Lecture 2
Image formation

bring:
gumby, big flash light, apertures, straw camera, one paper bag camera
lens, holder, laser pointer, corner camera pieces of paper, tape measure (for lens focal length).
Imaging

• Forming images with pinholes and edges
• Forming images with lenses
• More general imaging devices
Light striking a surface

- Bidirectional reflectance distribution function
  
  \[ I_{\text{out}} = F(I_{\text{in}}, \hat{n}, \lambda, \hat{p}, \hat{q}) \]

- For a Lambertian surface:
  
  \[ I_{\text{out}} = F_L(I_{\text{in}}, \hat{n}, \hat{p}) = AI_{\text{in}}(\lambda) \cos(\hat{n} \cdot \hat{p}) \]
The structure of ambient light
The structure of ambient light
All light rays
Why don’t we generate an image when an object is in front of a white piece of paper?

Why is there no picture appearing on the paper?
Let’s check, do we get an image?
Let’s check, do we get an image? No
To make an image, we need to have only a subset of all the rays strike the sensor or surface.

The camera obscura
The pinhole camera
Let’s try putting different occluders in between
light on wall past pinhole

Light through hole A
grocery bag pinhole camera

1. grocery bag

2. Put newspaper inside bag to block light better

3. Insert white paper on which to show image

- Poke hole through grocery bags and newspapers
- Add 2nd grocery bag on top of newspapers
grocery bag pinhole camera

- Side with white paper
- Side with hole

Put grocery bags over your head, outside in sun, with a grown up watching you.
grocery bag pinhole camera

view from outside the bag

http://www.youtube.com/watch?v=FZyCFxsyx8o

view from inside the bag

http://youtu.be/-rhZaAM3F44
Perspective projection
Cartesian coordinates:

We have, by similar triangles, that

\( (x, y, z) \rightarrow (\frac{dx'}{z}, \frac{dy'}{z}, df) \)

Ignore the third coordinate, and get

\[
(x, y, z) \rightarrow (d \frac{x'}{z}, d \frac{y'}{z})
\]
Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image or half-planes.
- Polygons go to polygons
- Degenerate cases
  - line through focal point to point
  - plane through focal point to line

Figure from: Forsyth and Ponce.
Vanishing point for this 3-d line
Line in 3-space

\[ x(t) = x_0 + at \]
\[ y(t) = y_0 + bt \]
\[ z(t) = z_0 + ct \]

Perspective projection of that line

\[ x'(t) = \frac{fx}{z} = \frac{f(x_0 + at)}{z_0 + ct} \]
\[ y'(t) = \frac{fy}{z} = \frac{f(y_0 + bt)}{z_0 + ct} \]

In the limit as \( t \to \pm \infty \) we have (for \( c \neq 0 \)):

\[ x'(t) \to \frac{fa}{c} \]
\[ y'(t) \to \frac{fb}{c} \]

This tells us that any set of parallel lines (same \( a, b, c \) parameters) project to the same point (called the vanishing point).
Vanishing points

• Each set of parallel lines (=direction) meets at a different point
  – The vanishing point for this direction

• Sets of parallel lines on the same plane lead to collinear vanishing points.
  – The line is called the horizon for that plane
http://www.ider.herts.ac.uk/school/courseware/graphics/two_point_perspective.html
What if you photograph a brick wall head-on?
Brick wall line in 3-space
\[ x(t) = x_0 + at \]
\[ y(t) = y_0 \]
\[ z(t) = z_0 \]

Perspective projection of that line
\[ x'(t) = \frac{f \cdot (x_0 + at)}{z_0} \]
\[ y'(t) = \frac{f \cdot y_0}{z_0} \]

All bricks have same \( z_0 \). Those in same row have same \( y_0 \)

Thus, a brick wall, photographed head-on, gets rendered as set of parallel lines in the image plane.
Straw camera
Straw camera
Other projection models:
Orthographic projection

\[(x, y, z) \rightarrow (x, y)\]
Other projection models: Weak perspective

- **Issue**
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: only approximate

\[
(x, y, z) \rightarrow \left(\frac{fx}{z_0}, \frac{fy}{z_0}\right)
\]
Three camera projections

(1) Perspective: \((x, y, z) \rightarrow \left(\frac{fx}{z}, \frac{fy}{z}\right)\)

(2) Weak perspective: \((x, y, z) \rightarrow \left(\frac{fx}{z_0}, \frac{fy}{z_0}\right)\)

(3) Orthographic: \((x, y, z) \rightarrow (x, y)\)
which is perspective, which orthographic?

Perspective projection

Parallel (orthographic) projection
Problem Set 2

http://www.foundphotography.com/PhotoThoughts/archives/2005/04/pinhole_camera_2.html
Example images from pinhole camera
Measuring distance

- Object size decreases with distance to the pinhole
- There, given a single projection, if we know the size of the object we can know how far it is.
- But for objects of unknown size, the 3D information seems to be lost.
Playing with pinholes
Two pinholes
Two pinholes

What is the minimal distance between the two projected images?
Anaglyph pinhole camera
Anaglyph pinhole camera

front of camera

image of a point of light
Anaglyph pinhole camera
Synthesis of new views

Anaglyph
Problem set

- Build the device
- Take some pictures and put them in the report
- Take anaglyph images
- Work out the geometry
- Recover depth for some points in the image
A problem: pinhole camera images are dark, or require long exposures.
Large aperture gives a brighter image, but at the price of sharpness.
A lens allows a large aperture and a sharp image
Let’s try putting different occluders in between the scene and the sensor plane.
Influence of aperture size: with a small aperture, the image is sharp, but dim. A large aperture gives a bright, but blurry image.
A lens can focus light from one point in the world to one point on the sensor plane.
Images through large aperture, with and without lens present
Images through large aperture, with and without lens present.
Light at a material interface

$\alpha_1$  

indices of refraction

$n_1$  

$n_2$
Light at a material interface

\[
\lambda_1 = \frac{c}{\omega n_1}\quad \text{wavelength is speed/ freq}
\]

\[
\lambda_1 = L \sin(\alpha_1)\quad \text{from the geometry at right}
\]

\[
n_1 \sin(\alpha_1) = \frac{c}{\omega L} = n_2 \sin(\alpha_2)
\]

rearranging the first two equations

\[
n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)
\]

indices of refraction

Speed, and thus wavelength of light, scales inversely with \( n \). This requires that plane waves bend, according to

\[
n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)
\]
Refraction: Snell’s law

\[ n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2) \]

For small angles, \( n_1 \alpha_1 \approx n_2 \alpha_2 \)
Spherical lens
For a spherical lens surface, we can define the relevant angles, apply Snell’s law, and find an expression telling how the lens focusses light.

\[ \theta_1, \theta_2, \beta_1, \beta_2, \alpha_1, \alpha_2 \]

\[ P, P_1, P_2, C \]

\[ d_1, d_2, R, h, \gamma \]

Forsyth and Ponce
That is easiest to do under the assumptions of “first order optics”: small bending angles, and a thin lens

$$\sin(\theta) \approx \theta$$
Paraxial refraction equation

\[ \alpha_1 = \gamma + \beta_1 \approx h \left( \frac{1}{R} + \frac{1}{d_1} \right) \]

Relates distance from sphere and sphere radius to bending angles of light ray, for lens with one spherical surface.

\[ n_1 \alpha_1 \approx n_2 \alpha_2 \]
Deriving the lensmaker’s formula

\[ \alpha_1 = h \left( \frac{1}{R} + \frac{1}{d'_1} \right) \quad \text{small angle approx.} \]

\[ n_1 \alpha_1 \approx n_2 \alpha_2 \quad \text{Snell's law} \]

\[ \alpha_2 = 2 \gamma - \alpha_3 \quad \text{geometry} \]

\[ n_2 \alpha_3 \approx n_1 \alpha_4 \quad \text{Snell's law} \]

\[ \alpha_4 = h \left( \frac{1}{R} + \frac{1}{d'_2} \right) \quad \text{small angle} \]

\[ \gamma = \frac{h}{R} \]

\[ n_1 \alpha_1 = n_2 \left( \frac{2h}{R} - \frac{n_1}{n_2} \alpha_4 \right) = h \left( \frac{1}{R} + \frac{1}{d'_1} \right) \]

Let \( n_1 = 1 \), \( n_2 = n \)

\[ n \left( \frac{2}{R} - \frac{1}{n} \left( \frac{1}{R} + \frac{1}{d'_2} \right) \right) = \frac{1}{R} + \frac{1}{d'_1} \]

\[ \frac{2n}{R} - \frac{1}{R} - \frac{1}{d'_2} = \frac{1}{R} + \frac{1}{d'_1} \]

\[ \frac{2(n-1)}{R} = \frac{1}{d'_1} + \frac{1}{d'_2} \]

“Lens maker’s formula”
The thin lens, first order optics

The lensmaker’s equation:

\[ \frac{1}{z'} + \frac{1}{z} = \frac{1}{f} \]

\[ f = \frac{R}{2(n-1)} \]
How does the mapping from the 3-d world to the image plane compare for a lens and for a pinhole camera?

The perspective projection of a pinhole camera. But note that many more of the rays leaving from P arrive at P'.

[Diagram showing the perspective projection with labeled points P, P', F, F', O, z, z', y, and f.]
Lens demonstration

- Verify:
  - Focusing property
  - Lens maker’s equation \((f = 25 \text{ inches})\)
  - The relationship between distances in the world and distances in the sensor plane
more general cameras
general imagers
(from imaging chapter of course notes)

\[ \hat{y} = A \hat{x} \] (1.9)

For the case of conventional cameras, where the observed intensities, \( \hat{y} \) are an image of the reflected intensities in the scene, \( \hat{x} \), then \( A \) is approximately an identity matrix.

For more general cameras, \( A \) may be very different from an identity matrix, and we will need to estimate \( \hat{x} \) from \( \hat{y} \). In the presence of noise, there may not be a solution \( \hat{x} \) that exactly satisfies Eq. (1.9), so we often seek to satisfy it in a least squares sense. In most cases, \( A \) is either not invertable, or is poorly conditioned. It is often useful to introduce a regularizer, an additional term in the objective function to be minimized. If the regularization term favors small \( \hat{x} \), then the objective term to minimize, \( E \), could be

\[ E = |\hat{y} - A \hat{x}|^2 + \lambda |\hat{x}|^2 \] (1.10)
general imagers
(from imaging chapter of course notes)

Setting the derivative of Eq. (1.10) with respect to the elements of the vector $\tilde{x}$ equal to zero, we have

$$0 = \nabla_{\tilde{x}}|y - A\tilde{x}|^2 + \nabla_{\tilde{x}}|\tilde{x}|^2$$  \hspace{1cm} (1.11)
$$= A^T A \tilde{x} - A^T y + \lambda \tilde{x}$$  \hspace{1cm} (1.12)

or

$$\tilde{x} = (A^T A + \lambda I)^{-1} A^T y$$  \hspace{1cm} (1.14)

See, e.g.: https://en.wikipedia.org/wiki/Matrix_calculus
system matrix, $A$, for pinhole imager

(a) Schematic drawing of a small-hole 1-d pinhole camera. (b) Visualization of imaging matrices: The imaging matrix relating scene intensities to sensor readings; the inverse of that matrix; the regularized inverse. For the small-pinhole imager, all three matrices are approximately identity matrices.
system matrix, $A$, for large aperture pinhole imager

(a)

Figure 1.9
(a) Large-hole 1-d pinhole camera. (b) Visualization of imaging matrices: The imaging matrix relating scene intensities to sensor readings; the inverse of that matrix; the regularized inverse. For the small-pinhole imager, all three matrices are approximately identity matrices.
system matrix, $A$, for large aperture pinhole imager
Another occlusion-based camera: edge camera

show intensity demo with cards
Corner Camera 1-D Image Computation

Images you multiply the rectified ground plane images by to recover the input image around the corner (projected to 1d), for each different angle.
Experiment Proof of Concept
Experimental Proof of Concept
Experimental Proof of Concept
Experimental Proof of Concept
Video Corresponding to 1-D Camera
1-D Corner Camera Output

- How many people?
- Where slowed down, where moved quickly?
1-D Corner Camera Output

- How many people?
- How fast is each person moving?
More Corner Camera Videos

1 Person Walking in Circles
1 Person Randomly Walking
2 People Walking in Circles

Distribution Statement
Additional Results

Paper ID: 1983
Summary

• Want to make images
• Pinhole camera models the geometry of perspective projection
• Lenses make it work in practice
• Models for lenses
  – Thin lens, spherical surfaces, first order optics
• Cameras as linear systems to invert