Color

6.819/6.869, MIT Bill Freeman Antonio Torralba

Sept. 21, 2017

Why does a visual system need color?



http://www.hobbylinc.com/gr/pll/pll5019.jpg

Why does a visual system need color? (an incomplete list...)

- To tell what food is edible.
- To distinguish material changes from shading changes.
- To group parts of one object together in a scene.
- To find people's skin.
- Check whether a person's appearance looks normal/healthy.



http://www.pouted.com/know-10points-information-unicorn/sick-child/





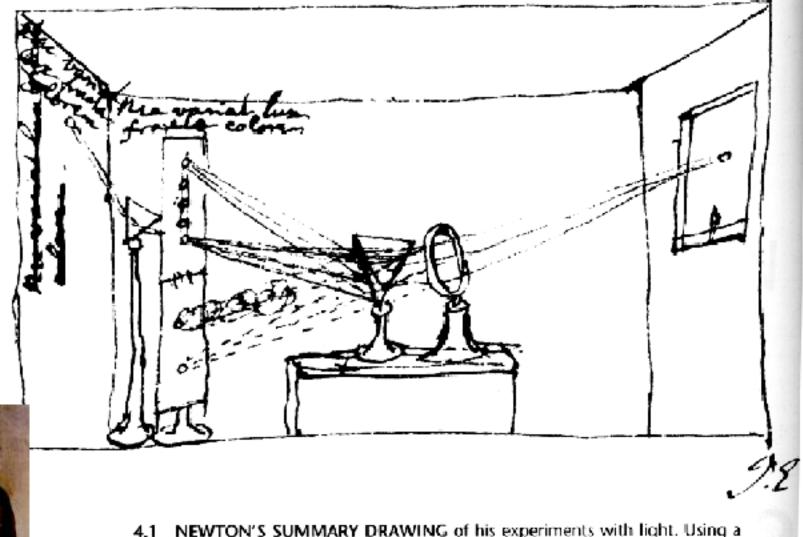
Lecture outline

- Color physics.
- Color perception.

Lecture outline

- Color physics.
- Color perception.

Color

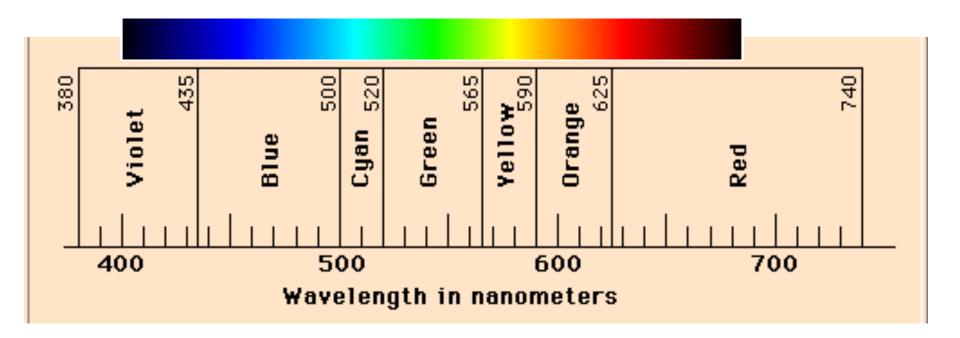


www.popularpersons.org

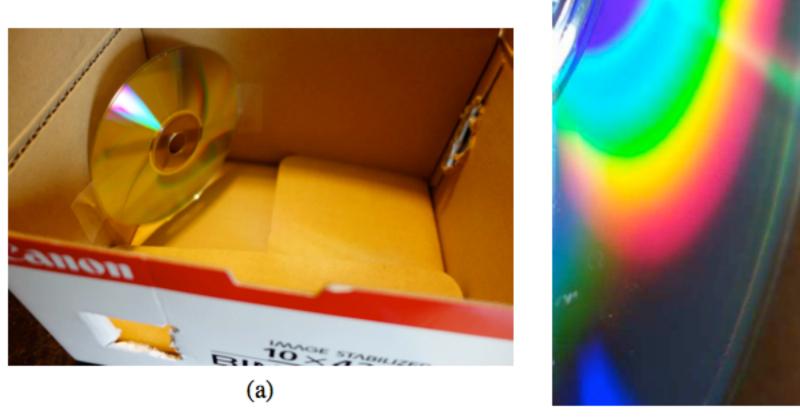
4.1 NEWTON'S SUMMARY DRAWING of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

From Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Spectral colors



http://hyperphysics.phy-astr.gsu.edu/hbase/vision/specol.html#c2



(b)

e 6.3: (a) A spectrograph constructed using a compact disk (CD). Light enters through a slit at diffracting from the narrowly spaced lines of the CD. (b) Photograph of diffraction pattern f ght, seen thorugh hole at bottom left.

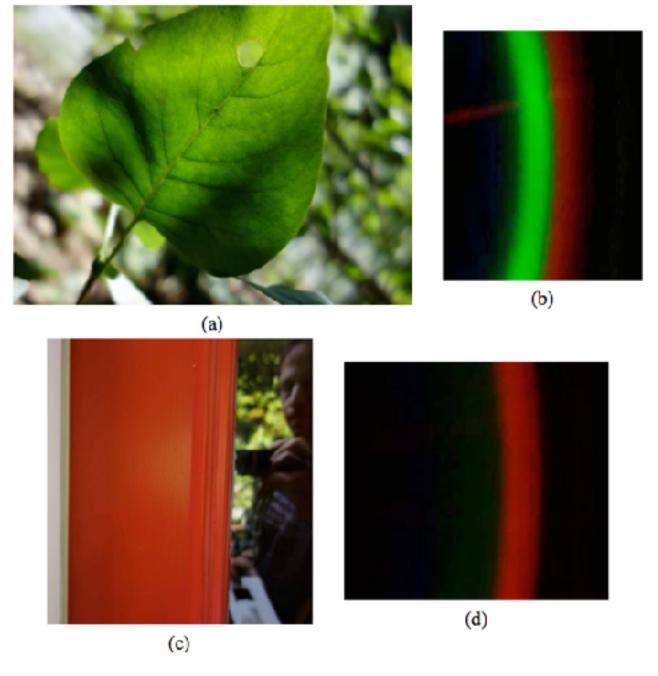
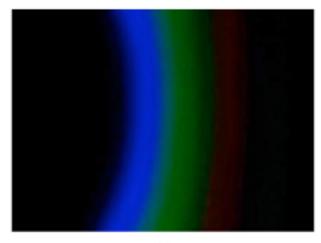


Figure 6.5: Some real-world objects and the reflected light spectra (photographed using Fig. (6.3) (a)) from outdoor viewing. (a) Leaf and (b) its reflected spectrum. (c) A red door and (d) its reflected





(a)

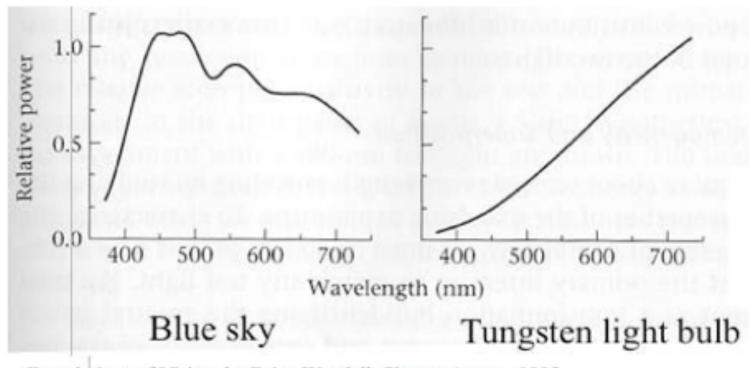




Figure 6.6: More real-world objects and the reflected light spectra. (a) Blue-green chair and (b) its reflected light. (c) Toby the dog and (d) his reflected spectrum.

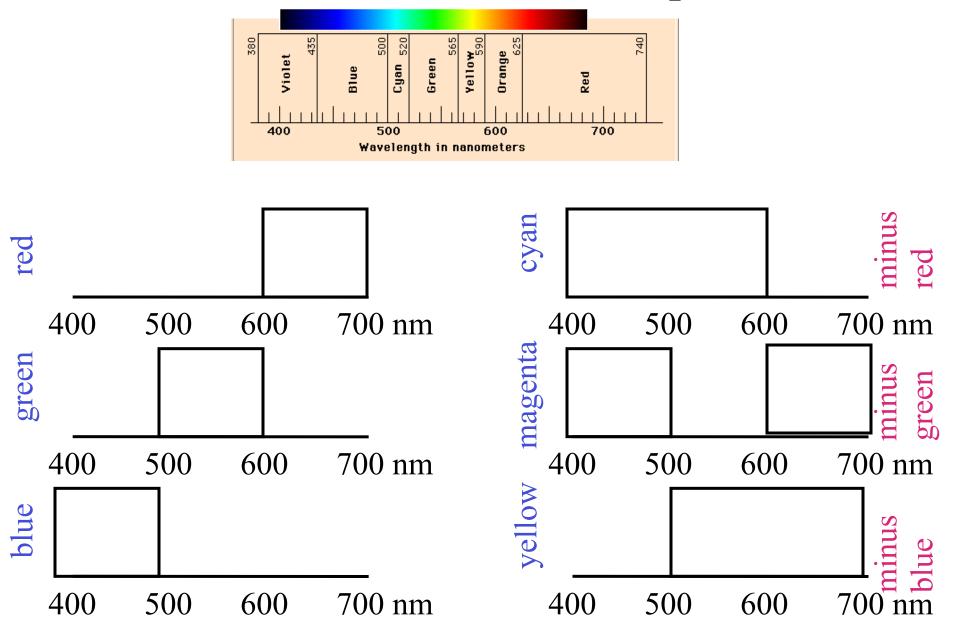




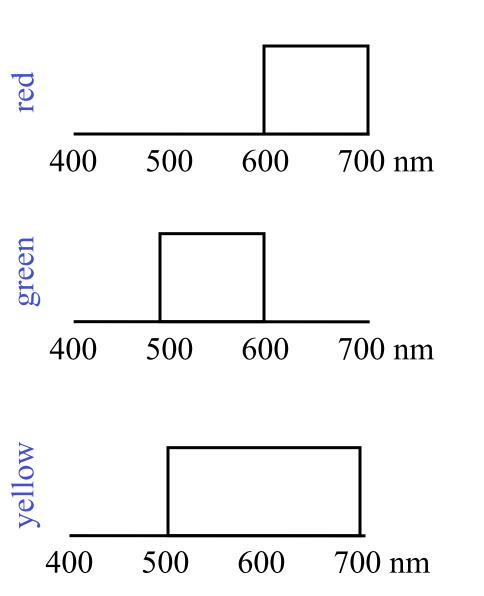


Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Color names for cartoon spectra



Additive color mixing

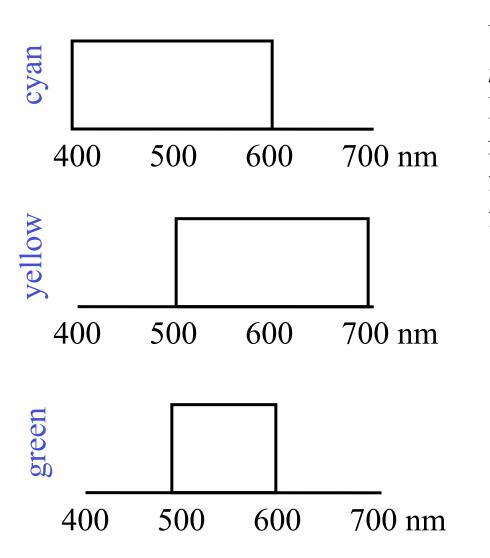


When colors combine by *adding* the color spectra. Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.

Red and green make...

Yellow!

Subtractive color mixing



When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

Cyan and yellow (in crayons, called "blue" and yellow) make...

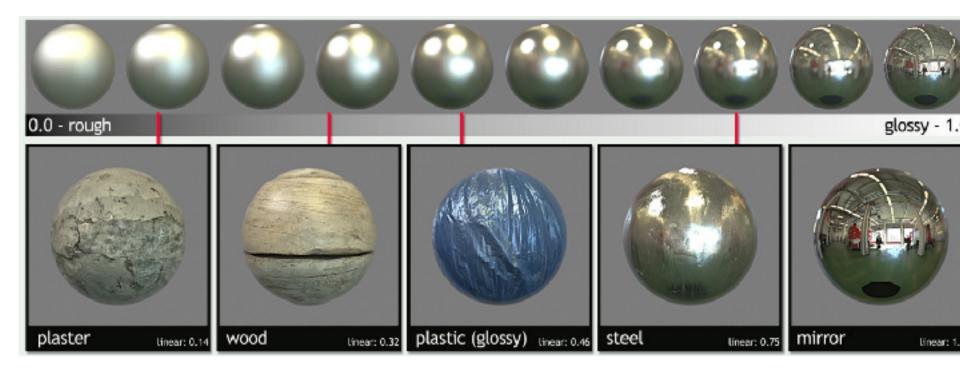
Green!

Overhead projector demo

Subtractive color mixing

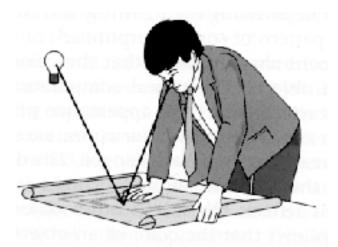
The interaction of light with surfaces $\theta_i, \phi_i, \lambda$ θ_e, ϕ_e, γ Horn, 1986 Figure 10-7. The bidirectional reflectance distribution function is the ratio of the radiance of the surface patch as viewed from the direction (θ_e, ϕ_e) to the irradiance resulting from illumination from the direction (θ_i, ϕ_i) . Spectral radiance: power in a specified direction, per unit area, per unit solid angle, per unit wavelength $BRDF = f(\theta_i, \phi_i, \theta_e, \phi_e, \lambda) = \frac{L(\theta_e, \phi_e, \lambda)}{E(\theta_i, \phi_i, \lambda)}$ Spectral irradiance: incident power per unit area, per unit wavelength

Effect of BRDF on sphere rendering

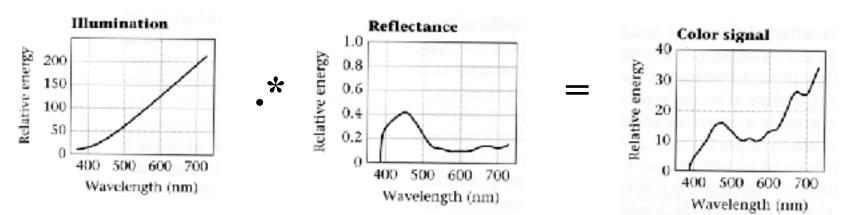


http://www.marmoset.co/toolbag/learn/pbr-practice

Simplified rendering models: BRDF → reflectance

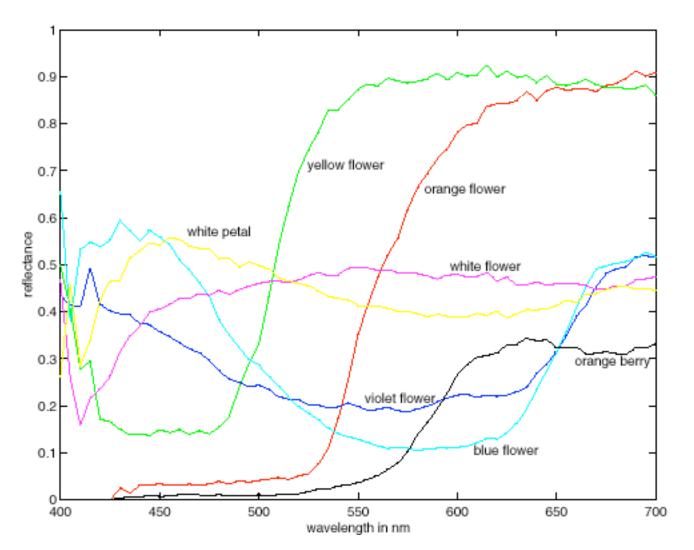


For diffuse reflections, we replace the BRDF calculation with a wavelength-by-wavelength scalar multiplication



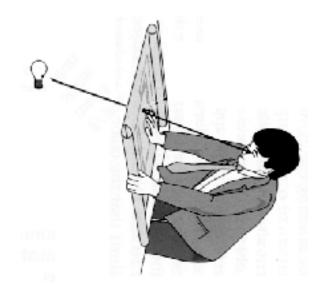
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

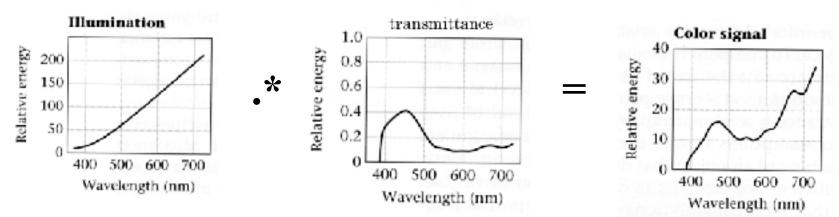
Some reflectance spectra



Spectral albedoes for several different leaves, with color names attached. Notice that different colours typically have different spectral albedo, but that different spectral albedoes may result in the same perceived color (compare the two whites). Spectral albedoes are typically quite smooth functions. Measurements by E.Koivisto.

Simplified rendering models: transmittance





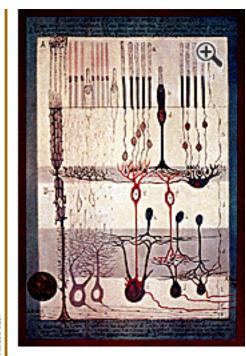
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Lecture outline

- Color physics.
- Color perception.

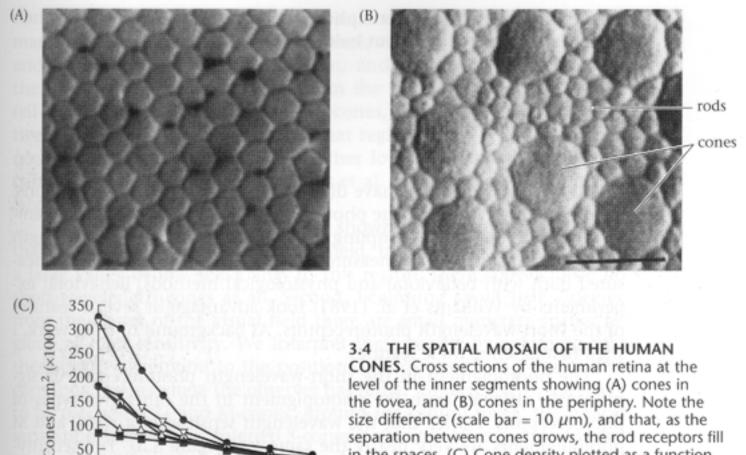
What's the machinery in the eye?

Eye Photoreceptor responses



The intricate layers and connections of nerve cells in the retina were drawn by the famed Spanish anatomist Santiago Ramón y Cajal around 1900. Rod and cone cells are at the top. Optic nerve fibers leading to the brain may be seen at bottom right. (Where do you think the light comes in?)

Human Photoreceptors



separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

0.1

0.2

Eccentricity (mm)

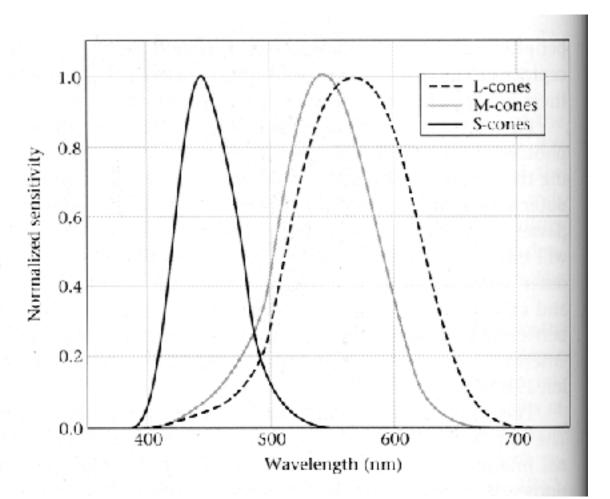
0.3

0.4

0.5

50

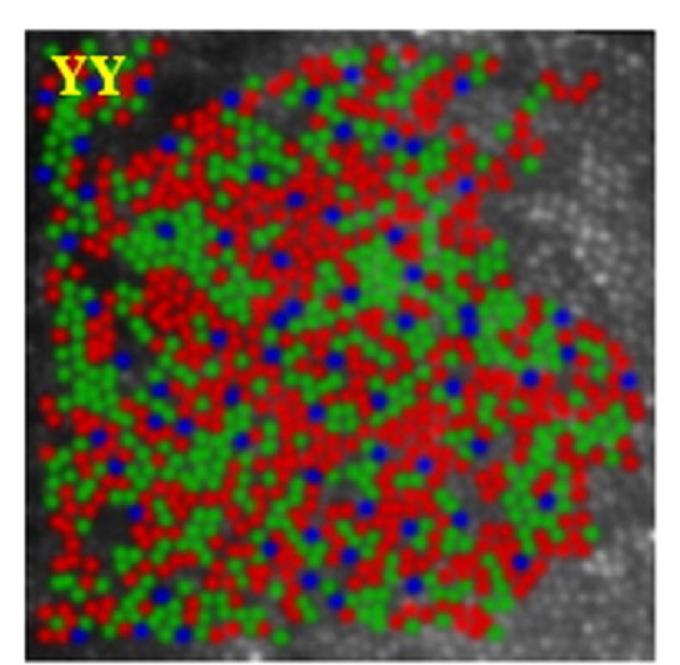
Human eye photoreceptor spectral sensitivities



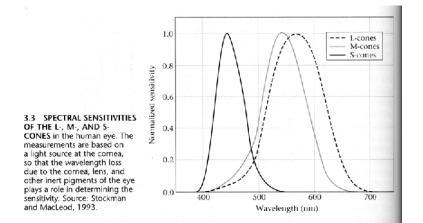
3.3 SPECTRAL SENSITIVITIES OF THE L-, M-, **AND S-CONES** in the human eye. The measurements are based on a light source at the cornea, so that the wavelength loss due to the cornea; lens, and other inert pigments of the eye plays a role in determining the sensitivity. Source: Stockman and MacLeod, 1993.

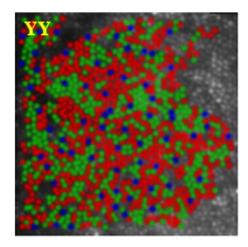
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

L, M, and S cone receptor types colored as R, G, B

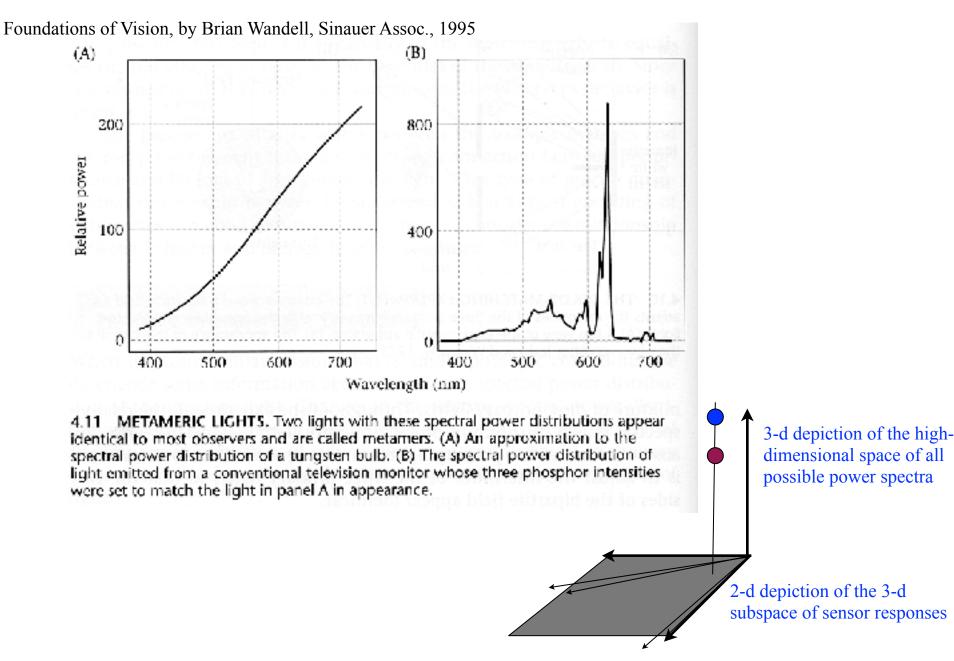


What are some color artifacts we might expect our visual system to experience, based on this way of measuring the light spectra falling on our eye?





A property of our visual system: these two spectra look the same

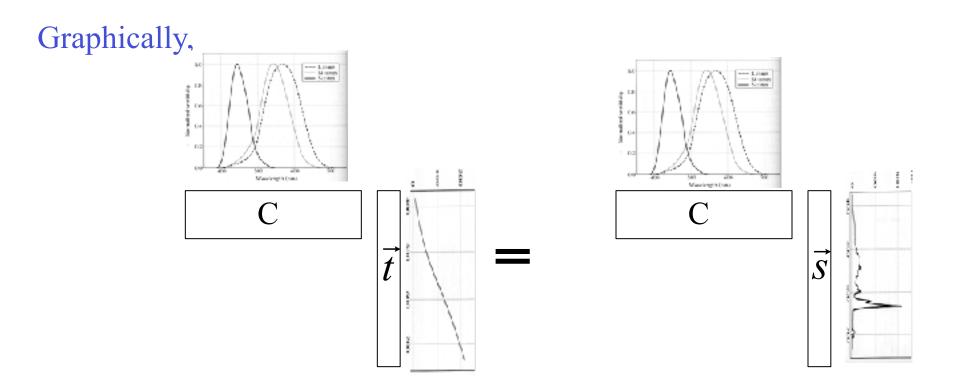


Color metamerism: different spectra looking the same color

Two spectra, t and s, perceptually match when

$$C\vec{t} = C\vec{s}$$

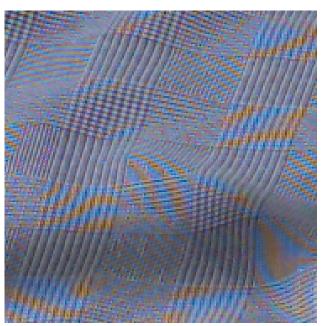
where C are the cone response curves.



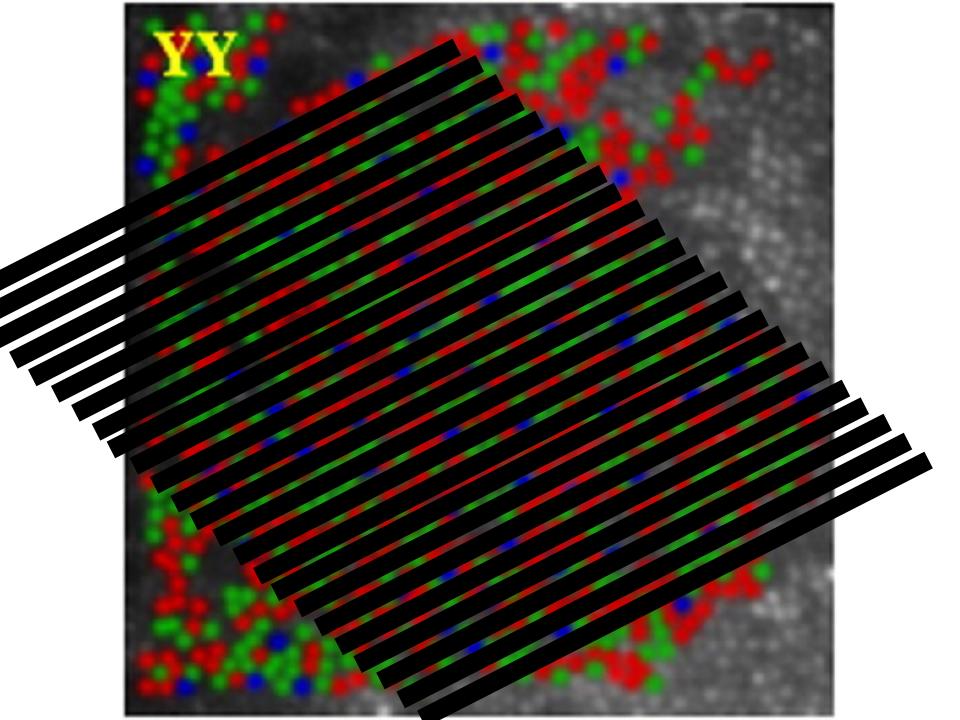
Evidence of spatially offset color sampling in an old digital camera

sensor color sampling pattern

• Color fringes or jaggies







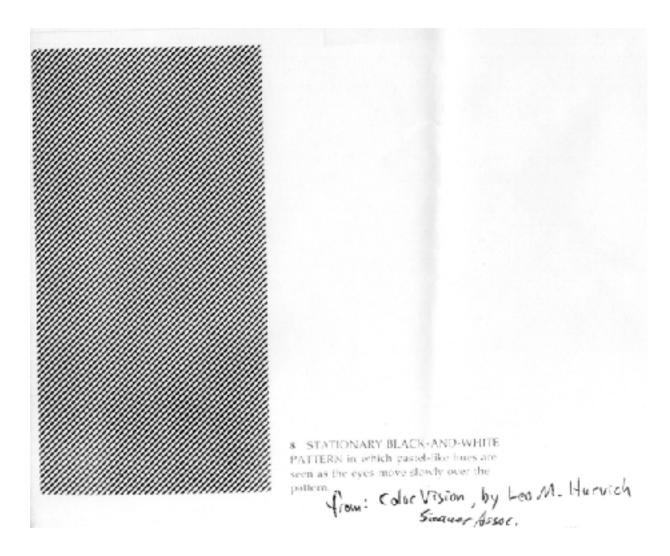
Where you can see color fringe reconstruction artifacts from your own eye

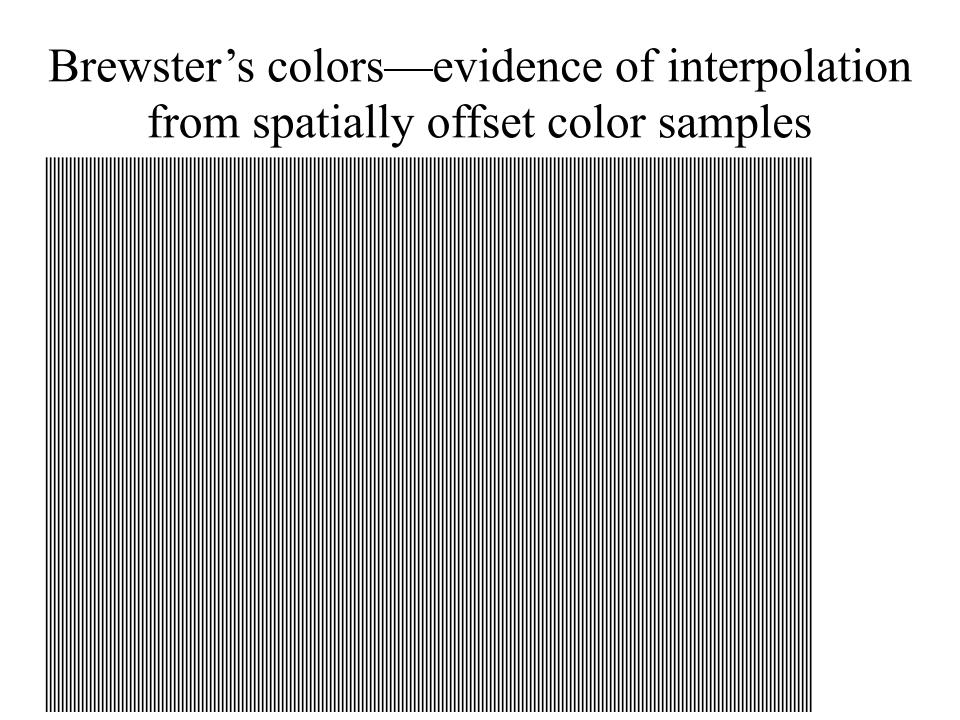


http://static.flickr.com/21/31393422_23013da003.jpg

Brewster's colors—evidence of interpolation from spatially offset color samples

Scale relative to human photoreceptor size: each line covers about 7 photoreceptors.





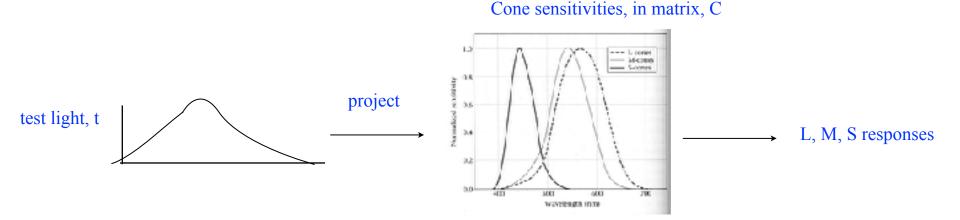
Lecture outline

- Color physics.
- Color perception
 - part 1: assume perceived color only depends on light spectrum.
 - part 2: the more general case.

The assumption for color perception, part 1

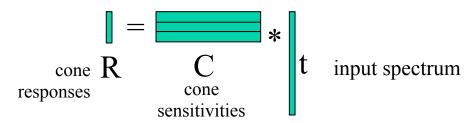
- We know color appearance really depends on:
 - The illumination
 - Your eye's adaptation level
 - The colors and scene interpretation surrounding the observed color.
- But for now we will assume that <u>the spectrum of</u> <u>the light arriving at your eye completely</u> <u>determines the perceived color</u>.

How we sense light spectra

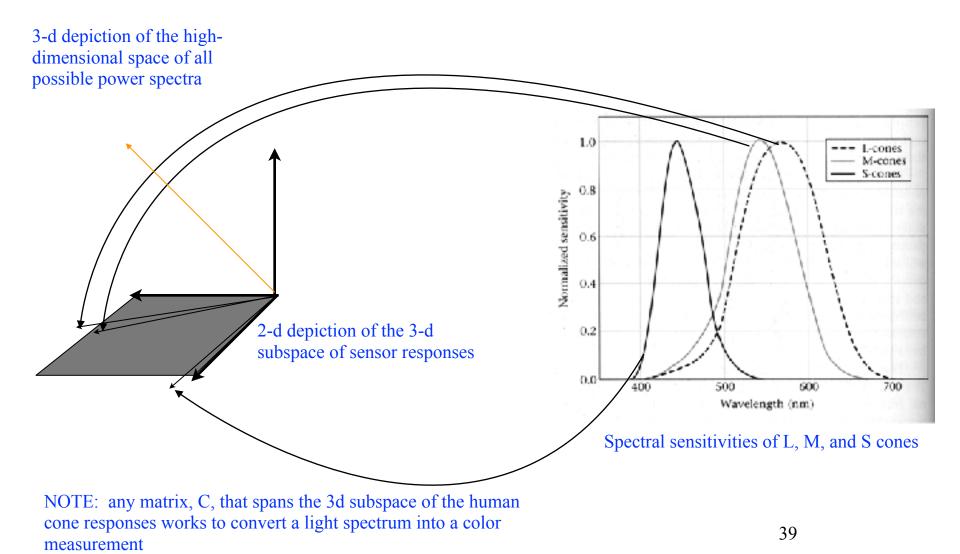


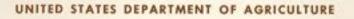
<u>biophysics</u>: integrate the response over all wavelengths, weighted by the photosensor's sensitivity at each wavelength.

<u>mathematically</u>: take dot product of input spectrum with the cone sensitivity basis vectors. Project the high-dimensional test light into a 3-d space. R = C t



Cone response curves as basis vectors in a 3-d subspace of light power spectra





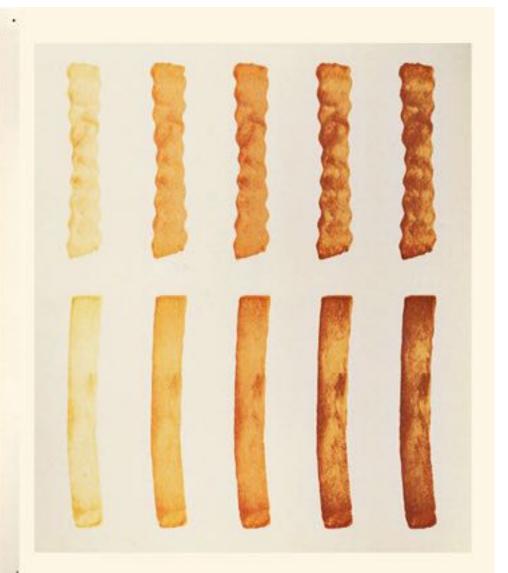
COLOR STANDARDS

for

FROZEN FRENCH FRIED POTATOES



FOURTH EDITION, THEE CTHEE KOLLMORGEN CORPORATION MUNIFILE COLOR BALTIMORE, MARYLAND 64-1



Color trademarks

CURRENTLY REGISTERED COLOR TRADEMARKS

http://blog.patents-tms.com/?p=52

A color trademark is a non-conventional trademark where at least one color is used to identify the commercial origin of a product or service. A color trademark must meet the same requirements of a conventional trademark. Thus, the color trademark must either be inherently distinctive or have acquired secondary meaning. To be inherently distinctive, the color must be arbitrarily or suggestively applied to a product or service. In contrast, to acquire secondary meaning, consumers must associate the color used on goods or services as originating from a single source. Below is a selection of some currently registered color trademarks in the U.S. Trademark Office:

MARK/COLOR(S)/OWNER:

BANK OF AMERICA 500 blue, red & grey Bank of America Corporation

NATIONAL CAR RENTAL green NCR Affiliate Servicer, Inc.

FORD blue Ford Motor Company

VISTEON

orange Ford Motor Company

76 red & blue ConocoPhillips Company

VW silver, metallic blue, black and white Volkswagen Aktiengesellschaft Corp THE HOME DEPOT orange Homer TLC, Inc.

HONDA red Honda Motor Co., Ltd.

M MARATHON brown, orange, yellow Marathon Oil Company

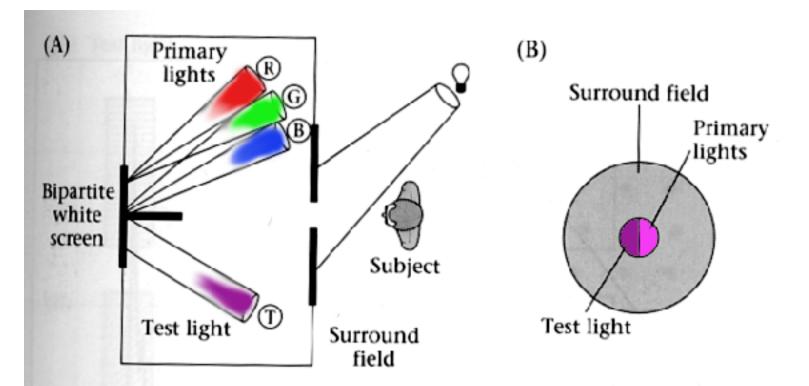
M MARATHON gray, black & white Marathon Oil Company

COSTCO red Costco Wholesale Membership, Inc.

TEENAGE MUTANT NINJA TURTLES MUTANTS & MONSTERS red, green, yellow, black, grey and white Mirage Studios, Inc. 41

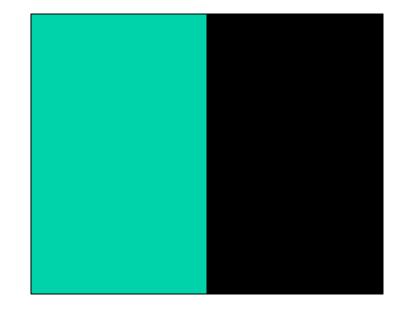
TARGET

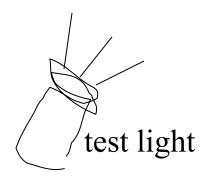
- How do we measure colors?
- How do we make systems that match colors?



4.10 THE COLOR-MATCHING EXPERIMENT. The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.

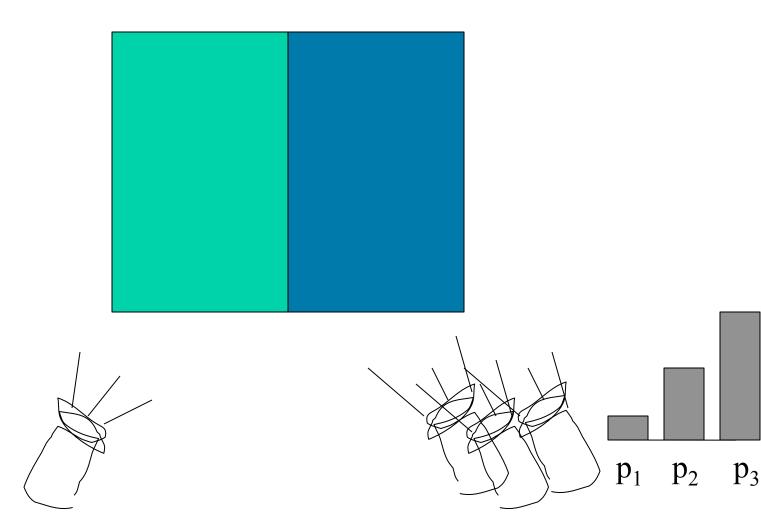
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

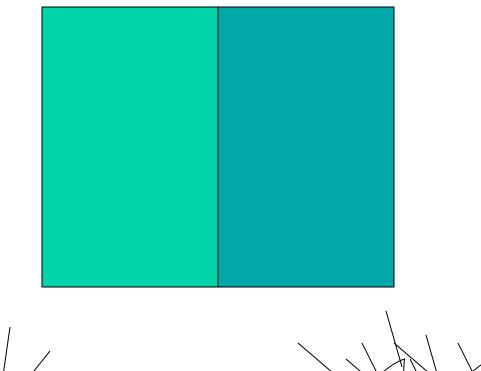


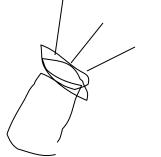


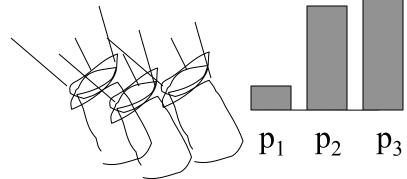


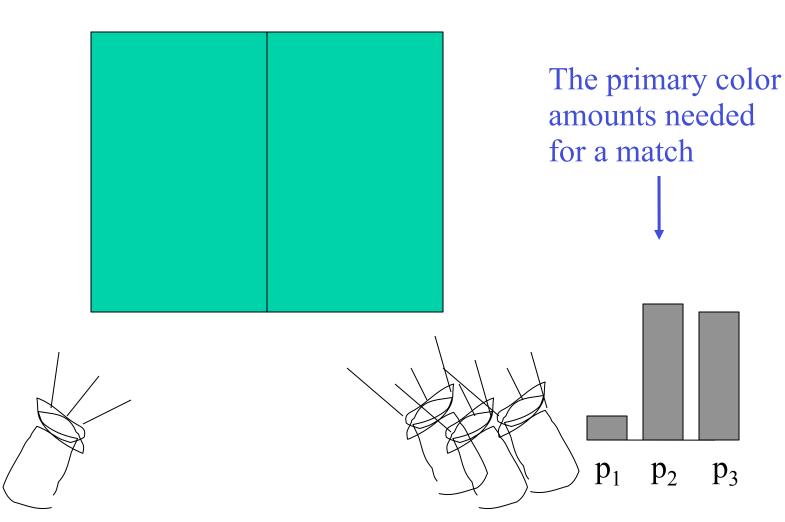
primary lights











p₃

Relevant to color matching experiments, solve this puzzle:

uart4kids.com/wp-content/uploads/ jmomys-about-us-picture-027.jpg



Want to measure out 7 lbs of clay

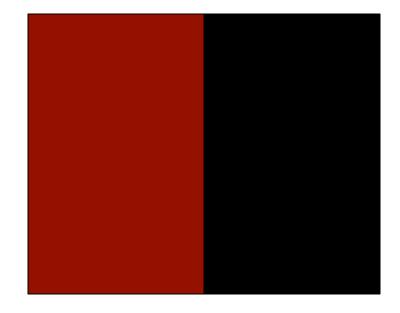
http://commons.wikimedia.org/wiki/ File:Balanced_scale_of_Justice.svg

(we wish we could add a -1 lb mass to the 5 and 3 lb masses to weigh out 7 lbs of clay. But we don't have negative mass. Instead, we just add the 1 lb mass to the other side, where the clay is, to weigh out 7 lbs of clay) http://www.retonthen vintage-metric-brassweights-50-20--10-gr

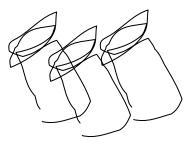


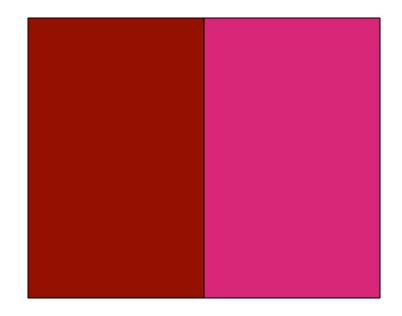
using 5, 3, 1 lb weights ⁴⁸ Relevant to color matching experiments, solve this puzzle:



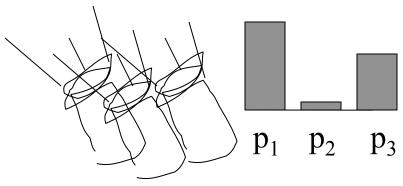


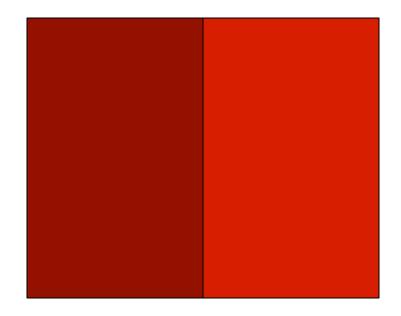




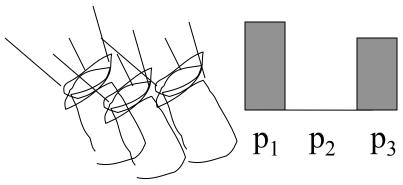






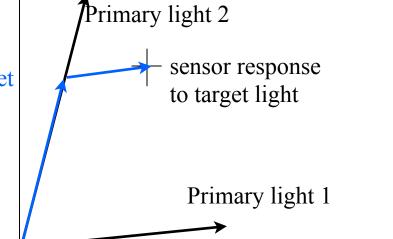




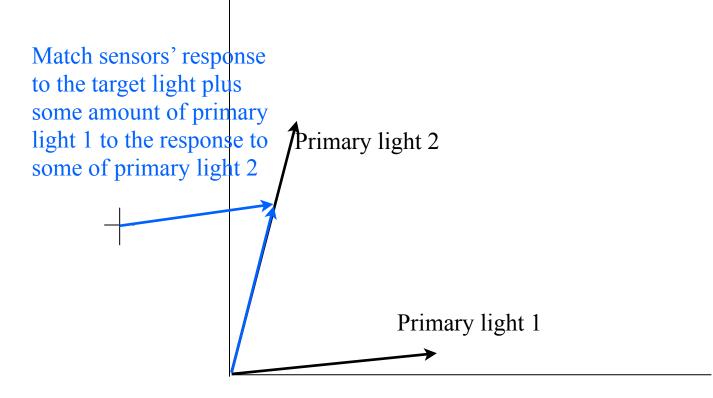


Color matching with positive amounts of the primaries

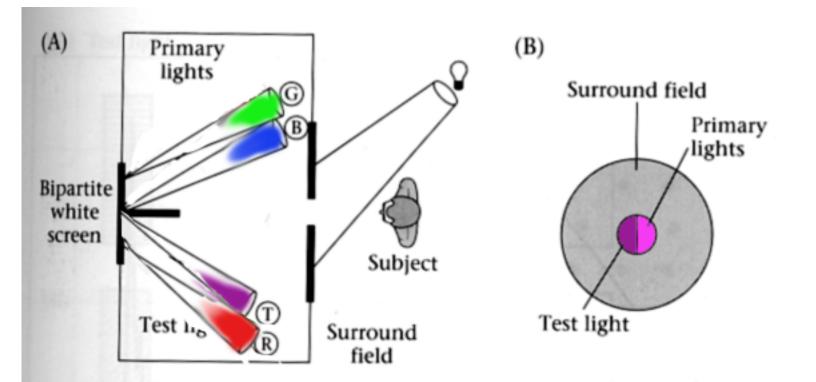
Match the sensors' response to the target light to the sum of responses to the primary lights



Color matching with a negative amount of primary 1



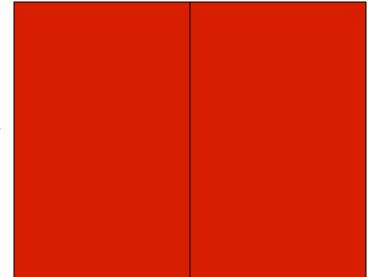
Color matching experiment--handle negative light by adding light to the test.



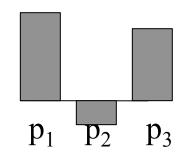
4.10 THE COLOR-MATCHING EXPERIMENT. The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.

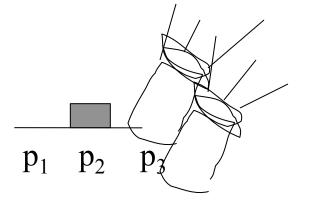
Foundations of vision, by difait wander, Sinauer Assoc., 1993

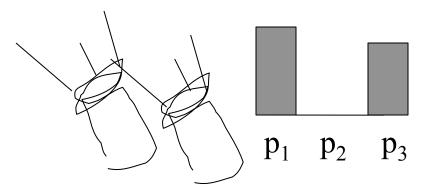
We say a "negative" amount of p_2 was needed to make the match, because we added it to the test color's side.



The primary color amounts needed for a match:



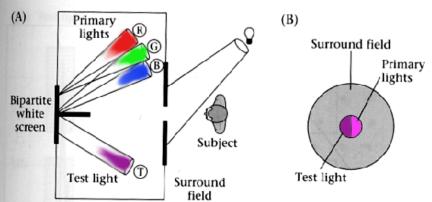




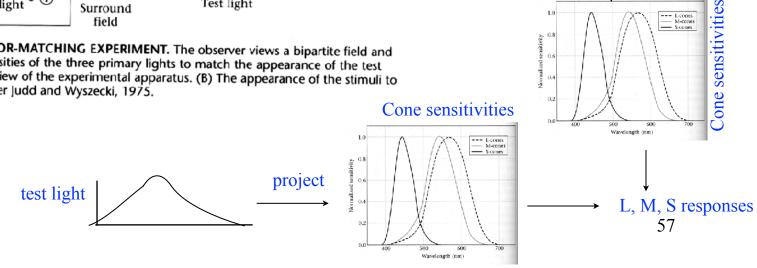
To measure a color

Primarie

- 1. Choose a set of 3 primary colors (three power spectra).
- 2. Determine how much of each primary needs to be added to a probe signal to match the test light. $a_1 a_2$



4.10 THE COLOR-MATCHING EXPERIMENT. The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After ludd and Wyszecki, 1975.



a3

weighted

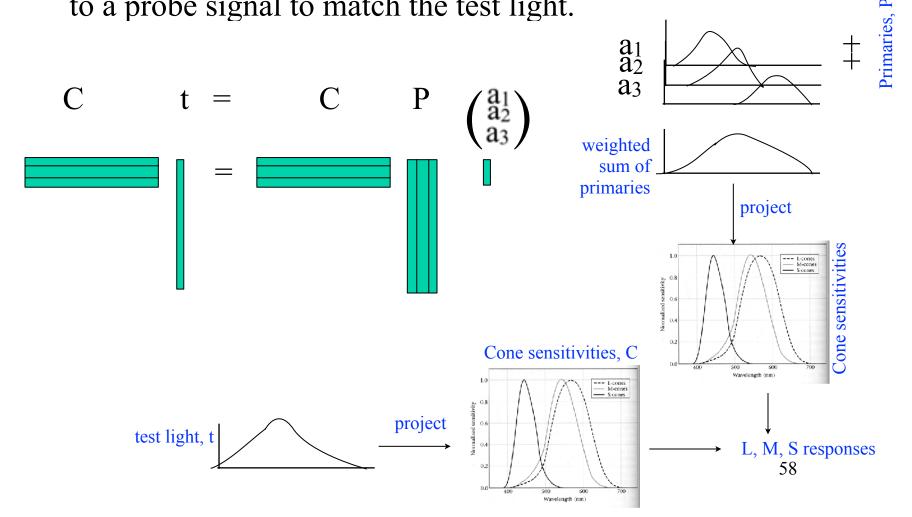
primaries

sum of

project

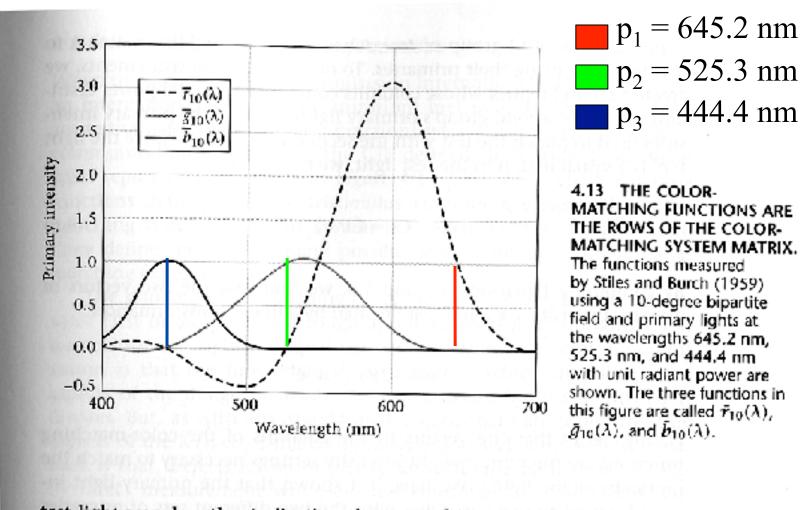
To measure a color

- 1. Choose a set of 3 primary colors (three power spectra).
- 2. Determine how much of each primary needs to be added to a probe signal to match the test light.



- We can measure a color by measuring how much of each primary is needed to match that color.
- Can we measure color without having to make psychophysical experiments each time?
- We'd like to find a matrix, C, that we can project a spectrum onto, to tell us how much of each primary, in the columns of P, , to use to match the spectrum.

"Color matching functions" tell us how to control primary lights in order to perceptually match a given spectrum

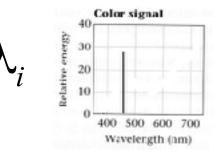


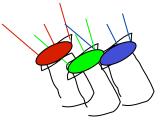
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Using the color matching functions to predict the primary match to a new spectral signal

We know that a monochromatic light of wavelength will be matched by the amounts $c_1(\lambda_i), c_2(\lambda_i), c_3(\lambda_i)$

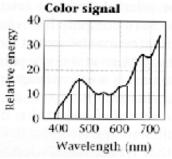
of each primary.





And any spectral signal can be thought of as a linear combination of very many monochromatic lights, with the linear coefficient given by the spectral power at each wavelength.

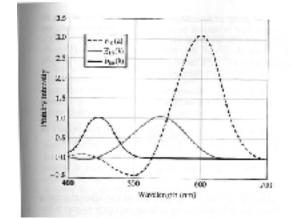
$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$



Using the color matching functions to predict the primary match to a new spectral signal

Store the color matching functions in the rows of the matrix, C

$$C = \begin{pmatrix} c_1(\lambda_1) & \cdots & c_1(\lambda_N) \\ c_2(\lambda_1) & \cdots & c_2(\lambda_N) \\ c_3(\lambda_1) & \cdots & c_3(\lambda_N) \end{pmatrix}$$



Let the new spectral signal be described by the vector t.

$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$

Using the color matching functions, measured from a set of primaries, to predict how to match any new spectrum, t, with those primaries

Then the amounts of each primary needed to match t are:

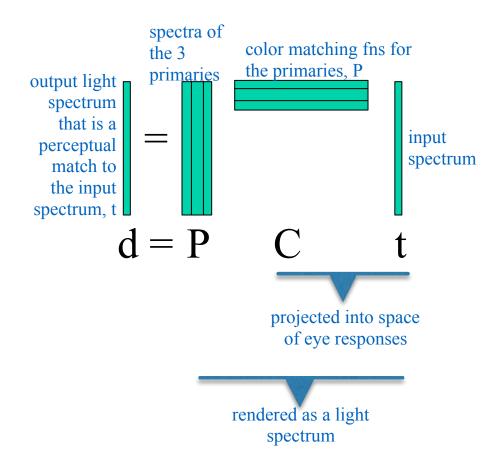
$$\sum_{j} \begin{pmatrix} c_1(\lambda_j)t(\lambda_j) \\ c_2(\lambda_j)t(\lambda_j) \\ c_3(\lambda_j)t(\lambda_j) \end{pmatrix} = C\vec{t} = \begin{bmatrix} \vec{t} \\ \vec{t} \end{bmatrix}$$

color matching functions for a

given set of primaries

input spectrum

How the color matching functions, C, and the corresponding primary spectra, P, relate to each other.



If the primaries, P, correspond to the color matching functions, C, then t and s are perceptual matches. When projected down by C they must give the same answer, so we must have,

C t = C d

Requirements on C, P to form a color matching system:

(1) the rows of C must be some (non-degenerate) linear combination of the eye photosensor response curves.
 That ensures that if two spectra match when projected into the subspace spanned by C, they will match when projected into the subspace of the eye

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(2) C, P must satisfy:
```

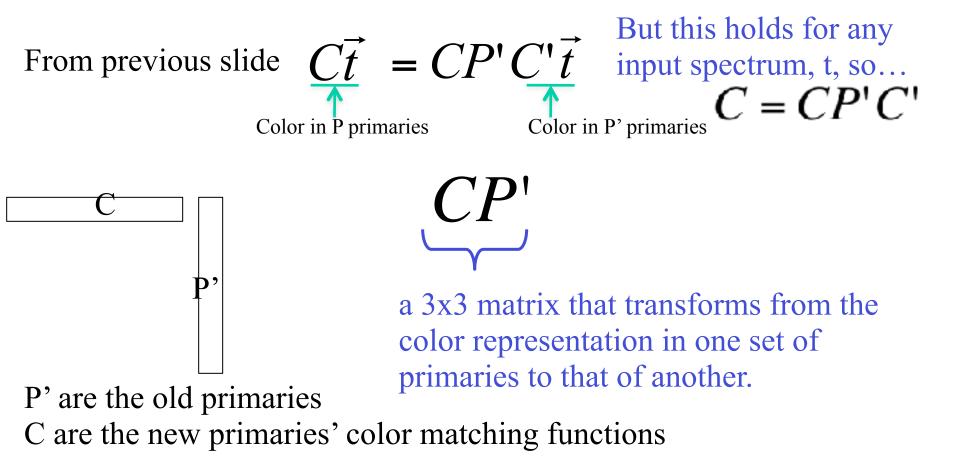
C P = I

why must this hold? Because the amounts of the 3 primaries needed to match the spectrum of each primary (the columns of P) must be [1;0;0], [0;1;0], [0;0;1].

response curves.

If those conditions hold, then the spectrum PCt will be a perceptual match to t, because Ct = CPCt

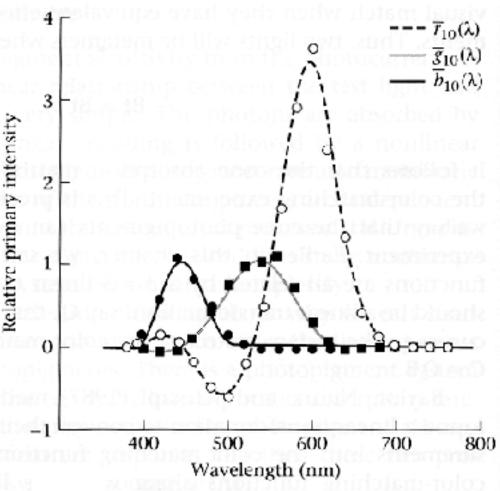
How do color coordinates translate between different sets of primaries?



Comparison of color matching functions with best linear combination of cone response curves

4.20 COMPARISON OF CONE PHOTOCURRENT RESPONSES AND THE COLOR-MATCHING FUNCTIONS. The cone photocurrent spectral responsivities are within a linear transformation of the color-matching functions, after a correction has been made for the optics and inert pigments in the eye. The smooth curves show the Stiles and Burch (1959) colormatching functions. The symbols show the matches predicted from the photocurrents of the three types of macaque cones. The predictions included a correction for absorption by the lens and other inert pigments in the eye. Source: Baylor, 1987.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

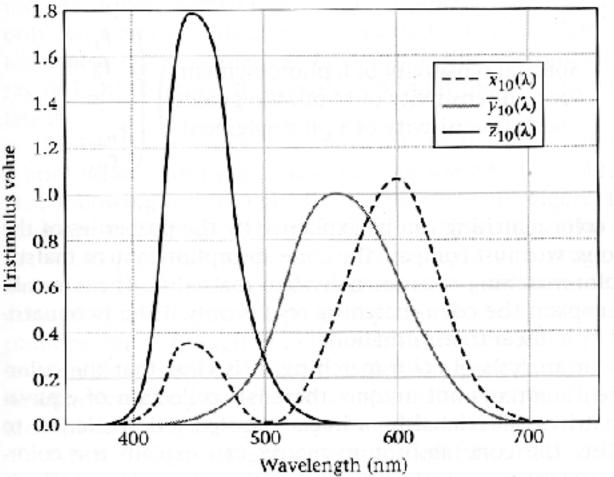


Standardization

- Now we know, for any given set of primaries, P, how to measure the color matching functions, C, corresponding to those primaries. And, knowing C, we know how to control the primaries P to match any given color spectrum. And we know how to translate from one set of color matching functions to another.
- Now we just need to standardize on a set of color matching functions, C, so that our color measurements are compatible.

CIE XYZ color space

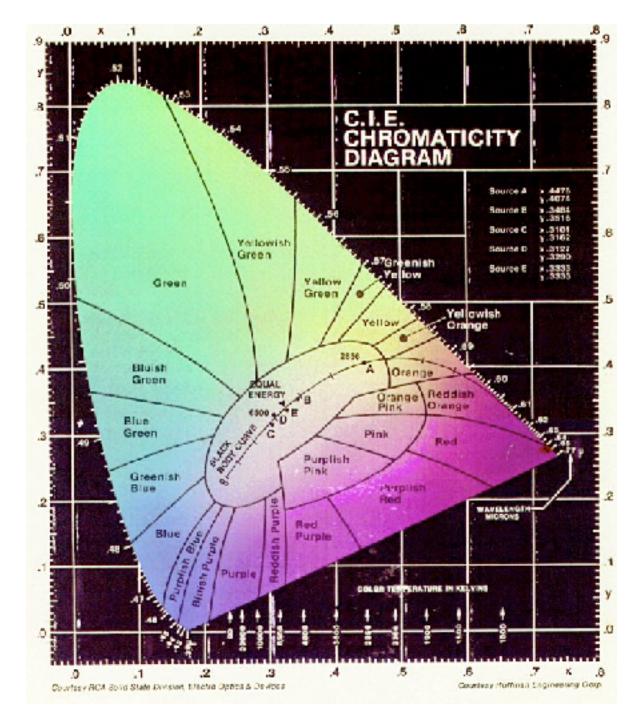
- Commission Internationale d'Eclairage, 1931 (International Commission on Illumination).
- "...as with any standards decision, there are some irratating aspects of the XYZ color-matching functions as well...no set of physically realizable primary lights that by direct measurement will yield the color matching functions."
- "Although they have served quite well as a technical standard, and are understood by the mandarins of vision science, they have served quite poorly as tools for explaining the discipline to new students and colleagues outside the field."



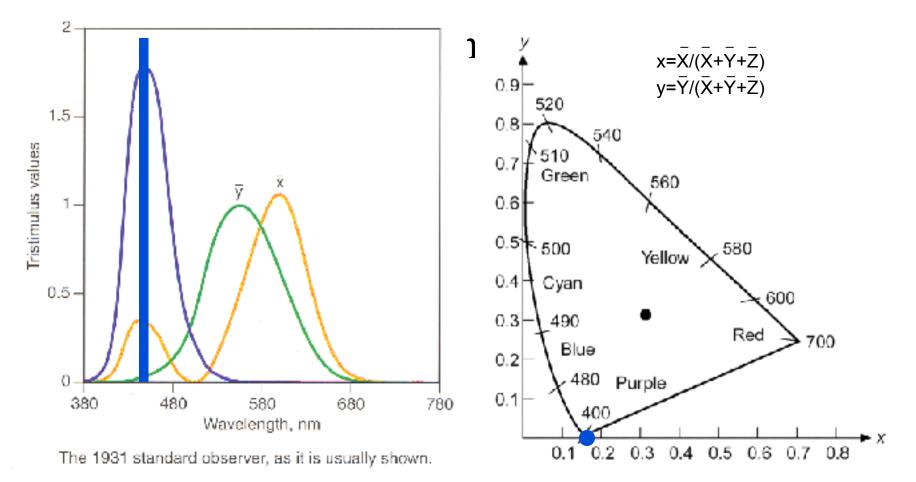
4.14 THE XYZ STANDARD COLOR-MATCHING FUNCTIONS. In 1931 the CIE standardized a set of color-matching functions for image interchange. These color-matching functions are called $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$. Industrial applications commonly describe the color properties of a light source using the three primary intensities needed to match the light source that can be computed from the XYZ color-matching functions.

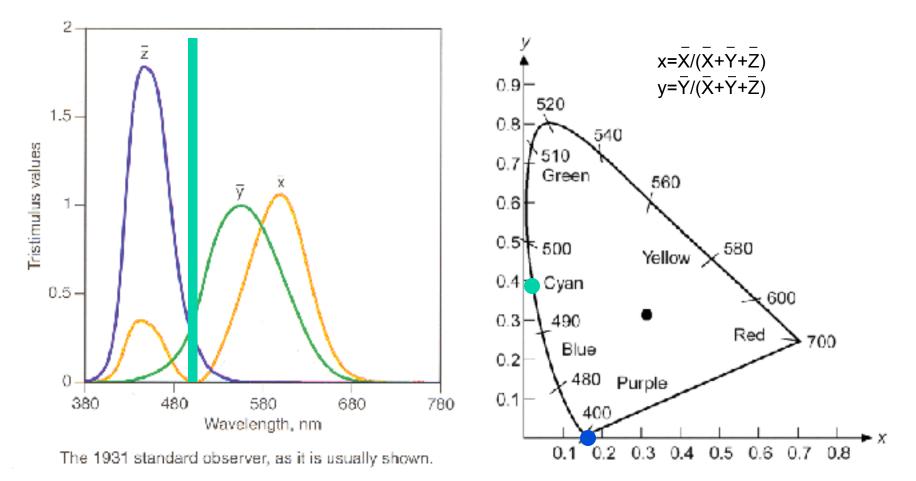
CIE XYZ: Color matching functions are positive everywhere, but primaries are "imaginary" (require adding light to the test color's side in a color matching experiment). Usually compute x, y, where x=X/(X+Y+Z)y=Y/(X+Y+Z)

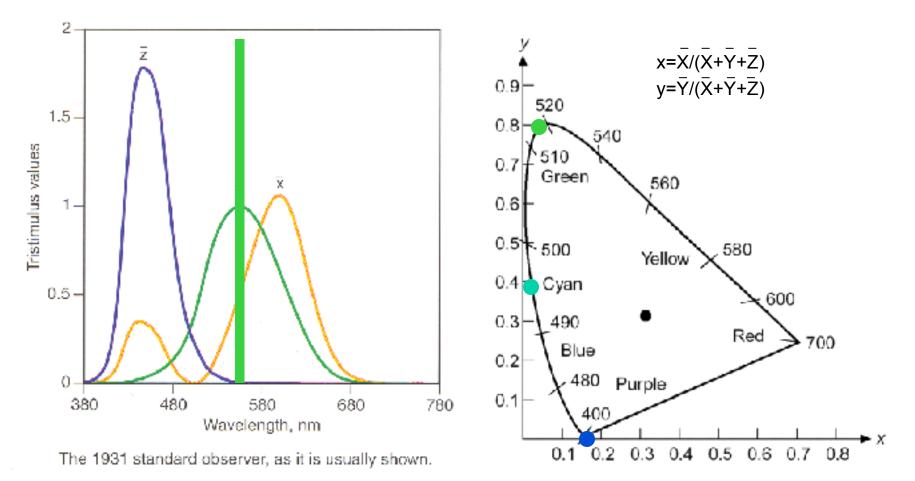
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

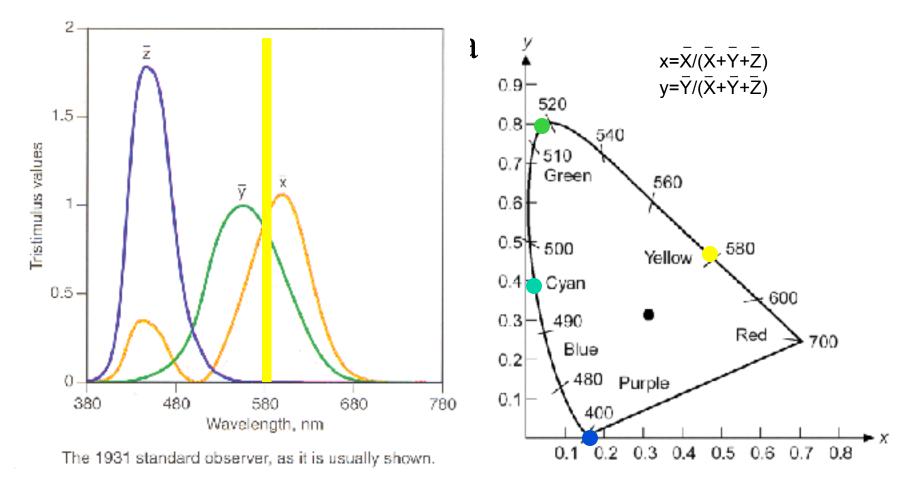


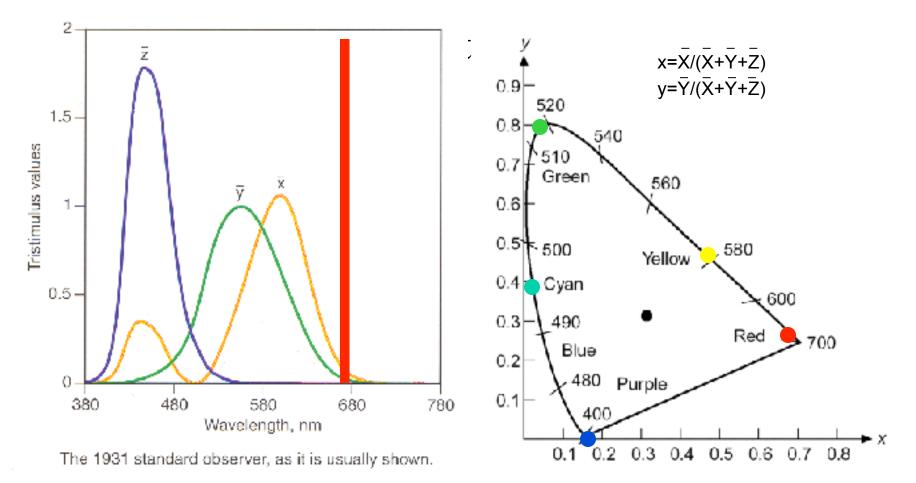
Pure wavelength in chromaticity diagram











XYZ vs. RGB

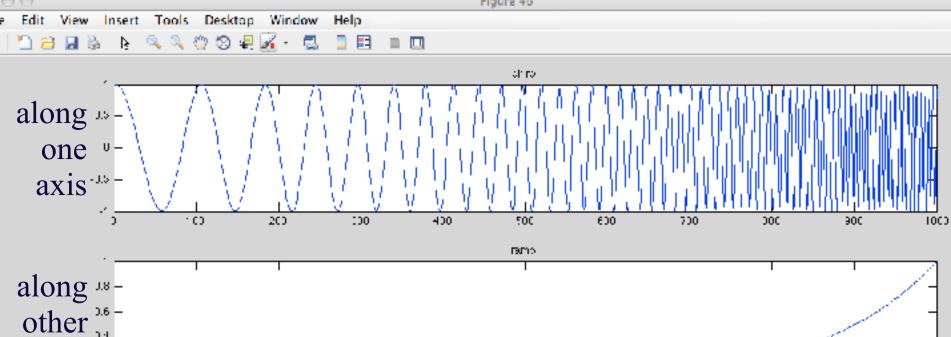
- Linear transform
- XYZ is rarely used for storage
- There are tons of flavors of RGB
 - sRGB, Adobe RGB
 - Different matrices!
- XYZ is more standardized
- XYZ can reproduce all colors with positive values
- XYZ is not realizable physically !!
 - What happens if you go "off" the diagram
 - In fact, the orthogonal (synthesis) basis of XYZ requires negative values.

0.04(0.41-0.36-0.18)

Concepts in color measurement

- What are colors?
 - Arise from power spectrum of light.
- How represent colors:
 - Pick primaries
 - Measure color matching functions (CMF's)
 - Matrix mult the test color's power spectrum by CMF's to find color in terms of the 3 primary color values which will give a perceptual match to the test color's power spectrum.
- How share color descriptions between people?
 - Standardize on a few sets of primaries.
 - Translate colors between systems of primaries (3x3 matrix multiplications).

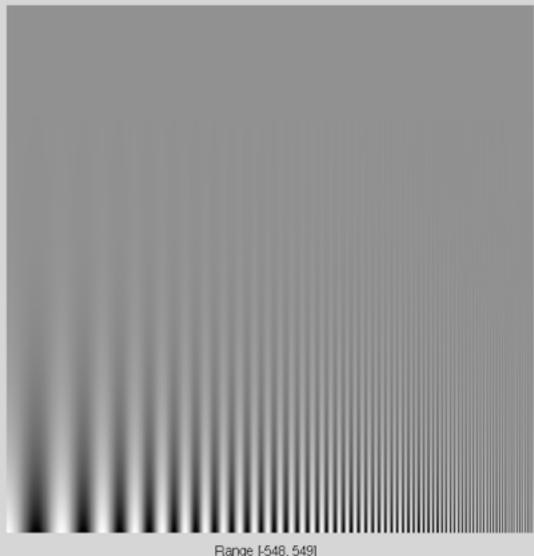




axis ...-

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						Figure 43
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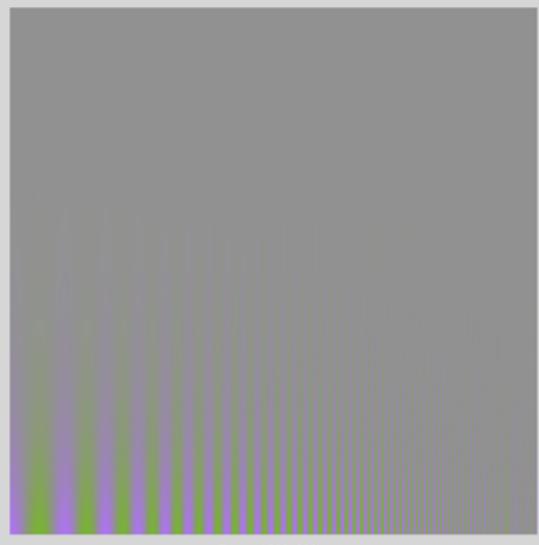
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						Figure 42
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Range [-605, 606] Dims (1000, 1000]

						Figure 44	
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Range [-932, 933] Dims [1000, 1000] Another psychophysical fact: luminance and chrominance channels in the brain

> From W. E. Glenn, in Digital Images and Human Vision, MIT Press, edited by Watson, 1993

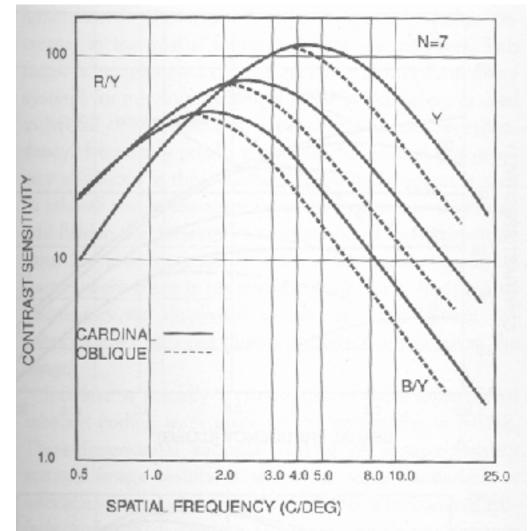


Figure 6.1

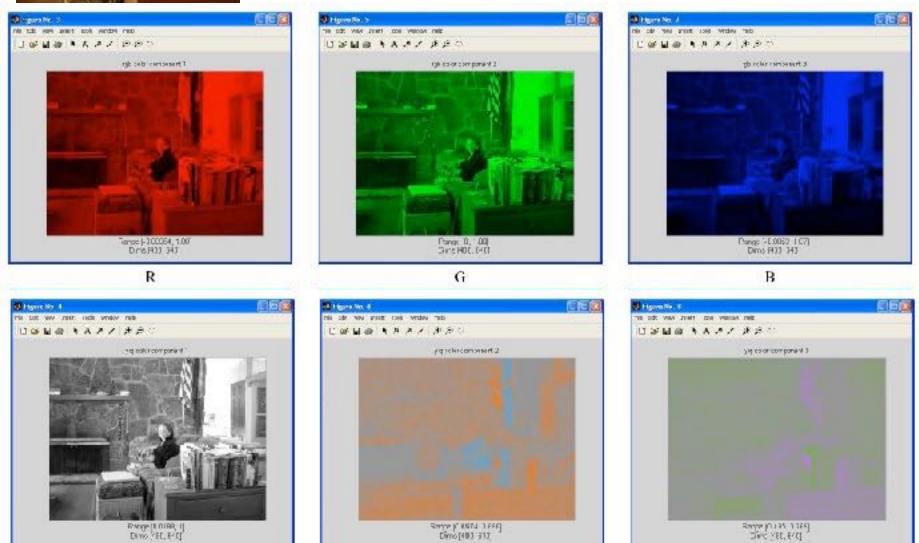
Contrast sensitivity threshold functions for static luminance gratings (Y) and isoluminance chromaticity gratings (R/Y,B/Y) averaged over seven observers.

NTSC color components: Y, I, Q

$\begin{pmatrix} Y \\ I \\ Q \end{pmatrix} = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$



NTSC - RGB



Spatial resolution and color



original



R

G

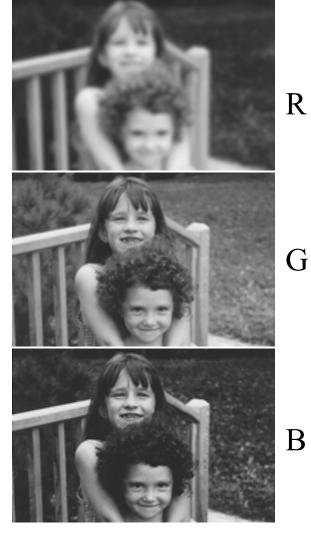
В

Blurring the R component

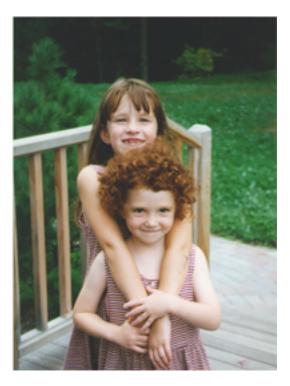


processed





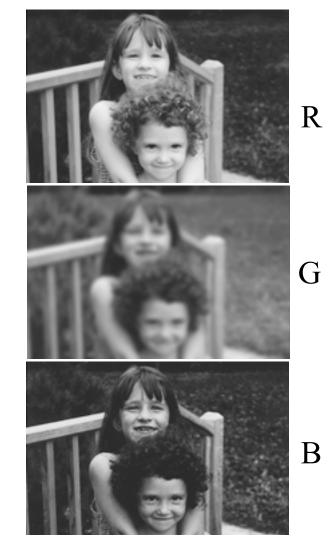
Blurring the G component





original

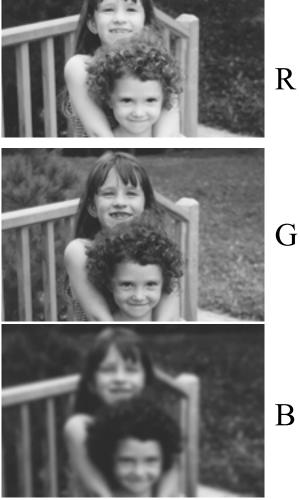
processed



Blurring the B component







original

В

From W. E. Glenn, in Digital Images and Human Vision, MIT Press, edited by Watson, 1993

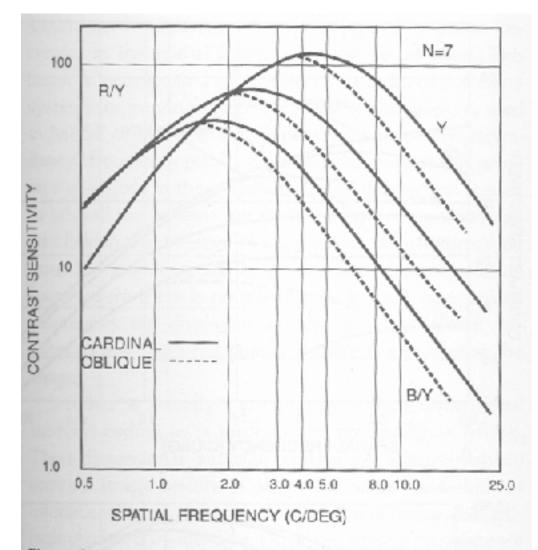


Figure 6.1

Contrast sensitivity threshold functions for static luminance gratings (Y) and isoluminance chromaticity gratings (R/Y,B/Y) averaged over seven observers.

Lab color components





L

a

b

A rotation of the color
coordinates into directions that
are more
perceptually
meaningful:
L: luminance,
a: red-green,
b: blue-yellow

Blurring the L Lab component



original



processed



a

b

L

Blurring the a Lab component



original



processed

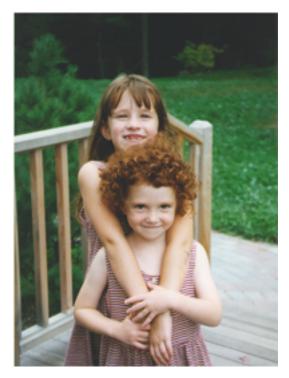




b

L

Blurring the b Lab component



original



processed

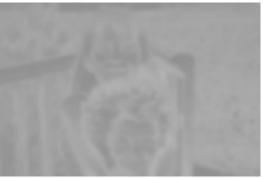


L

a

b





Lecture outline

- Color physics.
- Color perception
 - part 1: assume perceived color only depends on light spectrum.
 - part 2: the more general case.

Color constancy demo

• We assumed that the spectrum impinging on your eye determines the object color. That's often true, but not always. Here's a counter-example...

Low-dimensional models for color spectra

$$\begin{pmatrix} \vdots \\ a(\lambda) \\ \vdots \end{pmatrix} \approx \begin{pmatrix} \vdots & \vdots & \vdots \\ a_1(\lambda) & a_2(\lambda) & a_3(\lambda) \\ \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

How to find a linear model for color spectra:

--form a matrix, D, of measured spectra, 1 spectrum per column.

--[u, s, v] = svd(D) satisfies D = u*s*v'

--the first n columns of u give the best (least-squares optimal) n-dimensional linear bases for the data, D:

 $D \approx u(:,1:n) * s(1:n,1:n) * v(1:n,:)'$

Macbeth Color Checker





My Macbeth Color Checker Tattoo

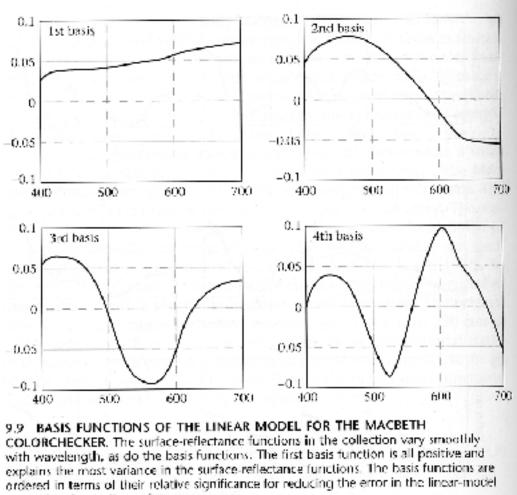
I think I have all the other color checker photos beat...

Yes, the tattoo is real. No, it is not a rubik's cube.

THIS PHOTOGRAPH IS COPYRIGHT 2007 THE X-RITE CORPORATION!

A photograph from this session can be viewed on the X-Rite Website: <u>www.xrite.com/</u> <u>top_munsell.aspx</u>

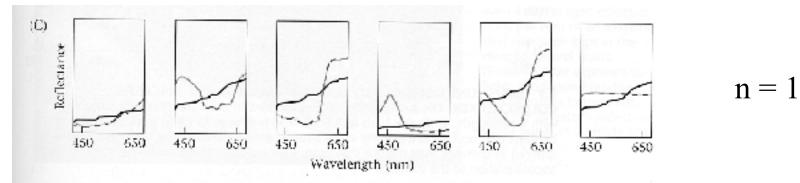
Basis functions for Macbeth color checker



approximation to the surfaces.

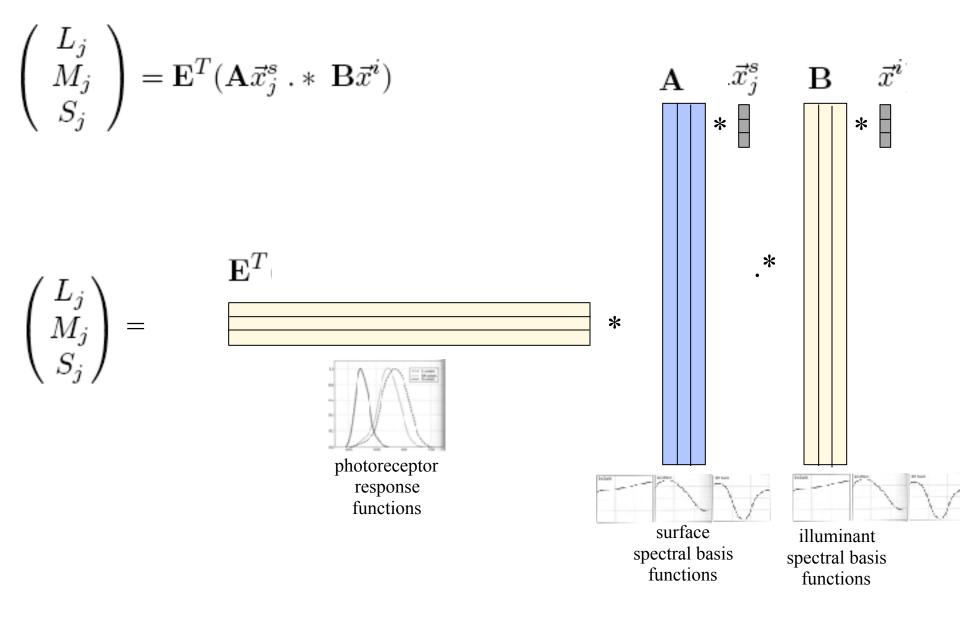
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Fitting color spectra with low-dimensional linear models



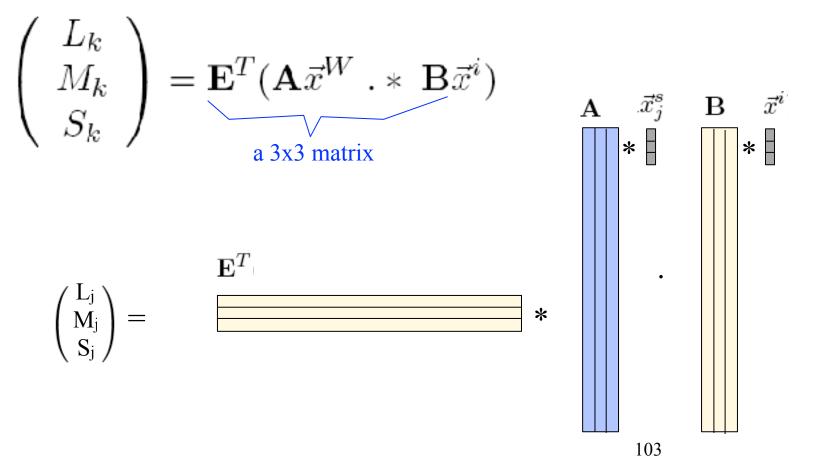
9.8 A LINEAR MODEL TO APPROXIMATE THE SURFACE REFLECTANCES IN THE MACBETH COLORCHECKER. The panels in each row of this figure show the surfacereflectance functions of six colored surfaces (shaded lines) and the approximation to these functions using a linear model (solid lines). The approximations using linear models with (A) three, (B) two, and (C) one dimension are shown. Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Rendering equation for jth observation



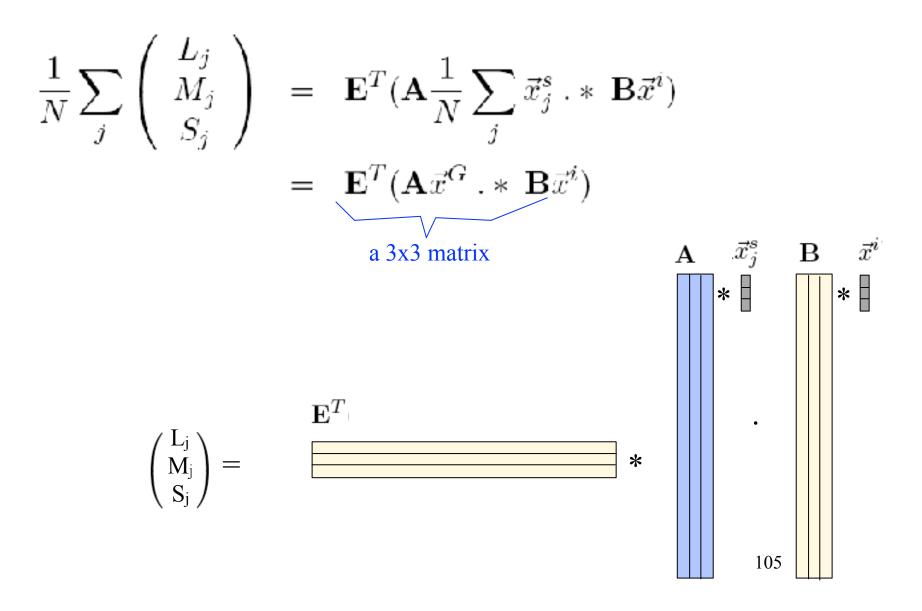
Color constancy solution 1: find white in the scene

Let the kth patch be the white one, with surface coefficients assumed to be \vec{x}^W . Then we can solve for the illuminant coefficient, \vec{x}^i





Color constancy solution 2: assume scene colors average to grey, then solve for the illuminant, \vec{x}^i

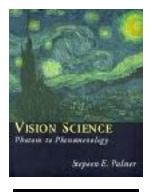


an image that violates both assumptions



TSb5RVWDM9Q/s1600/NATURE-GreenForest_1024x768.jpeg

Selected Bibliography



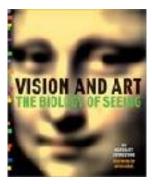
Vision Science

by Stephen E. Palmer MIT Press; ISBN: 0262161834 760 pages (May 7, 1999)



Billmeyer and Saltzman's Principles of Color Technology, 3rd Edition

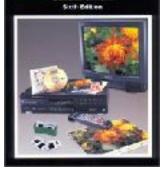
by Roy S. Berns, Fred W. Billmeyer, Max Saltzman Wiley-Interscience; ISBN: 047119459X 304 pages 3 edition (March 31, 2000)



Vision and Art : The Biology of Seeing

by Margaret Livingstone, David H. Hubel Harry N Abrams; ISBN: 0810904063 208 pages (May 2002)

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HE REPRODUCTION OF

Dr. R.W. G. Hunt

The Reproduction of Color

by R. W. G. Hunt Fountain Press, 1995



Color Appearance Models

by Mark Fairchild Addison Wesley, 1998

Other color references

- Reading:
 - Chapter 6, Forsyth & Ponce
 - Chapter 4 of Wandell, Foundations of Vision, Sinauer, 1995 has a good treatment of this.