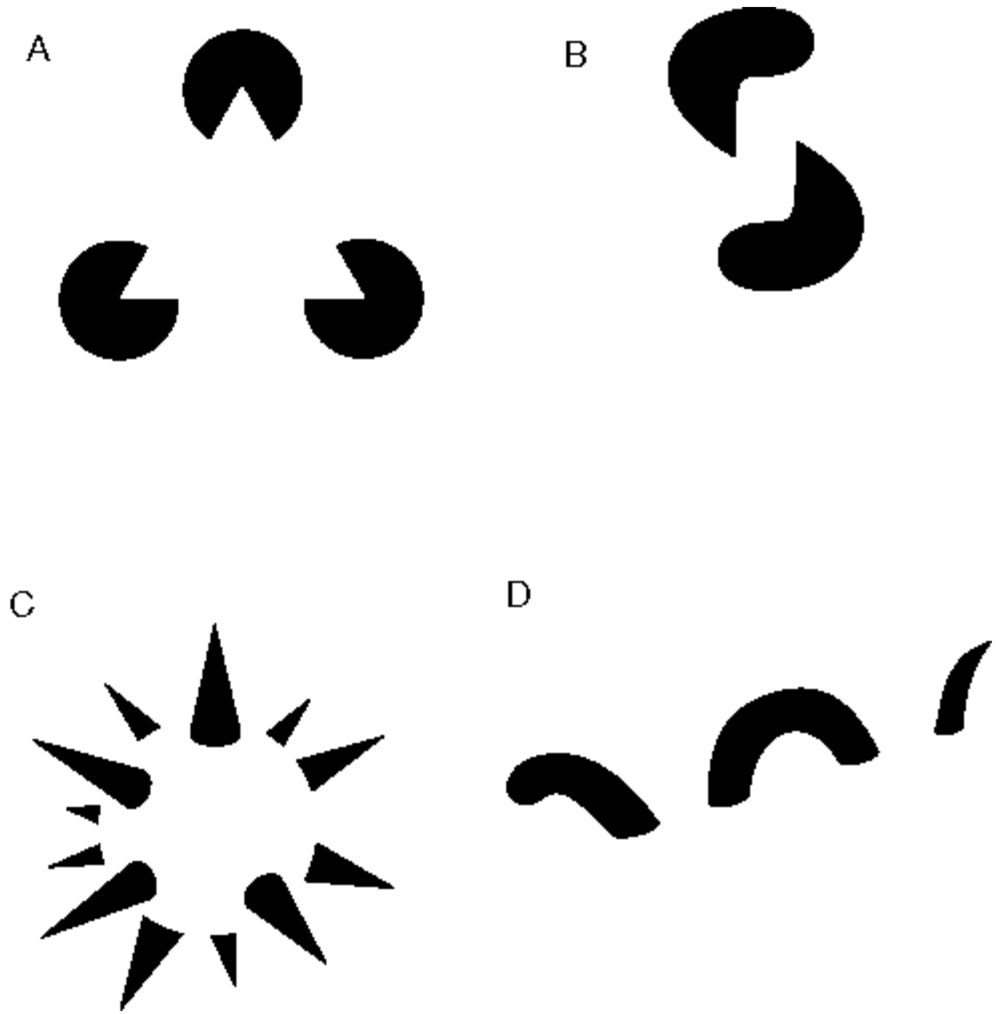
What are the occluded numbers?

Occlusion is an important cue in grouping.

… but not too global



Groupings by Invisible Completions



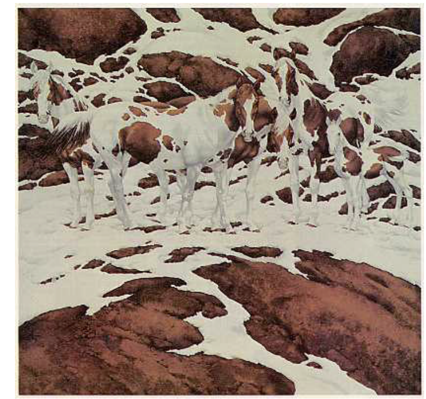
* Images from Steve Lehar's Gestalt papers

Emergence

http://en.wikipedia.org/wiki/Gestalt_psychology

Perceptual organization

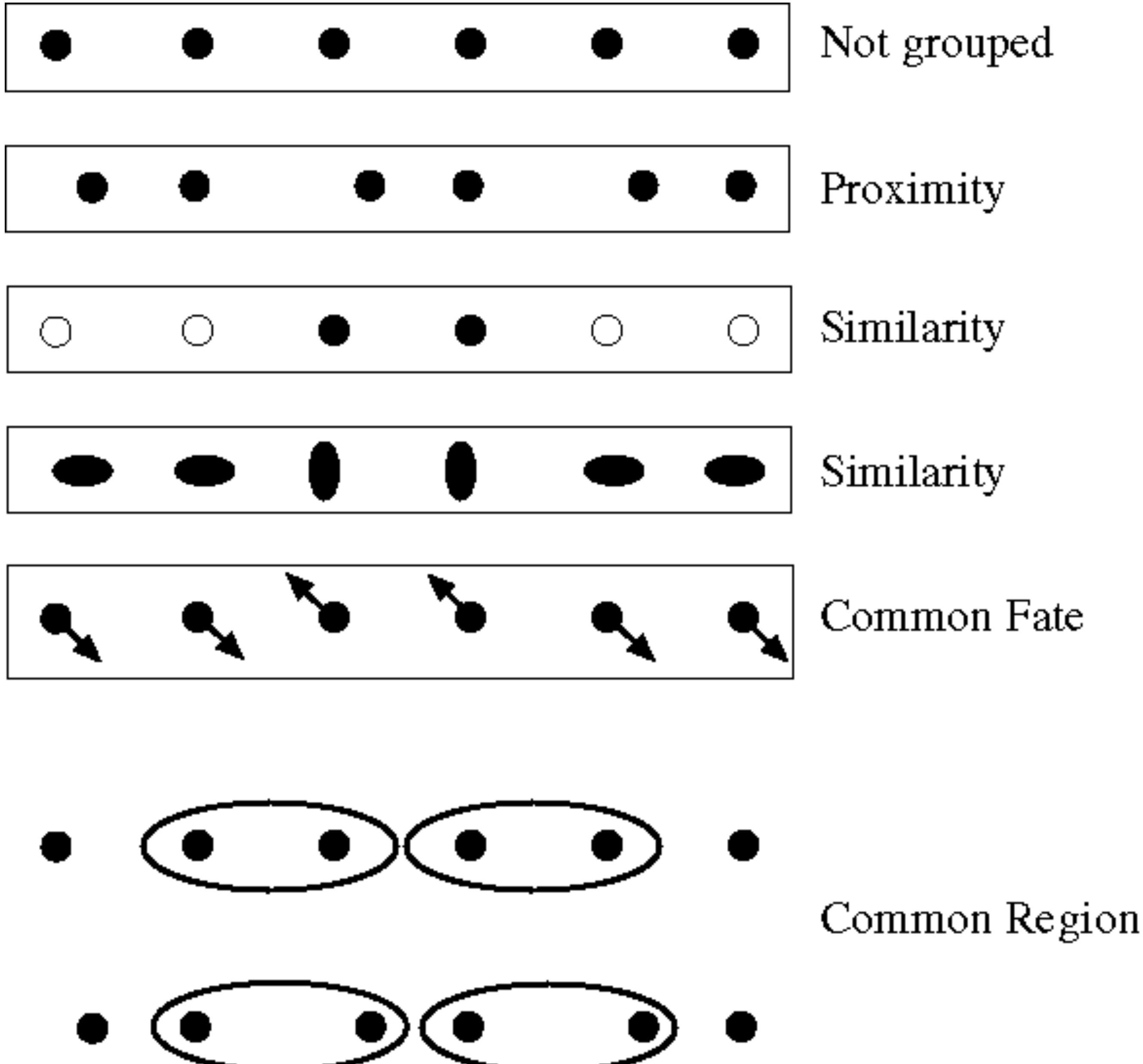
"…the processes by which the bits and pieces of visual information that are available in the retinal image are structured into the larger units of perceived objects and their interrelations"

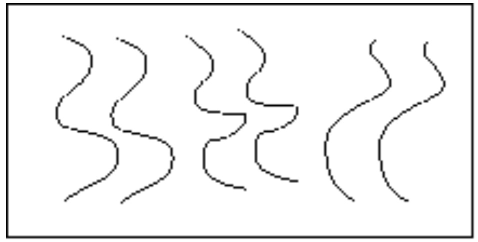


Stephen E. Palmer, *Vision Science, 1999*

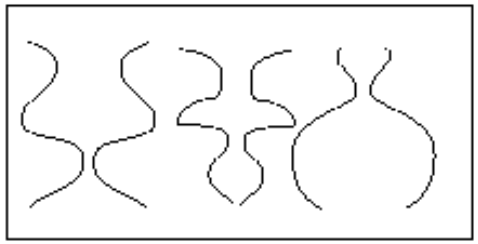
Gestalt principles

There are hundreds of different grouping laws

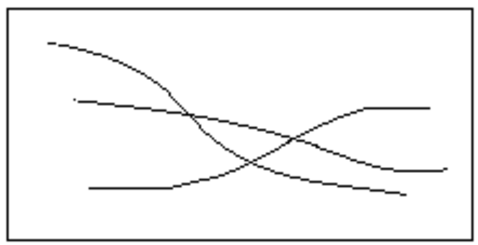

Not grouped
Proximity
Similarity
Similarity
Common Fate
Common Region



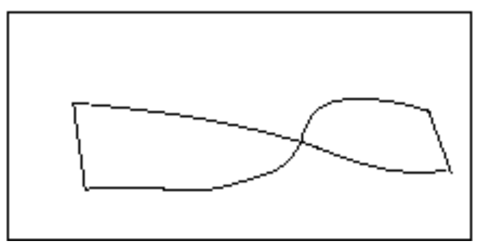
Parallelism



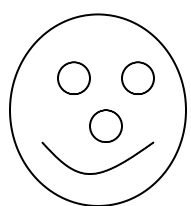
Symmetry



Continuity

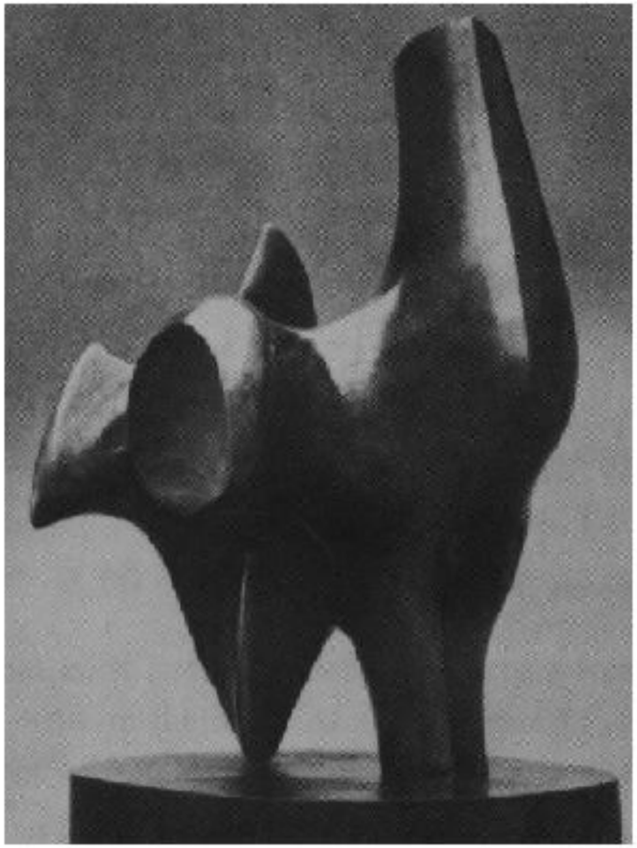


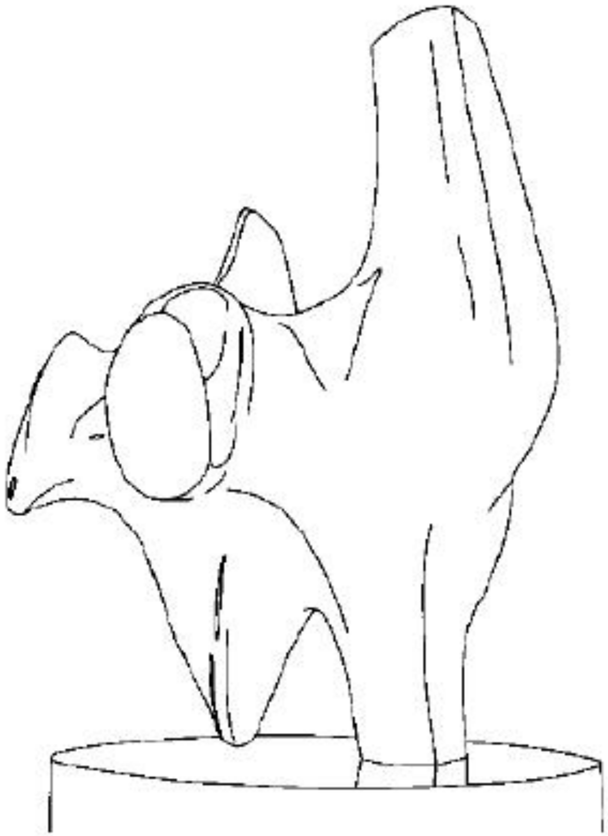
Closure



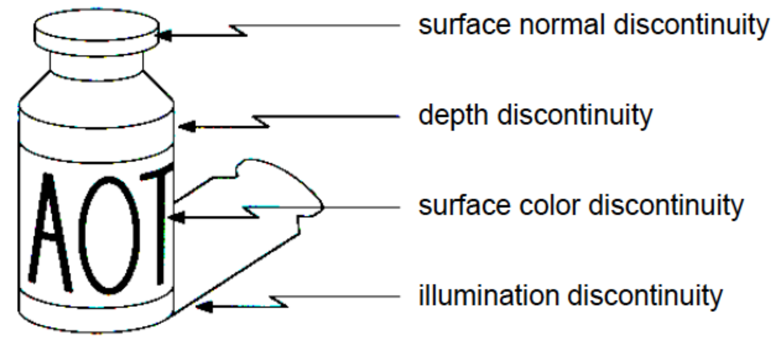
Familiar configuration

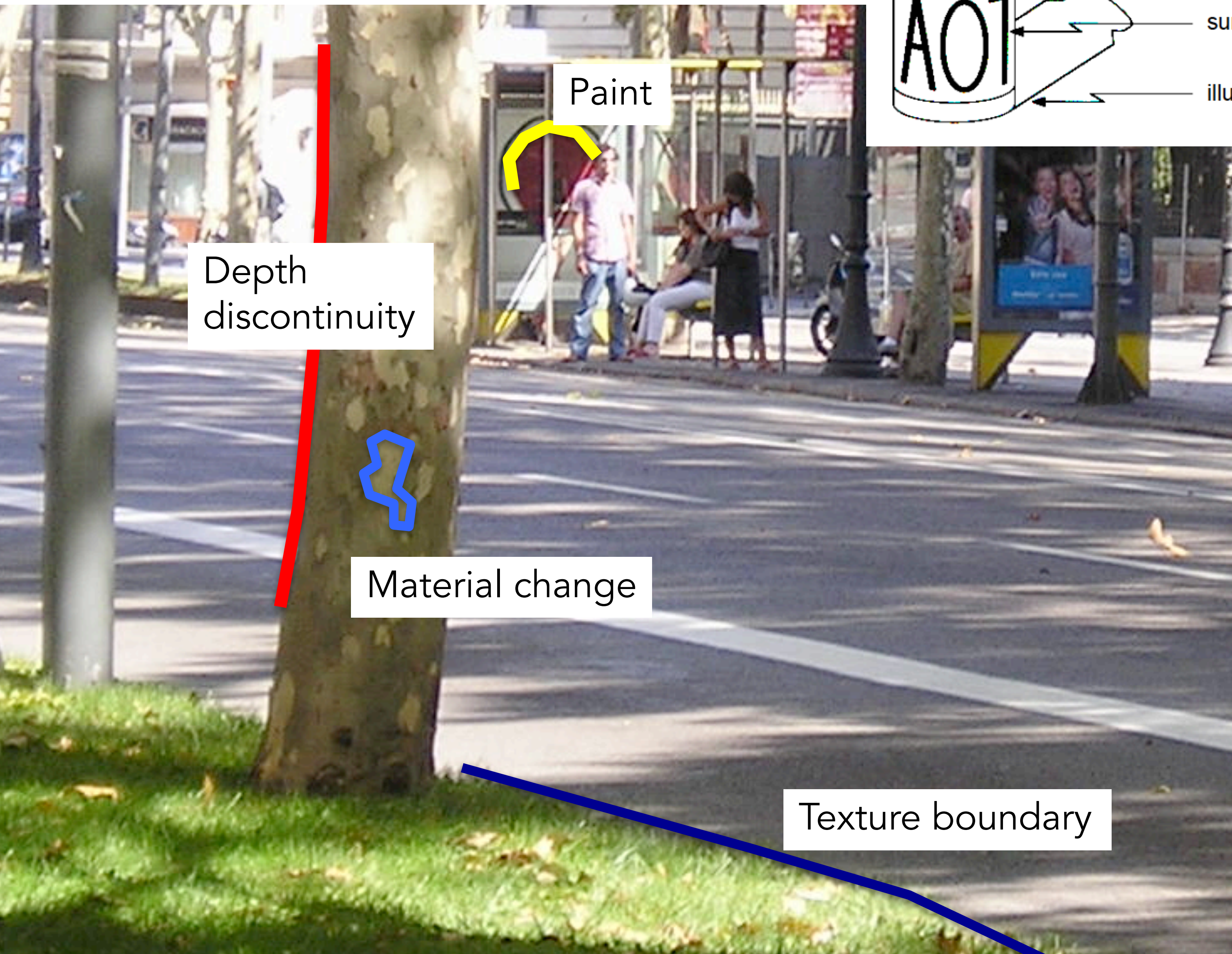
I. Edges



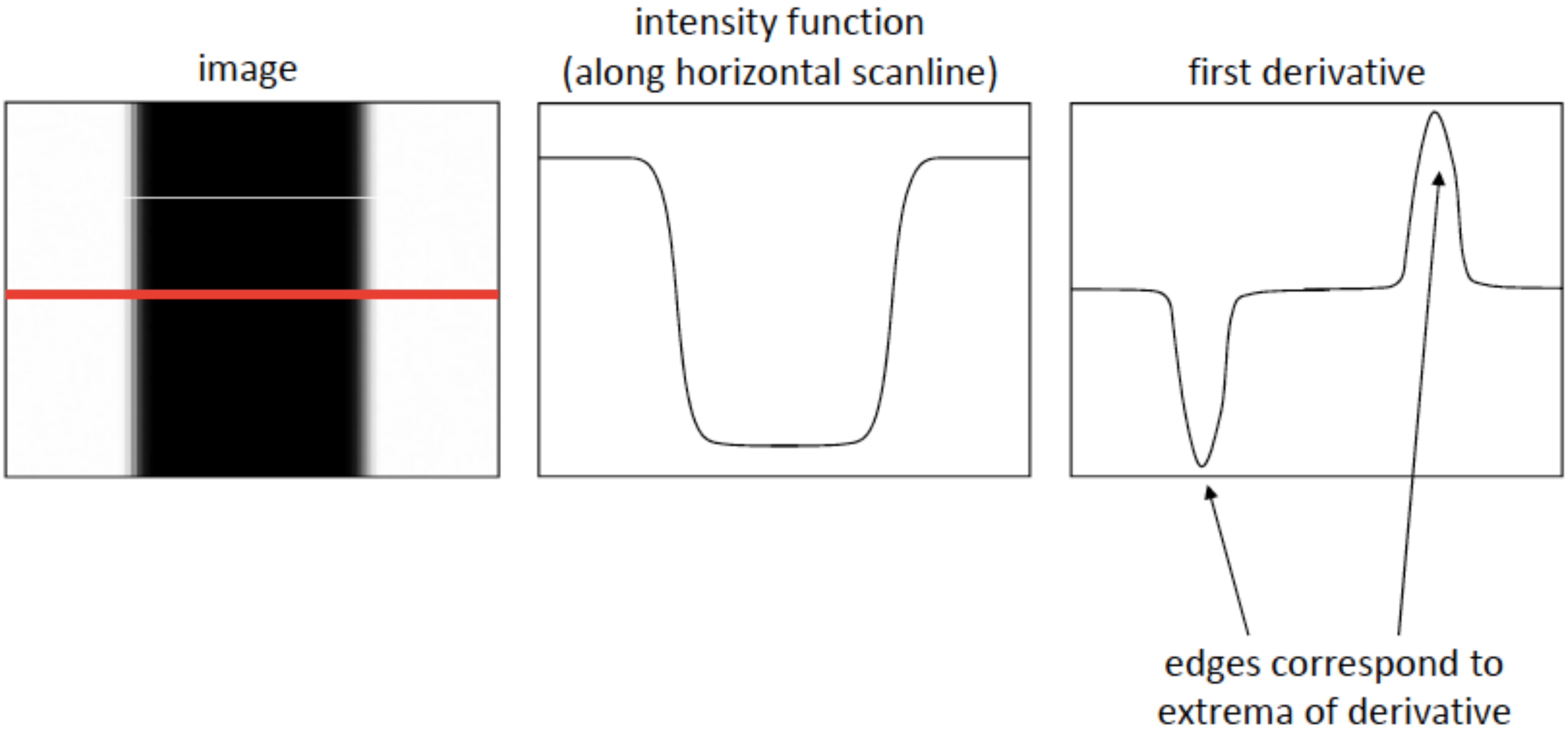


What is an edge?





Finding edges: Computing derivatives



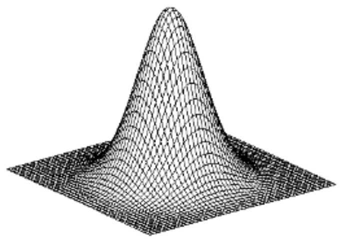
Canny edge detector

edge(image,'canny')

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient

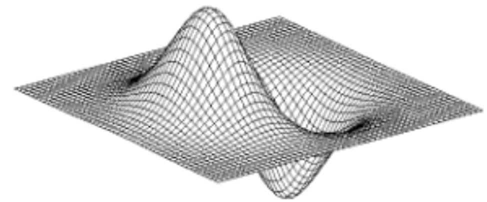
3. Non-maximum suppression
4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

1: Filter Image with derivatives of Gaussian
2D edge detection filters



Gaussian

$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



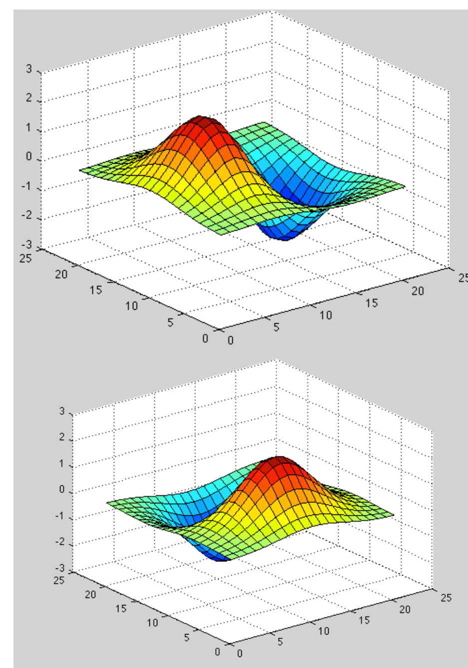
derivative of Gaussian (x)

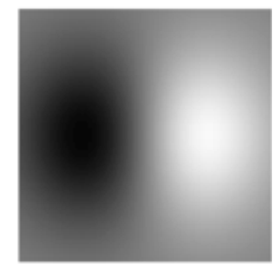
$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

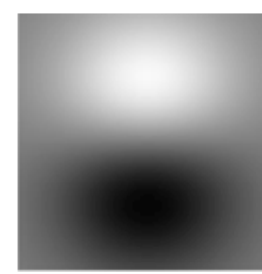
$$h_x(x,y) = \frac{\partial h(x,y)}{\partial x} = \frac{-x}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$h_y(x,y) = \frac{\partial h(x,y)}{\partial x} = \frac{-x}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

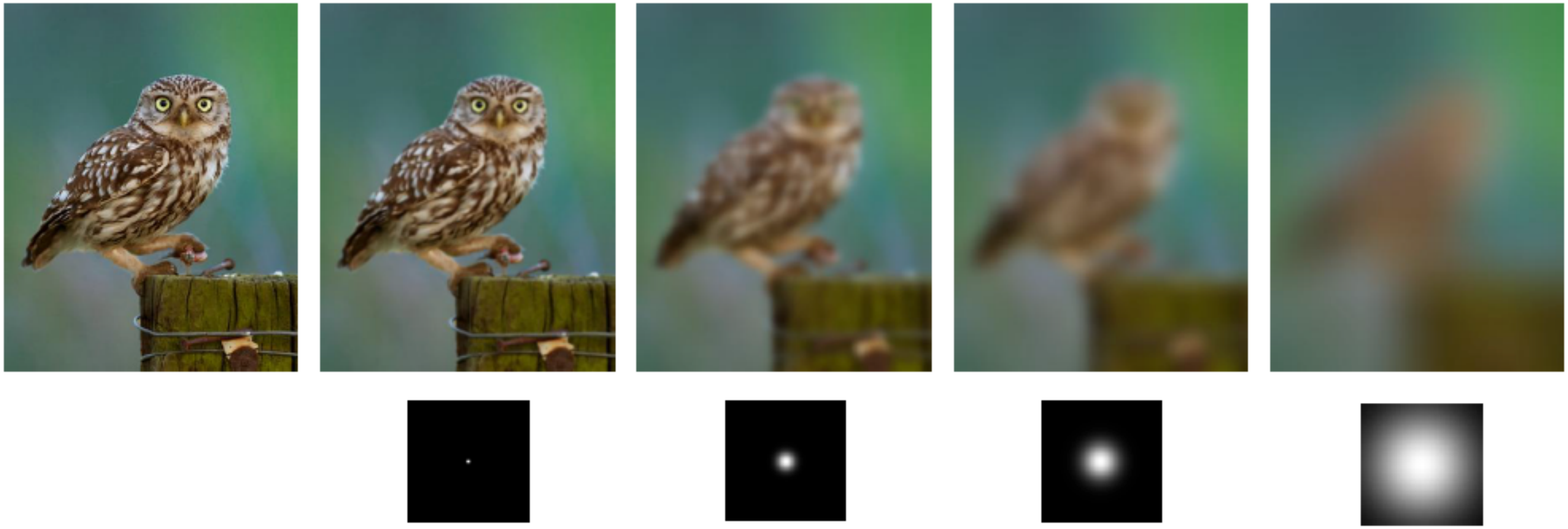
Scale



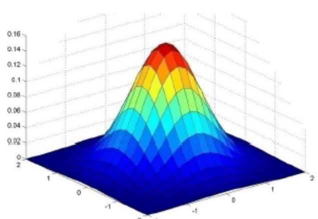


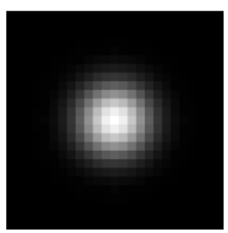


Gaussian filters



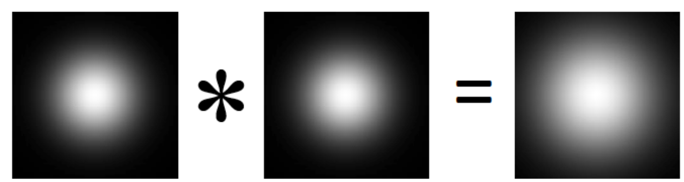
σ = 1 pixel σ = 5 pixels σ = 10 pixels σ = 30 pixels





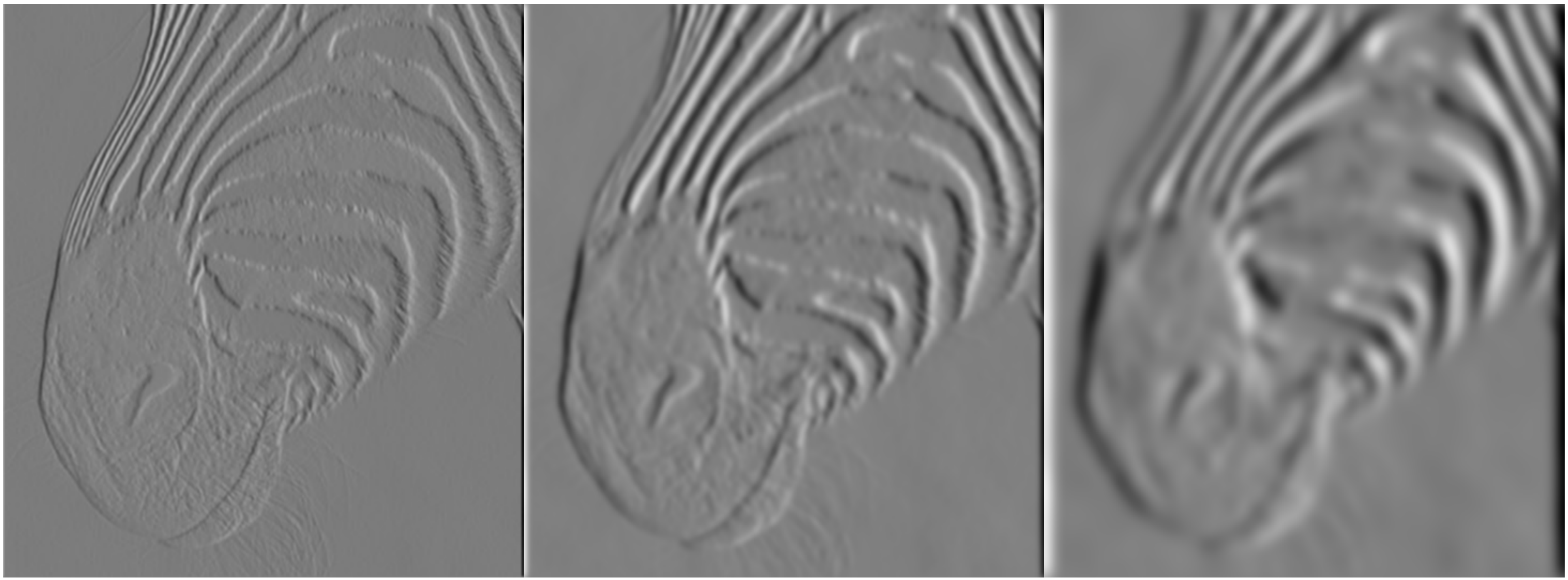
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Convolution with self is another Gaussian



– Convolving two times with Gaussian kernel of width σ = convolving once with kernel of width $\sigma\sqrt{2}$

1: Filter Image with derivatives of Gaussian
2D edge detection filters



1 pixel 3 pixels 7 pixels

Smoothing filters with different scales

The Sobel Operator: A common approximation of derivative of gaussian

- Common approximation of derivative of Gaussian

$$s_x = \frac{1}{8}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline\end{array} \qquad s_y = \frac{1}{8}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline\end{array}$$

- The standard defn. of the Sobel operator omits the 1/8 term
  - doesn't make a difference for edge detection
  - the 1/8 term **is** needed to get the right gradient value

Canny edge detector

edge(image,'canny')

1. Filter image with derivative of Gaussian

2. Find magnitude and orientation of gradient

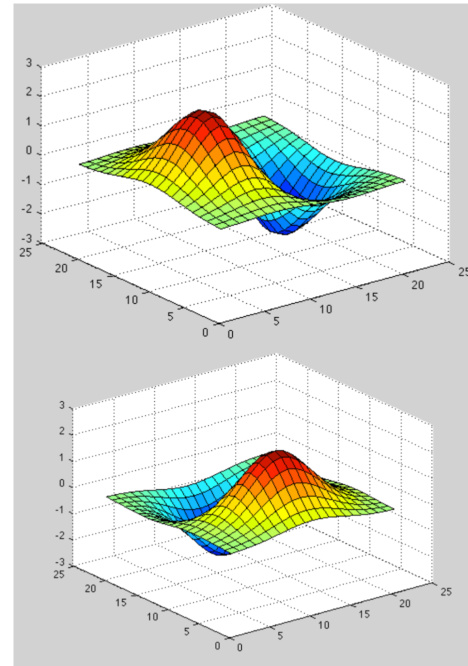
3. Non-maximum suppression

4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

2: Gradient: Find edge strength (magnitude) and direction (angle) of gradient

$$h_x(x,y) = \frac{\partial h(x,y)}{\partial x} = \frac{-x}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$h_y(x,y) = \frac{\partial h(x,y)}{\partial x} = \frac{-x}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



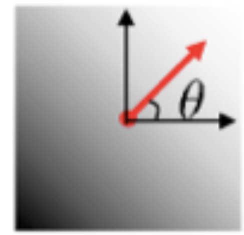
Magnitude: $h_x(x,y)^2 + h_y(x,y)^2$ *Edge strength*

Angle: $\arctan\left(\frac{h_y(x,y)}{h_x(x,y)}\right)$ *Edge normal*

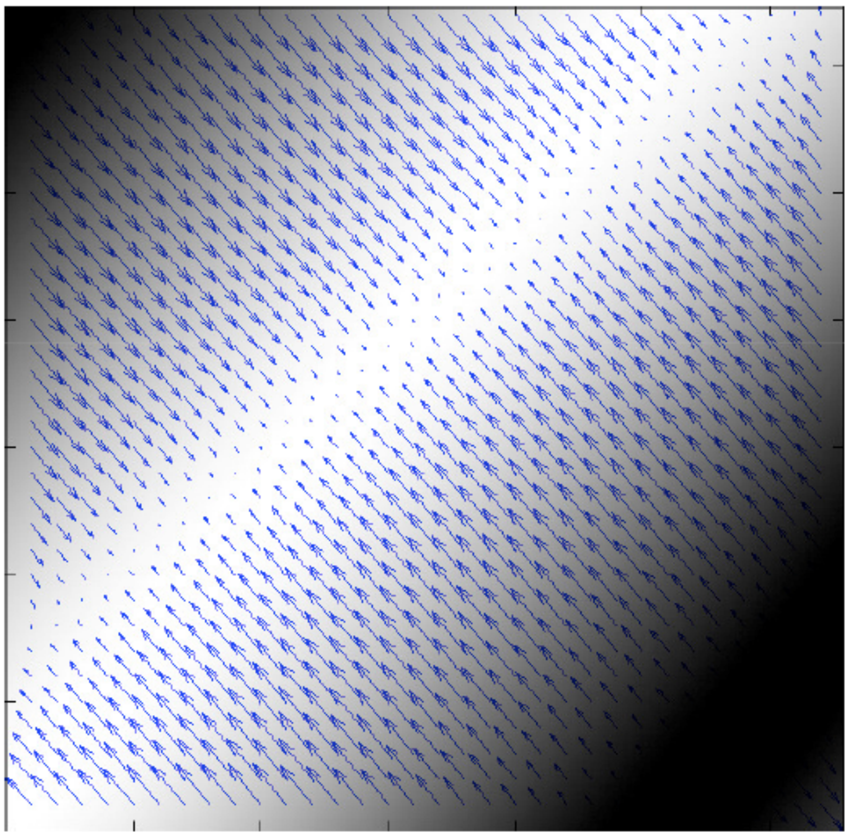
Image Gradient: gradient points in the direction of most rapid increase in intensity

$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$

$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$



Can think of it as the slope of a 3D surface

Gradient at a single point (x,y) is a vector:

- Direction is the direction of maximum slope:

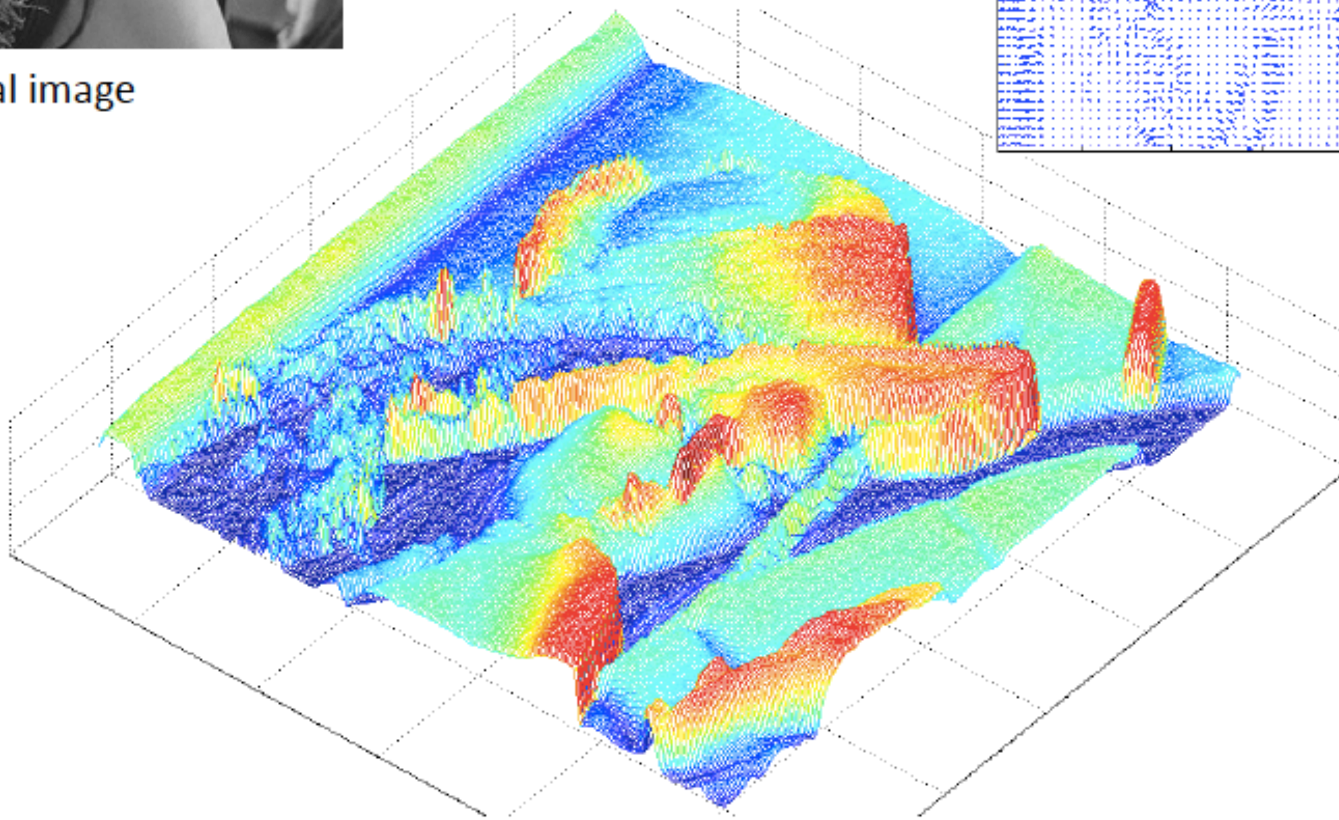
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

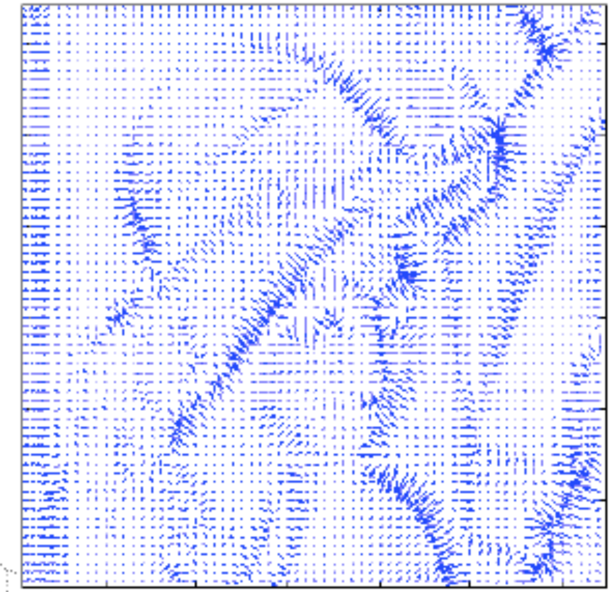
- Length is the magnitude (steepness) of the slope

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

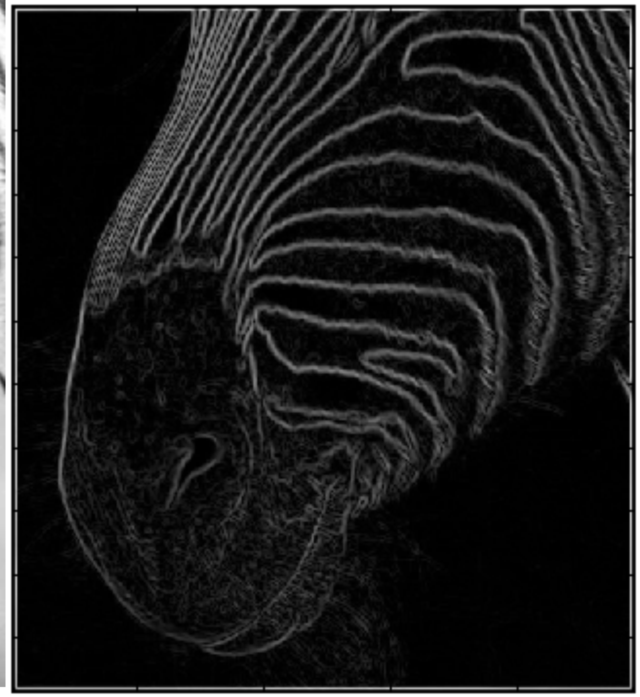
Original image

3D plot of luminance

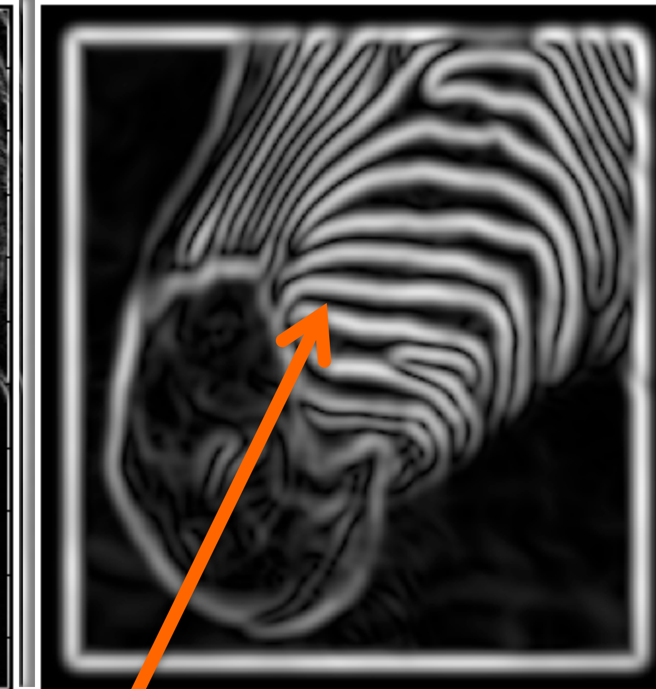




Gradient



Gradient magnitudes at scale 1



Gradient magnitudes at scale 2

Issues:

1) The gradient magnitude at different scales is different; which should we choose?
2) **The gradient magnitude is large along thick trails; how do we identify the significant points?**
3) How do we link the relevant points up into curves?
4) Noise.

The scale of the smoothing filter affects derivative estimates, and also the semantics of the edges recovered.

Canny edge detector

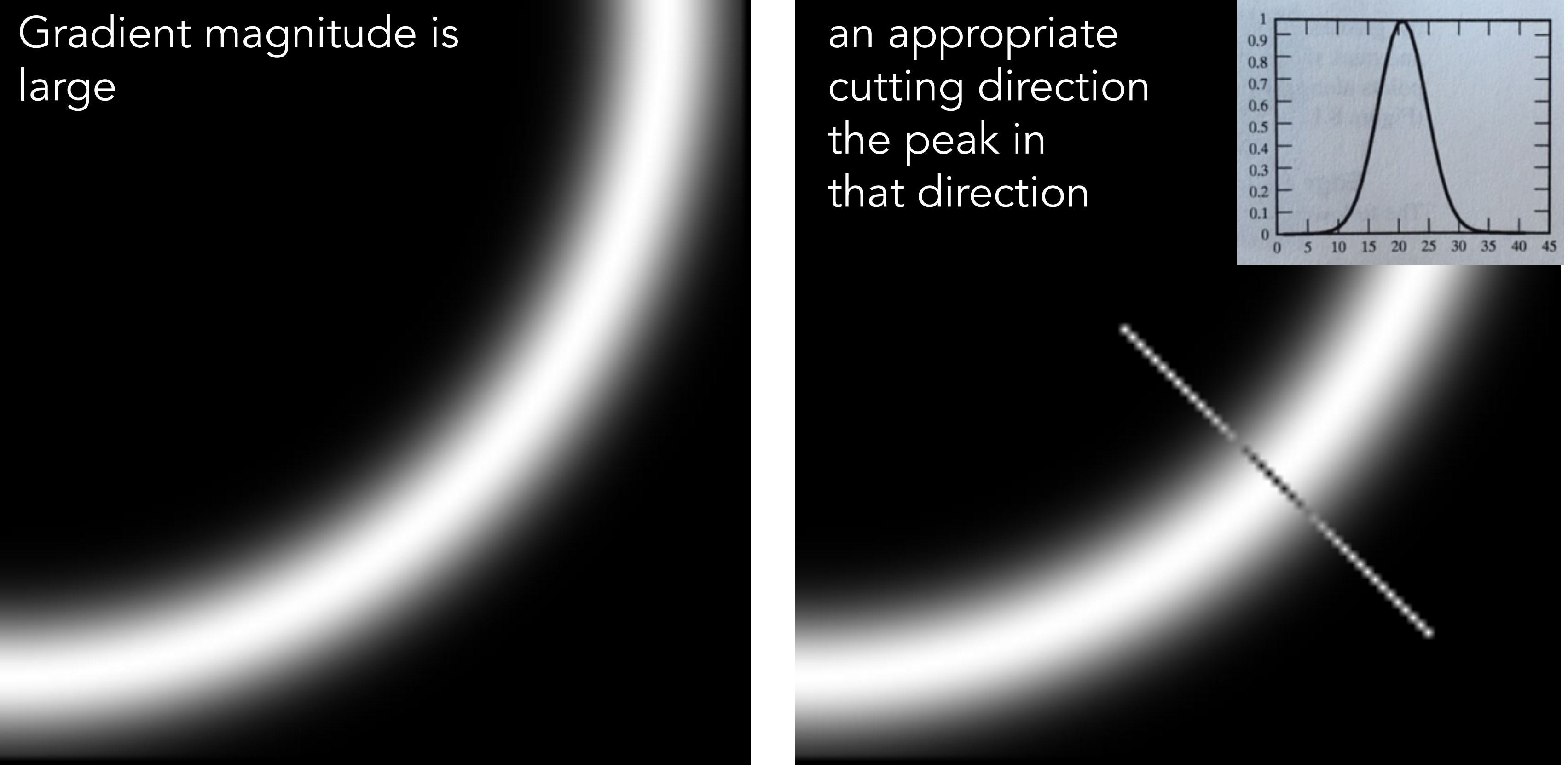
edge(image,'canny')

1. Filter image with derivative of Gaussian

2. Find magnitude and orientation of gradient

3. Non-maximum suppression

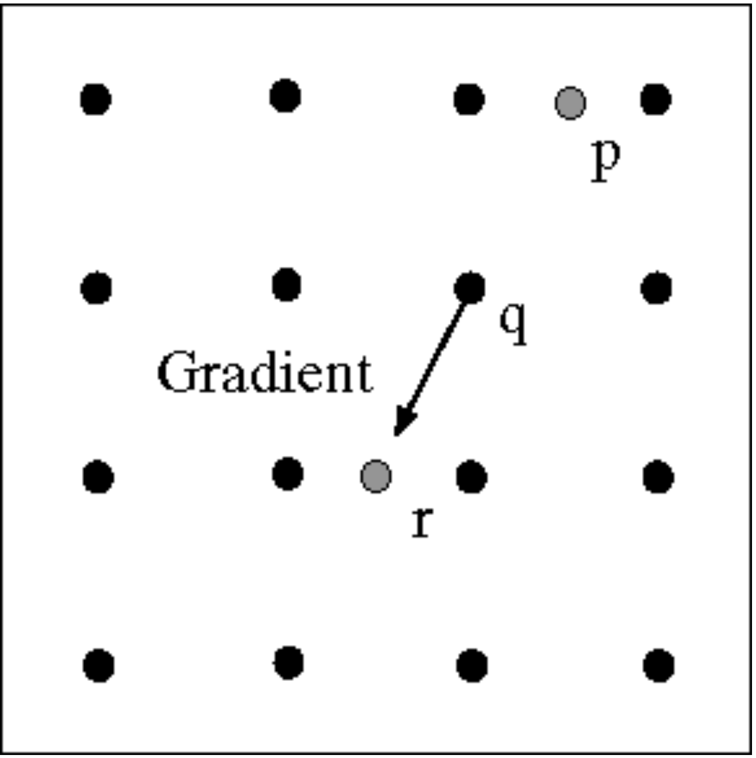
4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

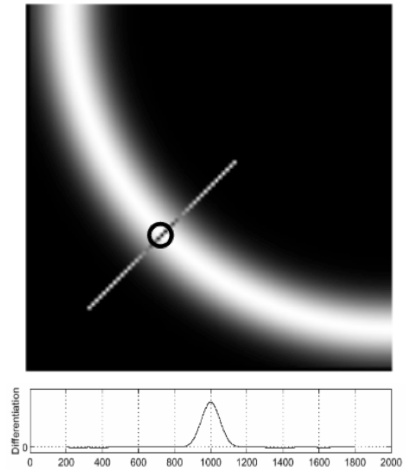


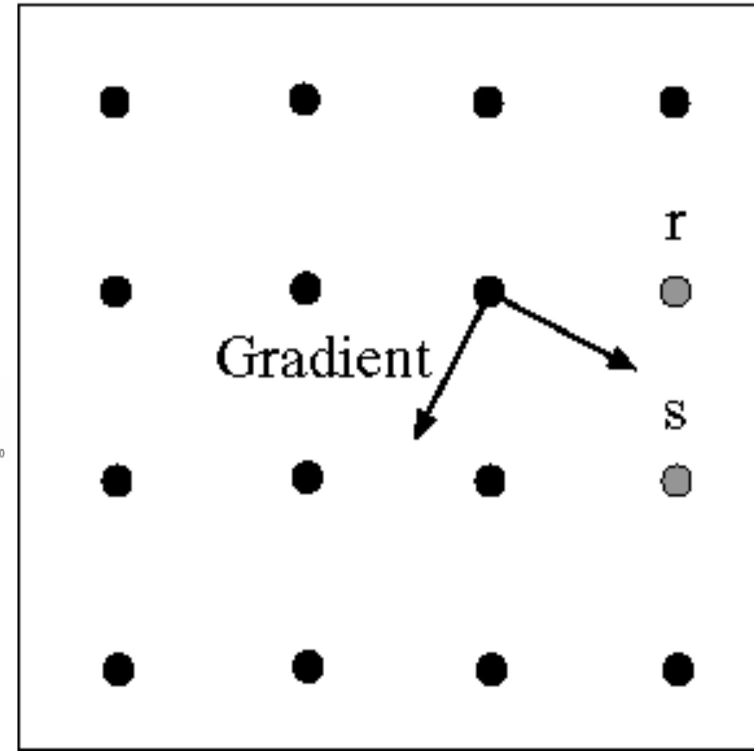
Goal: mark points along the curve where the magnitude is biggest.
How? looking for a maximum along a slice normal to the curve (non-maximum suppression). These points should form a curve. There are then two algorithmic issues:
-at which point is the maximum
-where is the next one?

Forsyth, 2002

Non maximum suppression: check if pixel is local maximum along gradient direction







At q, we have a maximum (1) if the value is larger than those at both *p* and at *r*.
Interpolate between *p* and *r* to get these values.

<u>Predicting the next edge point:</u> Assume the marked point is an edge point. Then we construct the tangent to the edge curve (which is normal to the gradient at that point) and use this to predict the next points (here either *r* or *s*).

Examples:
Non-Maximum Suppression

Original image

Gradient magnitude

But some edges are broken

Non-maxima
Suppressed
(remaining pixels are the local
Maximum)

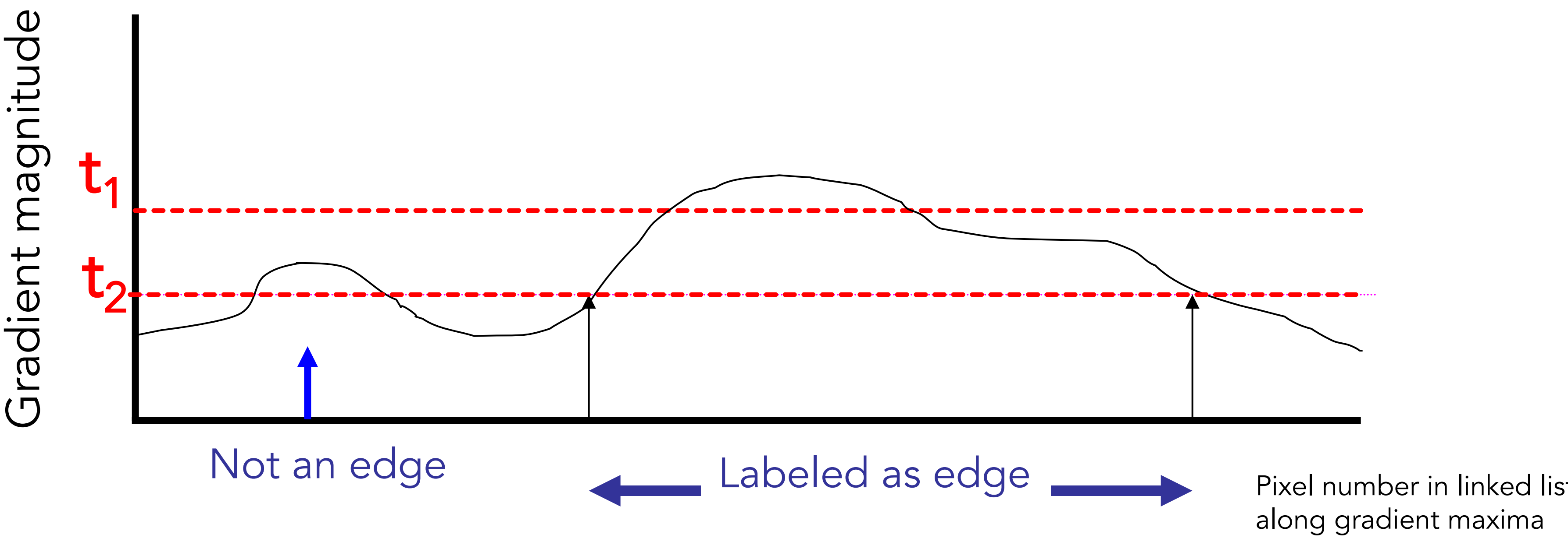
Canny edge detector

edge(image,'canny')

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression
4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

Closing edge gaps

- Check that maximum value of gradient value is sufficiently large
  - drop-outs? use **hysteresis**
    - use a high threshold to start edge curves and a low threshold to continue them.



Example: Canny Edge Detection

Original image

gap is gone

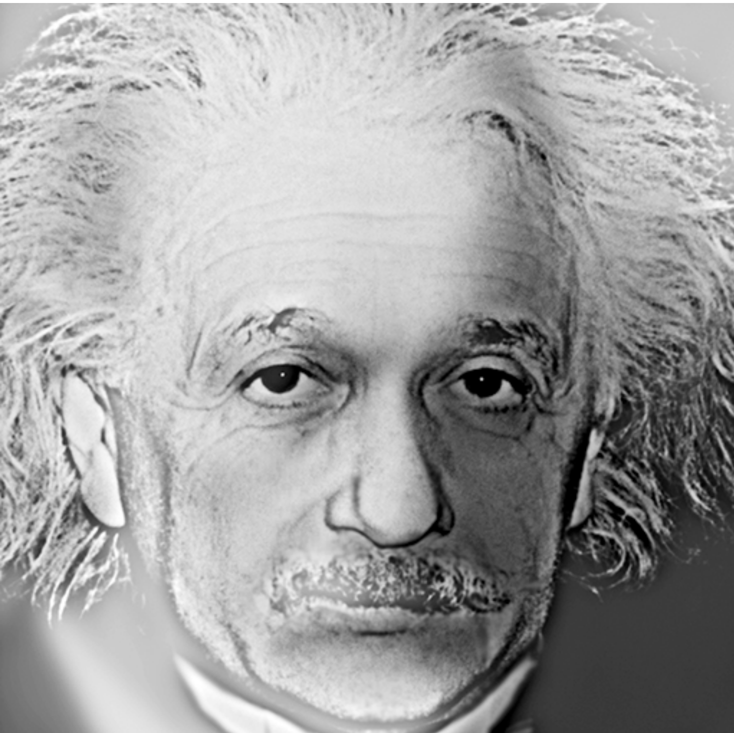
Strong + connected weak edges

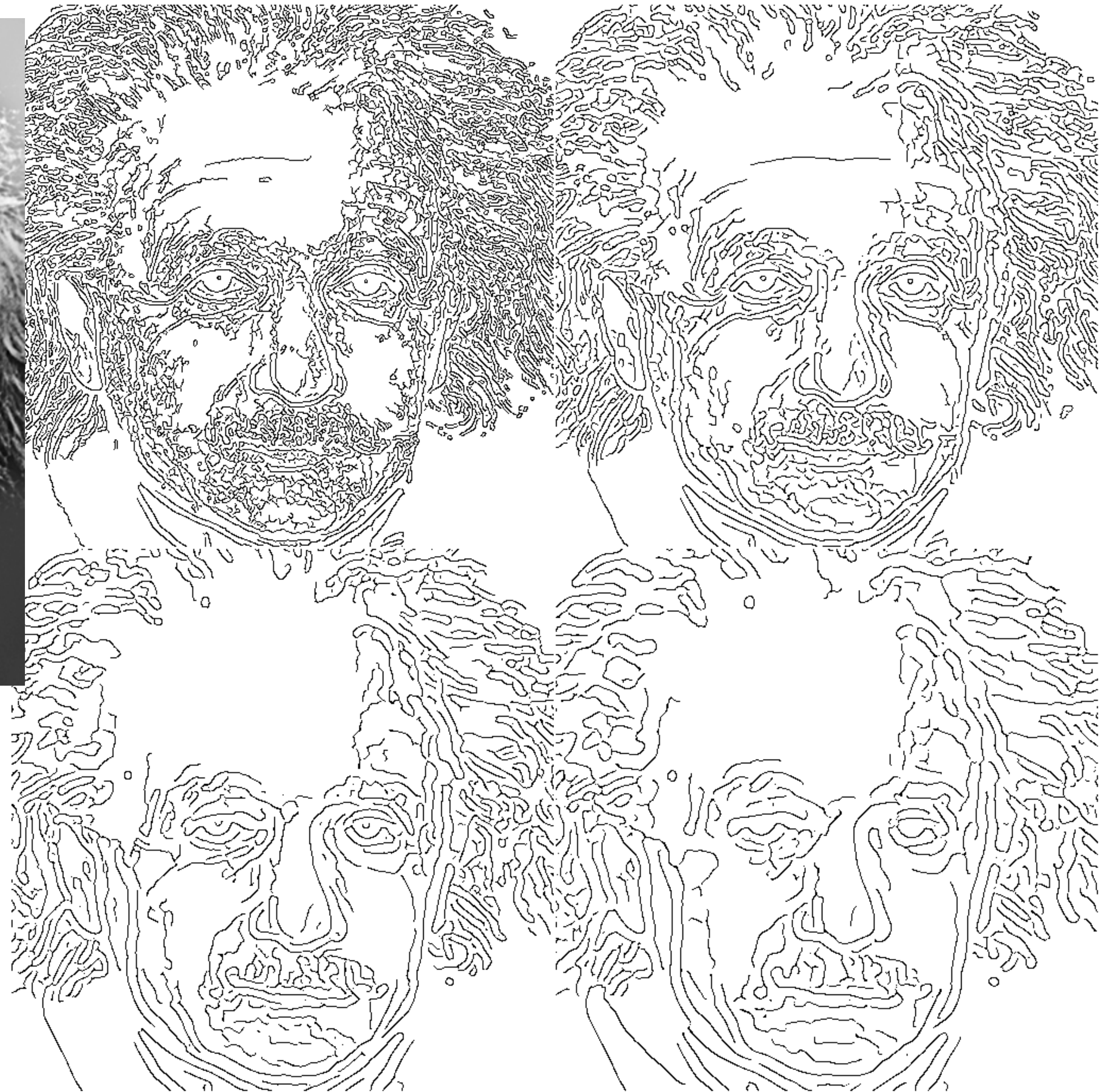
Strong edges only

Weak edges

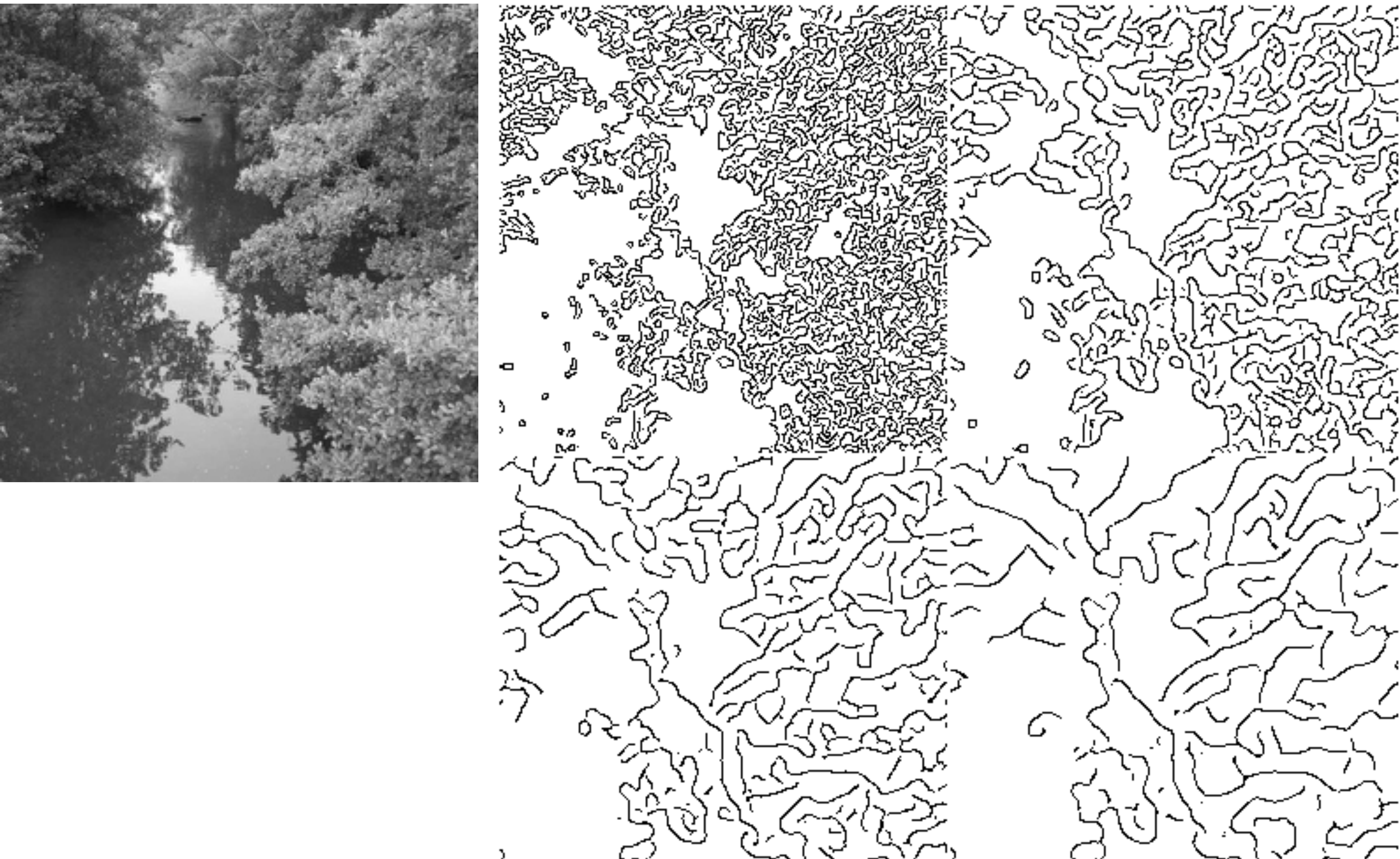
courtesy of G. Loy

Example: Canny Edge Detection





Example: Canny Edge Detection

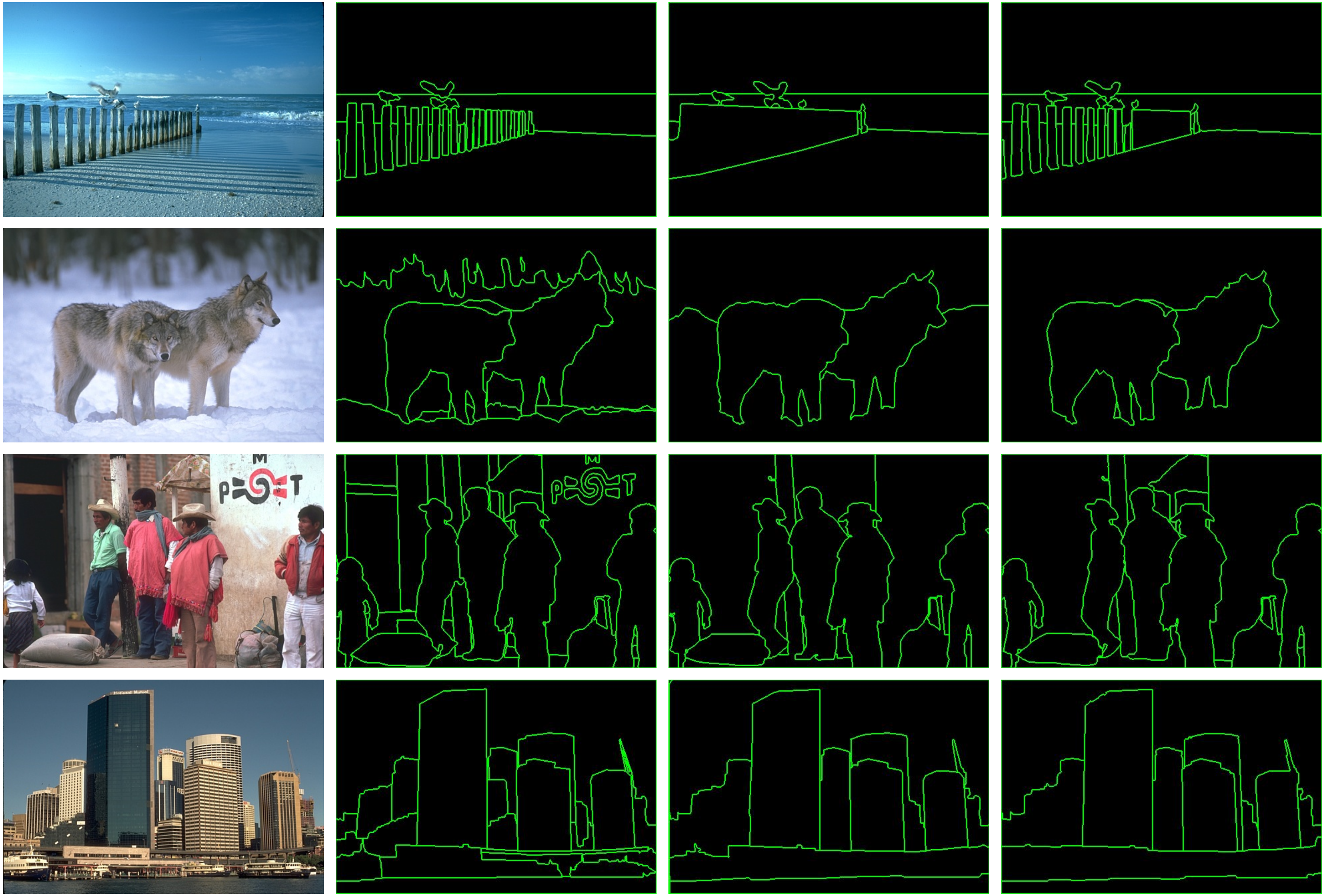


Learning to Detect Natural Image Boundaries Using Local Brightness, Color, and Texture Cues

David R. Martin, *Member*, *IEEE*, Charless C. Fowlkes, and Jitendra Malik, *Member*, *IEEE*

Abstract—The goal of this work is to accurately detect and localize boundaries in natural scenes using local image measurements. We formulate features that respond to characteristic changes in brightness, color, and texture associated with natural boundaries. In order to combine the information from these features in an optimal way, we train a classifier using human labeled images as ground truth. The output of this classifier provides the posterior probability of a boundary at each image location and orientation. We present precision-recall curves showing that the resulting detector significantly outperforms existing approaches. Our two main results are 1) that cue combination can be performed adequately with a simple linear model and 2) that a proper, explicit treatment of texture is required to detect boundaries in natural images.

https://www2.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/papers/mfm-pami-boundary.pdf



Slides credit: Jitendra Malik

Slides credit: Jitendra Malik

Slides credit: Jitendra Malik

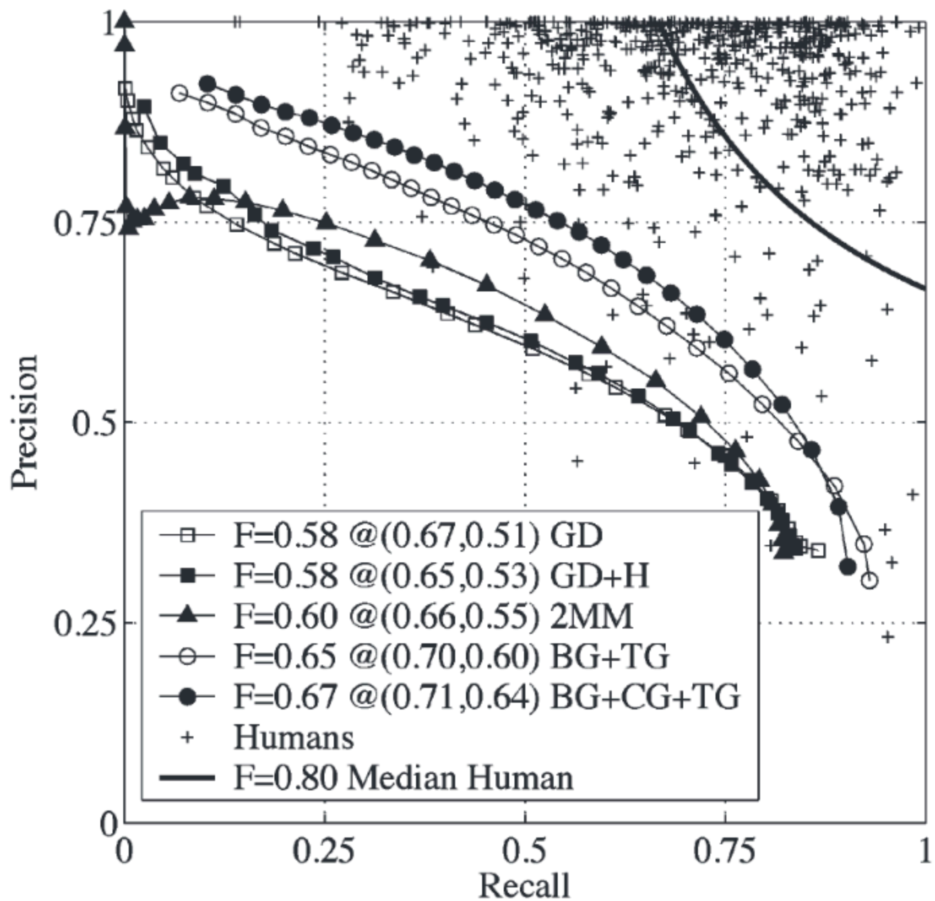


Fig. 3. Two Decades of Boundary Detection. The performance of our boundary detector compared to classical boundary detection methods and to the human subjects' performance. A precision-recall curve is shown for each of five boundary detectors: 1) Gaussian derivative (GD), 2) Gaussian derivative with hysteresis thresholding (GD+H), the Canny detector, 3) A detector based on the second moment matrix (2MM), 4) our gray-scale detector that combines brightness and texture (BG+TG), and 5) our color detector that combines brightness, color, and texture (BG+CG+TG). Each detector is represented by its *precision-recall* curve, which measures the trade off between accuracy and noise as the detector's threshold varies. Shown in the caption is each curve's F-measure, valued from zero to one. The F-measure is a summary statistic for a precision-recall curve. The points marked by a "+" on the plot show the precision and recall of each ground truth human segmentation when compared to the other humans. The median F-measure for the human subjects is 0.80. The solid curve shows the F=0.80 curve, representing the frontier of human performance for this task.

Color Gray InvNeg

Slides credit: Jitendra Malik

DeepEdge: A Multi-Scale Bifurcated Deep Network for Top-Down Contour Detection

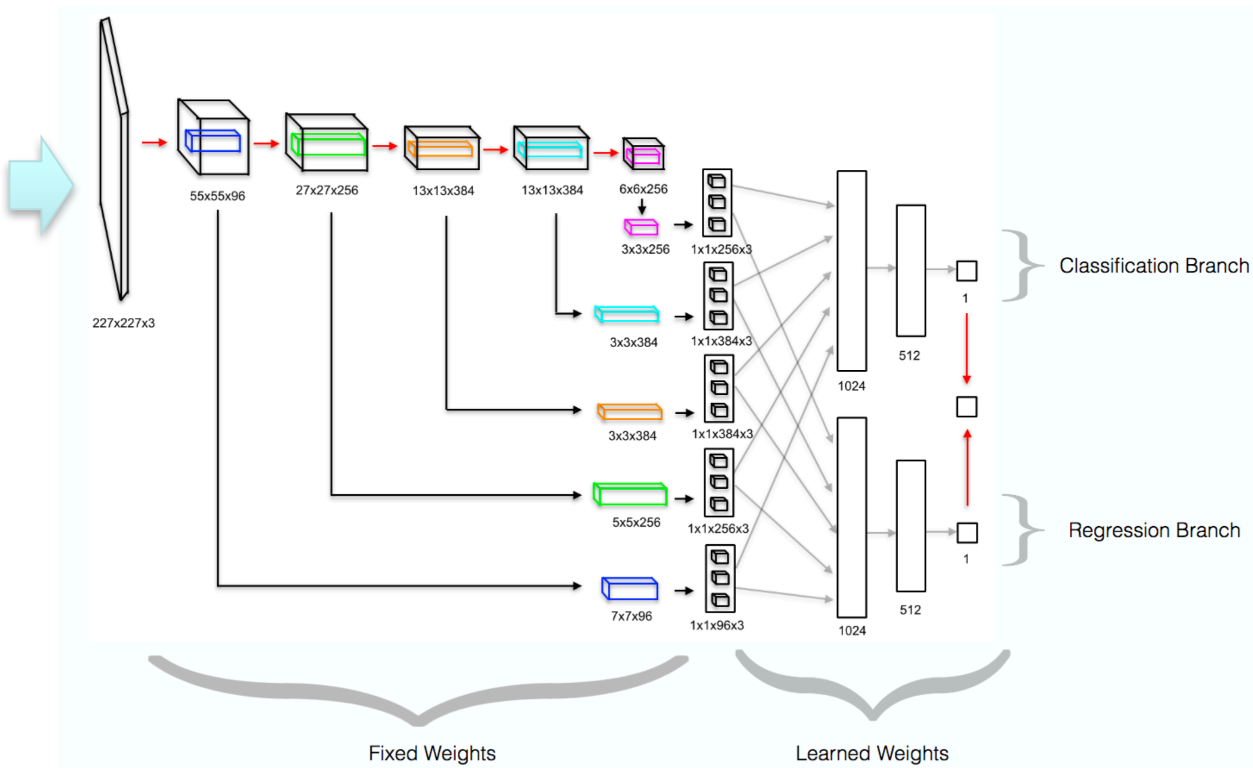
Submitted on 2 Dec 2014

Gedas Bertasius
University of Pennsylvania
gberta@seas.upenn.edu

Jianbo Shi
University of Pennsylvania
jshi@seas.upenn.edu

Lorenzo Torresani
Dartmouth College
lt@dartmouth.edu

Sample patches on canny edges



https://arxiv.org/abs/1412.1123

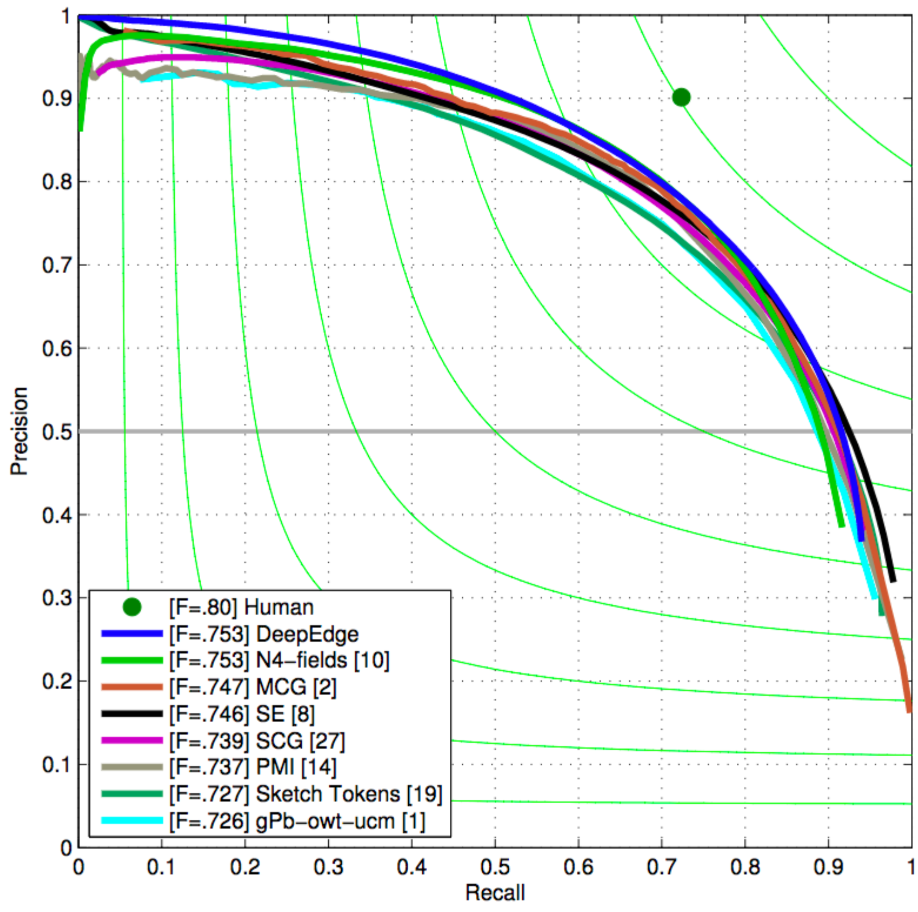
DeepEdge: A Multi-Scale Bifurcated Deep Network for Top-Down Contour Detection

Submitted on 2 Dec 2014

Gedas Bertasius
University of Pennsylvania
gberta@seas.upenn.edu

Jianbo Shi
University of Pennsylvania
jshi@seas.upenn.edu

Lorenzo Torresani
Dartmouth College
lt@dartmouth.edu

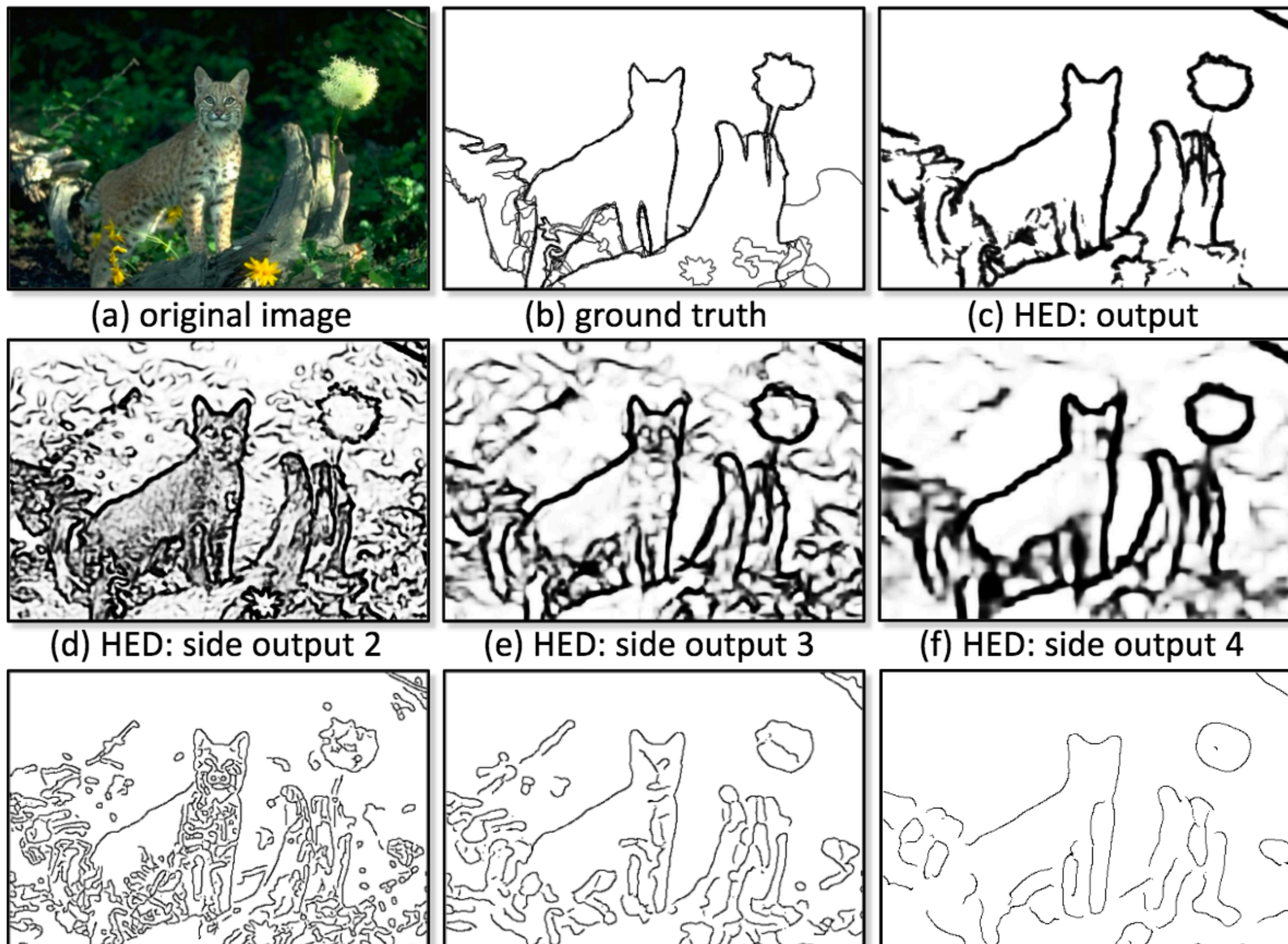


https://arxiv.org/abs/1412.1123

Holistically-Nested Edge Detection

Saining Xie
Dept. of CSE and Dept. of CogSci
University of California, San Diego
9500 Gilman Drive, La Jolla, CA 92093
s9xie@eng.ucsd.edu

Zhuowen Tu
Dept. of CogSci and Dept. of CSE
University of California, San Diego
9500 Gilman Drive, La Jolla, CA 92093
ztu@ucsd.edu



(a) original image, (b) ground truth, (c) HED: output

(d) HED: side output 2, (e) HED: side output 3, (f) HED: side output 4

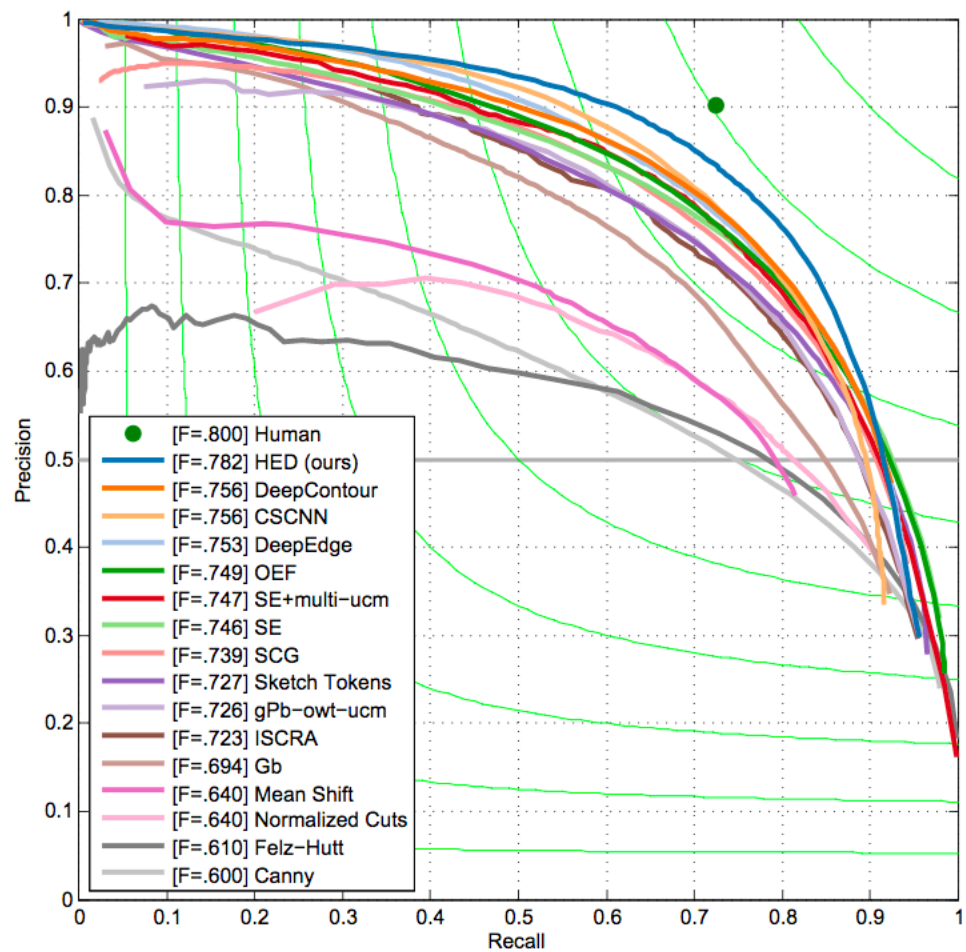
(g) Canny: $\sigma = 2$, (h) Canny: $\sigma = 4$, (i) Canny: $\sigma = 8$

https://arxiv.org/pdf/1504.06375.pdf

Holistically-Nested Edge Detection

Saining Xie
Dept. of CSE and Dept. of CogSci
University of California, San Diego
9500 Gilman Drive, La Jolla, CA 92093
s9xie@eng.ucsd.edu

Zhuowen Tu
Dept. of CogSci and Dept. of CSE
University of California, San Diego
9500 Gilman Drive, La Jolla, CA 92093
ztu@ucsd.edu

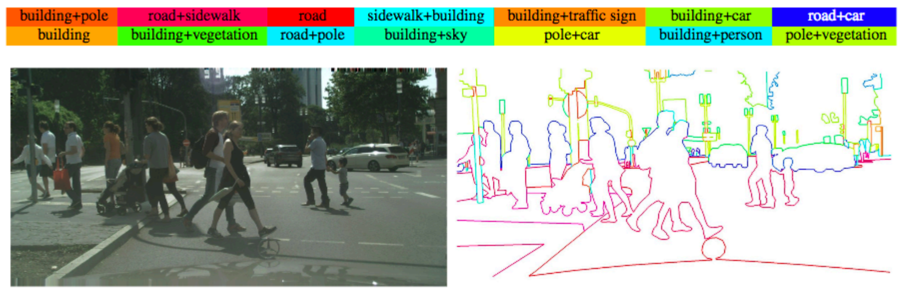


https://arxiv.org/pdf/1504.06375.pdf

CASENet: Deep Category-Aware Semantic Edge Detection

Zhiding Yu[*]
Carnegie Mellon University
yzhiding@andrew.cmu.edu

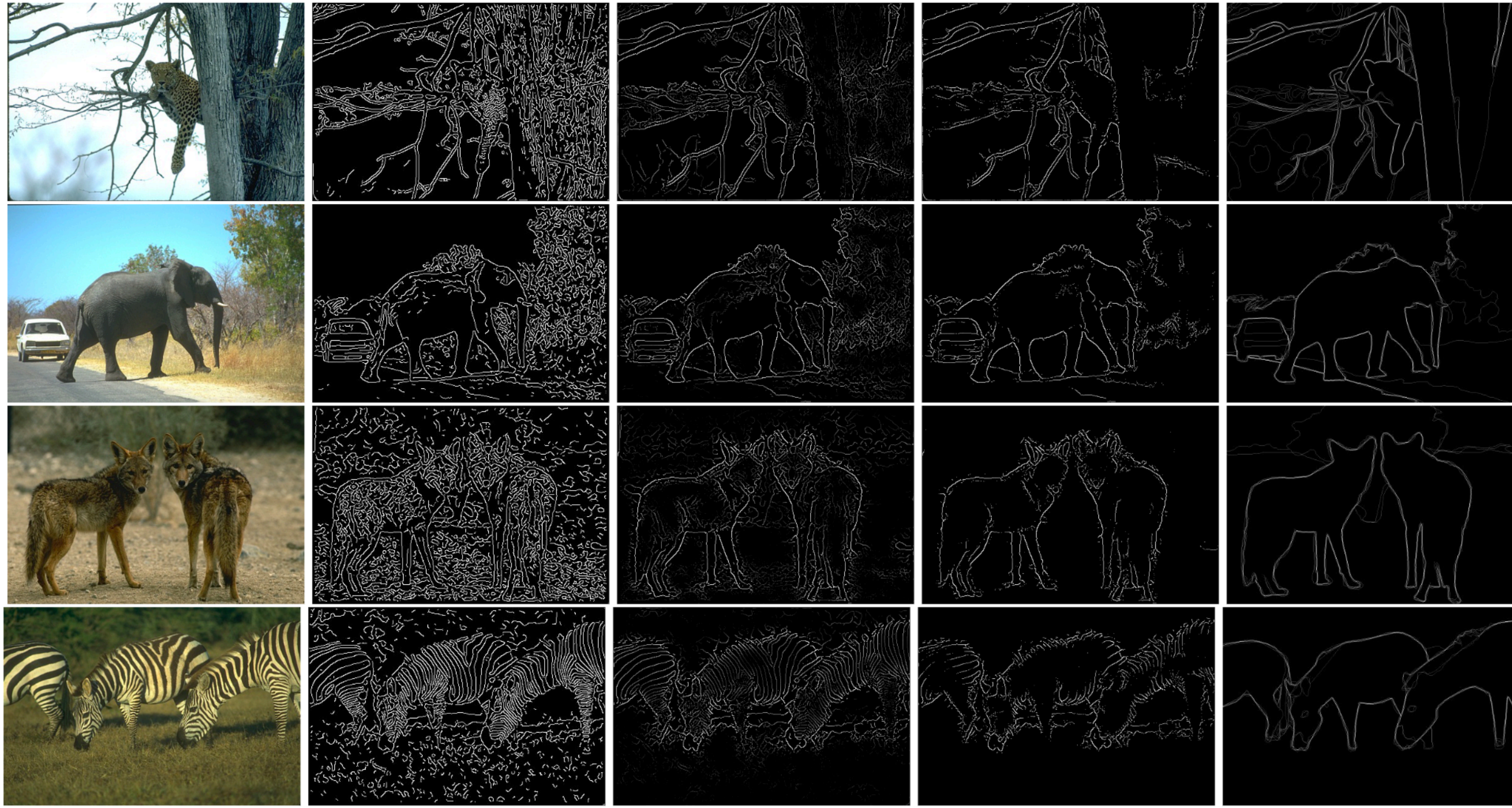
Chen Feng[*] Ming-Yu Liu[†] Srikumar Ramalingam[†]
Mitsubishi Electric Research Laboratories (MERL)
cfeng@merl.com, mingyul@nvidia.com, srikumar@cs.utah.edu



(a) Input image (b) Ground truth

(c) CASENet output

https://arxiv.org/pdf/1705.09759.pdf



Input Image Canny Edges Raw DeepEdges Thresholded DeepEdges Ground Truth Edges

II. Segmentation

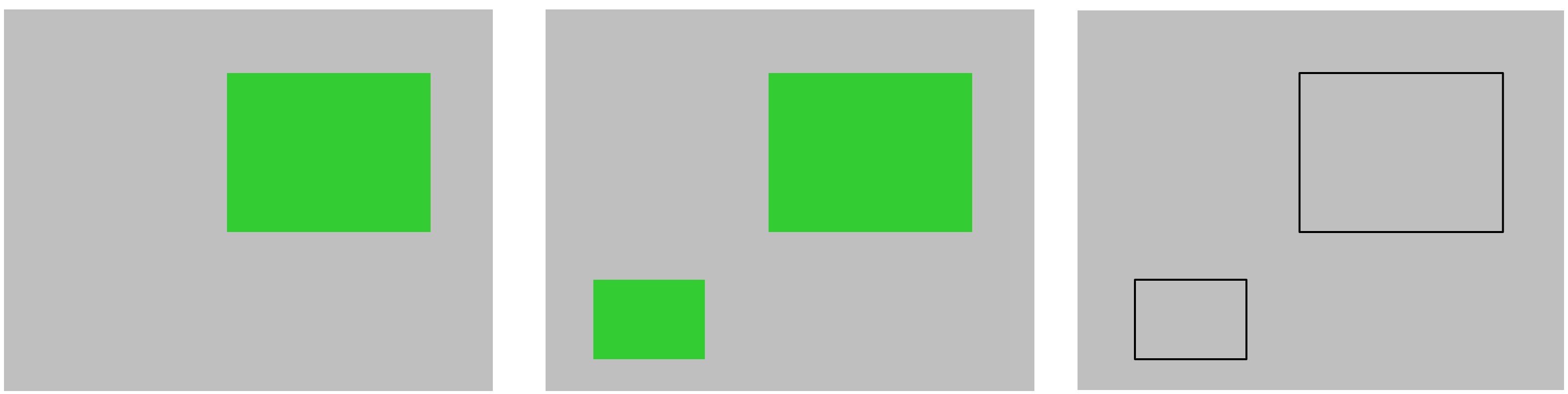
II.1 Bottom-up segmentation

- Group together similar-looking pixels
  - "Bottom-up" process
  - Unsupervised
- Bottom-up segmentation
  - Clustering
  - Mean shift
  - Graph-based

"superpixels"

Issues

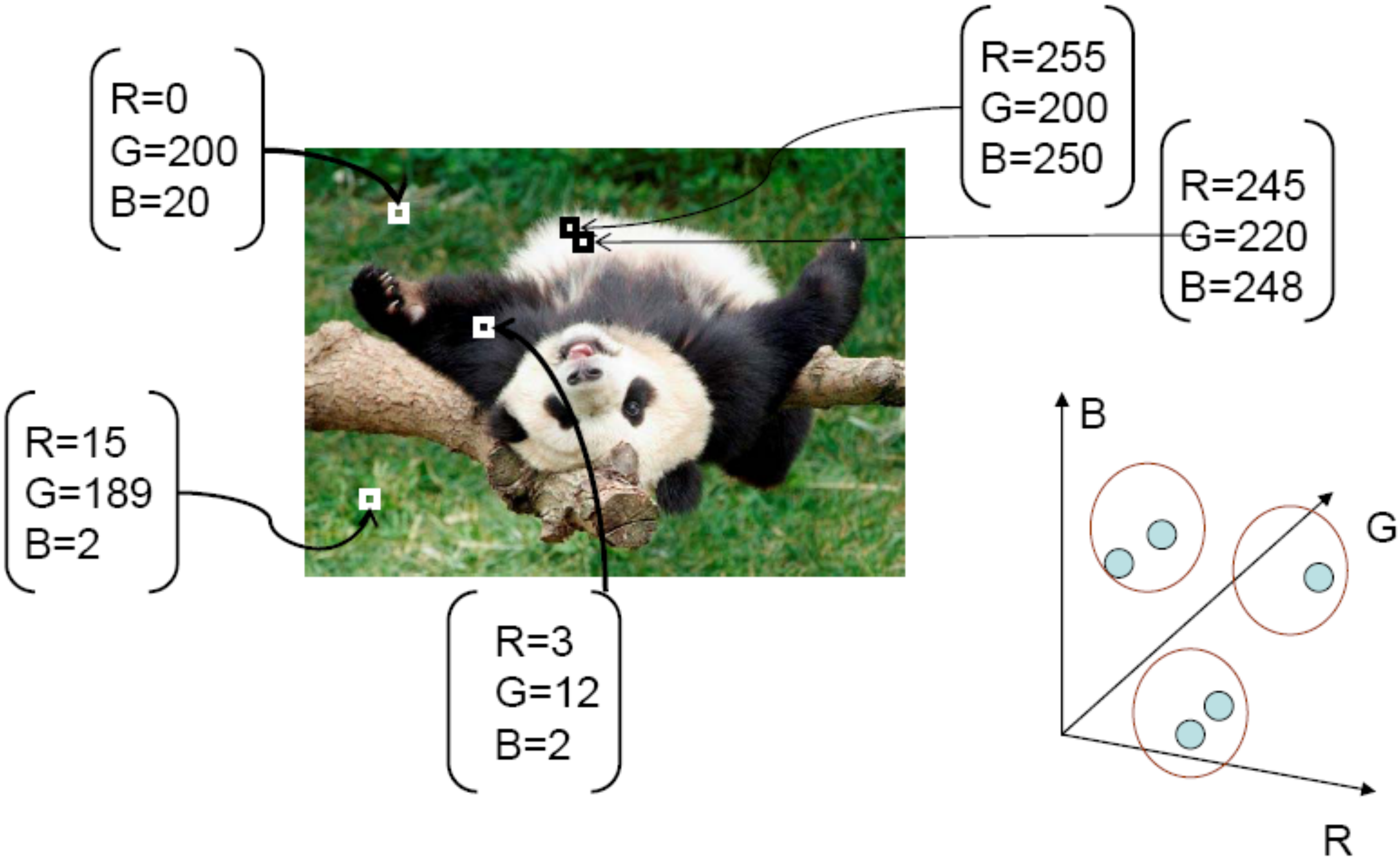
- How do we decide that two pixels are likely to belong to the same region?



- How many regions are there?

Method 1: Clustering

- Cluster similar pixels (features) together



Source: K. Grauman

Segmentation as clustering

- Cluster together (pixels, tokens, etc.) that belong together…
- Agglomerative clustering
  - attach closest to cluster it is closest to
  - repeat
- Divisive clustering
  - split cluster along best boundary
  - repeat
- Dendrograms
  - yield a picture of output as clustering process continues

Chapter – Forsyth & Ponce

A simple segmentation algorithm

- Each pixel is described by a vector

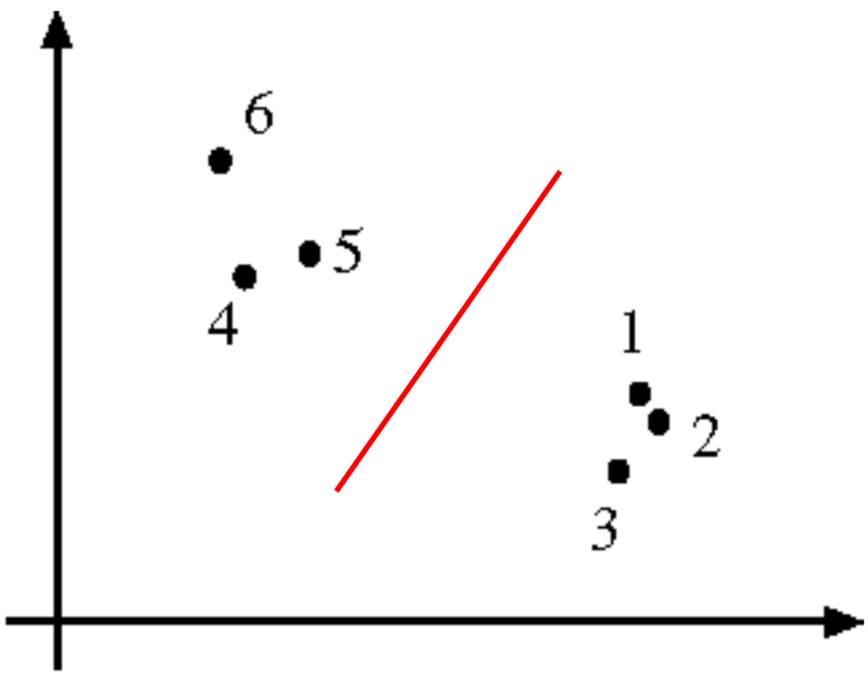
$$z = [r, g, b] \text{ or } [Y\ u\ v], \ldots$$

- Run a clustering algorithm (e.g. k-means) using some distance between pixels:

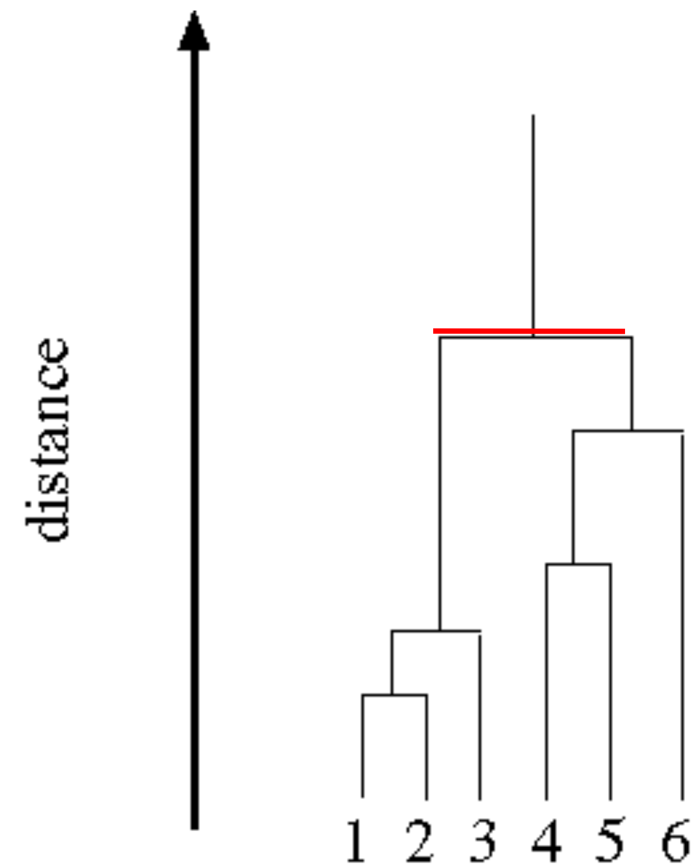
$$D(\text{pixel}_i, \text{pixel}_j) = \| z_i - z_j \|^2$$

Dendogram

Data set



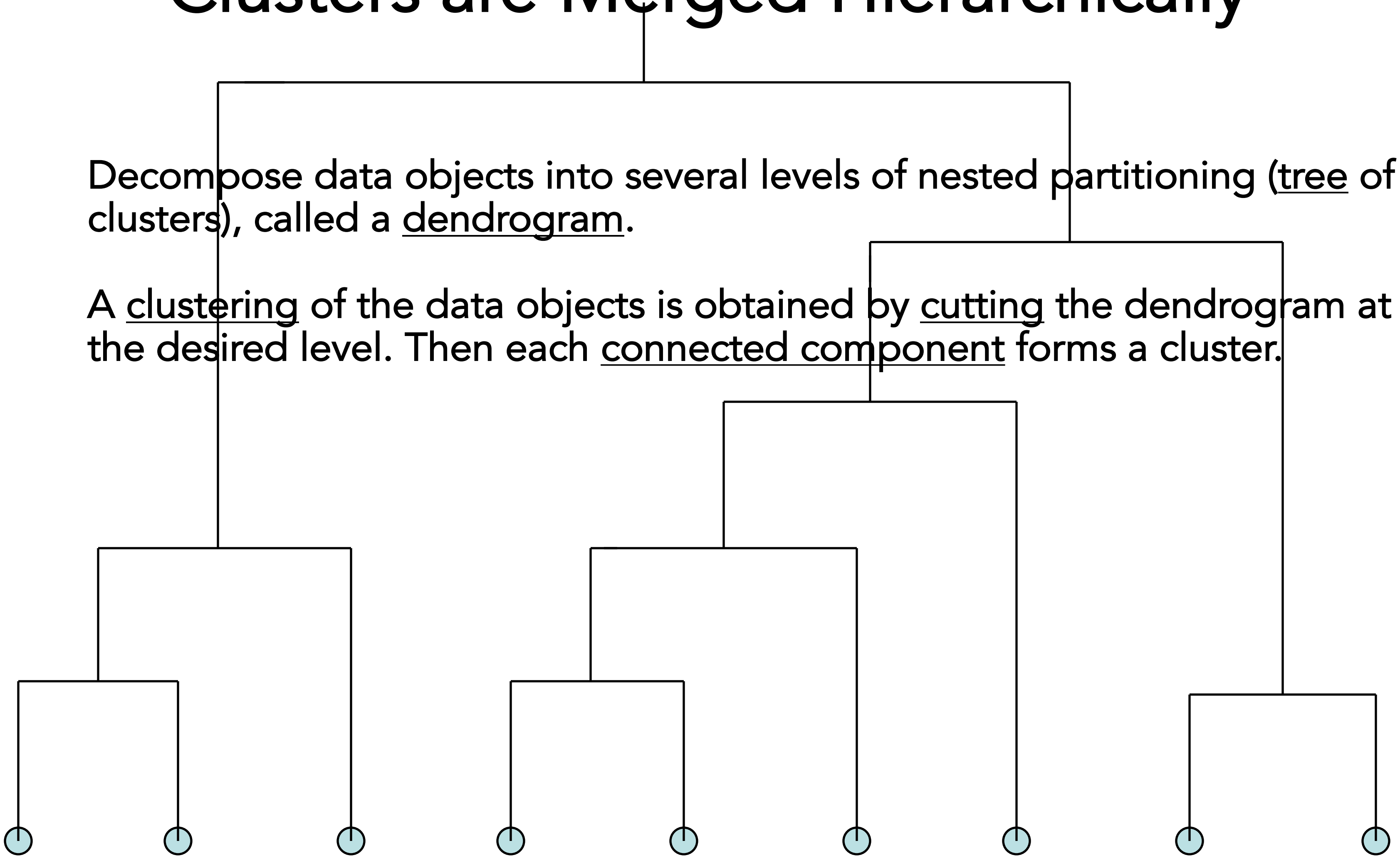
Dendrogram obtained by agglomerative clustering



A *Dendrogram* Shows How the Clusters are Merged Hierarchically

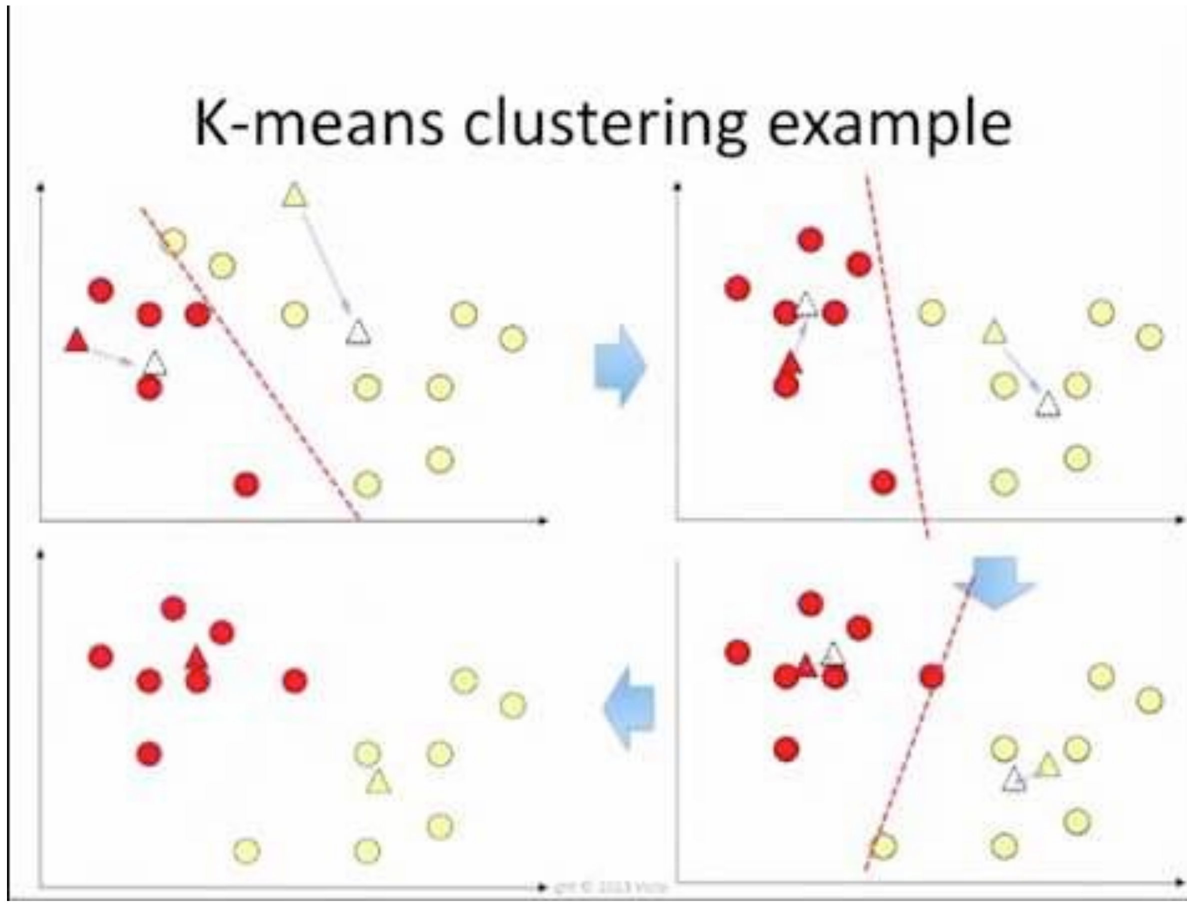
Decompose data objects into several levels of nested partitioning (tree of clusters), called a dendrogram.

A clustering of the data objects is obtained by cutting the dendrogram at the desired level. Then each connected component forms a cluster.



K-Means Clustering

- Given *k*, the *k-means* algorithm consists of four steps:
  - Select initial centroids at random.
  - Assign each object to the cluster with the nearest centroid.
  - Compute each centroid as the mean of the objects assigned to it.
  - Repeat previous 2 steps until no change.



- K-means (k=5) clustering based on intensity (middle) or color (right) is essentially vector quantization of the image attributes
  – Clusters don’t have to be spatially coherent

Image

Intensity-based clusters

Color-based clusters

See pdf chapter 14

each pixel is replaced with the mean value of its cluster

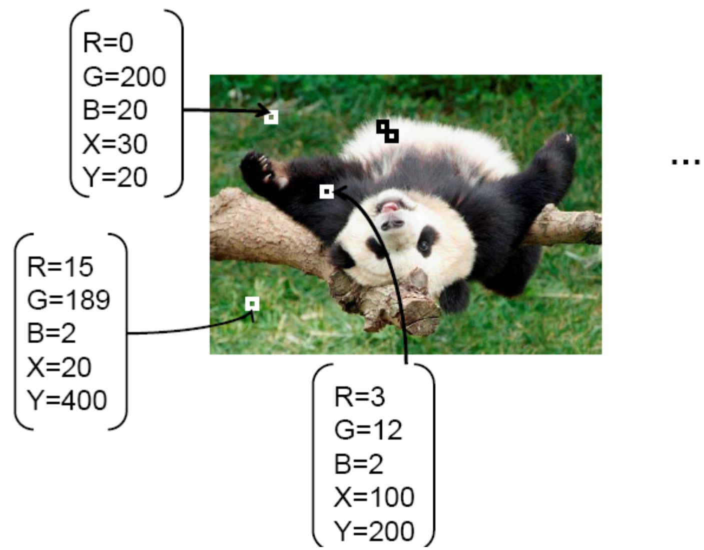
K-means using color alone (k=11 clusters)
Showing 4 of the segments, (not necessarily connected)
Some are good, some meaningless

Including spatial relationships

Augment data to be clustered with spatial coordinates.

$$z = \begin{pmatrix} Y \\ u \\ v \\ x \\ y \end{pmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{color coordinates (or r,g,b)} \\ \left.\begin{matrix} \\ \end{matrix}\right\} \text{spatial coordinates} \end{matrix}$$

- Cluster similar pixels (features) together



- Clustering based on (r,g,b,x,y) values enforces more spatial coherence

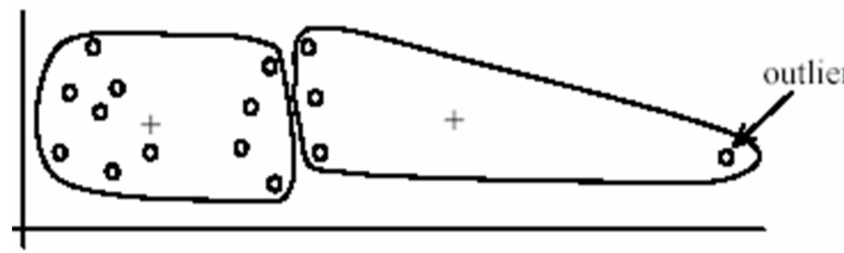
K-means using colour and position, 20 segments

Still misses goal of perceptually pleasing or useful segmentation
No measure of texture

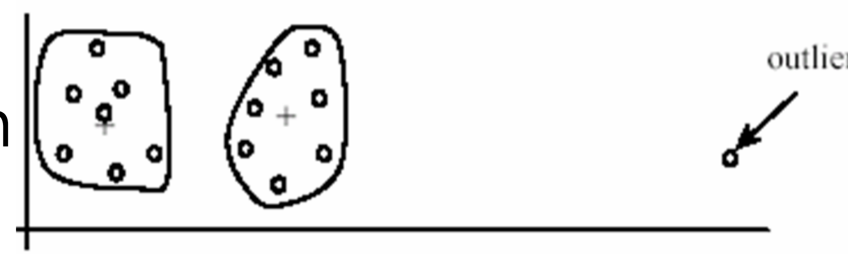
Hard to pick K…

K-Means for segmentation

- Pros
  - Very simple method
  - Converges to a local minimum of the error function
- Cons
  - Memory-intensive
  - Need to pick K
  - Sensitive to initialization
  - Sensitive to outliers



(A): Undesirable clusters

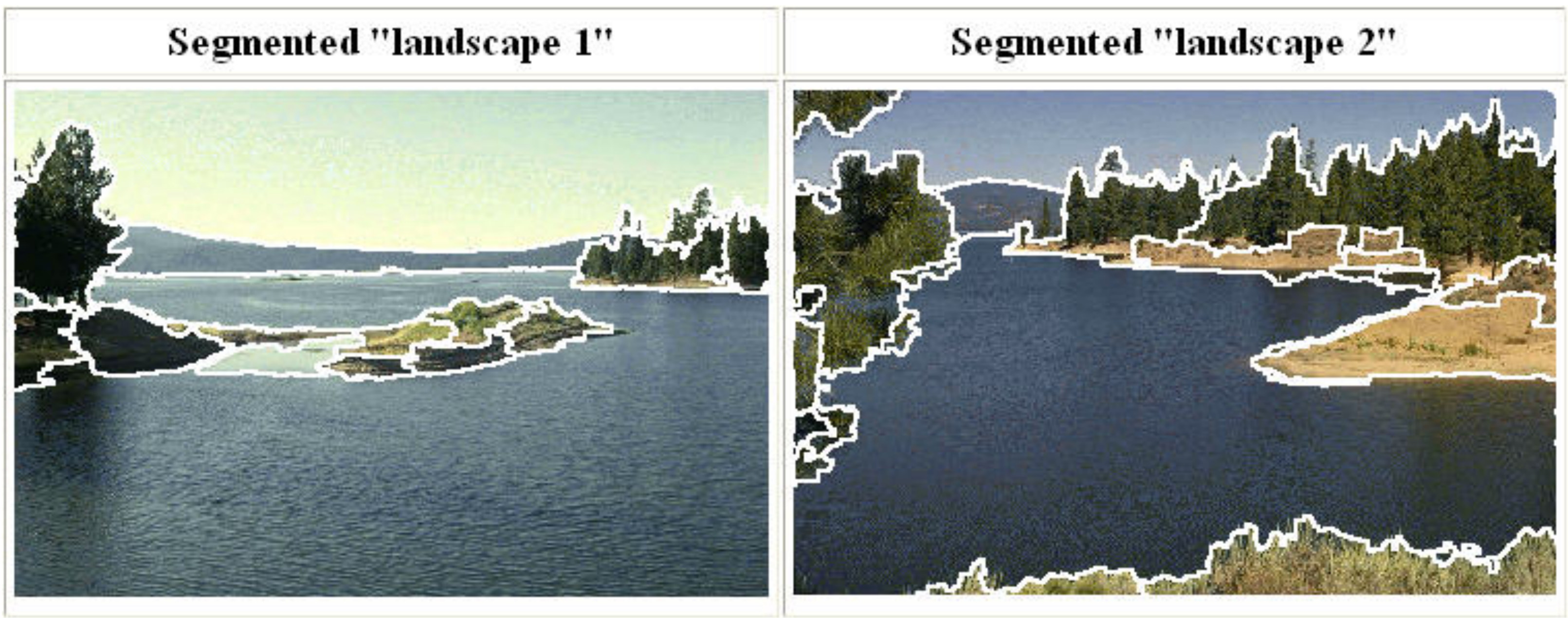


(B): Ideal clusters

Slide credit: S. Lazebnik

Method 2: Mean shift clustering

- An advanced and versatile technique for clustering-based segmentation



http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

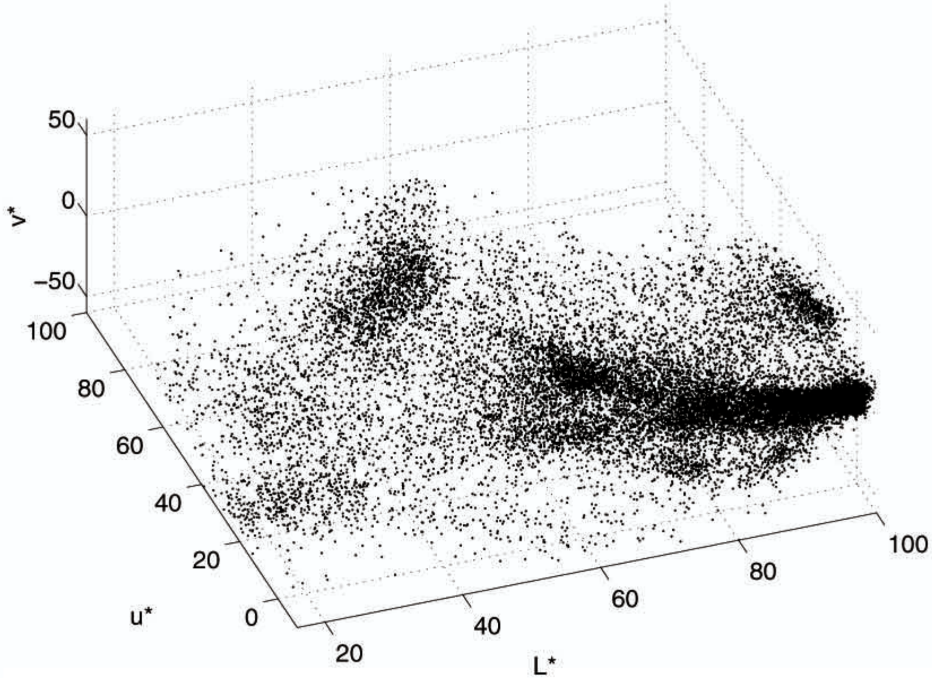
D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Mean shift algorithm

The mean shift algorithm seeks *modes* or local maxima of density in the feature space

image

Feature space (L*u*v* color values)

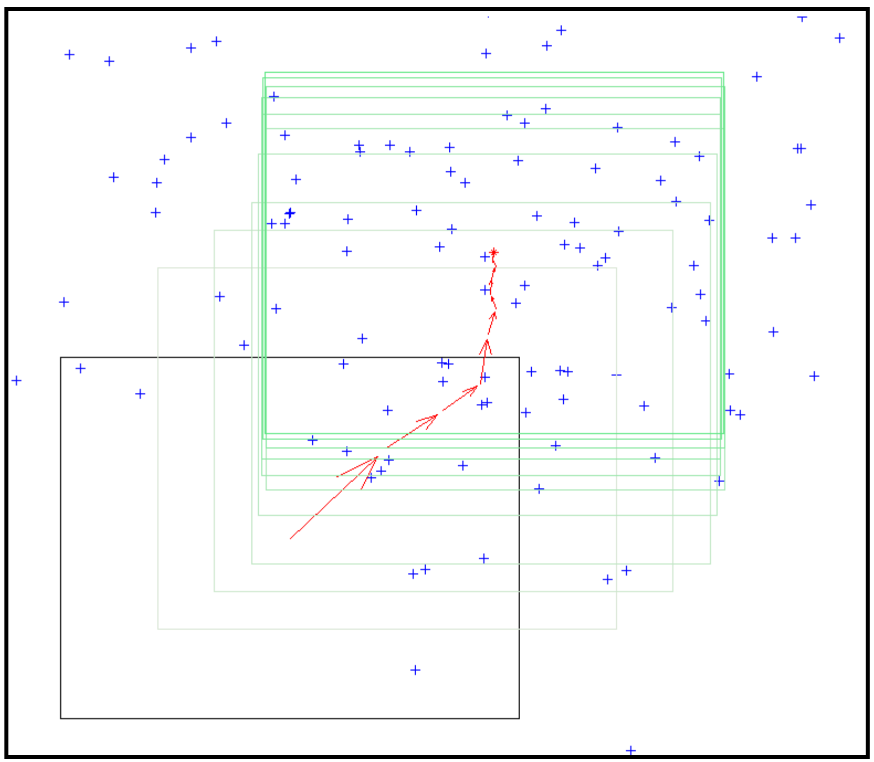


Mean Shift Algorithm

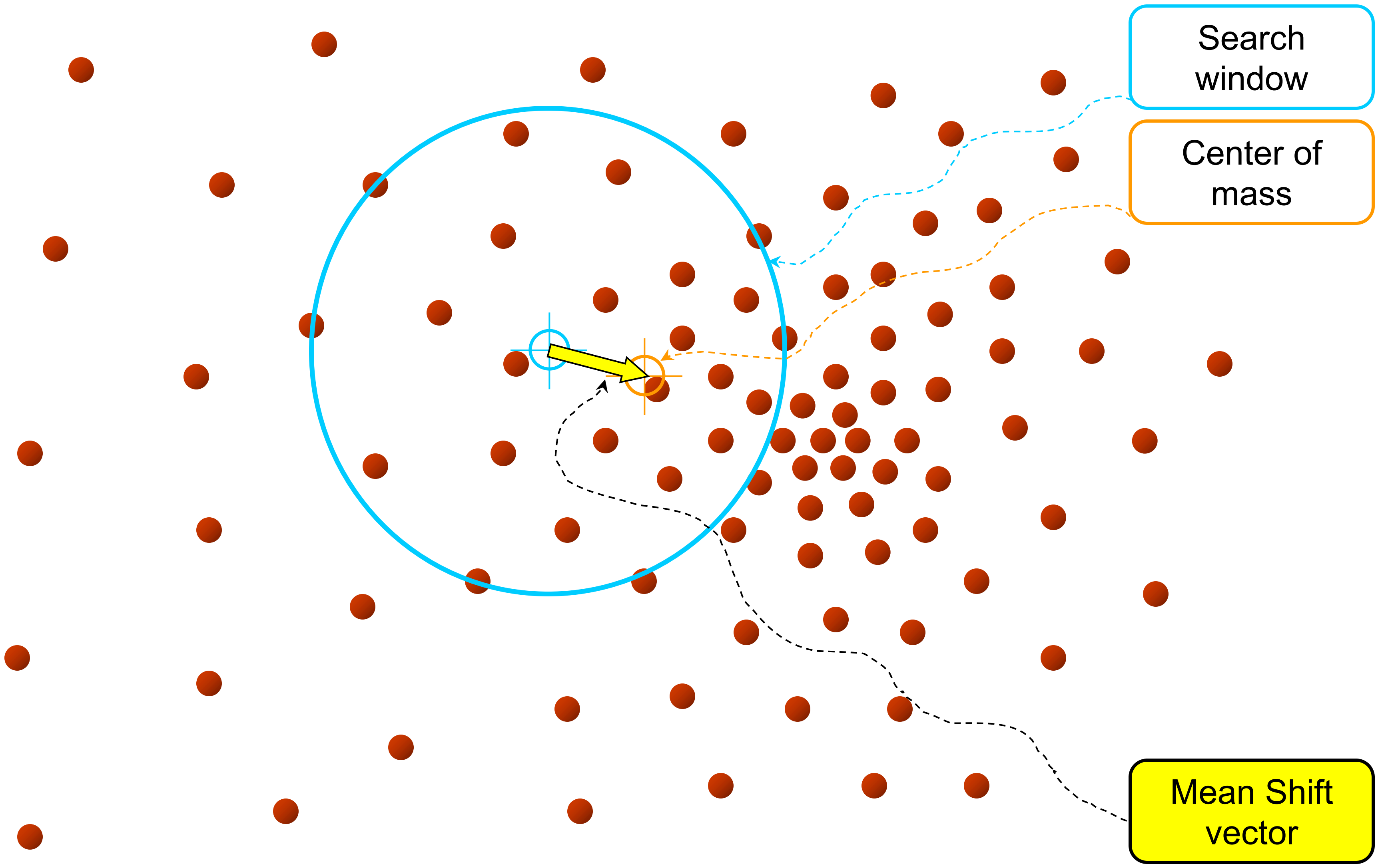
Mean Shift Algorithm

1. Choose a search window size.
2. Choose the initial location of the search window.
3. Compute the mean location (centroid of the data) in the search window.
4. Center the search window at the mean location computed in Step 3.
5. Repeat Steps 3 and 4 until convergence.

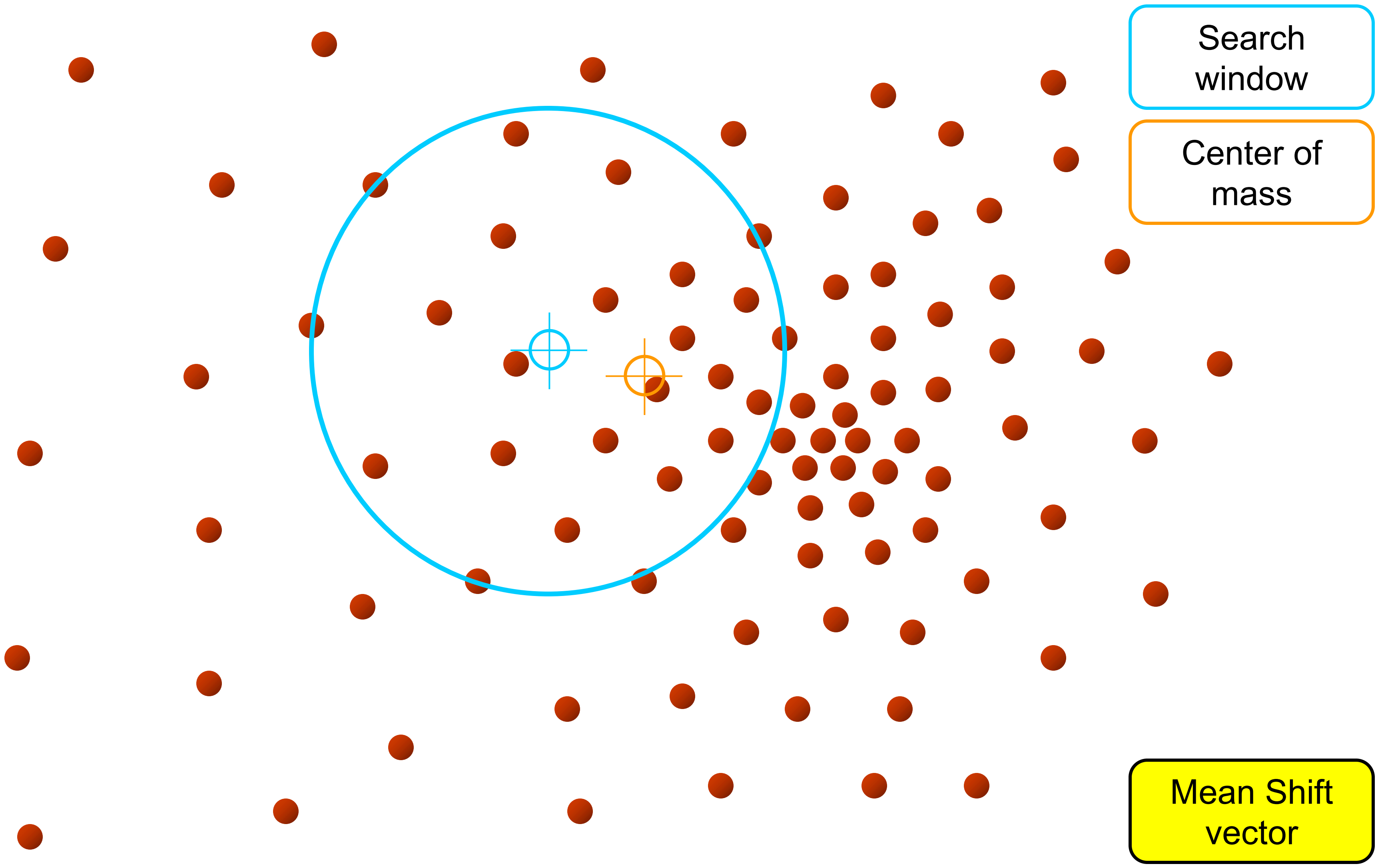
The mean shift algorithm seeks the “mode” or point of highest density of a data distribution:



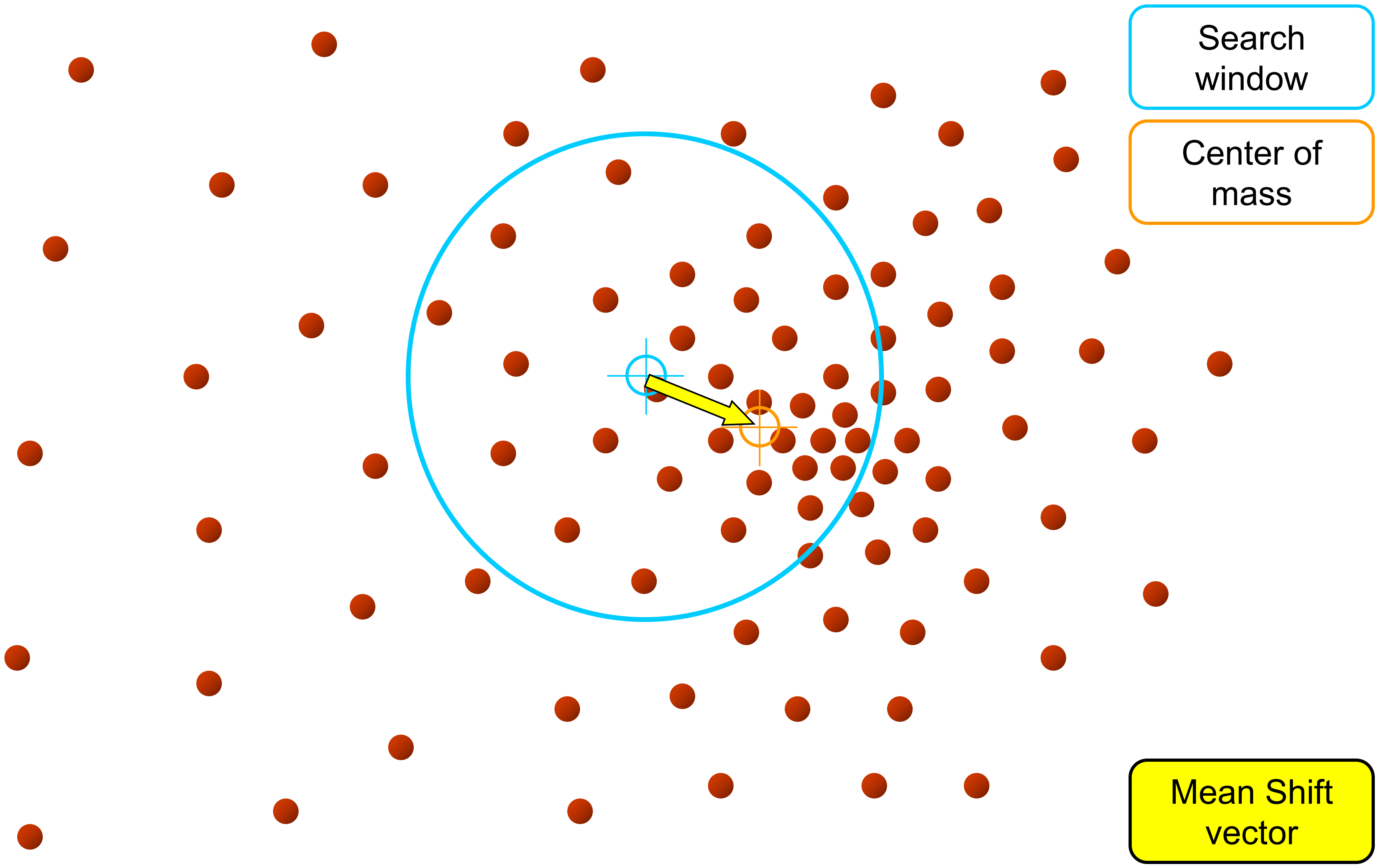
Two issues:
(1) Kernel to interpolate density based on sample positions.
(2) Gradient ascent to mode.



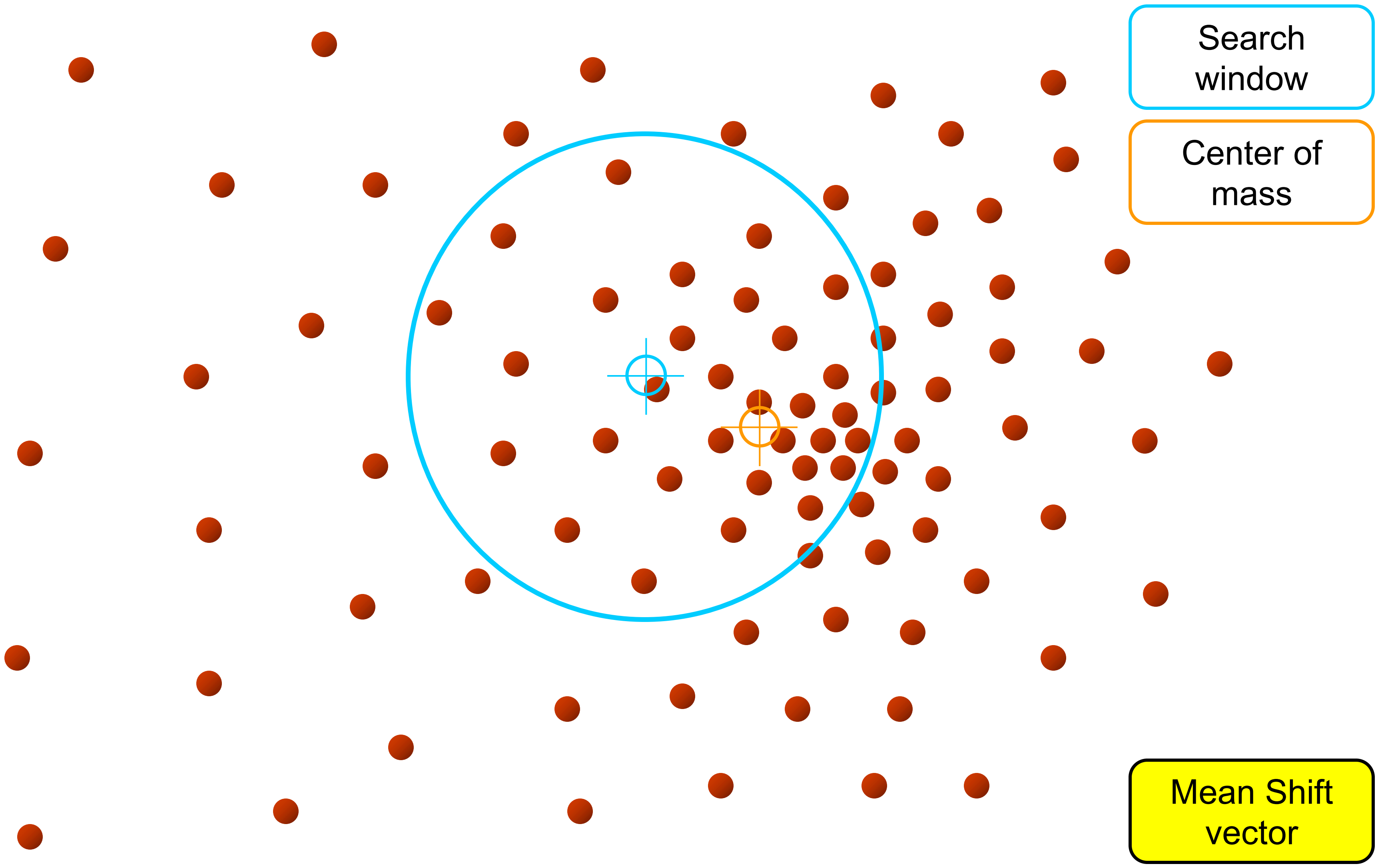
Slide by Y. Ukrainitz & B. Sarel



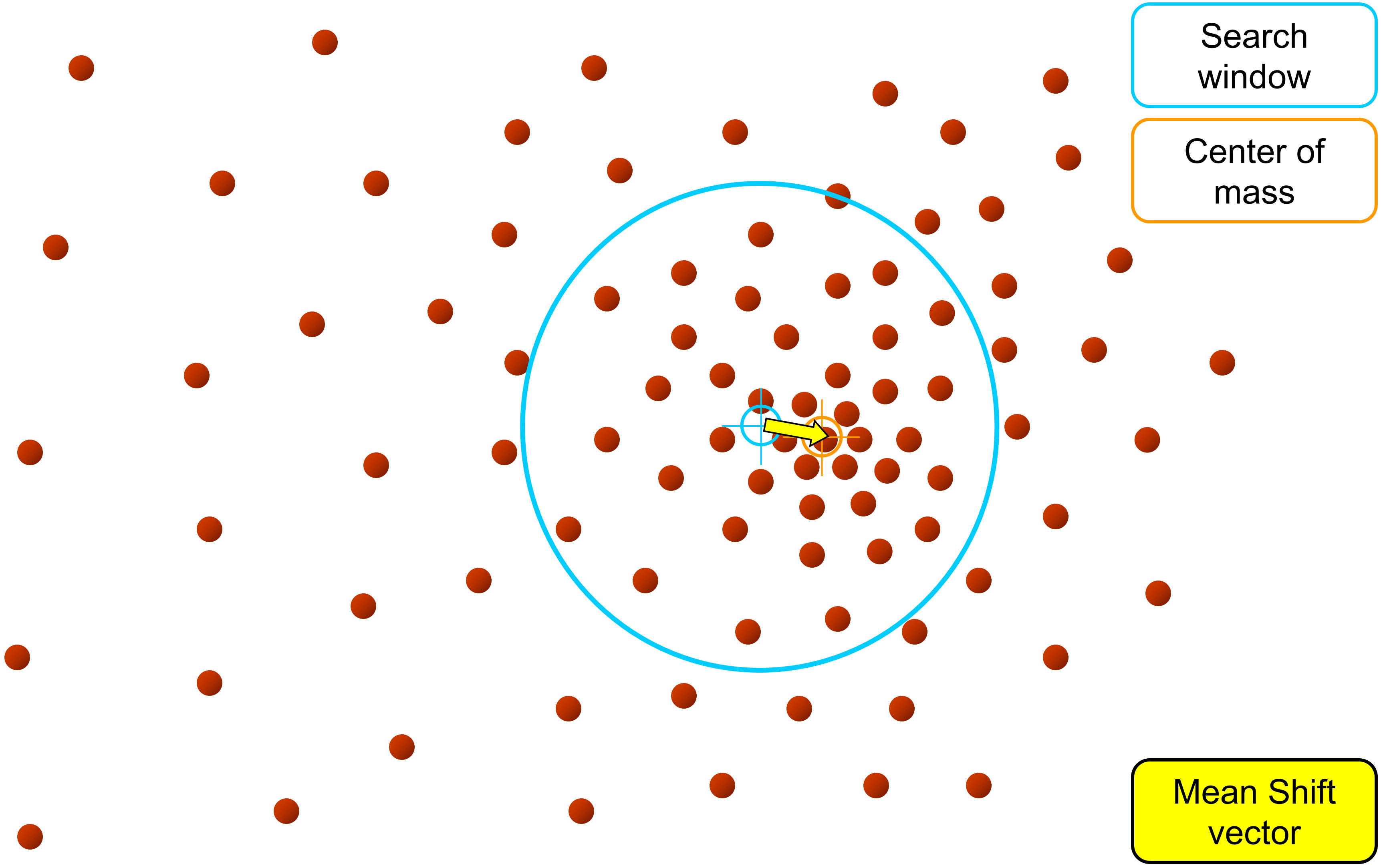
Slide by Y. Ukrainitz & B. Sarel



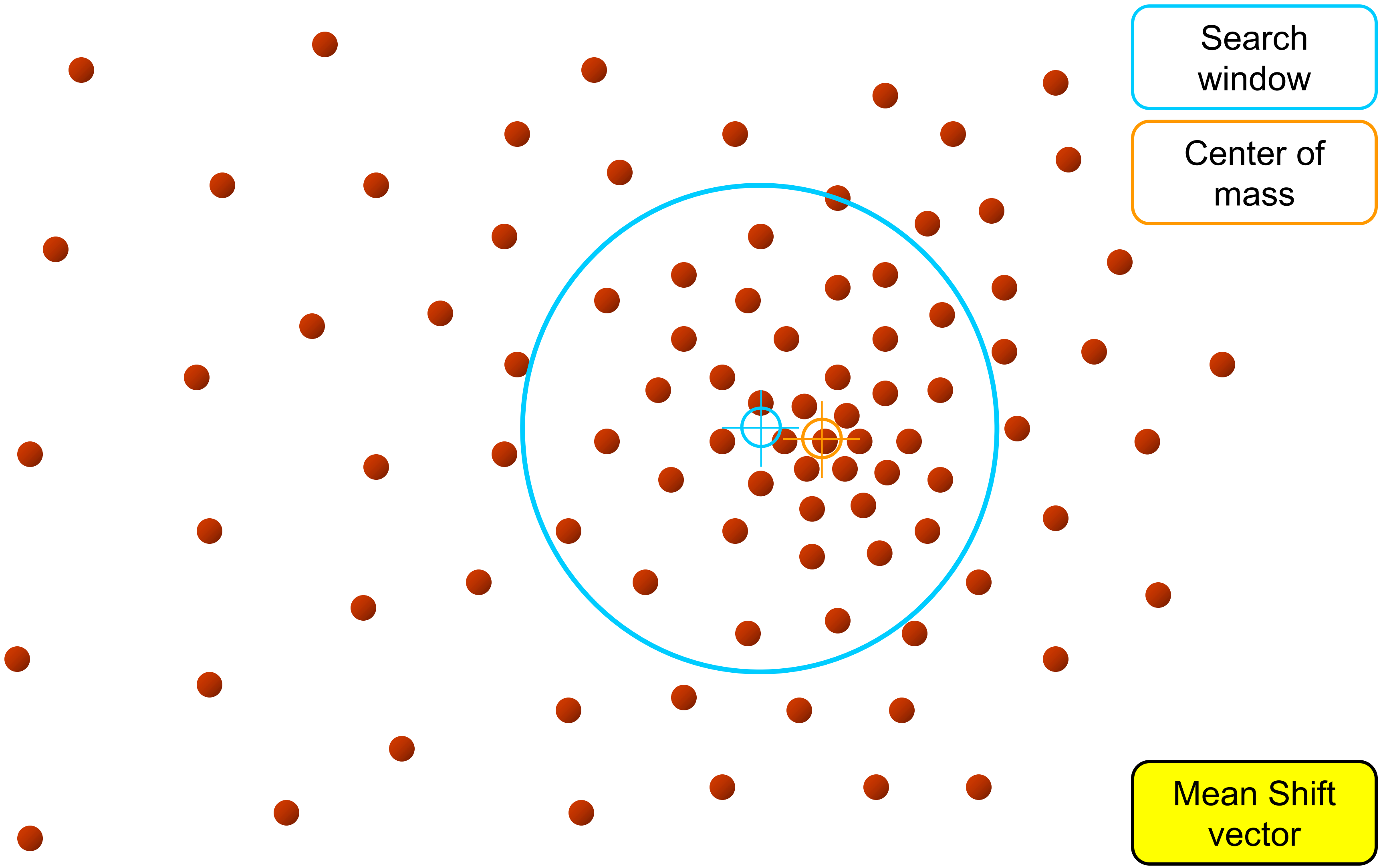
Slide by Y. Ukrainitz & B. Sarel



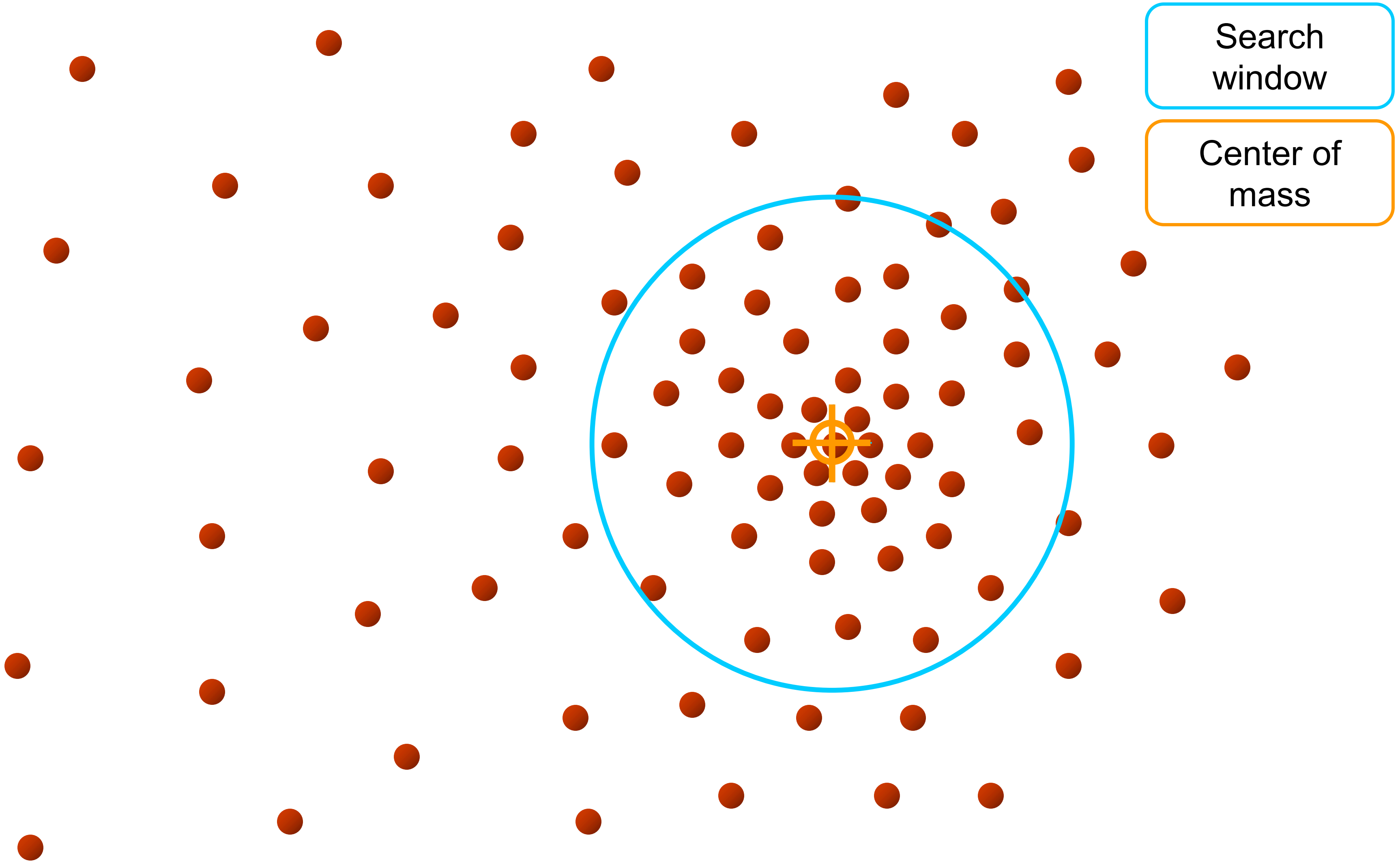
Slide by Y. Ukrainitz & B. Sarel



Slide by Y. Ukrainitz & B. Sarel



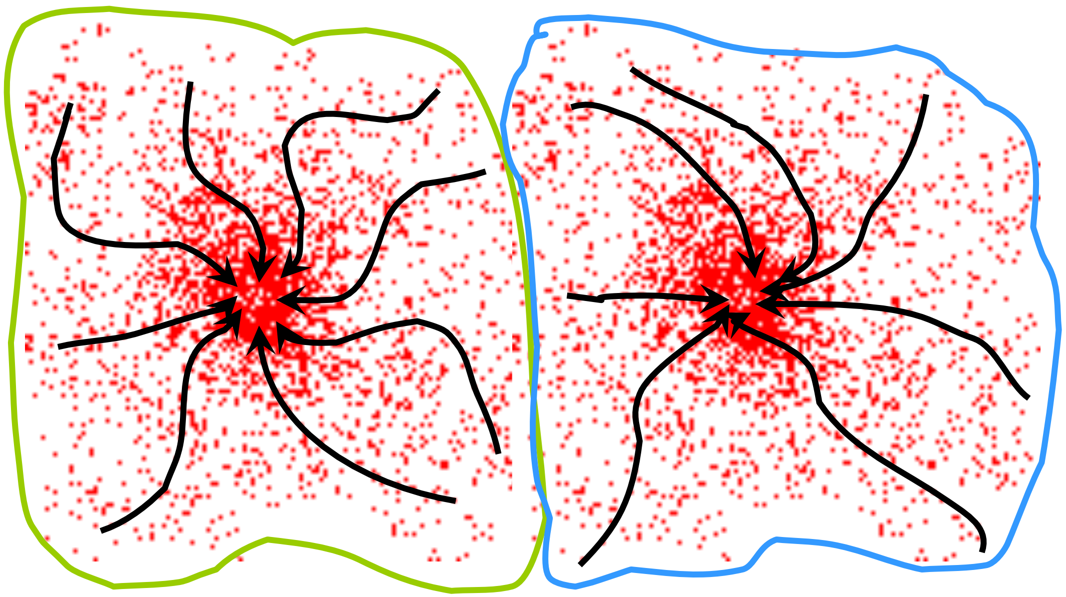
Slide by Y. Ukrainitz & B. Sarel



Slide by Y. Ukrainitz & B. Sarel

Mean shift clustering

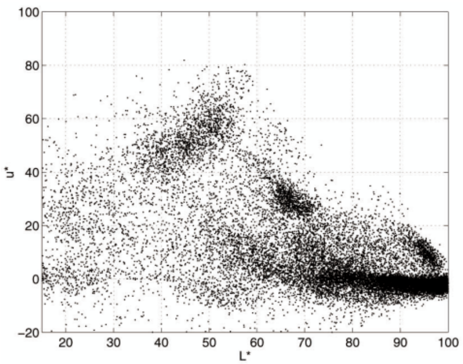
- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode



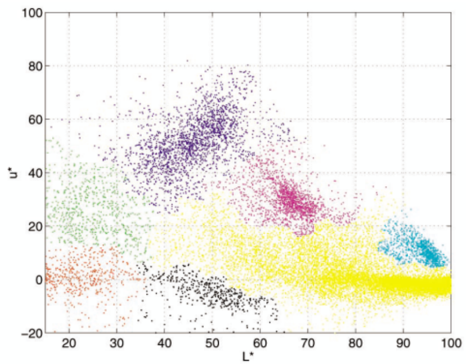
Slide by Y. Ukrainitz & B. Sarel

Mean Shift Segmentation

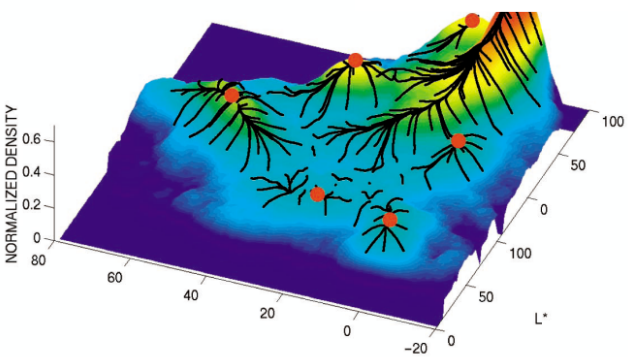
1. Convert the image into tokens (via color, gradients, texture measures etc).
2. Choose initial search window locations uniformly in the data.
3. Compute the mean shift window location for each initial position.
4. Merge windows that end up on the same "peak" or mode.
5. The data these merged windows traversed are clustered together.



Pixels in L*u* space



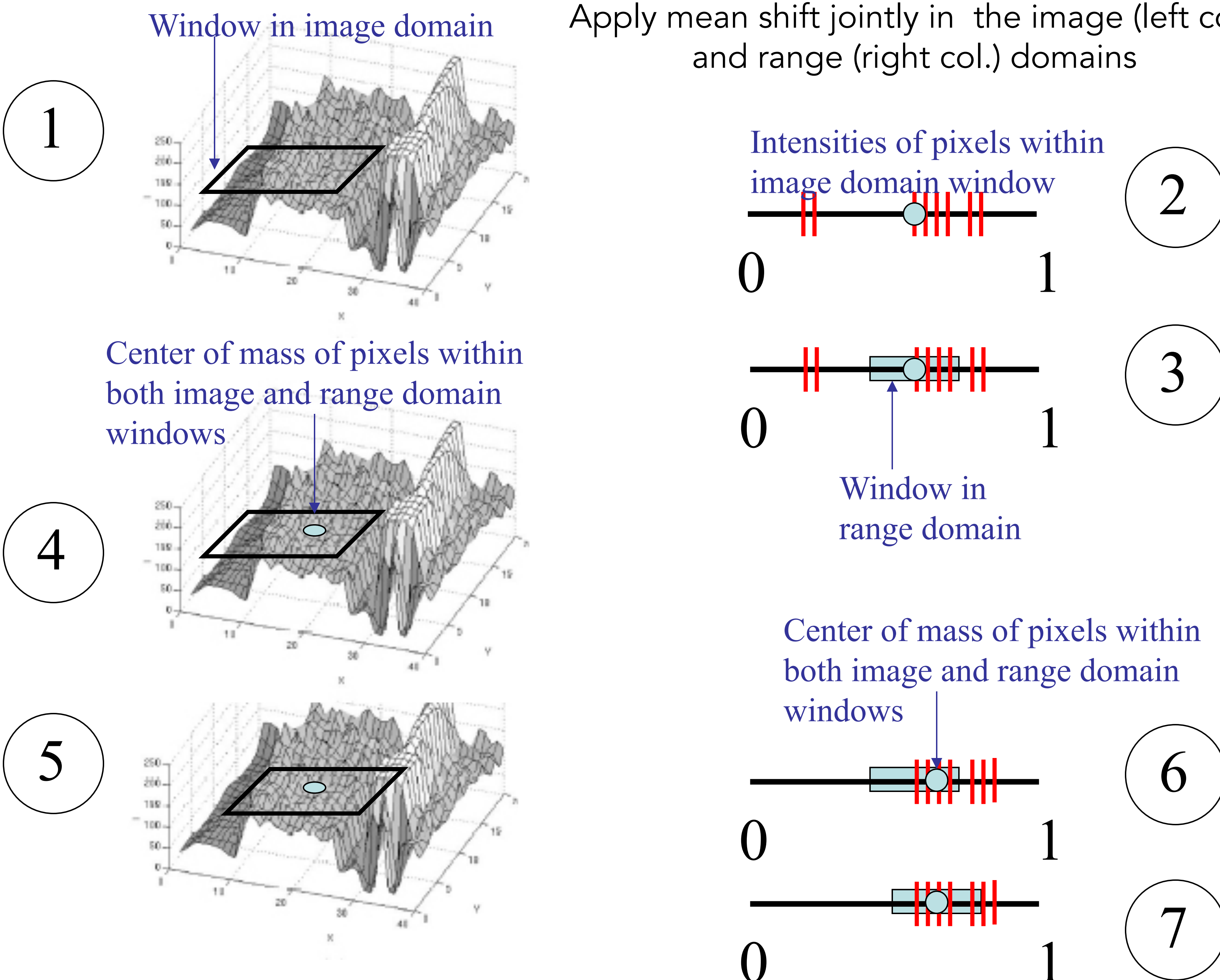
Clustering results after
~160 mean shift procedures



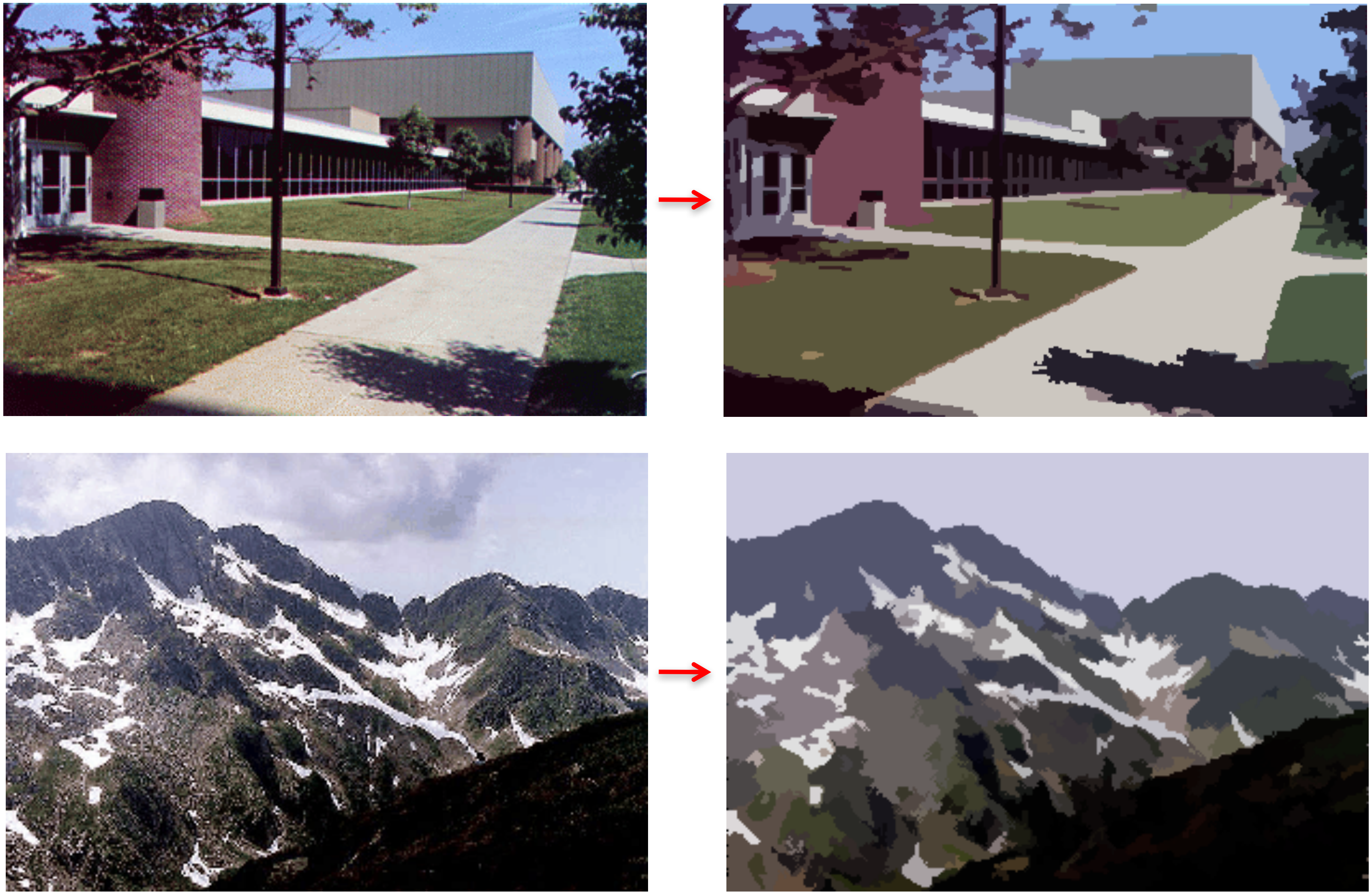
Corresponding trajectories with peaks marked as red dots

Szeliski – Chapter 5 – Segmentation, 5.3 Mean shift

Apply mean shift jointly in the image (left col.) and range (right col.) domains



Mean Shift color & spatial Segmentation Results:



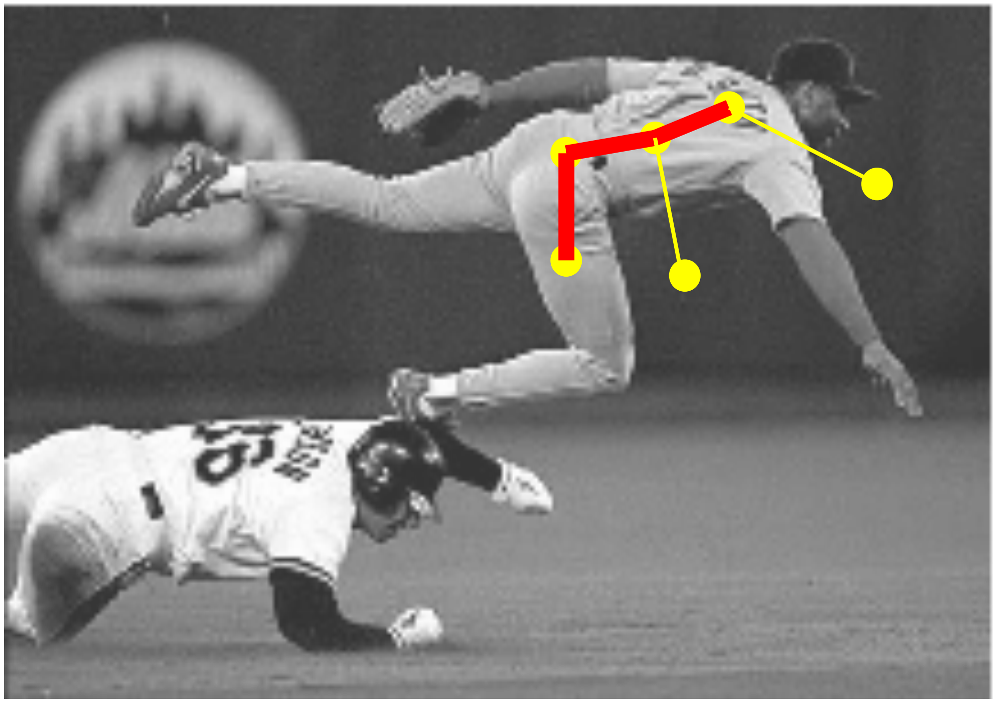
Mean shift pros and cons

- Pros
  - Clusters are places where data points tend to be close together
  - Just a single parameter (window size)
  - Finds variable number of modes
  - Robust to outliers
- Cons
  - Output depends on window size
  - Computationally expensive
  - Does not scale well with dimension of feature space

Slide credit: S. Lazebnik

Method 3: Graph-Theoretic Image Segmentation

Build a **weighted graph G**=(V,E) from image



A different way of thinking about segmentation…

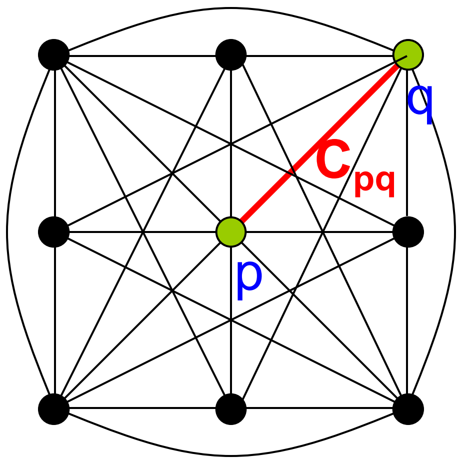
V: image pixels

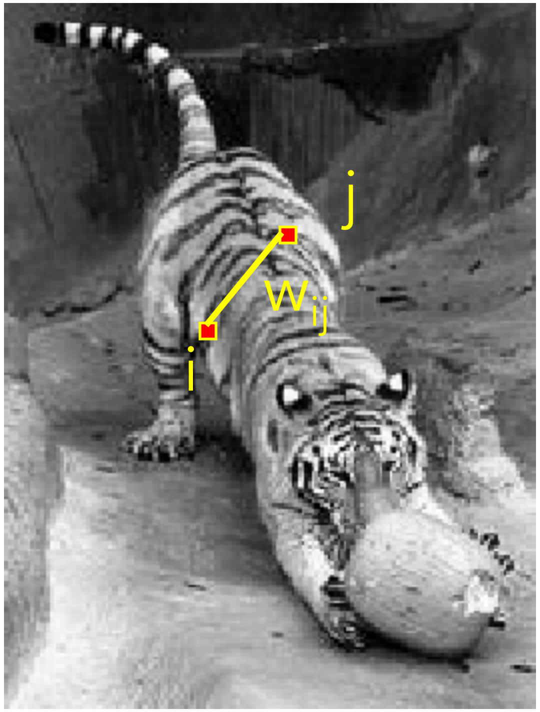
E: connections between pairs of nearby pixels

W_{ij} : probability that i &j belong to the same region

Segmentation = graph partition

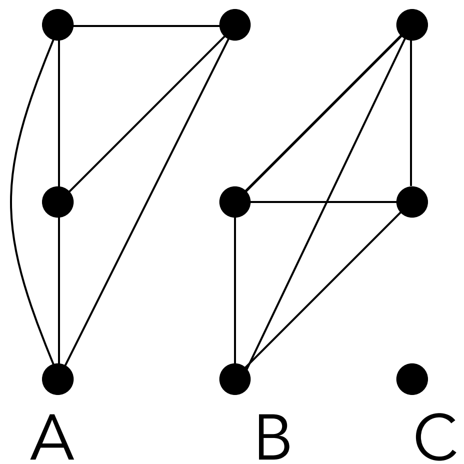
Segmentation by graph cut





- Fully connected graph (node for every pixel i,j)
- Edge/link between every pair of pixels: p,q
- Each edge is weighted by the *affinity* or similarity of the two nodes:
  - cost c_{pq} for each link: c_{pq} measures *similarity (or affinity)*
  - similarity is *inversely proportional* to difference in color and position

Segmentation by graph cut



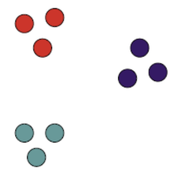
- Break Graph into Segments
  - Delete links that cross between segments
  - Easiest to break links that have low cost (similarity or affinity)
    - similar pixels should be in the same segments
    - dissimilar pixels should be in different segments

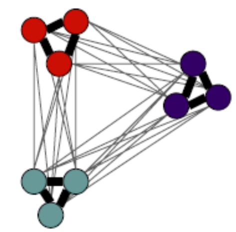
Source: S. Seitz

Graph-based Image Segmentation

Goal: Given data points X1, …, Xn and similarities w(Xi,Xj), partition the data into groups so that points in a group are similar and points in different groups are dissimilar.

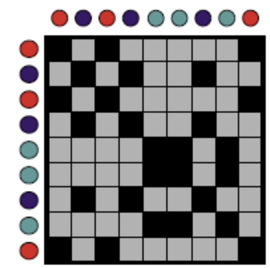
1- Get vectors of data





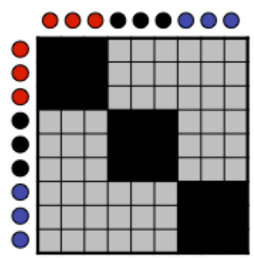
2a- Build a similarity graph

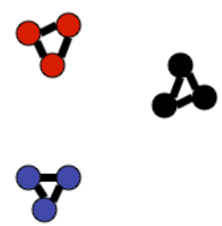
2b- Build a similarity/affinity matrix



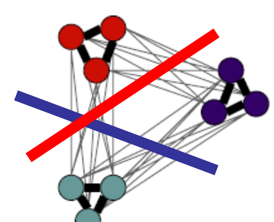
Similarities

3- Calculate eigenvectors





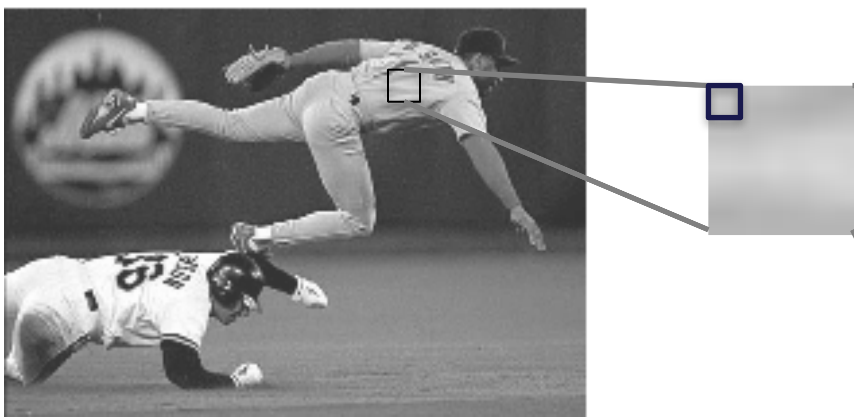
4- Cut the graph:
apply threshold to eigenvectors



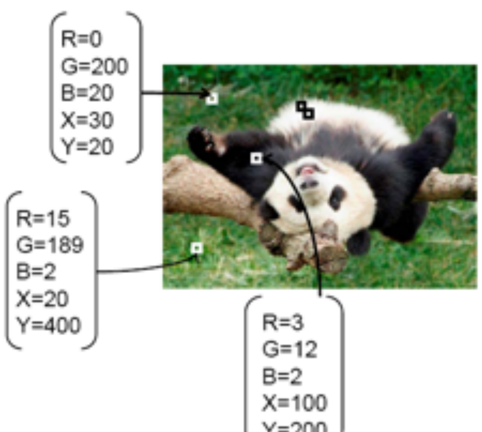
1- Vectors of data

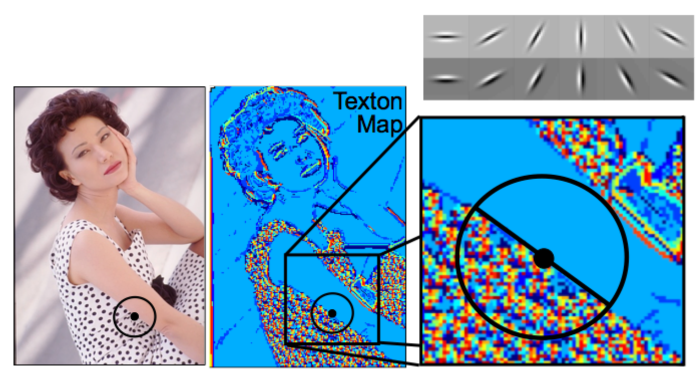
We represent each pixel by a feature vector **x**, and define a distance function appropriate for this feature representation (e.g. euclidean distance).

Features can be brightness value, color– RGB, L*u*v; texton histogram, etc- and calculate distances between vectors (e.g. Euclidean distance)



191	187	179	176	176
186	180	169	165	165
181	177	171	172	172
178	178	178	184	188
194	193	191	195	200
201	201	199	201	205
188	186	183	185	191
172	171	172	174	175
157	158	163	164	162
163	163	165	165	163





Textons

Computing distance

- We represent each pixel by a feature vector $\mathbf{x}$, and define a distance function appropriate for this feature representation
- Then we can convert the distance between two feature vectors into an affinity/similarity measure with the help of a generalized Gaussian kernel:

$$\exp\left(-\frac{1}{2\sigma^2}\mathrm{dist}(\mathbf{x}_i,\mathbf{x}_j)^2\right)$$

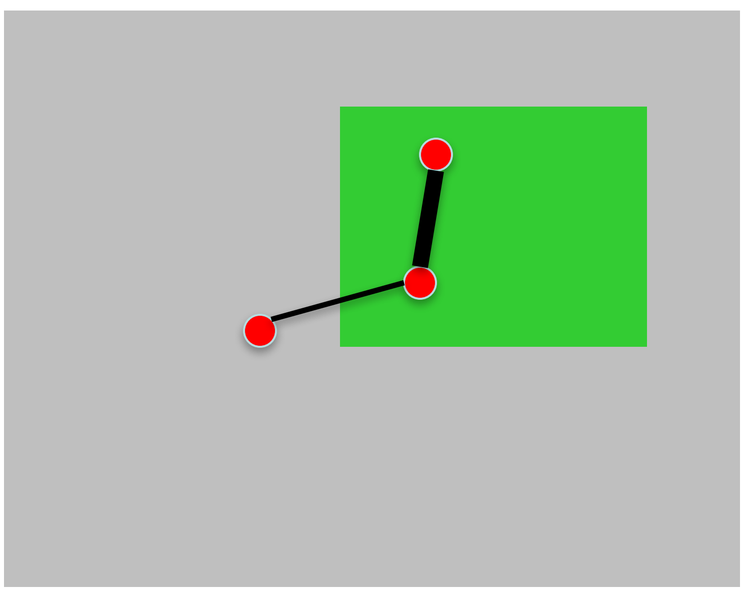
Slide credit: S. Lazebnik

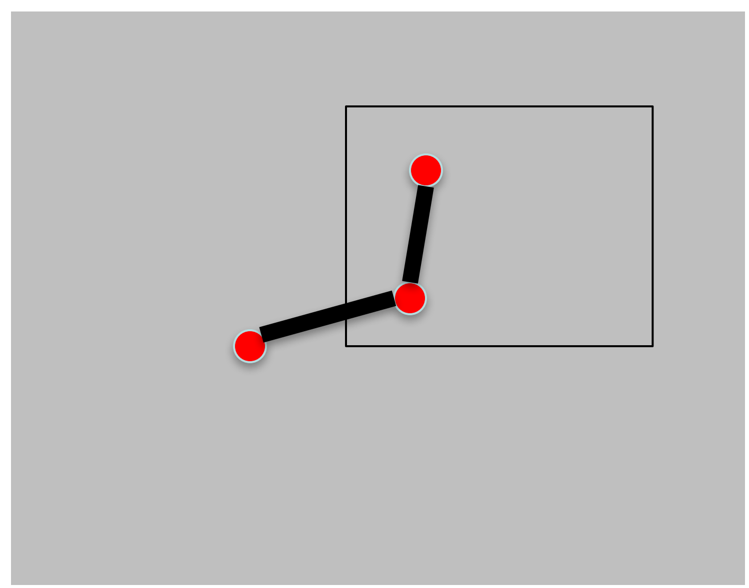
Affinity between pixels

Similarities among pixel descriptors

$$W_{ij} = \exp(-\| z_i - z_j \|^2 / \sigma^2)$$

σ = Scale factor…
it will hunt us later





Affinity between pixels

Similarities among pixel descriptors

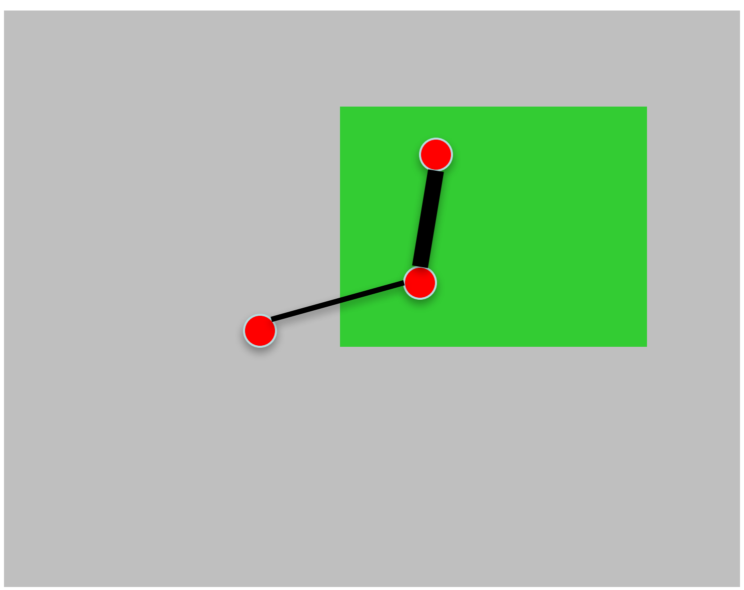
$$W_{ij} = \exp(-\| z_i - z_j \|^2 / \sigma^2)$$

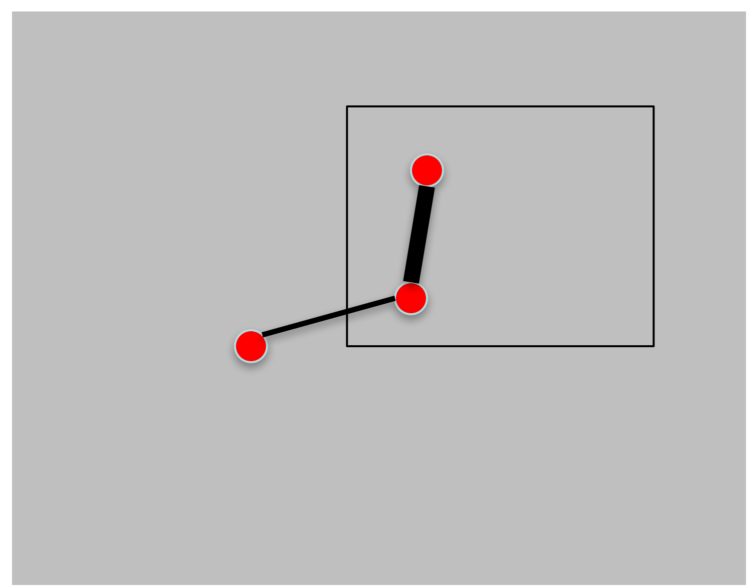
σ = Scale factor…
it will hunt us later

Interleaving edges

$$W_{ij} = 1 - \max_{\text{Line between i and j}} Pb$$

With Pb = probability of boundary

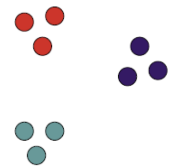




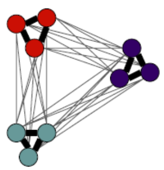
Graph-based Image Segmentation

Goal: Given data points X1, …, Xn and similarities w(Xi,Xj), partition the data into groups so that points in a group are similar and points in different groups are dissimilar.

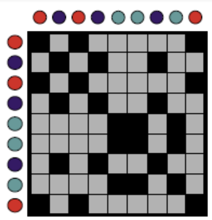
1- Get vectors of **data**



2a- Build a **similarity** graph

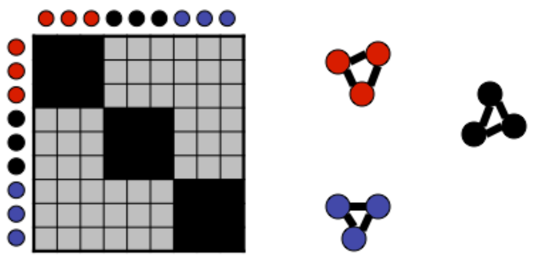


2b- Build a similarity/affinity **matrix**

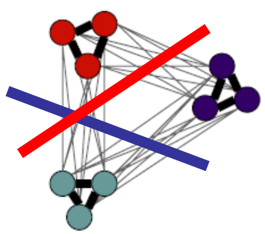


Similarities

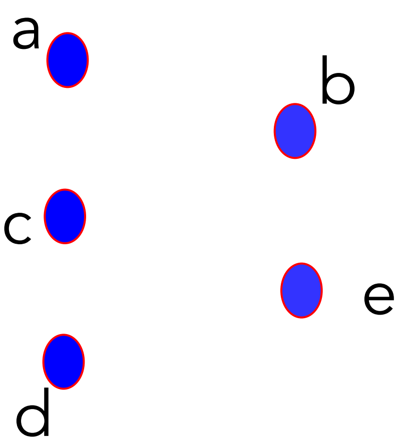
3- Calculate eigenvectors



4- Cut the graph:
apply threshold to eigenvectors



2a- What is a graph?

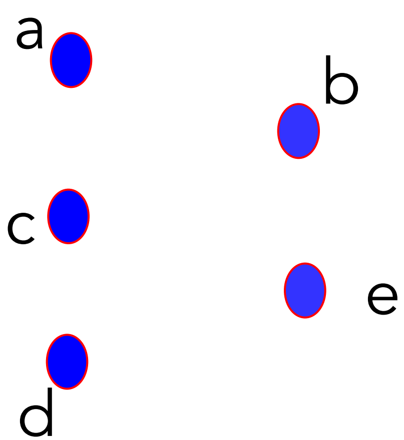


$$
\begin{array}{c c}
 & \begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \left[\begin{array}{ccccc} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{array}\right]
\end{array}
$$

Adjacency Matrix

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

2a- What is a graph?

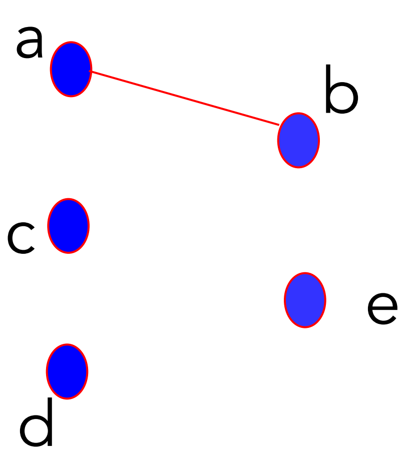


$$
\begin{array}{c|c}
 & \begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} &
\begin{bmatrix}
0 & - & - & - & - \\
- & 0 & - & - & - \\
- & - & 0 & - & - \\
- & - & - & 0 & - \\
- & - & - & - & 0
\end{bmatrix}
\end{array}
$$

Adjacency Matrix

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

2a- What is a graph?

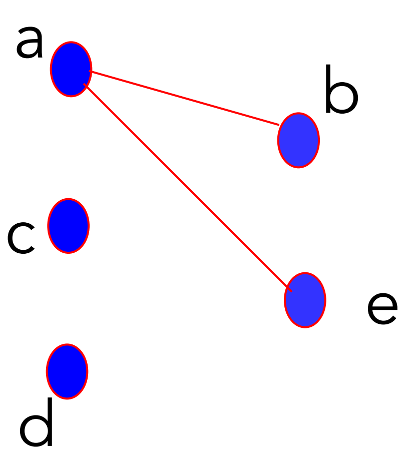


$$\begin{array}{c|c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 0 & 1 & - & - & - \\ 1 & 0 & - & - & - \\ - & - & 0 & - & - \\ - & - & - & 0 & - \\ - & - & - & - & 0 \end{bmatrix} \end{array}$$

Adjacency Matrix

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

2a- What is a graph?

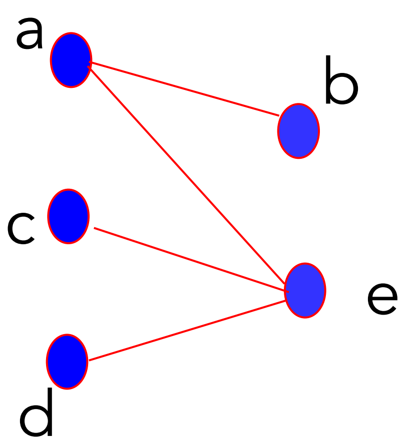


$$
\begin{array}{c c}
 & \begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} &
\left[\begin{array}{ccccc}
0 & 1 & - & - & 1 \\
1 & 0 & - & - & - \\
- & - & 0 & - & - \\
- & - & - & 0 & - \\
1 & - & - & - & 0
\end{array}\right]
\end{array}
$$

Adjacency Matrix

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

2a- What is a graph?



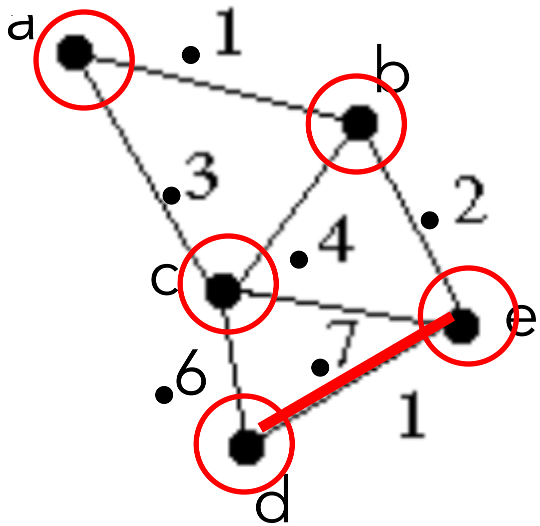
$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}
\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Adjacency Matrix

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

2a- What is a weighted graph?

Affinity Matrix represents the weighted links



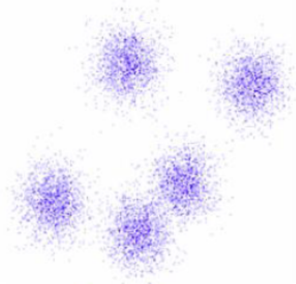
$$W = \begin{array}{c} \\ \\ \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \left[\begin{array}{ccccc} 1 & .1 & .3 & 0 & 0 \\ 0 & 1 & .4 & 0 & .2 \\ .3 & .4 & 1 & .6 & .7 \\ 0 & 0 & .6 & 1 & 1 \\ 0 & .2 & .7 & 1 & 1 \end{array}\right] \end{array}\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}$$

Diagonal: each point with itself is 1
Strong links/edges
Weak links/edges
No links/edges connected

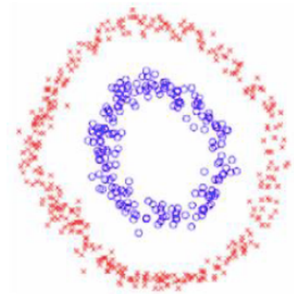
W_{ij} : probability that i &j belong to the same region

i,j are the pixels in the image

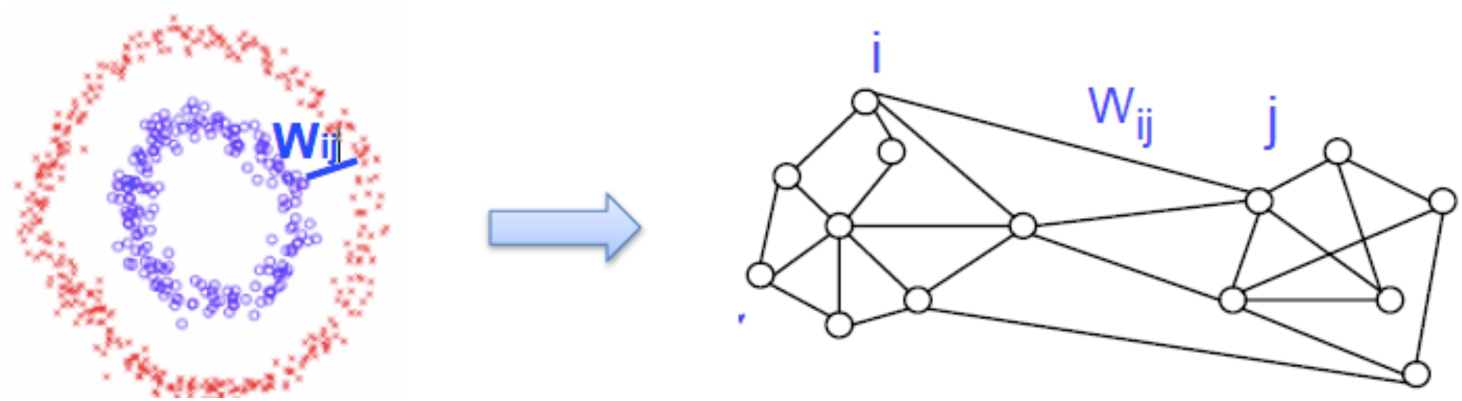
See Forsyth-Ponce chapter


Compactness

Similarity graph construction


Connectivity

Similarity Graphs: Model local neighborhood relations between data points



E.g. Gaussian kernel similarity function

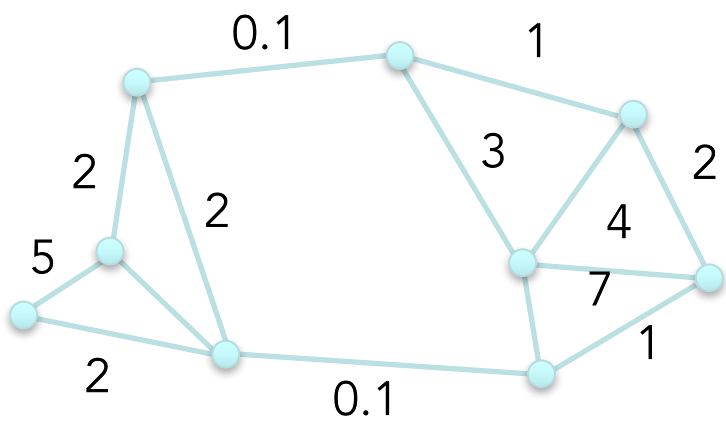
$$W_{ij} = e^{\frac{\|x_i - x_j\|^2}{2\sigma^2}}$$

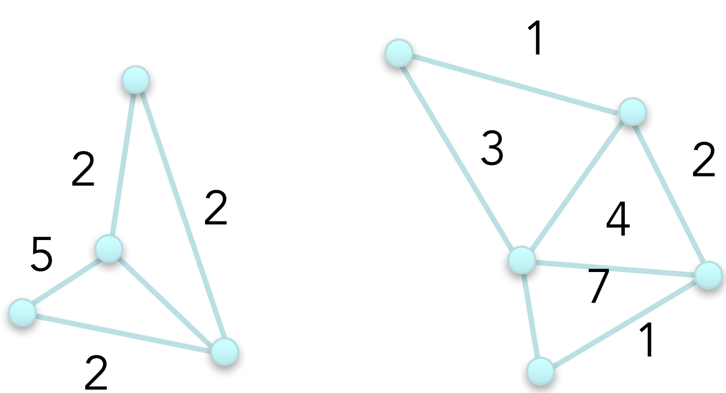
Controls size of neighborhood

See Forsyth-Ponce chapter

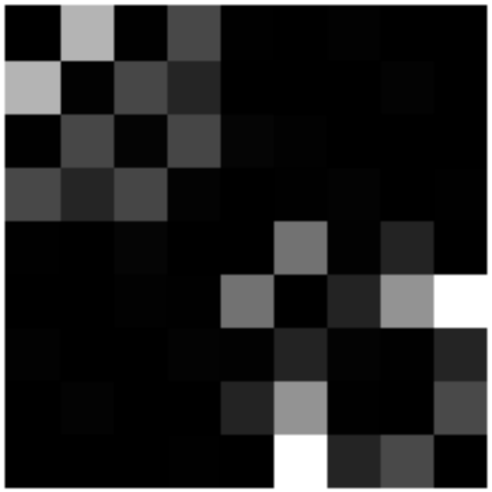
2b- Building Affinity Matrix

A weighted graph





Weight matrix associated with the graph (larger values are lighter)



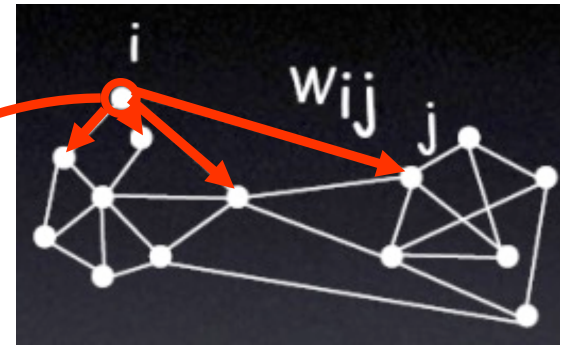
A cut of the graph: two tightly linked components. This cut decomposes the graph's matrix into two main blocks on the diagonal

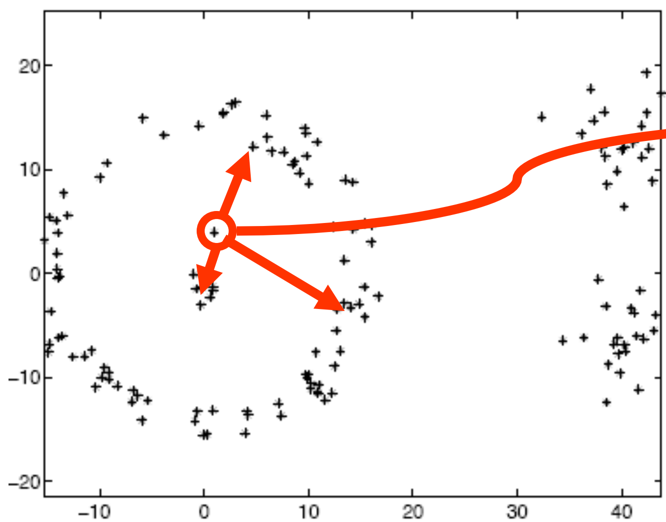
We can do segmentation by finding the *minimum cut* in a graph.

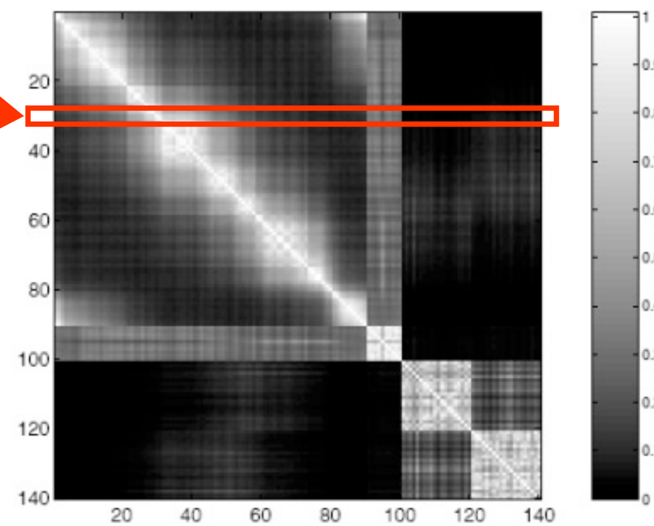
Graph terminology

- Similarity matrix: $W = \left[w_{i,j} \right]$

$$w_{i,j} = e^{\frac{-\left\| X_{(i)} - X_{(j)} \right\|_2^2}{\sigma_X^2}}$$







Weight matrix associated with the graph
(larger values are lighter)

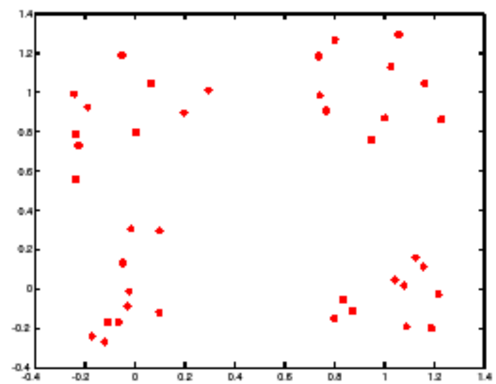
Slides from Jianbo Shi

Scale affects affinity

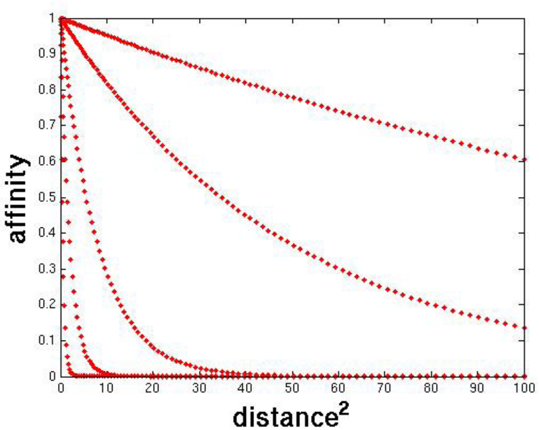
$$W_{ij} = \exp(-\| z_i - z_j \|^2 / \sigma^2)$$

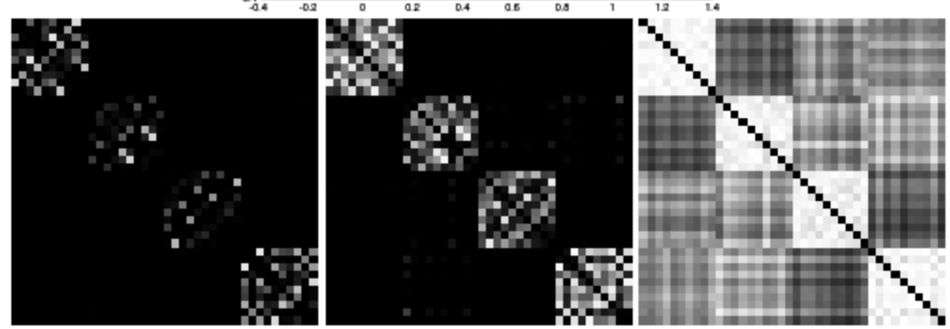
- Small σ: group only nearby points
- Large σ: group far-away points

Dataset of 4 groups of 10 points drawn from a normal distribution with four different means



σ=.2

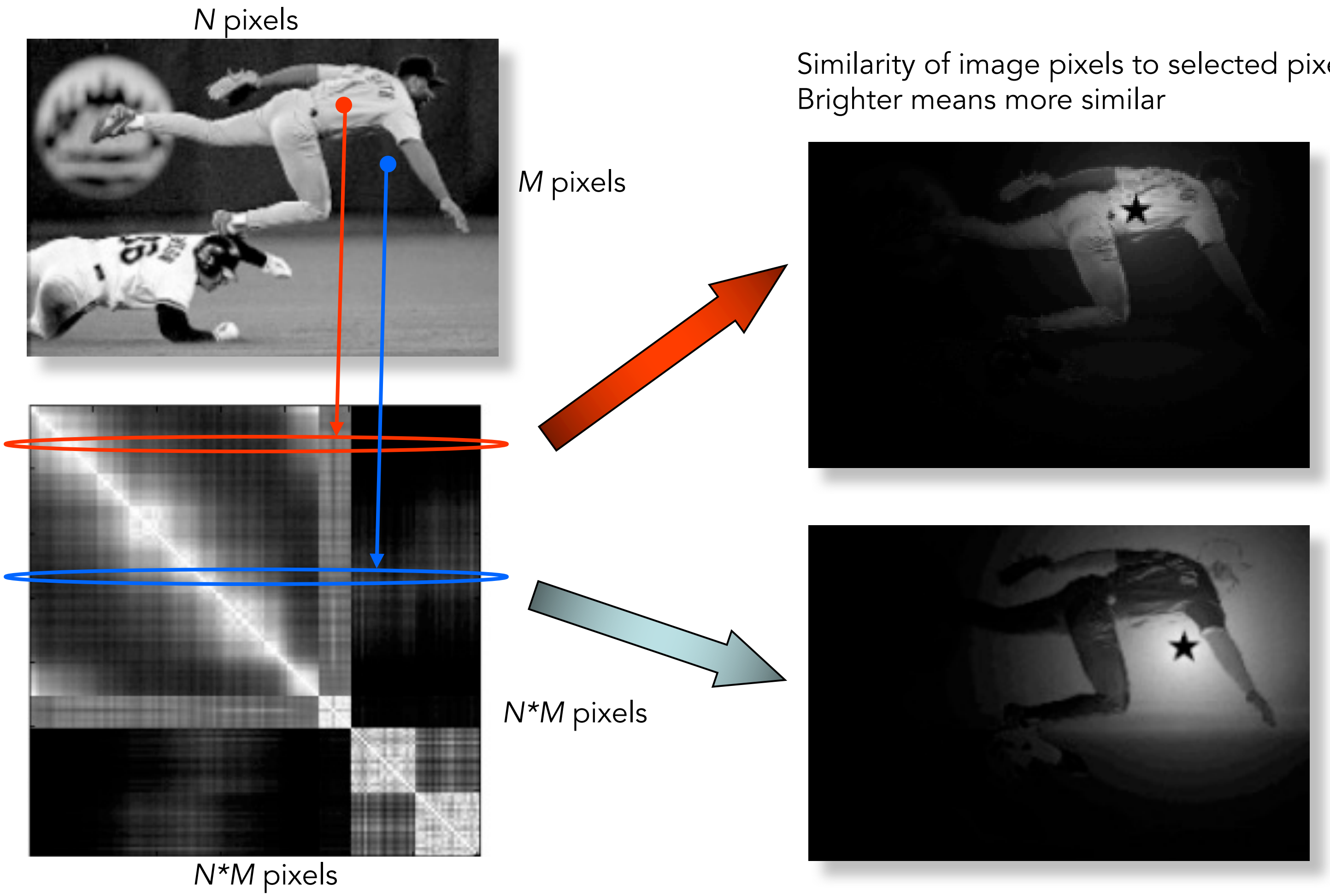




σ=.1 σ=.2 σ=1

See Forsyth-Ponce chapter

Affinity matrix of a natural image

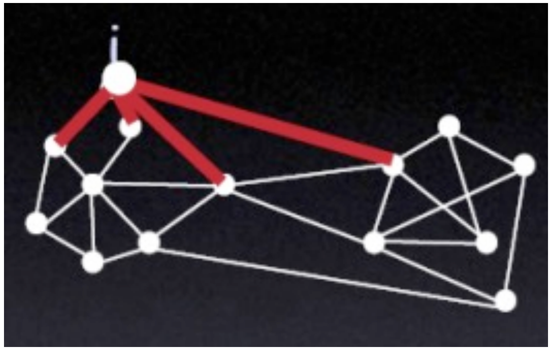


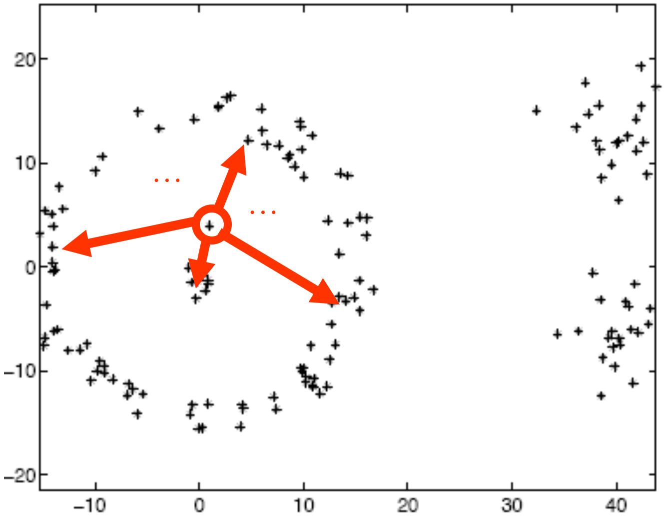
Similarity of image pixels to selected pixel
Brighter means more similar

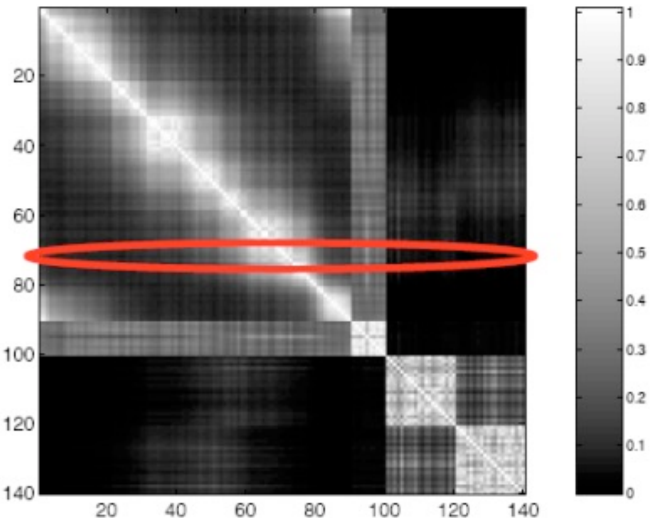
Graph terminology

- Degree of node:

$$d_i = \sum_j w_{i,j}$$





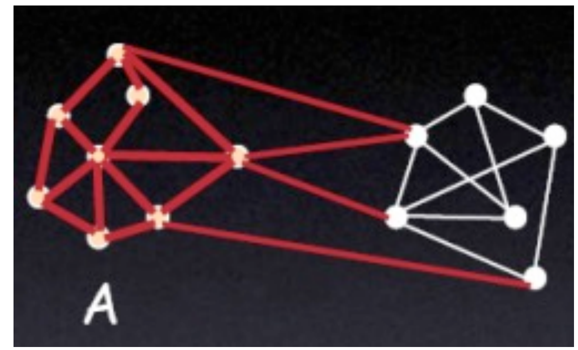


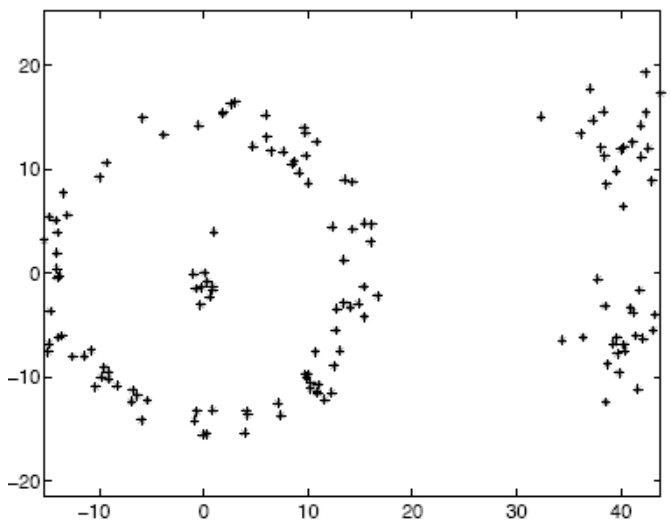
Slides from Jianbo Shi

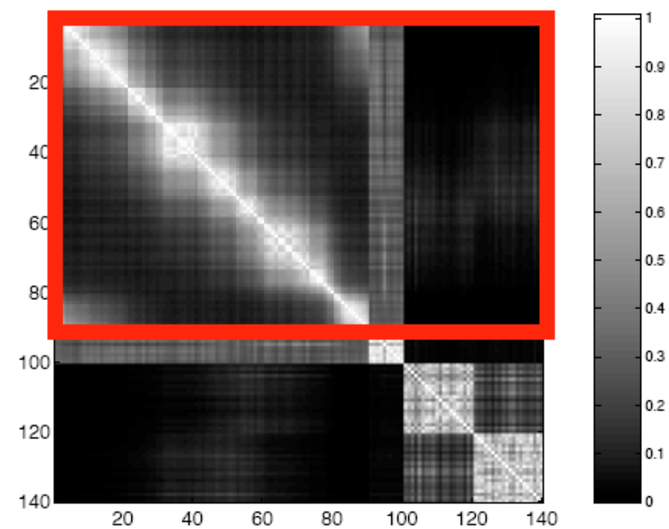
Graph terminology

- Volume of set:

$$vol(A) = \sum_{i \in A} d_i, A \subseteq V$$





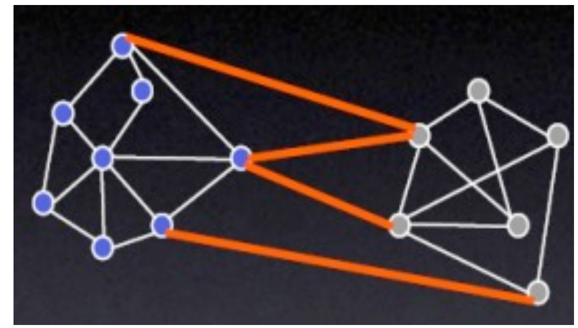


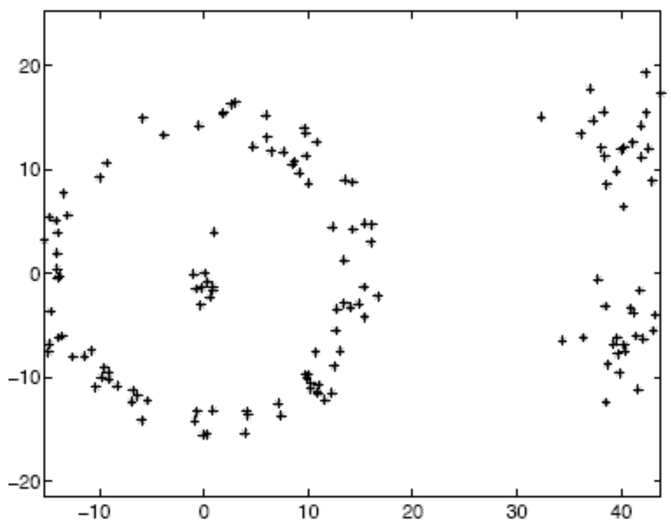
Slides from Jianbo Shi

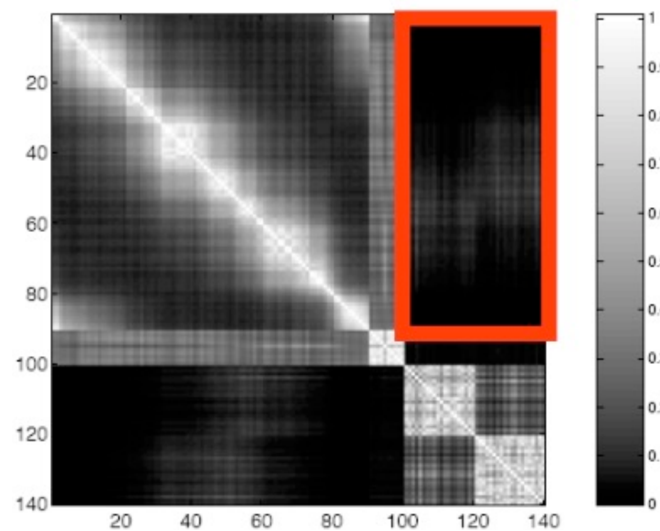
Graph terminology

Cuts in a graph:

$$cut(A,\overline{A}) = \sum_{i\in A, j\in \overline{A}} w_{i,j}$$





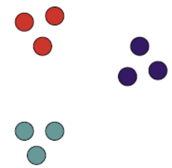


Slides from Jianbo Shi

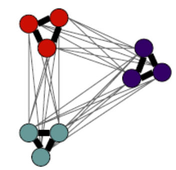
Graph-based Image Segmentation

Goal: Given data points X1, …, Xn and similarities w(Xi,Xj), partition the data into groups so that points in a group are similar and points in different groups are dissimilar.

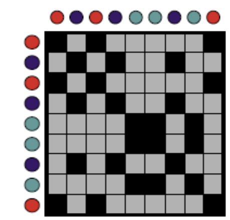
1- Get vectors of data



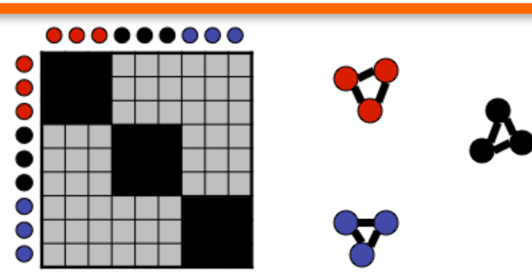
2a- Build a similarity graph



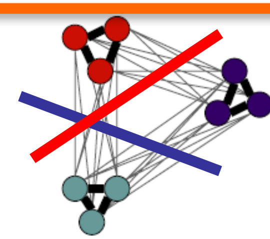
2b- Build a similarity matrix



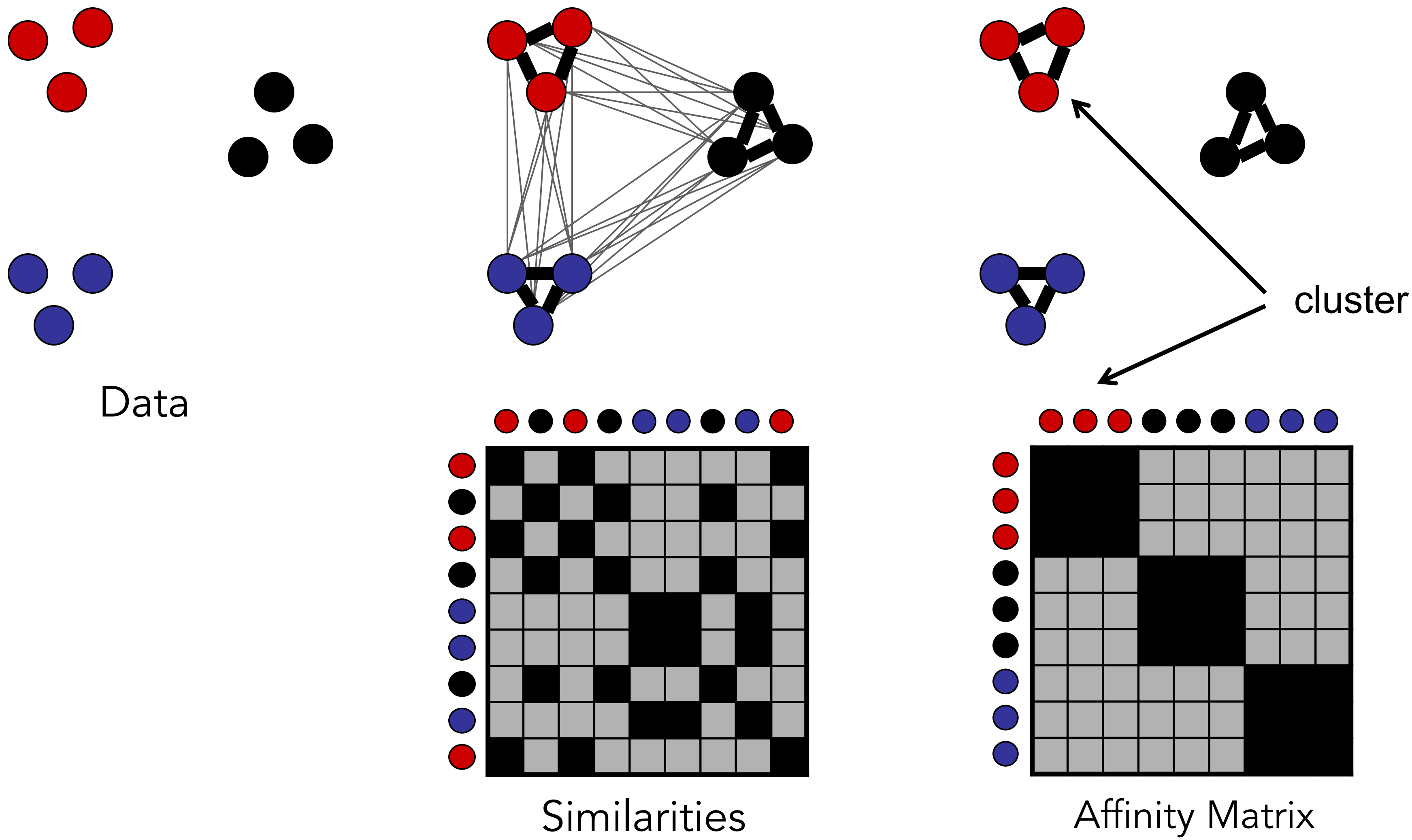
3- Calculate eigenvectors



4- Cut the graph:
apply threshold to eigenvectors



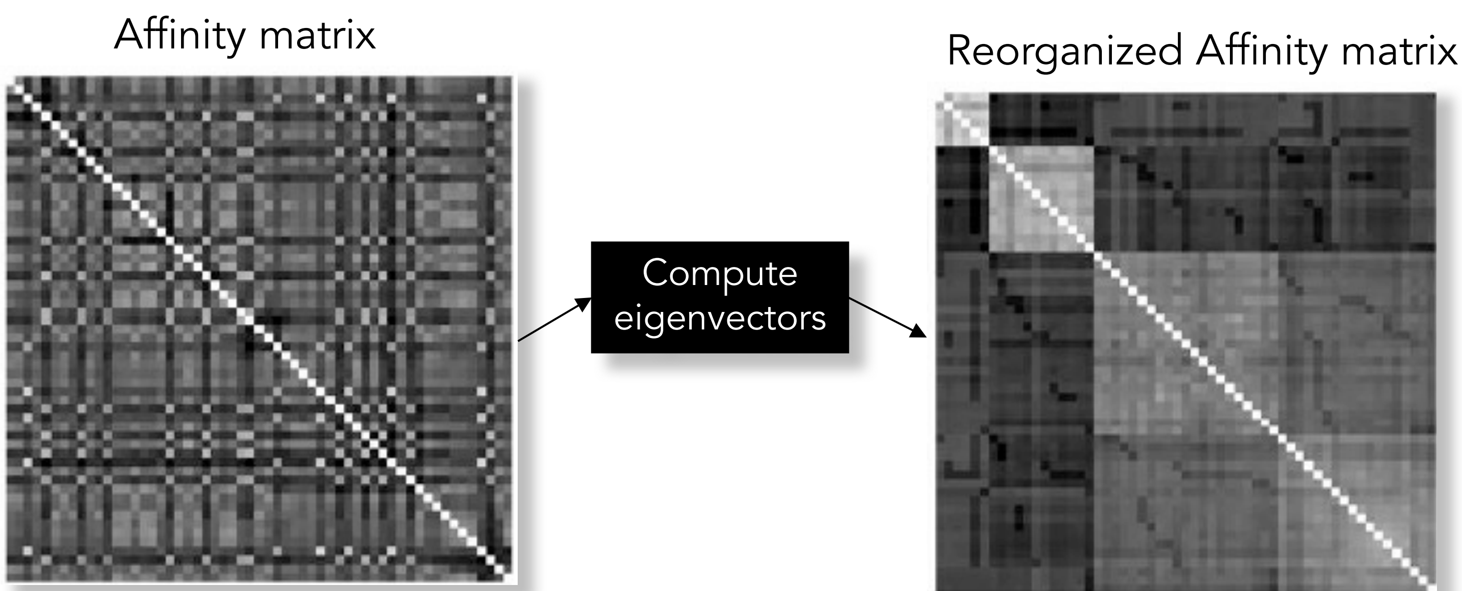
Spectral Clustering



* Slides from Dan Klein, Sep Kamvar, Chris Manning, Natural Language Group Stanford University

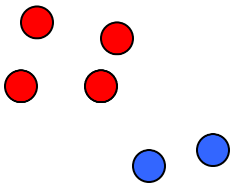
Spectral clustering: Using Eigenvalues of the matrix

- **spectral clustering** uses the eigenvalues of the similarity/affinity matrix of the data to perform dimensionality reduction before clustering in fewer dimensions



w(i,j)→ distance node i to node j

An ideal case



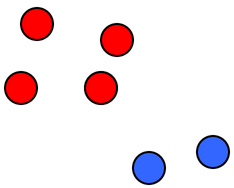
$$W=\begin{bmatrix}1&1&1&1&0&0\\1&1&1&1&0&0\\1&1&1&1&0&0\\1&1&1&1&0&0\\0&0&0&0&1&1\\0&0&0&0&1&1\end{bmatrix}$$

Affinity Matrix

What are the eigenvectors of this matrix?

On Spectral Clustering: Analysis and an algorithm. Andrew Y. Ng, Michael I. Jordan, Yair Weiss, NIPS 2001

An ideal case



Eigenvectors:

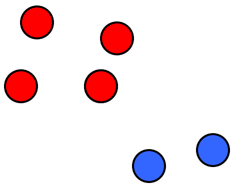
$$W=\begin{bmatrix}1&1&1&1&0&0\\1&1&1&1&0&0\\1&1&1&1&0&0\\1&1&1&1&0&0\\0&0&0&0&1&1\\0&0&0&0&1&1\end{bmatrix} \longrightarrow \begin{matrix}1&0\\1&0\\1&0\\1&0\\0&1\\0&1\\\lambda=4&\lambda=2\end{matrix} \quad \ldots$$

Affinity Matrix

On Spectral Clustering: Analysis and an algorithm. Andrew Y. Ng, Michael I. Jordan, Yair Weiss, NIPS 2001

An ideal case

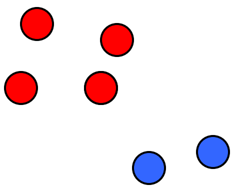
But we do not know the ordering, so W with have some random permutation:



On Spectral Clustering: Analysis and an algorithm. Andrew Y. Ng, Michael I. Jordan, Yair Weiss, NIPS 2001

An ideal case

But we do not know the ordering, so W with have some random permutation:



Eigenvectors:

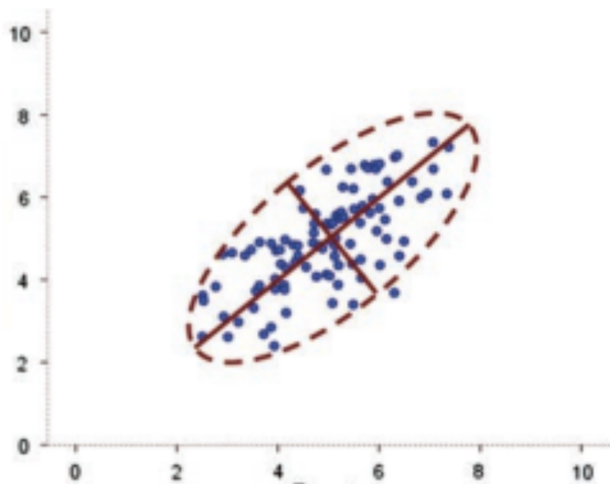
$$W=\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \longrightarrow \begin{matrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ \lambda=4 & \lambda=2 \end{matrix}$$

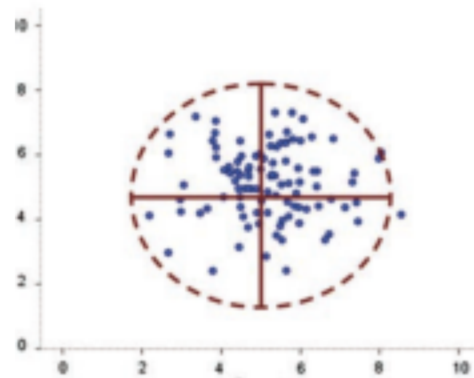
Affinity Matrix

On Spectral Clustering: Analysis and an algorithm. Andrew Y. Ng, Michael I. Jordan, Yair Weiss, NIPS 2001

What are eigenvectors?

Eigenvectors represent the dimensions of data
Eigenvalues are the length of eigenvectors

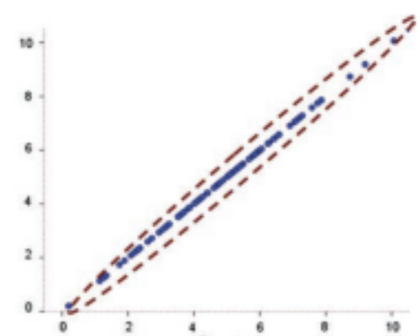




$$\frac{\text{largest eigenvalue}}{\text{smallest eigenvalue}} = 1$$

No relationship between variables

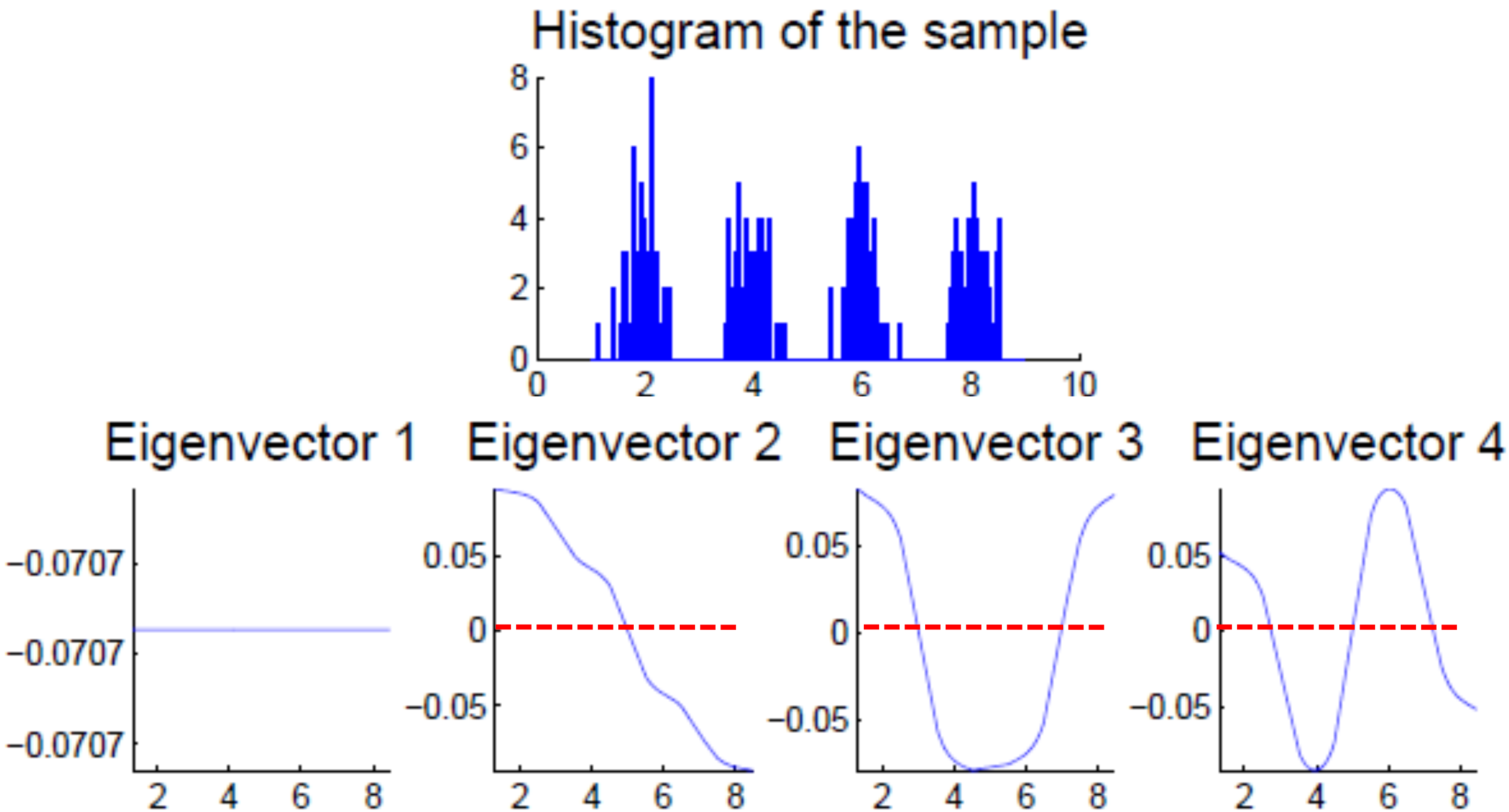
In a case of two variables,
Eigenvectors are the two lines
drawn in the scatterplot



$$\frac{\text{largest eigenvalue}}{\text{smallest eigenvalue}} = \infty$$

A linear relationship between variables

Eigenvectors example



1st Eigenvector is the all ones vector *1 (if graph is connected)*

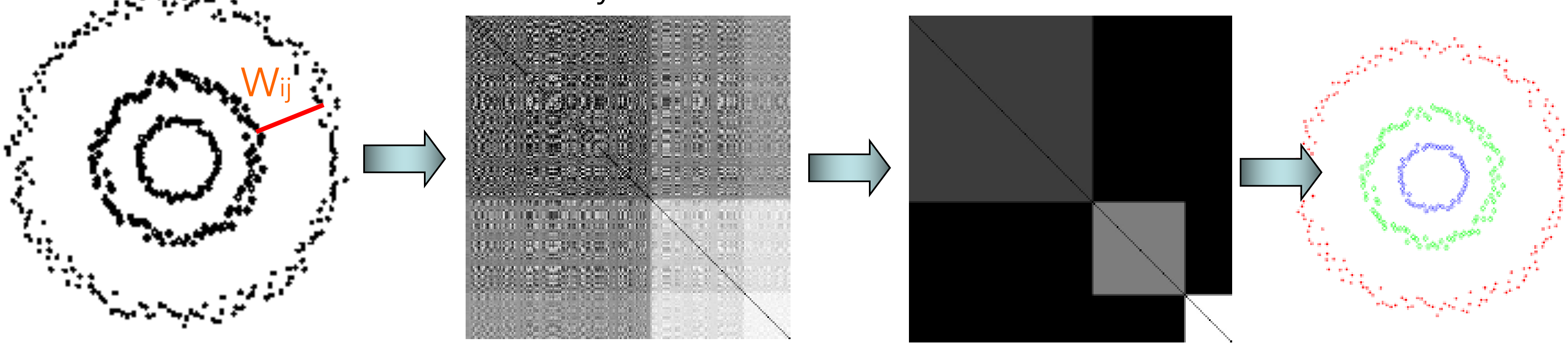
- 2nd Eigenvector thresholded at 0 separates first two clusters from last two
- k-means clustering of the 4 eigenvectors identifies all 4 clusters

Spectral Clustering pipeline

$$w_{i,j} = e^{\frac{-\left\|X_{(i)}-X_{(j)}\right\|_2^2}{\sigma_X^2}}$$

Affinity matrix M

Matrix V after Spectral Clustering



Data are projected into a lower-dimensional space (spectral/eigenvector domain) where they are easily separable

Given number *k* of clusters, compute the first k eigenvectors, V_1, …, V_k of the affinity matrix *M*
Build the matrix *V* with the eigenvectors as columns
Interpret the rows of V as new data points Z_i
Cluster the points Z_i with the k-means algorithms

	v_1	v_2	v_3
Z_1	v_{11}	v_{12}	v_{13}
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Z_n	v_{n1}	v_{n2}	v_{n3}

Dimensionality reduction
n x n → n x k

Eigenvectors and blocks

- Block weight matrices have block eigenvectors:

$\lambda_1 = 2$ $\lambda_2 = 2$ $\lambda_3 = 0$ $\lambda_4 = 0$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{\text{eigensolver}} \begin{bmatrix} .71 \\ .71 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ .71 \\ .71 \end{bmatrix}$$

- Near-block matrices have near-block eigenvectors:

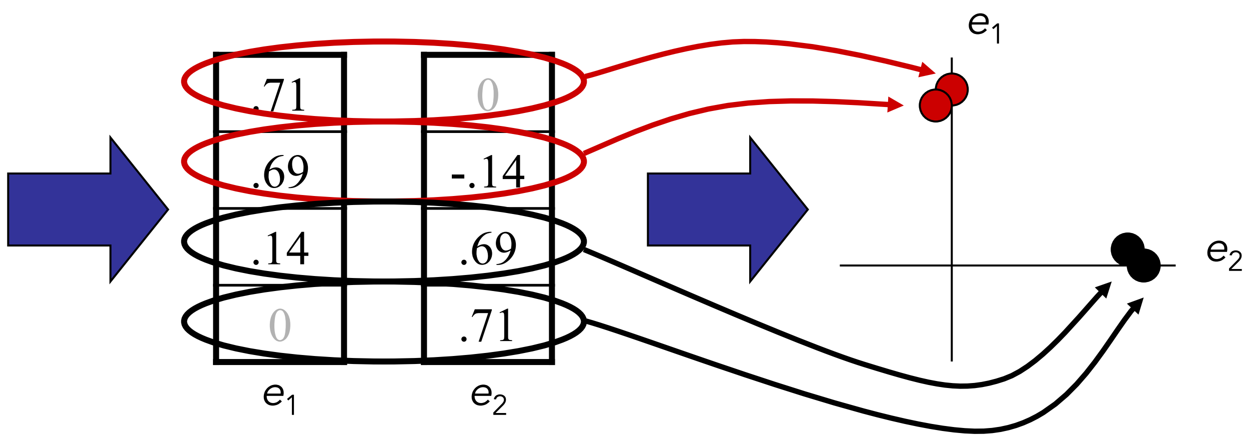
$\lambda_1 = 2.02$ $\lambda_2 = 2.02$ $\lambda_3 = -0.02$ $\lambda_4 = -0.02$

$$\begin{bmatrix} 1 & 1 & .2 & 0 \\ 1 & 1 & 0 & -.2 \\ .2 & 0 & 1 & 1 \\ 0 & -.2 & 1 & 1 \end{bmatrix} \xrightarrow{\text{eigensolver}} \underset{\text{e1}}{\begin{bmatrix} .71 \\ .69 \\ .14 \\ 0 \end{bmatrix}} \quad \underset{\text{e2}}{\begin{bmatrix} 0 \\ -.14 \\ .69 \\ .71 \end{bmatrix}}$$

Spectral Space

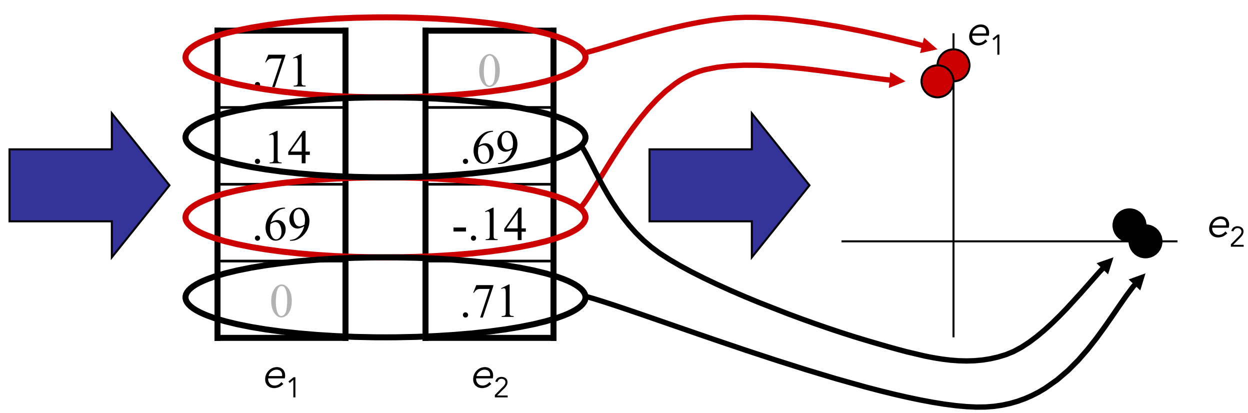
Can put items into blocks by eigenvectors:

1	1	.2	0
1	1	0	-.2
.2	0	1	1
0	-.2	1	1



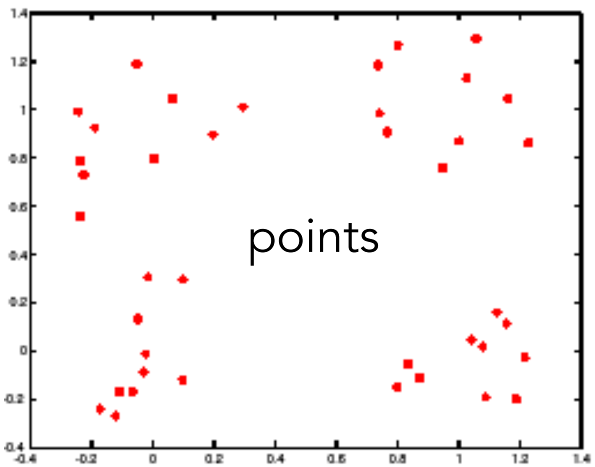
Clusters clear regardless of row ordering:

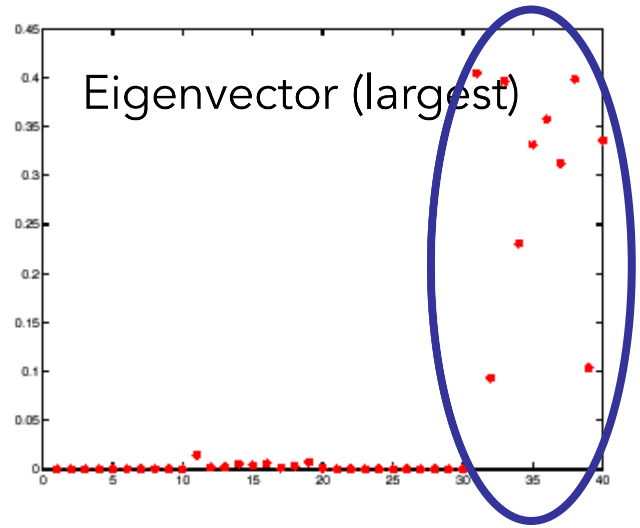
1	.2	1	0
.2	1	0	1
1	0	1	-.2
0	1	-.2	1

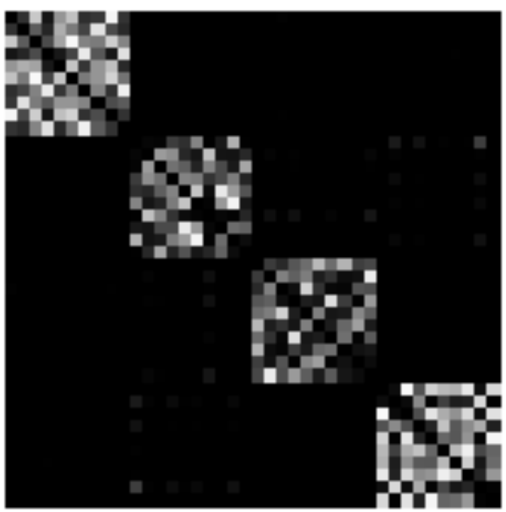


* Slides from Dan Klein, Sep Kamvar, Chris Manning, Natural Language Group Stanford University

Example eigenvector

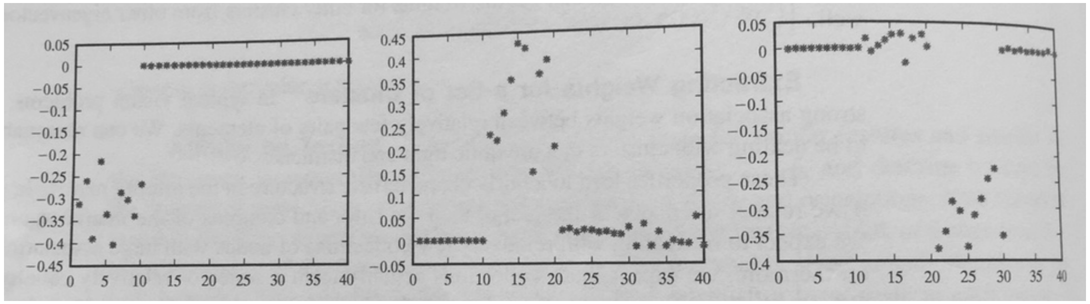






Affinity matrix

The eigenvector corresponding to the largest eigenvalue of the affinity matrix. Most values are small, but some, corresponding to the elements of the main cluster, are large



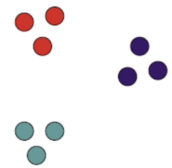
The 3 next eigenvectors corresponding to the next 3 largest eigenvalues of the affinity matrix. Most values are small but for (disjoint) sets of elements the values are large. This follows from the block structure of the affinity matrix

See Forsyth Ponce, Chapter 14 given

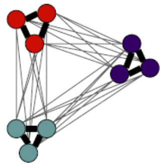
Graph-based Image Segmentation

Goal: Given data points X1, …, Xn and similarities w(Xi,Xj), partition the data into groups so that points in a group are similar and points in different groups are dissimilar.

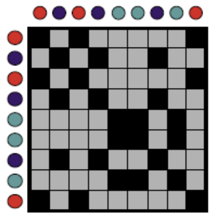
1- Get vectors of **data**



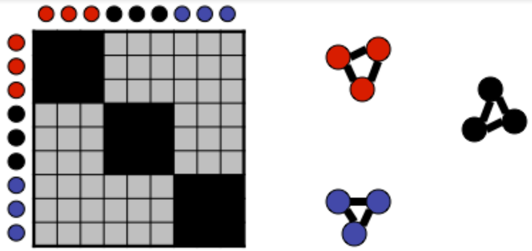
2a- Build a **similarity** graph



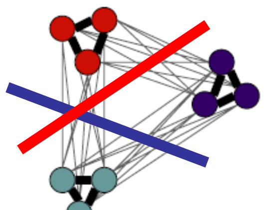
2b- Build a similarity **matrix**



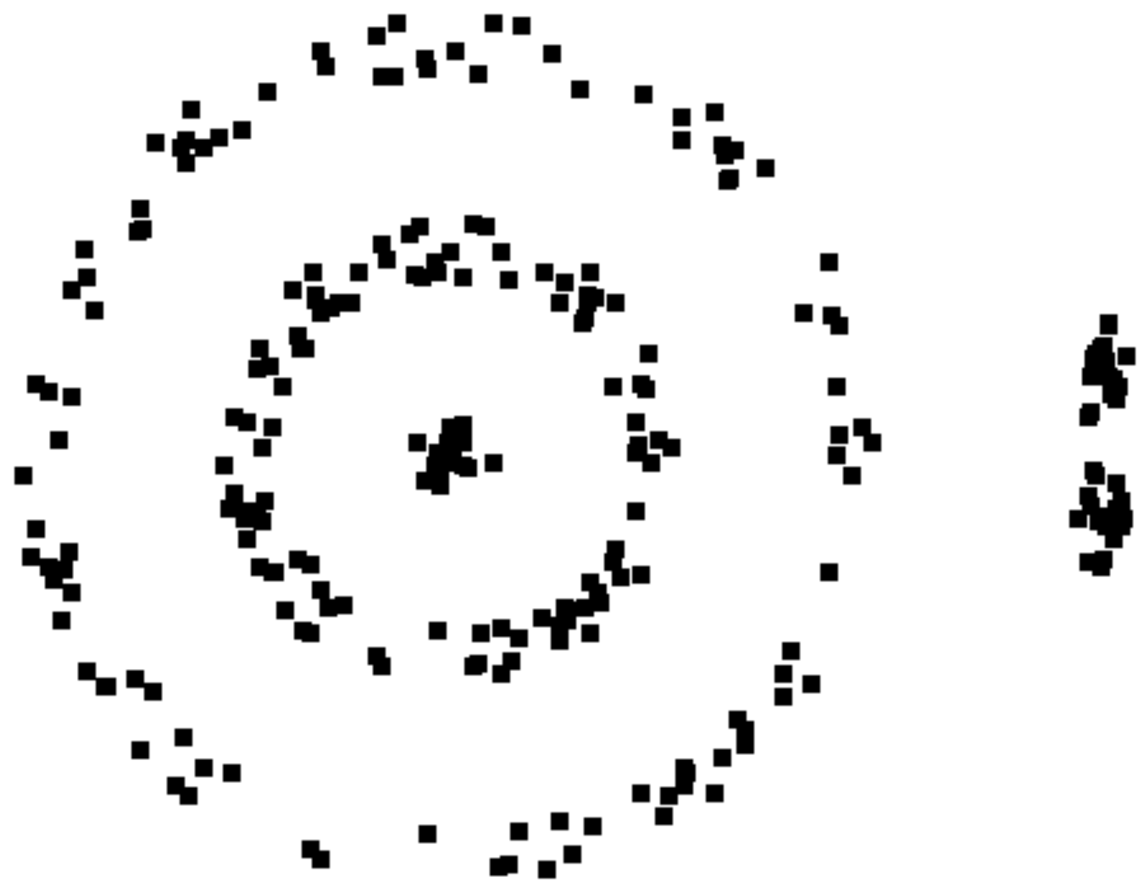
3- Calculate eigenvectors



4- Cut the graph:
apply threshold to eigenvectors



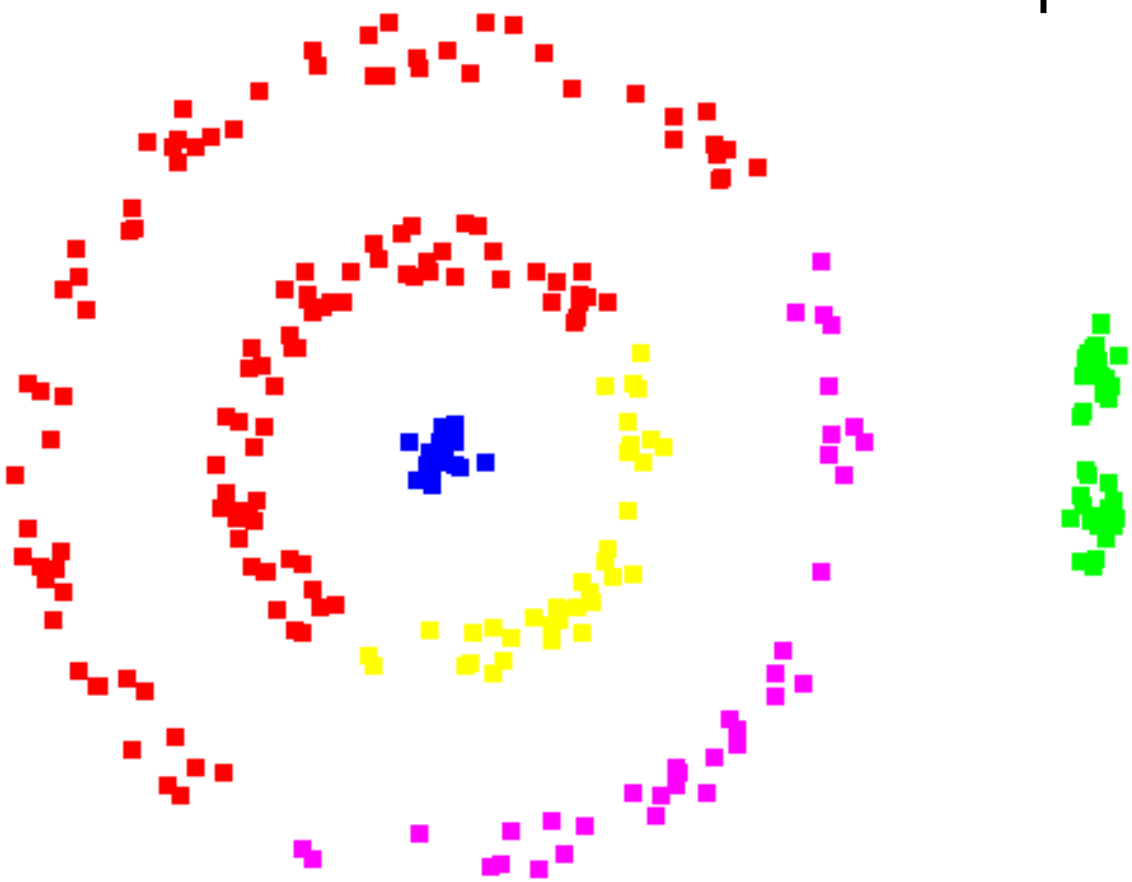
Clustering – How many groups are there?



Out of the various possible partitions, which is the correct one?

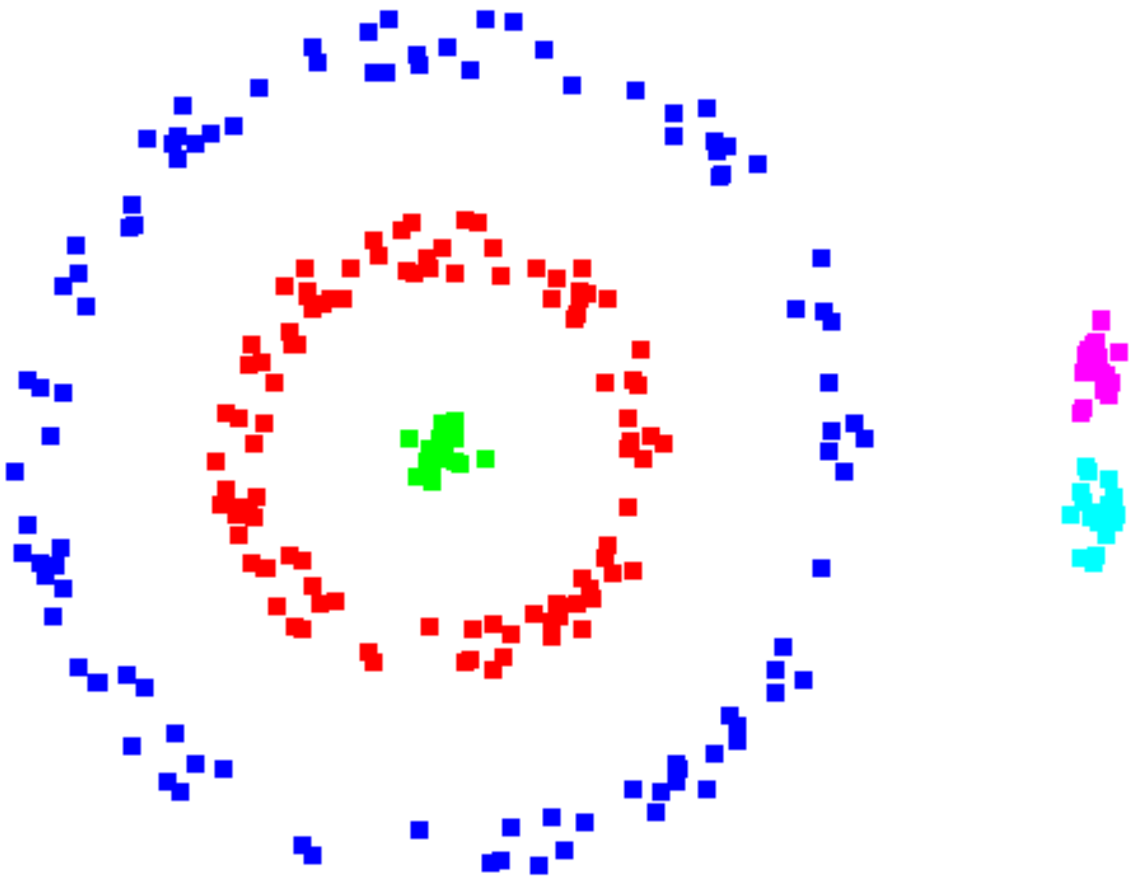
Clustering – 5 groups

Optimal?

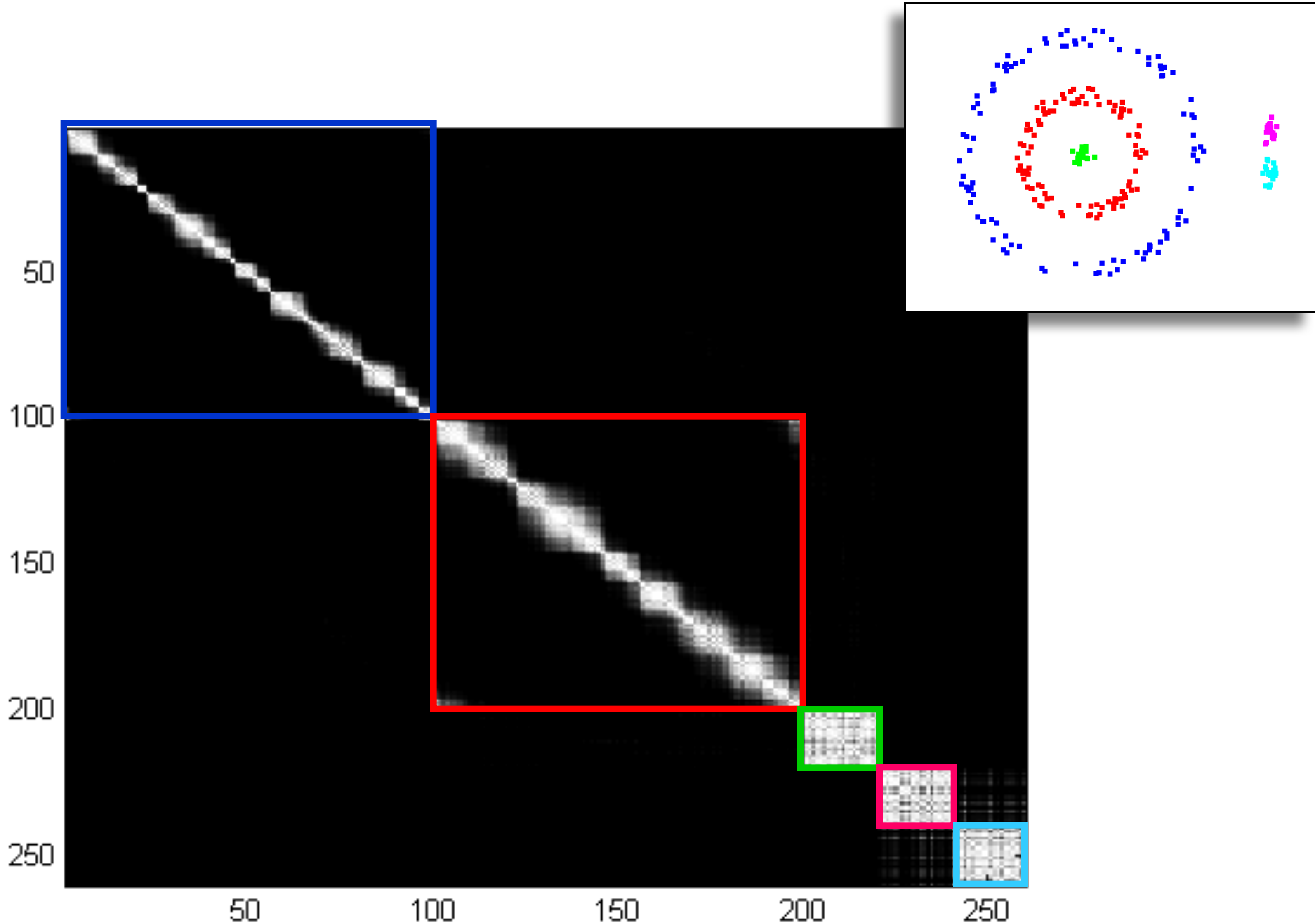


Clustering – 5 groups

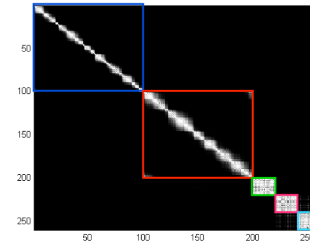
Looks optimal

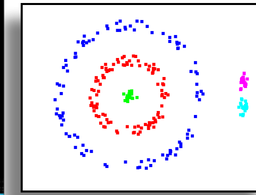


What does the Affinity Matrix Look Like?



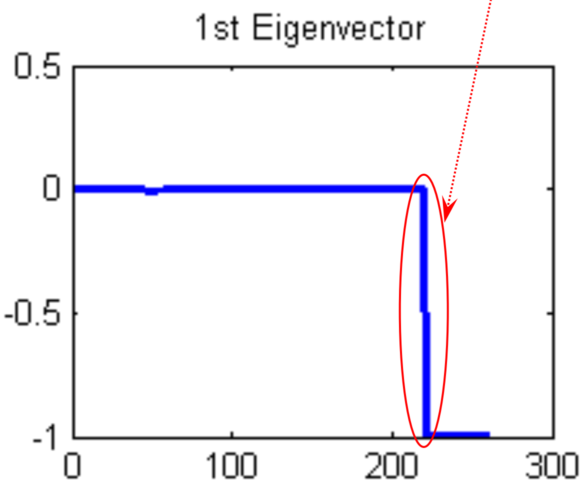
The Eigenvectors and the Clusters

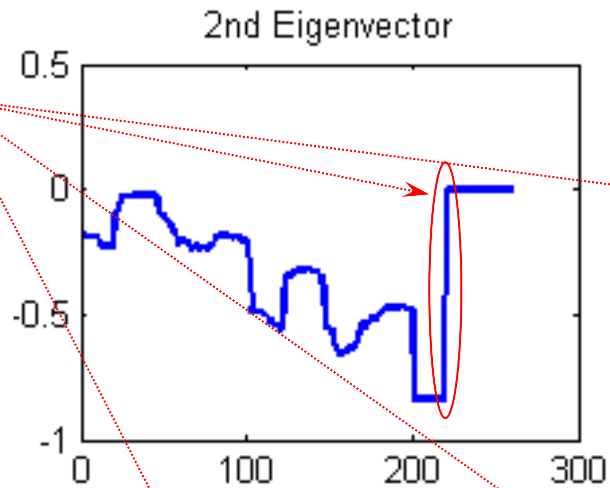


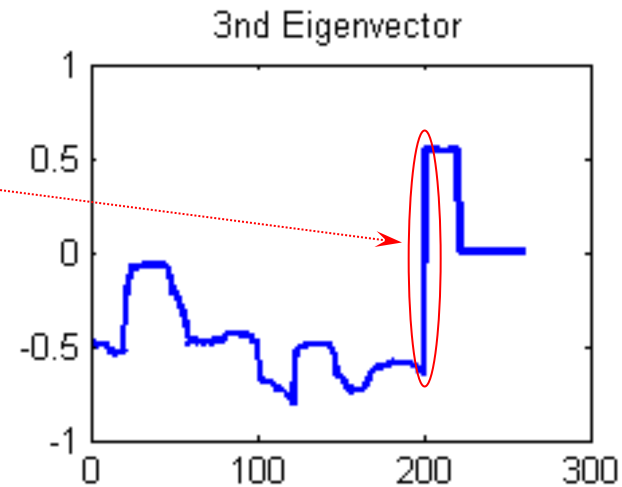


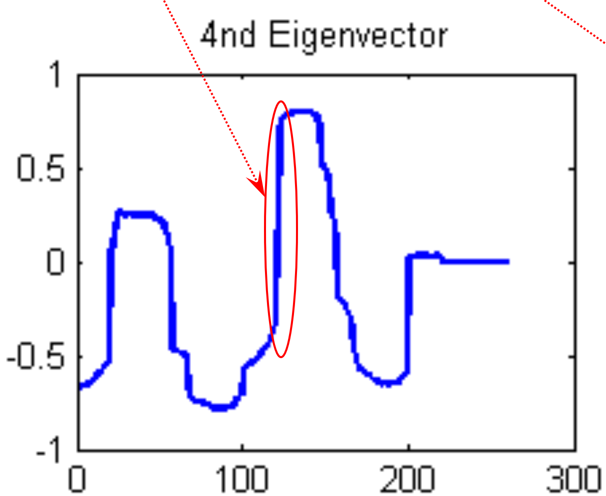
Step-Function like behavior preferred!

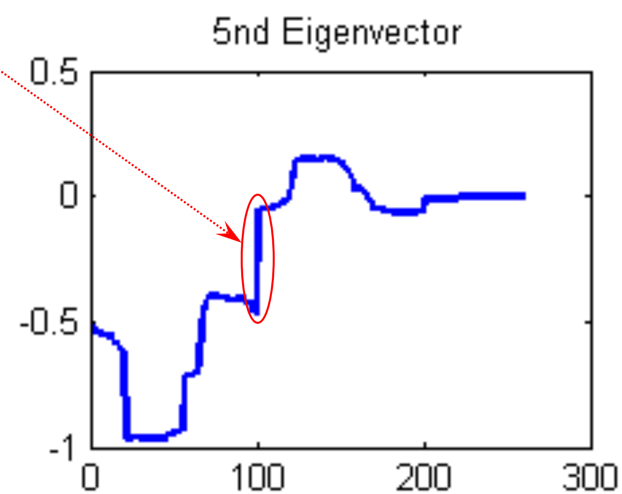
Makes Clustering Easier.



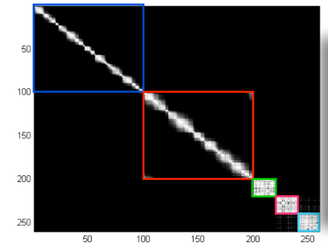


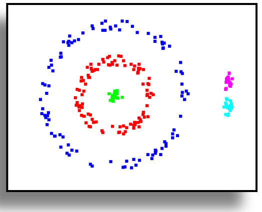


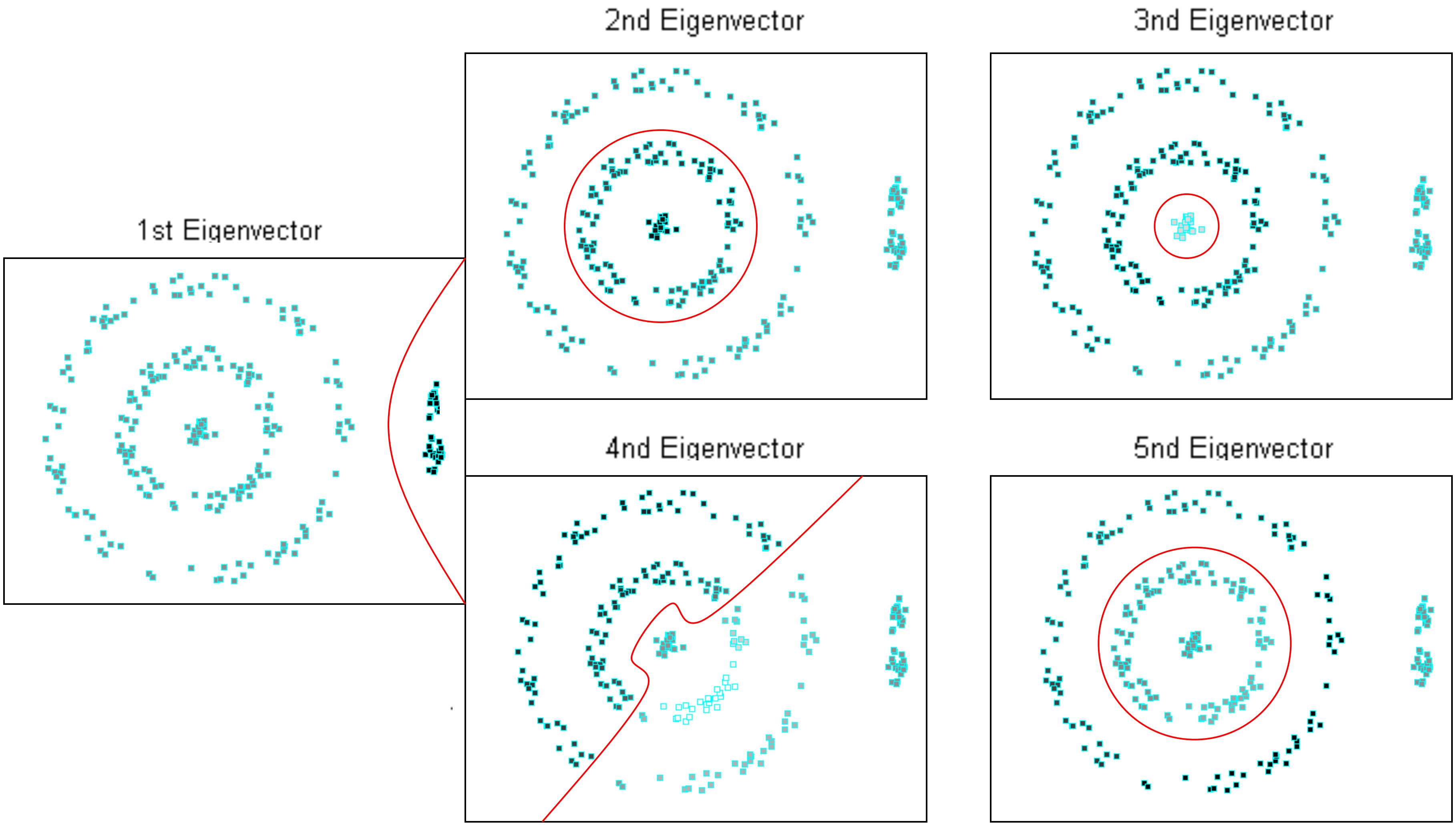




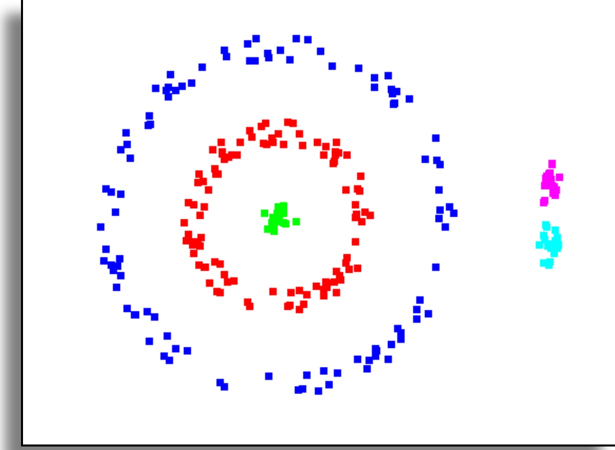
The Eigenvectors and the Clusters

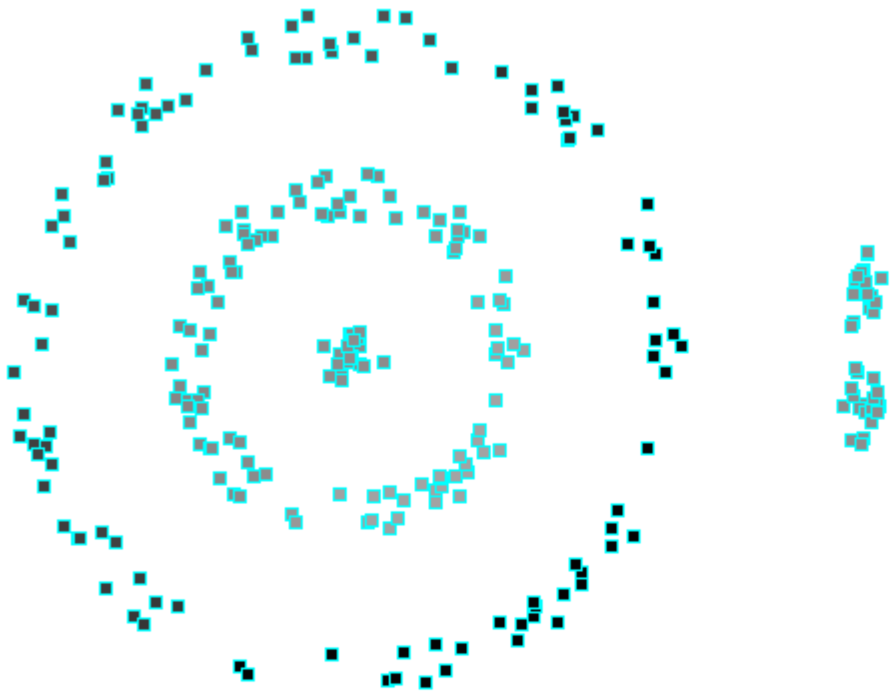




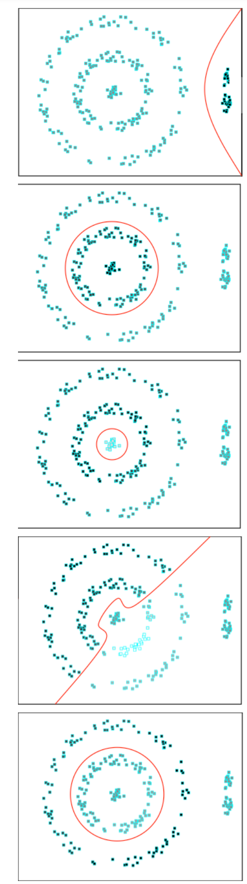


The Eigenvectors and the Clusters





Eigenvector #4



#1

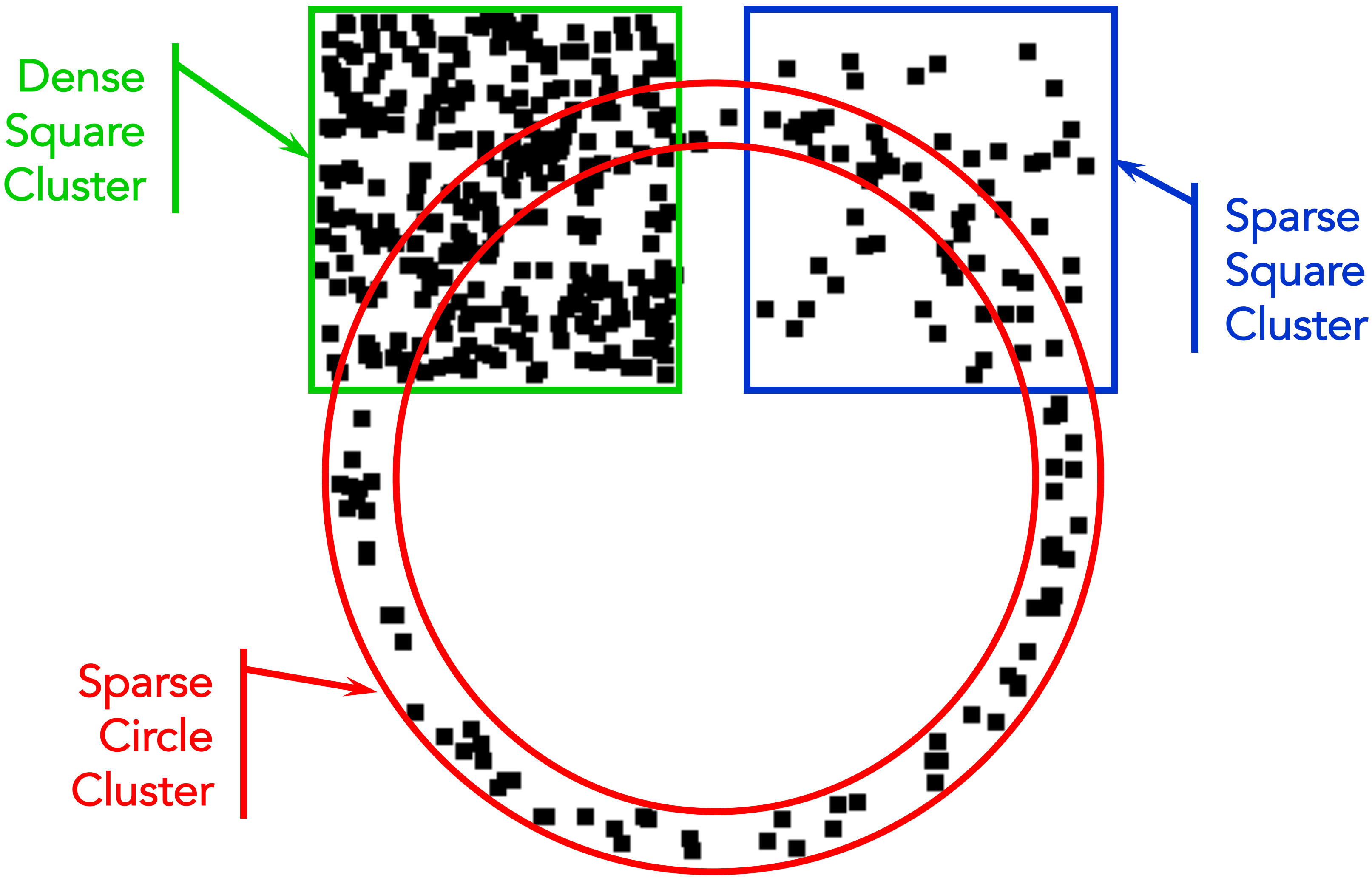
#2

#3

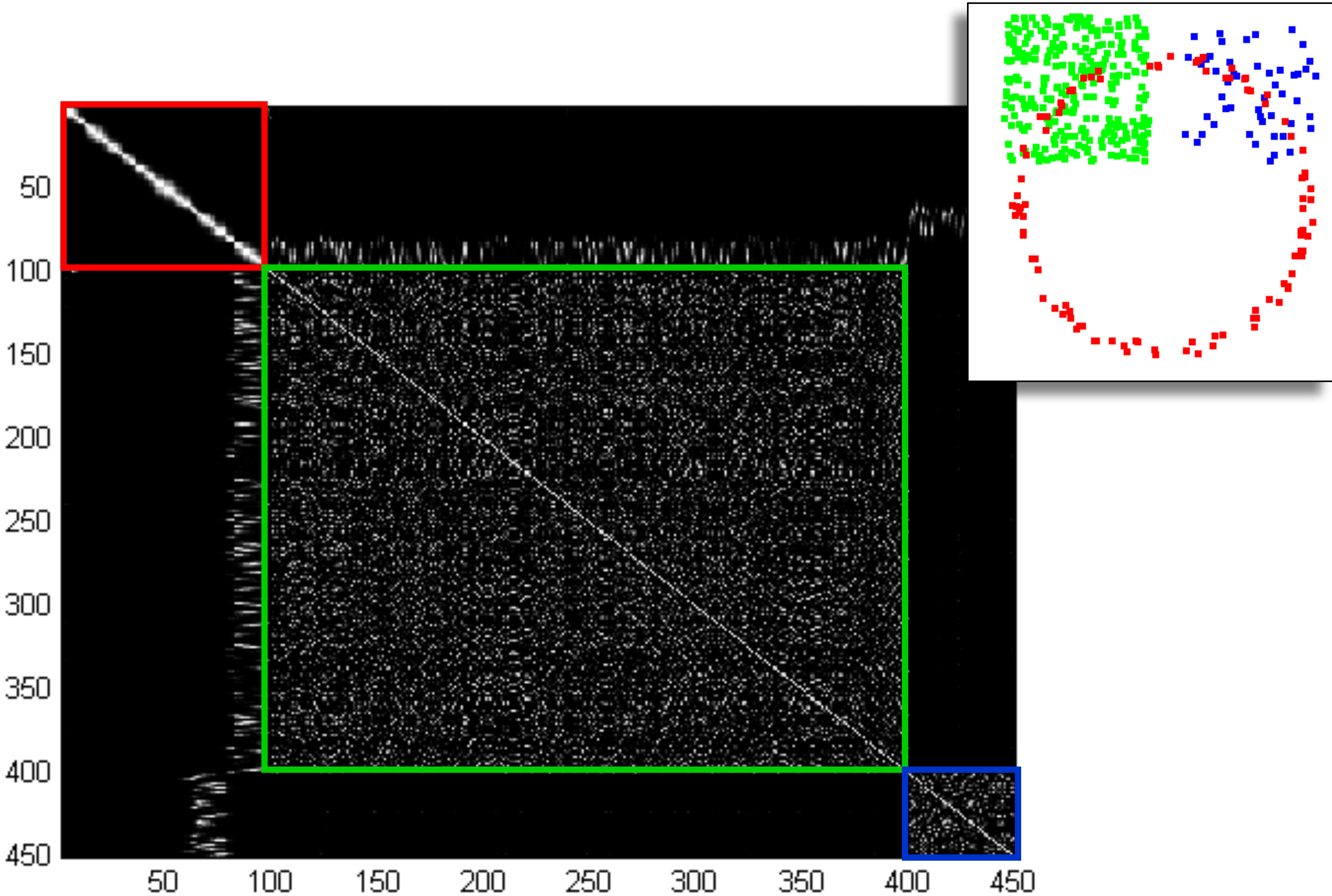
#4

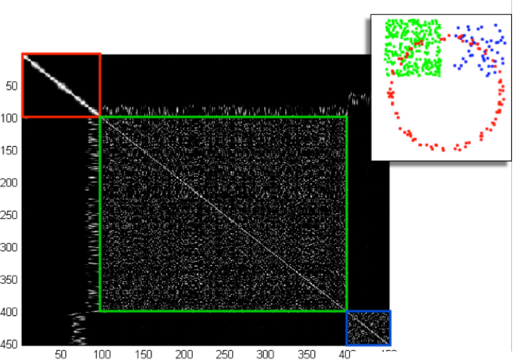
#5

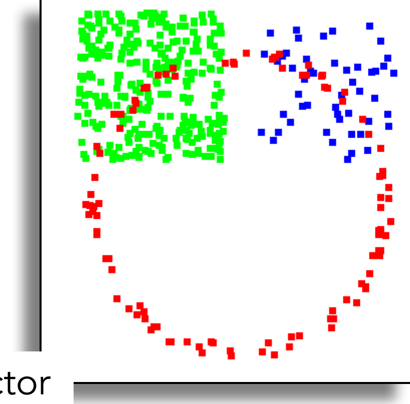
Clustering – Example 2



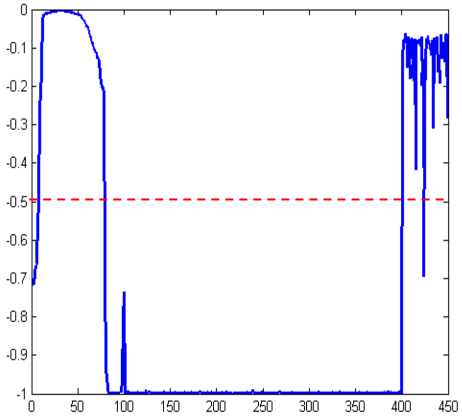
The Affinity Matrix



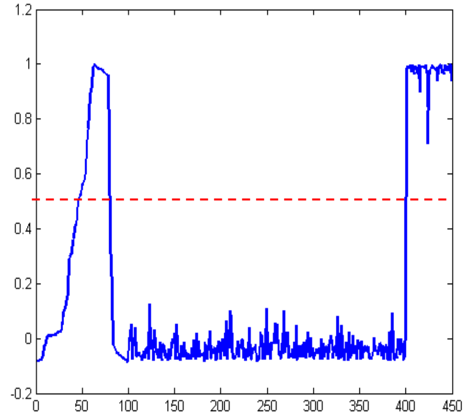




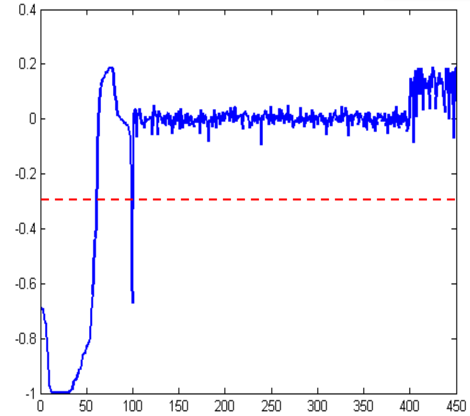
1st Eigenvector

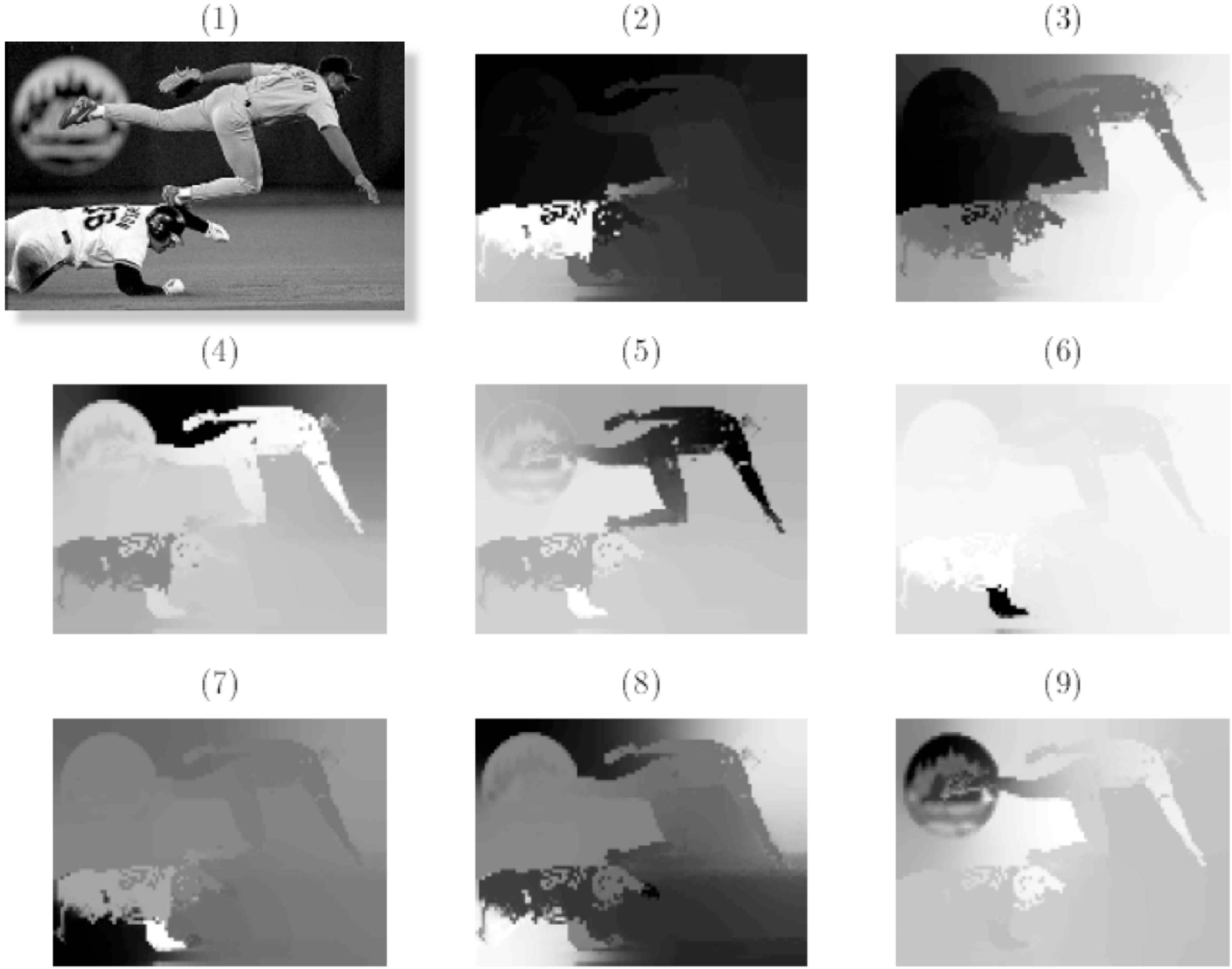


2nd Eigenvector



3rd Eigenvector

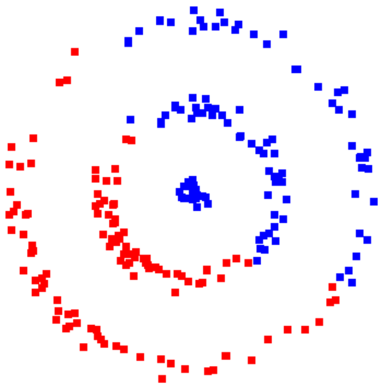




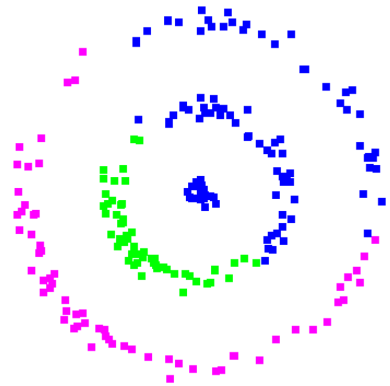
The eigenvectors correspond the 2nd smallest to the 9th smallest eigenvalues

Issue: Number of Clusters ?

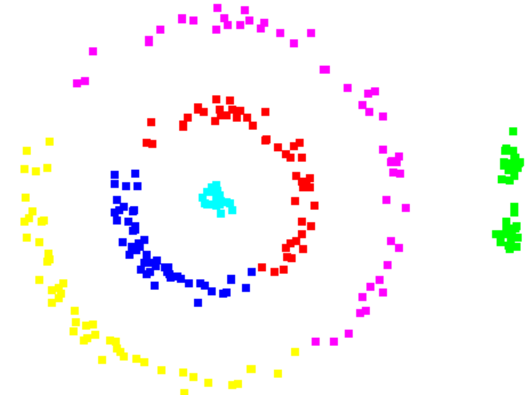
k = 3



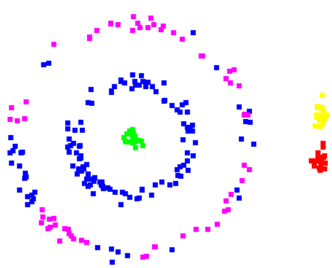
k = 4



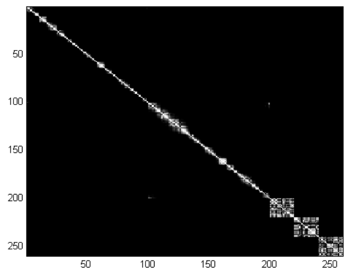
k = 6

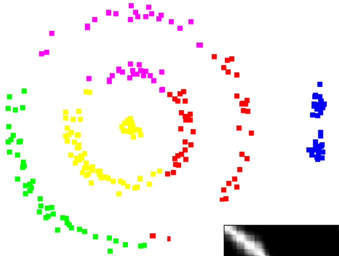


Issue: choice of kernel, for Gaussian kernels, choice of **σ**

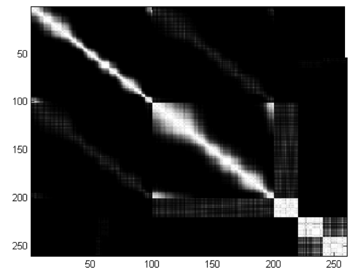


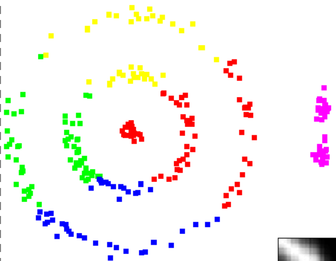
σ = 3



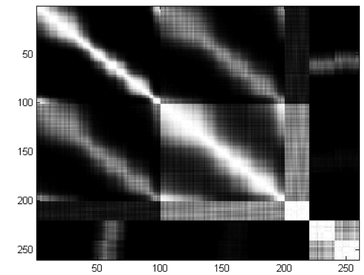


σ = 13





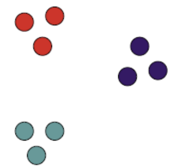
σ = 25



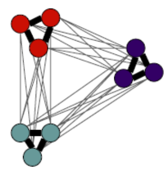
Graph-based Image Segmentation

Goal: Given data points X1, …, Xn and similarities w(Xi,Xj), partition the data into groups so that points in a group are similar and points in different groups are dissimilar.

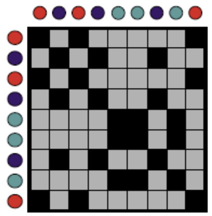
1- Get vectors of data



2a- Build a similarity graph

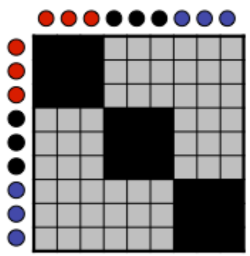


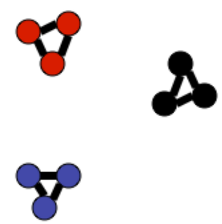
2b- Build a similarity matrix



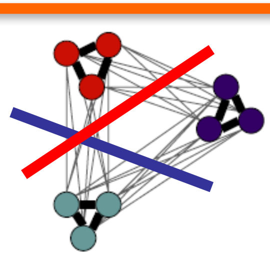
Similarities

3- Calculate eigenvectors





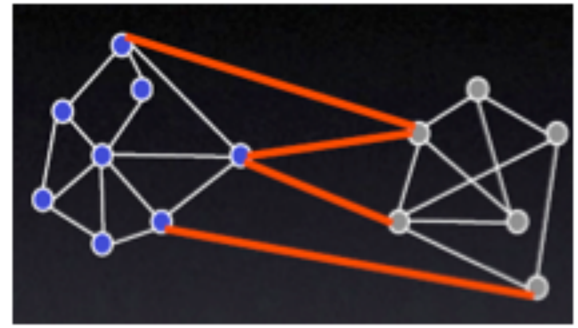
4- Cut the graph:
apply threshold to eigenvectors

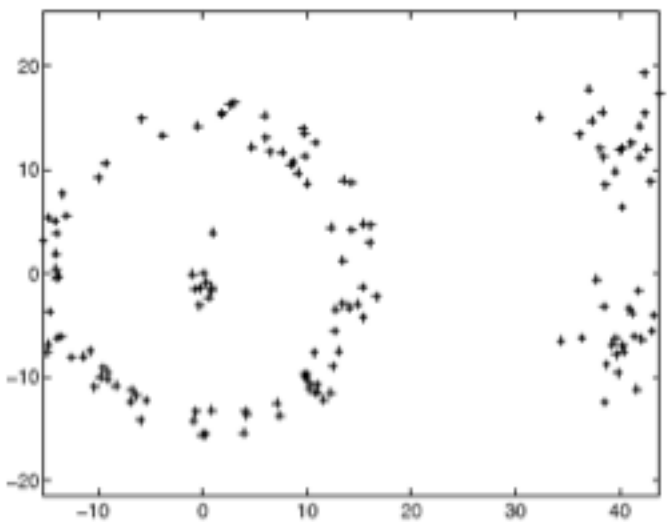


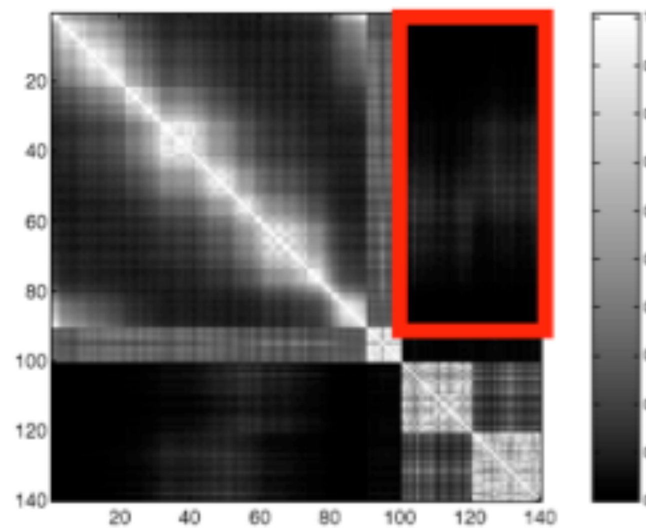
Graph cut

Cuts in a graph:

$$cut(A, \overline{A}) = \sum_{i \in A, j \in \overline{A}} w_{i,j}$$

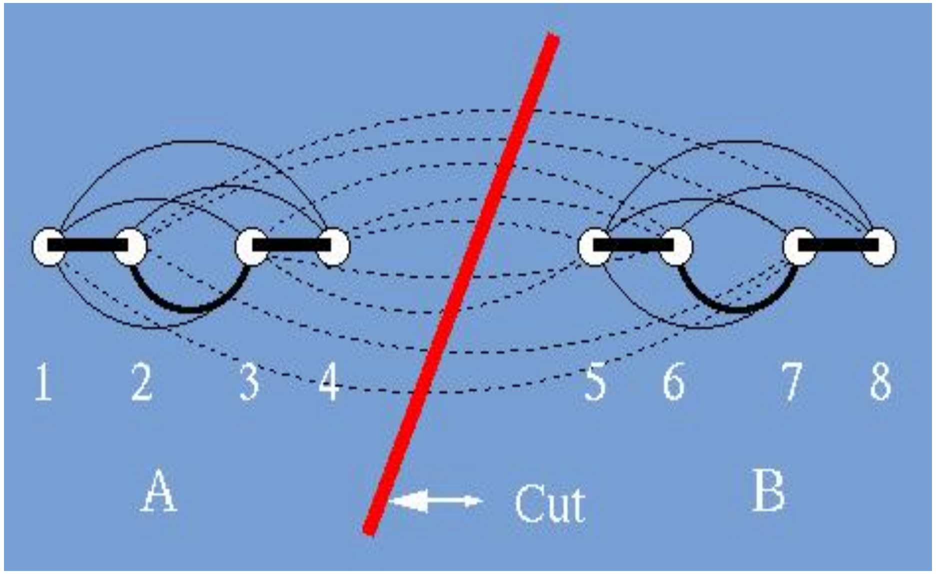






- Set of edges whose removal makes a graph disconnected
- Cost of a cut: sum of weights of cut edges
- A graph cut gives us a segmentation

Partition a graph with minimum cut

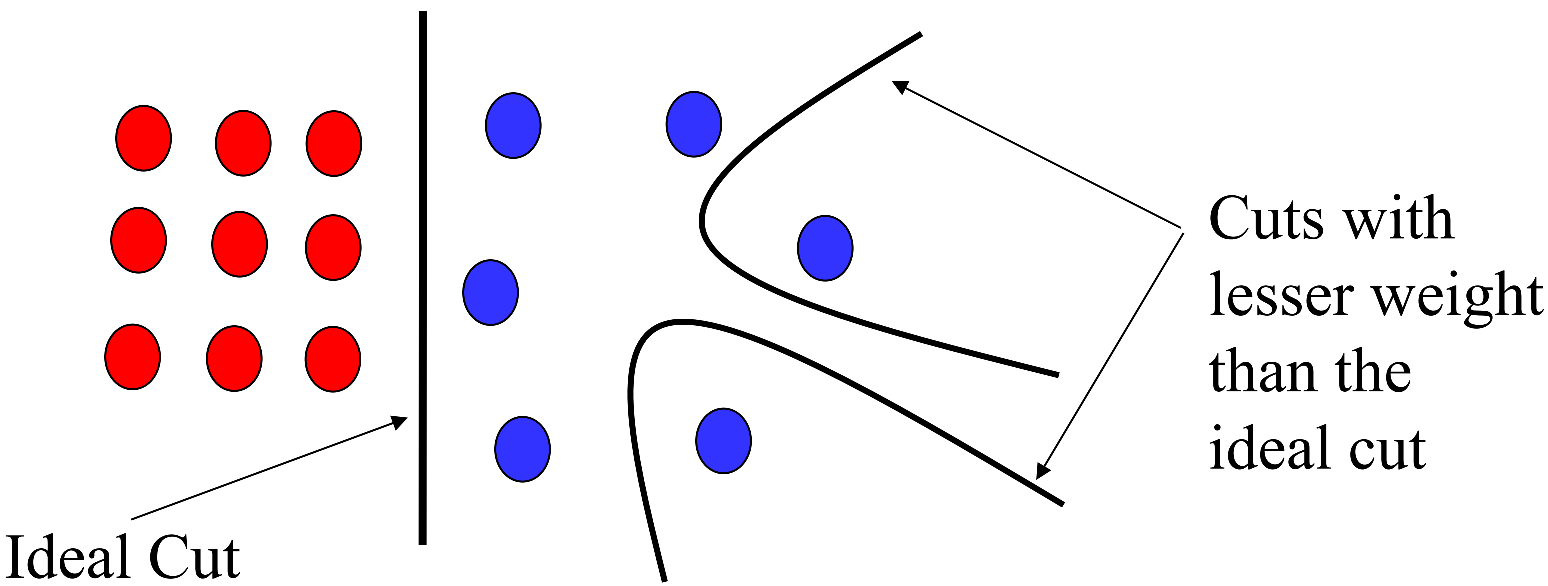


$$cut(A,B) = \sum_{u \in A, v \in B} w(u,v)$$

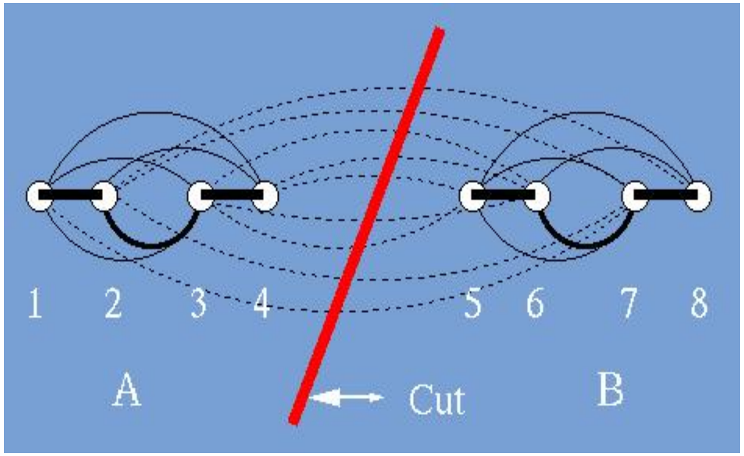
- **Cut**: sum of the weight of the cut edges:
- Minimum cut is the cut of minimum weight

Drawbacks of Minimum Cut

- Weight of cut is directly proportional to the number of edges in the cut.



Normalized Cut is a better measure ..



- We normalize by the total volume of connections

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

$$Ncut(\mathrm{A},\mathrm{B}) = \frac{cut(\mathrm{A},\mathrm{B})}{assoc(\mathrm{A},\mathrm{V})} + \frac{cut(\mathrm{A},\mathrm{B})}{assoc(B,\mathrm{V})}$$

where $assoc(A,V) = \sum_{u \in A, t \in V} w(u,t)$

Normalized Cut As Generalized Eigenvalue problem

$$D_{ii} = \sum_j W_{ij}$$

$$Ncut(\mathrm{A},\mathrm{B}) = \frac{cut(\mathrm{A},\mathrm{B})}{assoc(\mathrm{A},\mathrm{V})} + \frac{cut(\mathrm{A},\mathrm{B})}{assoc(\mathrm{B},\mathrm{V})}$$

$$= \frac{(1+x)^T(D-W)(1+x)}{k1^T D1} + \frac{(1-x)^T(D-W)(1-x)}{(1-k)1^T D1};\ k = \frac{\sum_{x_i>0} D(i,i)}{\sum_i D(i,i)}$$

$$= \ldots$$

after simplification, Shi and Malik derive

$$Ncut(A,B) = \frac{y^T(D-W)y}{y^T D y}, \quad \text{with } y_i \in \{1,-b\},\ y^T D1 = 0.$$

W = affinity matrix

For detailed derivation: http://www.cs.berkeley.edu/~malik/papers/SM-ncut.pdf

Normalized cuts

Minimize:

Binary Binary

$$Ncut(A,B) = \frac{y^T(D-W)y}{y^TDy}, \quad \text{with } y_i \in \{1,-b\}, y^TD1 = 0.$$

$$\max_y \left(y^T(D-W)y\right) \text{ subject to } \left(y^TDy = 1\right)$$

- Instead, solve the generalized eigenvalue problem

$$(D-W)y = \lambda Dy$$

- They show that the 2nd smallest eigenvector solution y is a good real-valued approx to the original normalized cuts problem. Then you look for a quantization threshold that maximizes the criterion --- i.e all components of y above that threshold go to one, all below go to -b

http://www.cs.berkeley.edu/~malik/papers/SM-ncut.pdf

Many different methods…

Goal: Given data points X1, …, Xn and similarities w(Xi,Xj), partition the data into groups so that points in a group are similar and points in different groups are dissimilar.

1- Get vectors of **data**

2- Build a **similarity** graph

3- Calculate eigenvectors

4- Apply threshold to largest eigenvectors

1- Get vectors of **data**

2- Build normalized cost matrix

3- Get eigenvectors with smallest eigenvalues

4- Apply threshold

Shi & Malik

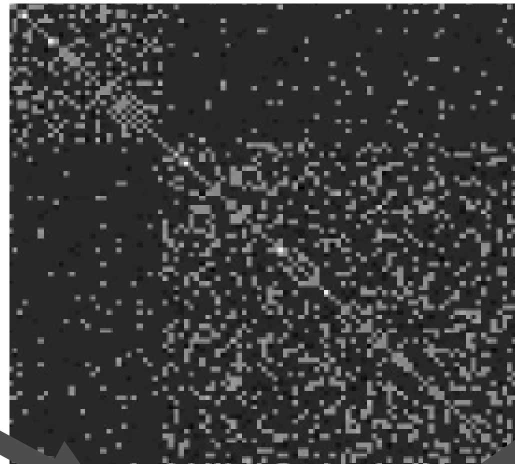
… etc

Global optimization

- In this formulation, the segmentation becomes a global process.
- Decisions about what is a boundary are not local (as in Canny edge detector)

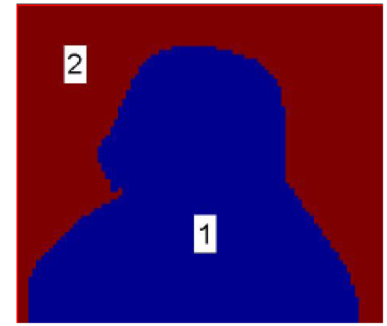
Graph-based Image Segmentation

Image (I)



Affinity matrix (W)

Eigenvector X(W)



Discretization/Thresholding

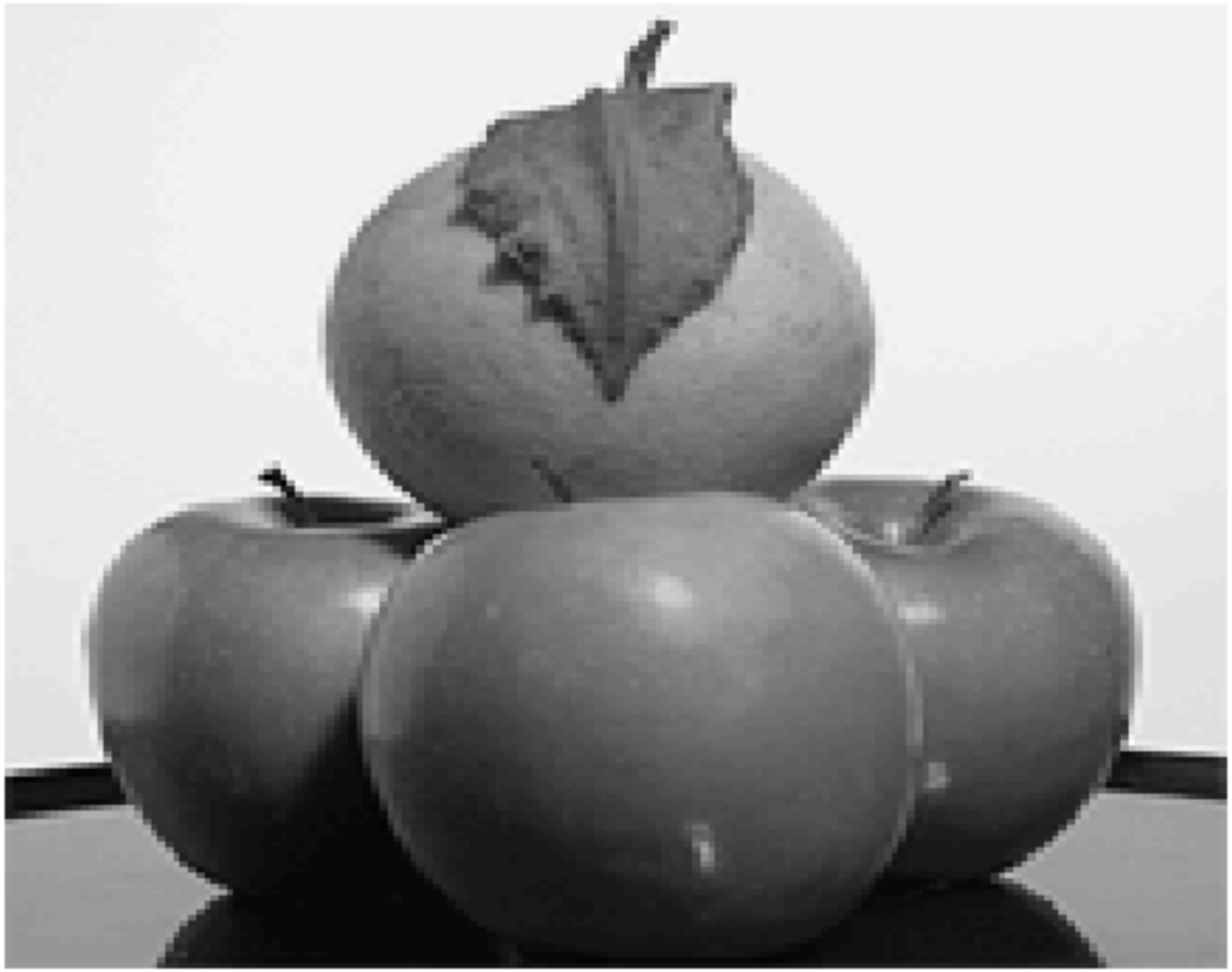
Intensity
Color
Edges
Texture

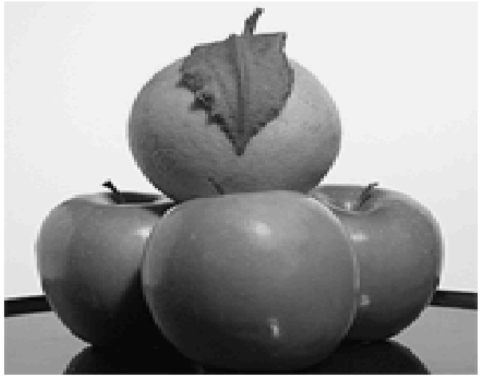
$$Ncut(A,B) = cut(A,B)\left(\frac{1}{vol(A)} + \frac{1}{vol(B)}\right)$$

$$(D-W)X = \lambda DX$$

$$X_A(i) = \begin{cases} 1 & if \quad i \in A \\ 0 & if \quad i \notin A \end{cases}$$

Slide from Timothee Cour

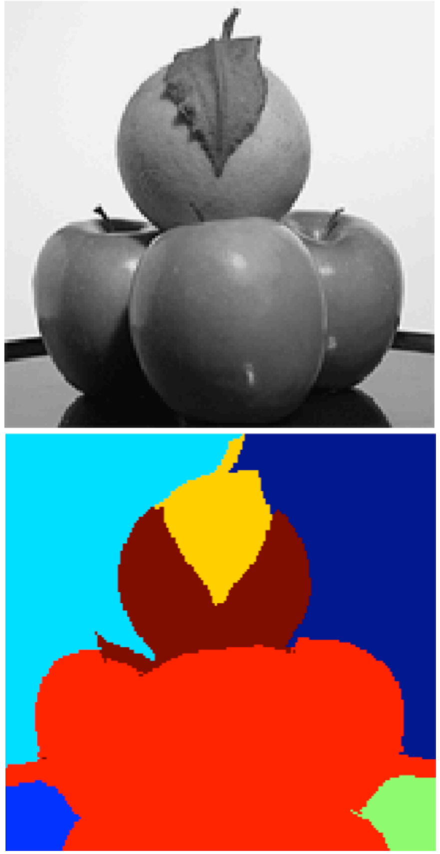


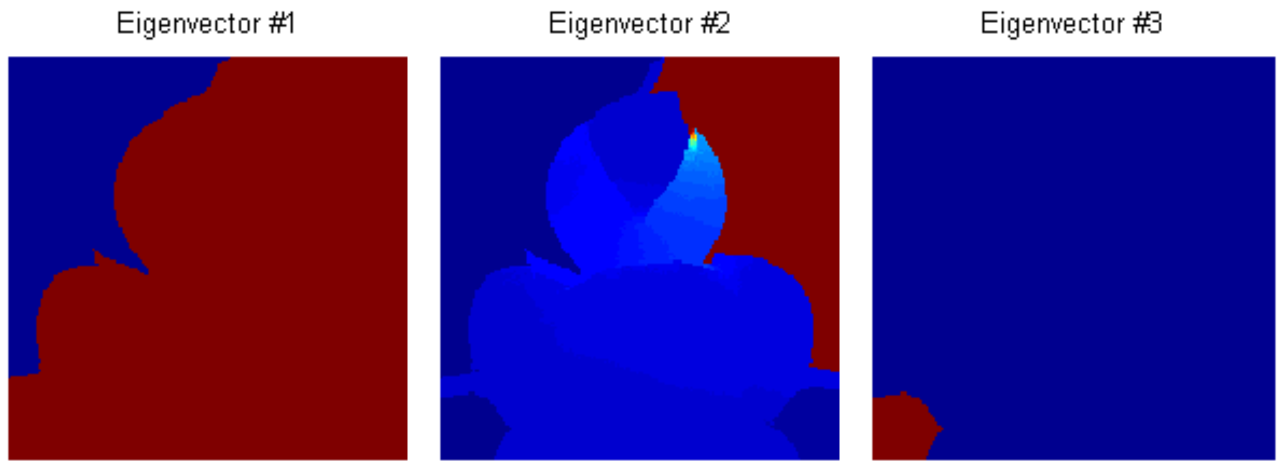


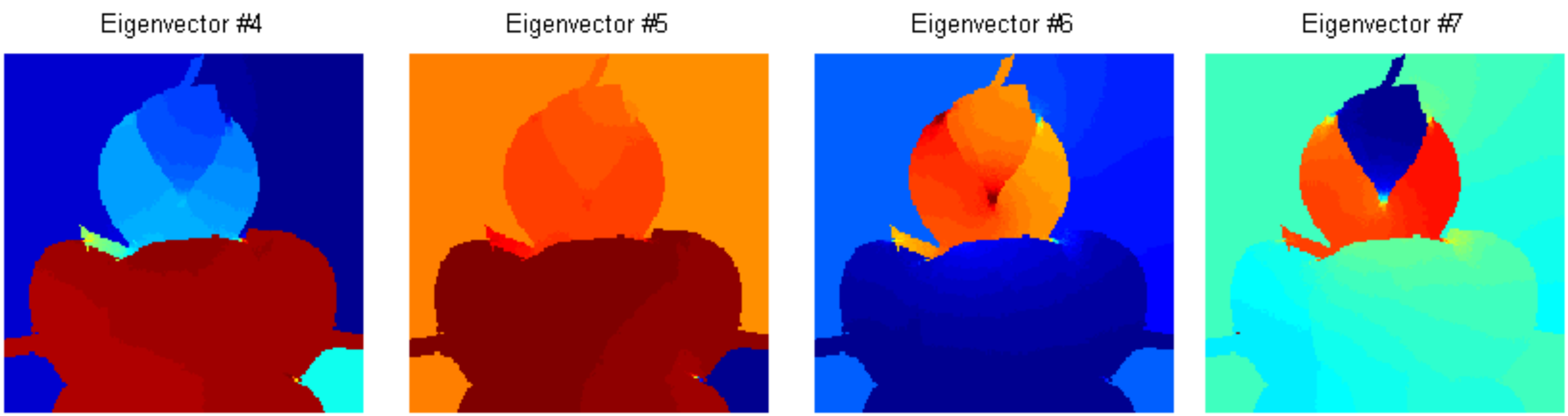
The Eigenvectors

Eigenvector #7

Normalized cut

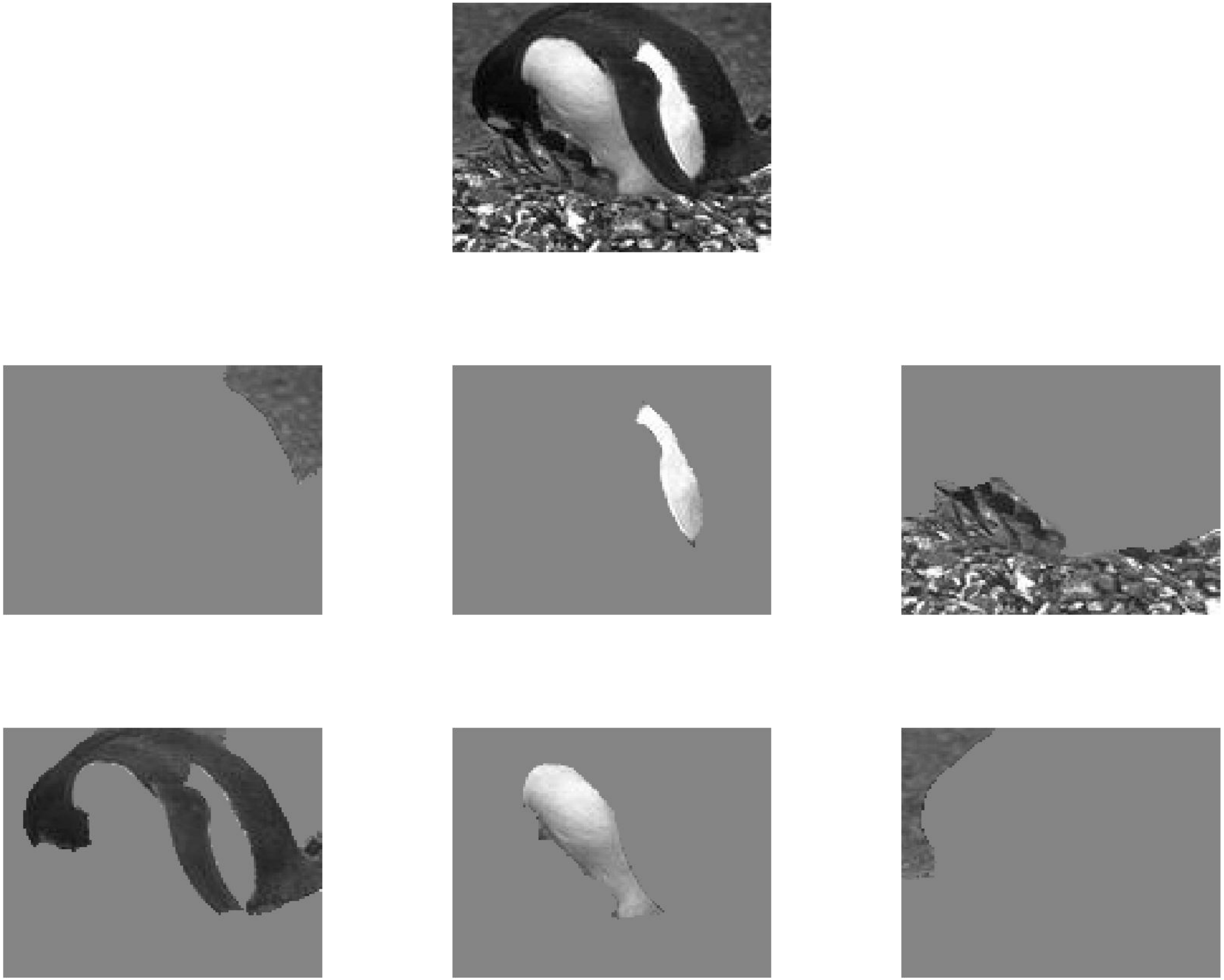


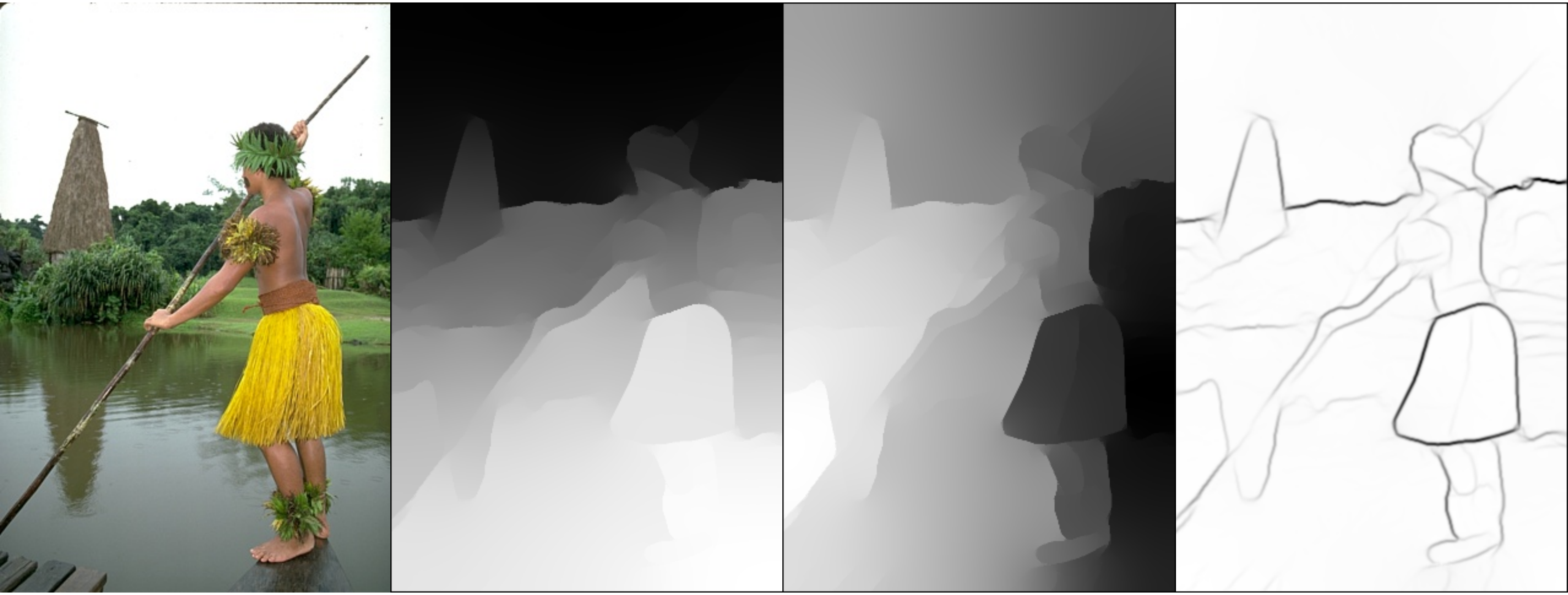




Normalized cut

Normalized cut





Fully Convolutional Networks for Semantic Segmentation

Jonathan Long* Evan Shelhamer* Trevor Darrell
UC Berkeley

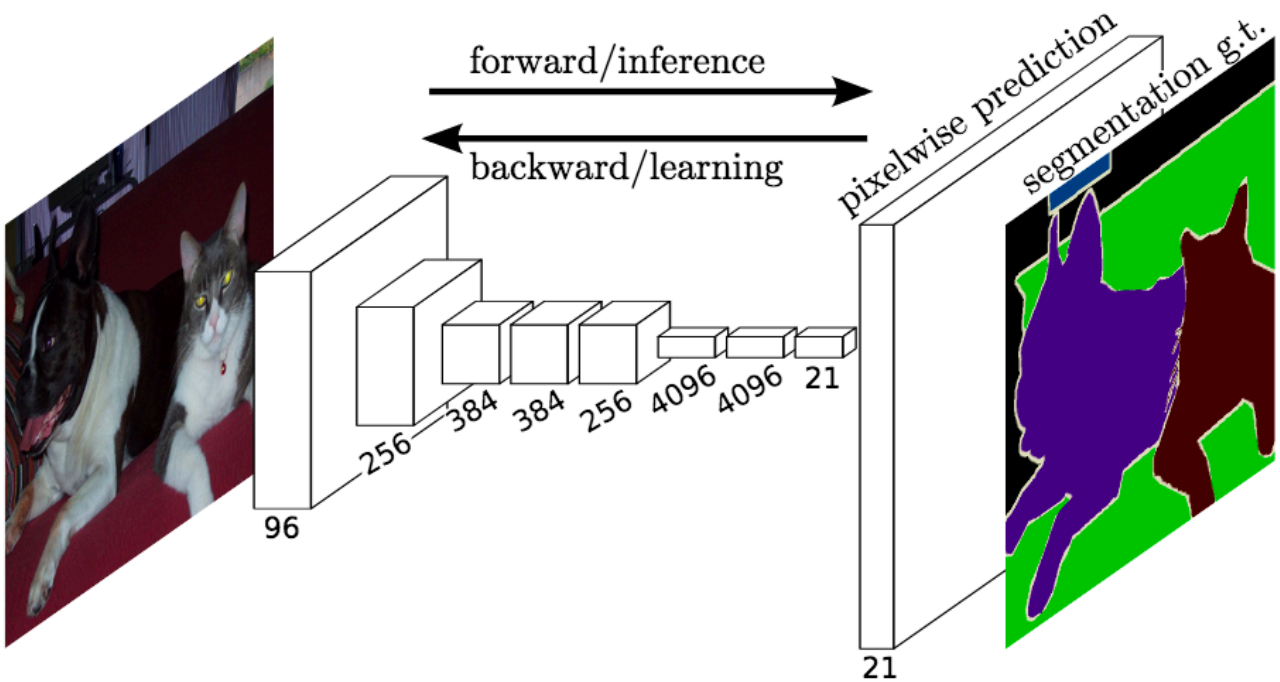


Figure 1. Fully convolutional networks can efficiently learn to make dense predictions for per-pixel tasks like semantic segmentation.