MIT CSAIL

6.869 Advances in Computer Vision

Fall 2017

Problem Set 5

Posted: Thursday, October 12, 2017 **Due:** Thursday 23:59, October 19, 2017

Please submit **two files:** 1) a **PDF** report named ⟨your_kerberos⟩.pdf, including your answers to all required questions with images and/or plots showing your results, and 2) a file named ⟨your_kerberos⟩.zip, containing relevant source code.

Late Submission Policy: We do not accept late submissions. The submission deadline has a 50-minute soft cut-off; after midnight Thursday, submissions are penalized 2% per minute late.

Problem 1 *Wiener Filter in Frequency Domain*

In this problem you will derive the Wiener Filter in frequency domain.

Assume we have a very simple image formation model: A perfect discrete image $X[k, l]$ was taken with a imperfect approach, which may introduce degradation of blurring and noise. We aim to find a filter G so that given the non-perfect measurement $Y[k, l]$, convolving G and Y will give you the best estimate of X in a least square sense.

To be specific, we assume $Y = H \star X + N$ ($\star$ as convolution), where N is additive white Gaussian noise with variance σ^2, and H is some **known** blurring kernel. Suppose our estimate is $\hat{X}$, which is given by $G \star Y$, we wish to derive the best filter G by minimizing the reconstruction error:

$$\epsilon^2 = E(|\hat{X}_F[u, v] - X_F[u, v]|^2) = \sum_{u,v} |\hat{X}_F[u, v] - X_F[u, v]|^2,$$

where $\hat{X}_F$ and X_F are the discrete Fourier transform of $\hat{X}$ and X; $E()$ denotes for taking expectation, since X and N are considered random signals.

Assuming the noise is completely uncorrelated with the input image, that is:

$$N_F^*[u, v]X_F[u, v] = X_F^*[u, v]N_F[u, v] = 0,$$

where * means taking conjugate.

To derive the frequency representation $G_F[u, v]$ of G that minimizes ϵ^2, you can suppose the energy spectral density of X, defined as:

$$P_{XX}[u, v] = |X_F[u, v]|^2,$$

and energy spectral density of H, defined as:

$$P_{HH}[u, v] = |H_F[u, v]|^2,$$

is known.

The best estimate G_F is given by $\frac{d\epsilon^2}{dG_F} = 0$. Since $\frac{d(z^* z)}{dz}, z \in \mathbb{C}$ is not well defined, such minimization should be done for the real and complex part of G_F respectively. That is, your final result should look like:

$$G_F^{Real}[u, v] = \textit{your expression}$$

$$G_F^{Complex}[u, v] = \textit{your expression}$$

Hint: for additive Gaussian white noise N, we have:

$$|N_F[u, v]|^2 = \sigma^2$$

Problem 2 *Eigenfaces*

In this problem, your goal is to compute a "face basis" for the Olivetti face dataset. Load the included face database given by `faces.mat`; the resulting Matlab variable is called `faces`. Compute the eigenvectors of the covariance matrix of these faces. These so-called "eigenfaces" form a basis for the set of faces. Show the top five basis images in your report (what do we mean by 'top'? Hint: PCA). Describe in your report how you chose the top basis images, and why. Give the coefficients of the first image in the dataset in terms of the top 20 basis images. Try to reconstruct the image from these coefficients, and include the result in your report.

References