Color

6.819/6.869, MIT
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Sept. 21, 2017
Why does a visual system need color?
Why does a visual system need color?
(an incomplete list…)

- To tell what food is edible.
- To distinguish material changes from shading changes.
- To group parts of one object together in a scene.
- To find people’s skin.
- Check whether a person’s appearance looks normal/healthy.

http://www.pouted.com/know-10-points-information-unicorn/sick-child/
Lecture outline

• Color physics.
• Color perception.
Lecture outline

• Color physics.
• Color perception.
4.1 **NEWTON’S SUMMARY DRAWING** of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

From *Foundations of Vision*, by Brian Wandell, Sinauer Assoc., 1995
Spectral colors

http://hyperphysics.phy-astr.gsu.edu/hbase/vision/specol.html#c2
Figure 6.3: (a) A spectrograph constructed using a compact disk (CD). Light enters through a slit at the bottom left, diffracting from the narrowly spaced lines of the CD. (b) Photograph of diffraction pattern for light, seen thorough hole at bottom left.
Figure 6.5: Some real-world objects and the reflected light spectra (photographed using Fig. (6.3) (a)) from outdoor viewing. (a) Leaf and (b) its reflected spectrum. (c) A red door and (d) its reflected spectrum.
Figure 6.6: More real-world objects and the reflected light spectra. (a) Blue-green chair and (b) its reflected light. (c) Toby the dog and (d) his reflected spectrum.
Blue sky

Tungsten light bulb

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995
Color names for cartoon spectra

- Red
- Green
- Blue
- Cyan
- Magenta
- Yellow
Additive color mixing

When colors combine by adding the color spectra. Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.

Red and green make…

Yellow!
When colors combine by multiplying the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

Cyan and yellow (in crayons, called “blue” and yellow) make…

Green!
Overhead projector demo

Subtractive color mixing
The interaction of light with surfaces

Horn, 1986

Figure 10-7. The bidirectional reflectance distribution function is the ratio of the radiance of the surface patch as viewed from the direction \((\theta_e, \phi_e)\) to the irradiance resulting from illumination from the direction \((\theta_i, \phi_i)\).

Spectral radiance: power in a specified direction, per unit area, per unit solid angle, per unit wavelength

\[
BRDF = f(\theta_i, \phi_i, \theta_e, \phi_e, \lambda) = \frac{L(\theta_e, \phi_e, \lambda)}{E(\theta_i, \phi_i, \lambda)}
\]

Spectral irradiance: incident power per unit area, per unit wavelength
Effect of BRDF on sphere rendering

http://www.marmoset.co/toolbag/learn/pbr-practice
Simplified rendering models: \( \text{BRDF} \rightarrow \text{reflectance} \)

For diffuse reflections, we replace the BRDF calculation with a wavelength-by-wavelength scalar multiplication.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995
Spectral albedoes for several different leaves, with color names attached. Notice that different colours typically have different spectral albedo, but that different spectral albedoes may result in the same perceived color (compare the two whites). Spectral albedoes are typically quite smooth functions. Measurements by E.Koivisto.

Forsyth, 2002
Simplified rendering models: transmittance

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995
Lecture outline

• Color physics.
• Color perception.
What’s the machinery in the eye?
Eye Photoreceptor responses

Where do you think the light comes in?

The intricate layers and connections of nerve cells in the retina were drawn by the famed Spanish anatomist Santiago Ramón y Cajal around 1900. Rod and cone cells are at the top. Optic nerve fibers leading to the brain may be seen at bottom right.
3.4 THE SPATIAL MOSAIC OF THE HUMAN CONES. Cross sections of the human retina at the level of the inner segments showing (A) cones in the fovea, and (B) cones in the periphery. Note the size difference (scale bar = 10 μm), and that, as the separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.
3.3 SPECTRAL SENSITIVITIES OF THE L-, M-, AND S-CONES in the human eye. The measurements are based on a light source at the cornea, so that the wavelength loss due to the cornea, lens, and other inert pigments of the eye plays a role in determining the sensitivity. Source: Stockman and MacLeod, 1993.
L, M, and S cone receptor types colored as R, G, B
What are some color artifacts we might expect our visual system to experience, based on this way of measuring the light spectra falling on our eye?

3.3 SPECTRAL SENSITIVITIES OF THE L-, M-, AND S-CONEs in the human eye. The measurements are based on a light source at the cornea, so that the wavelength loss due to the cornea, lens, and other lens pigments of the eye plays a role in determining the sensitivity. Source: Stockman and MacLeod, 1993.
A property of our visual system: these two spectra look the same

4.11 METAMERIC LIGHTS. Two lights with these spectral power distributions appear identical to most observers and are called metamers. (A) An approximation to the spectral power distribution of a tungsten bulb. (B) The spectral power distribution of light emitted from a conventional television monitor whose three phosphor intensities were set to match the light in panel A in appearance.
Color metamerism: different spectra looking the same color

Two spectra, $t$ and $s$, perceptually match when

$$C \vec{t} = C \vec{s}$$

where $C$ are the cone response curves.

Graphically,
Evidence of spatially offset color sampling in an old digital camera

- Color fringes or jaggies
Where you can see color fringe reconstruction artifacts from your own eye

http://static.flickr.com/21/31393422_23013da003.jpg
Brewster’s colors—evidence of interpolation from spatially offset color samples

Scale relative to human photoreceptor size: each line covers about 7 photoreceptors.
Brewster’s colors—evidence of interpolation from spatially offset color samples
Lecture outline

• Color physics.

• Color perception
  – part 1: assume perceived color only depends on light spectrum.
  – part 2: the more general case.
The assumption for color perception, part 1

• We know color appearance really depends on:
  – The illumination
  – Your eye’s adaptation level
  – The colors and scene interpretation surrounding the observed color.

• But for now we will assume that the spectrum of the light arriving at your eye completely determines the perceived color.
How we sense light spectra

**biophysics:** integrate the response over all wavelengths, weighted by the photosensor’s sensitivity at each wavelength.

**mathematically:** take dot product of input spectrum with the cone sensitivity basis vectors. Project the high-dimensional test light into a 3-d space. \( R = C \ t \)
Cone response curves as basis vectors in a 3-d subspace of light power spectra

NOTE: any matrix, C, that spans the 3d subspace of the human cone responses works to convert a light spectrum into a color measurement
UNITED STATES DEPARTMENT OF AGRICULTURE

COLOR STANDARDS
for
FROZEN
FRENCH FRIED POTATOES

FOURTH EDITION, 1966
© 1966 KOLLMORGAN CORPORATION
MUNSELL COLOR
BALTIMORE, MARYLAND 21201
Color trademarks

CURRENTLY REGISTERED COLOR TRADEMARKS

A color trademark is a non-conventional trademark where at least one color is used to identify the commercial origin of a product or service. A color trademark must meet the same requirements of a conventional trademark. Thus, the color trademark must either be inherently distinctive or have acquired secondary meaning. To be inherently distinctive, the color must be arbitrarily or suggestively applied to a product or service. In contrast, to acquire secondary meaning, consumers must associate the color used on goods or services as originating from a single source. Below is a selection of some currently registered color trademarks in the U.S. Trademark Office:

MARK/COLOR(S)/OWNER:

BANK OF AMERICA 500
blue, red & grey
Bank of America Corporation

NATIONAL CAR RENTAL
green
NCR Affiliate Servicer, Inc.

FORD
blue
Ford Motor Company

VISTEON
orange
Ford Motor Company

76
red & blue
ConocoPhillips Company

THE HOME DEPOT
orange
Homer TLC, Inc.

HONDA
red
Honda Motor Co., Ltd.

M MARATHON
brown, orange, yellow
Marathon Oil Company

M MARATHON
gray, black & white
Marathon Oil Company

COSTCO
red
Costco Wholesale Membership, Inc.

TEENAGE MUTANT NINJA TURTLES MUTANTS & MONSTERS
red, green, yellow, black, grey and white
Mirage Studios, Inc.

TARGET
red
Volkswagen Aktiengesellschaft Corp.
• How do we measure colors?
• How do we make systems that match colors?
4.10 THE COLOR-MATCHING EXPERIMENT. The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.
Color matching experiment 1

test light

primary lights
Color matching experiment 1
Color matching experiment 1

\[ p_1 \quad p_2 \quad p_3 \]
Color matching experiment 1

The primary color amounts needed for a match

\[ p_1 \quad p_2 \quad p_3 \]
Relevant to color matching experiments, solve this puzzle:

Want to measure out 7 lbs of clay using 5, 3, 1 lb weights

(we wish we could add a -1 lb mass to the 5 and 3 lb masses to weigh out 7 lbs of clay. But we don’t have negative mass. Instead, we just add the 1 lb mass to the other side, where the clay is, to weigh out 7 lbs of clay)


Relevant to color matching experiments, solve this puzzle:
Color matching experiment 2
Color matching experiment 2
Color matching experiment 2
Color matching with positive amounts of the primaries

Match the sensors’ response to the target light to the sum of responses to the primary lights
Color matching with a negative amount of primary 1

Match sensors’ response to the target light plus some amount of primary light 1 to the response to some of primary light 2
Color matching experiment--handle negative light by adding light to the test.

4.10 THE COLOR-MATCHING EXPERIMENT. The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.
Color matching experiment 2

We say a “negative” amount of $p_2$ was needed to make the match, because we added it to the test color’s side.

The primary color amounts needed for a match:

$p_1$  $p_2$  $p_3$
To measure a color

1. Choose a set of 3 primary colors (three power spectra).
2. Determine how much of each primary needs to be added to a probe signal to match the test light.

\[
a_1 + a_2 + a_3 \]

---

**4.10 THE COLOR-MATCHING EXPERIMENT.** The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.
To measure a color

1. Choose a set of 3 primary colors (three power spectra).
2. Determine how much of each primary needs to be added to a probe signal to match the test light.

\[
C_t = C_P \left( \frac{a_1}{a_2} \right) + \frac{a_3}{a_3}
\]

Weighted sum of primaries

Cone sensitivities

Test light, \( t \)

L, M, S responses

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• We can measure a color by measuring how much of each primary is needed to match that color.
• Can we measure color without having to make psychophysical experiments each time?
• We’d like to find a matrix, $C$, that we can project a spectrum onto, to tell us how much of each primary, in the columns of $P$, , to use to match the spectrum.
“Color matching functions” tell us how to control primary lights in order to perceptually match a given spectrum.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995
Using the color matching functions to predict the primary match to a new spectral signal

We know that a monochromatic light of wavelength $\lambda_i$ will be matched by the amounts $c_1(\lambda_i), c_2(\lambda_i), c_3(\lambda_i)$ of each primary.

And any spectral signal can be thought of as a linear combination of very many monochromatic lights, with the linear coefficient given by the spectral power at each wavelength.

$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$
Using the color matching functions to predict the primary match to a new spectral signal

Store the color matching functions in the rows of the matrix, $C$

$$C = \begin{pmatrix}
c_1(\lambda_1) & \cdots & c_1(\lambda_N) \\
c_2(\lambda_1) & \cdots & c_2(\lambda_N) \\
c_3(\lambda_1) & \cdots & c_3(\lambda_N)
\end{pmatrix}$$

Let the new spectral signal be described by the vector $t$.

$$\vec{t} = \begin{pmatrix}
t(\lambda_1) \\
\vdots \\
t(\lambda_N)
\end{pmatrix}$$
Using the color matching functions, measured from a set of primaries, to predict how to match any new spectrum, \( t \), with those primaries.

Then the amounts of each primary needed to match \( t \) are:

\[
\sum_j \begin{pmatrix}
    c_1(\lambda_j)t(\lambda_j) \\
    c_2(\lambda_j)t(\lambda_j) \\
    c_3(\lambda_j)t(\lambda_j)
\end{pmatrix} = C\vec{t} = C
\]
How the color matching functions, \( C \), and the corresponding primary spectra, \( P \), relate to each other.

If the primaries, \( P \), correspond to the color matching functions, \( C \), then \( t \) and \( s \) are perceptual matches. When projected down by \( C \) they must give the same answer, so we must have,

\[
C \ t = C \ d
\]
Requirements on $C$, $P$ to form a color matching system:

(1) the rows of $C$ must be some (non-degenerate) linear combination of the eye photosensor response curves. That ensures that if two spectra match when projected into the subspace spanned by $C$, they will match when projected into the subspace of the eye response curves.

(2) $C$, $P$ must satisfy:

$$C \, P = I$$

why must this hold? Because the amounts of the 3 primaries needed to match the spectrum of each primary (the columns of $P$) must be $[1;0;0]$, $[0;1;0]$, $[0;0;1]$.

If those conditions hold, then the spectrum $PCt$ will be a perceptual match to $t$, because

$Ct = CPCt$
How do color coordinates translate between different sets of primaries?

From previous slide:

\[ C \vec{t} = C P' C' \vec{t} \]

But this holds for any input spectrum, \( t \), so...

\[ C = C P' C' \]

\( C P' \)

a 3x3 matrix that transforms from the color representation in one set of primaries to that of another.

\( P' \) are the old primaries

\( C \) are the new primaries’ color matching functions
Comparison of color matching functions with best linear combination of cone response curves

4.20 COMPARISON OF CONE PHOTOCURRENT RESPONSES AND THE COLOR-MATCHING FUNCTIONS. The cone photocurrent spectral responsivities are within a linear transformation of the color-matching functions, after a correction has been made for the optics and inert pigments in the eye. The smooth curves show the Stiles and Burch (1959) color-matching functions. The symbols show the matches predicted from the photocurrents of the three types of macaque cones. The predictions included a correction for absorption by the lens and other inert pigments in the eye. Source: Baylor, 1987.
Now we know, for any given set of primaries, $P$, how to measure the color matching functions, $C$, corresponding to those primaries. And, knowing $C$, we know how to control the primaries $P$ to match any given color spectrum. And we know how to translate from one set of color matching functions to another.

Now we just need to standardize on a set of color matching functions, $C$, so that our color measurements are compatible.
CIE XYZ color space

• Commission Internationale d’Eclairage, 1931 (International Commission on Illumination).

• “...as with any standards decision, there are some irritating aspects of the XYZ color-matching functions as well...no set of physically realizable primary lights that by direct measurement will yield the color matching functions.”

• “Although they have served quite well as a technical standard, and are understood by the mandarins of vision science, they have served quite poorly as tools for explaining the discipline to new students and colleagues outside the field.”

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995
CIE XYZ: Color matching functions are positive everywhere, but primaries are "imaginary" (require adding light to the test color’s side in a color matching experiment). Usually compute $x, y$, where

$$ x = X/(X+Y+Z) $$

$$ y = Y/(X+Y+Z) $$

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995
Pure wavelength in chromaticity diagram

\[ x = \frac{X}{X + Y + Z} \]
\[ y = \frac{Y}{X + Y + Z} \]
Pure wavelength in chromaticity diagram

The 1931 standard observer, as it is usually shown.

\[ x = \frac{X}{X+Y+Z} \]
\[ y = \frac{Y}{X+Y+Z} \]
Pure wavelength in chromaticity diagram

The 1931 standard observer, as it is usually shown.

$x = \bar{X}/(\bar{X} + \bar{Y} + \bar{Z})$

$y = \bar{Y}/(\bar{X} + \bar{Y} + \bar{Z})$
Pure wavelength in chromaticity diagram

\[ x = \frac{\bar{X}}{\bar{X} + \bar{Y} + \bar{Z}} \]
\[ y = \frac{\bar{Y}}{\bar{X} + \bar{Y} + \bar{Z}} \]
Pure wavelength in chromaticity diagram

Tristimulus values

Wavelength, nm

The 1931 standard observer, as it is usually shown.

\[ x = \frac{\bar{X}}{\bar{X} + \bar{Y} + \bar{Z}} \]

\[ y = \frac{\bar{Y}}{\bar{X} + \bar{Y} + \bar{Z}} \]
XYZ vs. RGB

- Linear transform
- XYZ is rarely used for storage
- There are tons of flavors of RGB
  - sRGB, Adobe RGB
  - Different matrices!
- XYZ is more standardized
- XYZ can reproduce all colors with positive values
- XYZ is not realizable physically!!
  - What happens if you go “off” the diagram
  - In fact, the orthogonal (synthesis) basis of XYZ requires negative values.
Concepts in color measurement

• What are colors?
  – Arise from power spectrum of light.

• How represent colors:
  – Pick primaries
  – Measure color matching functions (CMF’s)
  – Matrix mult the test color’s power spectrum by CMF’s to find color in terms of the 3 primary color values which will give a perceptual match to the test color’s power spectrum.

• How share color descriptions between people?
  – Standardize on a few sets of primaries.
  – Translate colors between systems of primaries (3x3 matrix multiplications).
Displaying Contrast Sensitivity Function (CSF)

along one axis

along other axis
Figure 43

Range [-548, 548]
Dims [1000, 1000]
Contrast Sensitivity Function (CSF)
Another psychophysical fact: luminance and chrominance channels in the brain


Figure 6.1
Contrast sensitivity threshold functions for static luminance gratings (Y) and isoluminance chromaticity gratings (R/Y,B/Y) averaged over seven observers.
NTSC color components: Y, I, Q

\[
\begin{pmatrix}
Y \\
I \\
Q
\end{pmatrix}
= \begin{pmatrix}
0.299 & 0.587 & 0.114 \\
0.596 & -0.274 & -0.322 \\
0.211 & -0.523 & 0.312
\end{pmatrix}
\begin{pmatrix}
R \\
G \\
B
\end{pmatrix}
\]
NTSC - RGB
Spatial resolution and color

original
Blurring the R component

original

processed
Blurring the G component

original

processed
Blurring the B component

original

processed

Figure 6.1
Contrast sensitivity threshold functions for static luminance gratings (Y) and isoluminance chromaticity gratings (R/Y, B/Y) averaged over seven observers.
Lab color components

A rotation of the color coordinates into directions that are more perceptually meaningful:

L: luminance,
a: red-green,
b: blue-yellow
Blurring the L Lab component

original

processed

L

a

b
Blurring the a Lab component

original

processed
Blurring the b Lab component
Lecture outline

• Color physics.

• Color perception
  – part 1: assume perceived color only depends on light spectrum.
  – part 2: the more general case.
Color constancy demo

• We assumed that the spectrum impinging on your eye determines the object color. That’s often true, but not always. Here’s a counter-example…
Low-dimensional models for color spectra

How to find a linear model for color spectra:
--form a matrix, $D$, of measured spectra, 1 spectrum per column.
--$[u, s, v] = \text{svd}(D)$ satisfies $D = u*s*v'$
--the first $n$ columns of $u$ give the best (least-squares optimal) $n$-dimensional linear bases for the data, $D$:

$$D \approx u(:,1:n) * s(1:n,1:n) * v(1:n,:)'$$
Macbeth Color Checker
My Macbeth Color Checker Tattoo

I think I have all the other color checker photos beat...

Yes, the tattoo is real.
No, it is not a rubik's cube.

THIS PHOTOGRAPH IS COPYRIGHT 2007 THE X-RITE CORPORATION!

A photograph from this session can be viewed on the X-Rite Website: www.xrite.com/top_munsell.aspx
Basis functions for Macbeth color checker

9.9 BASIS FUNCTIONS OF THE LINEAR MODEL FOR THE MACBETH COLORCHECKER. The surface-reflectance functions in the collection vary smoothly with wavelength, as do the basis functions. The first basis function is all positive and explains the most variance in the surface-reflectance functions. The basis functions are ordered in terms of their relative significance for reducing the error in the linear-model approximation to the surfaces.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995
Fitting color spectra with low-dimensional linear models

9.8  A LINEAR MODEL TO APPROXIMATE THE SURFACE REFLECTANCES IN THE MACBETH COLORCHECKER. The panels in each row of this figure show the surface-reflectance functions of six colored surfaces (shaded lines) and the approximation to these functions using a linear model (solid lines). The approximations using linear models with (A) three, (B) two, and (C) one dimension are shown.

n = 1
Rendering equation for jth observation

\[
\begin{pmatrix}
L_j \\
M_j \\
S_j
\end{pmatrix}
= E^T (A \vec{x}_j^s \ast B \vec{x}_i^i)
\]
Color constancy solution 1: find white in the scene

Let the kth patch be the white one, with surface coefficients assumed to be \( \vec{x}^W \). Then we can solve for the illuminant coefficient, \( \vec{x}^i \).

\[
\begin{pmatrix}
L_k \\
M_k \\
S_k
\end{pmatrix} = E^T \left( A \vec{x}^W \times B \vec{x}^i \right)
\]

\[
\begin{pmatrix}
L_j \\
M_j \\
S_j
\end{pmatrix} = E^T
\]

\( A \ x^w_j \times B \ x^i \)

a 3x3 matrix

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Color constancy solution 2: assume scene colors average to grey, then solve for the illuminant, $\bar{x}^i$

$$\frac{1}{N} \sum_j \begin{pmatrix} L_j \\ M_j \\ S_j \end{pmatrix} = E^T (A \frac{1}{N} \sum_j \bar{x}_j^s \cdot B \bar{x}^i)$$

$$= E^T (A \bar{x}^G \cdot B \bar{x}^i)$$

a 3x3 matrix
an image that violates both assumptions
Selected Bibliography

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• Reading:
  – Chapter 6, Forsyth & Ponce
  – Chapter 4 of Wandell, Foundations of Vision, Sinauer, 1995 has a good treatment of this.