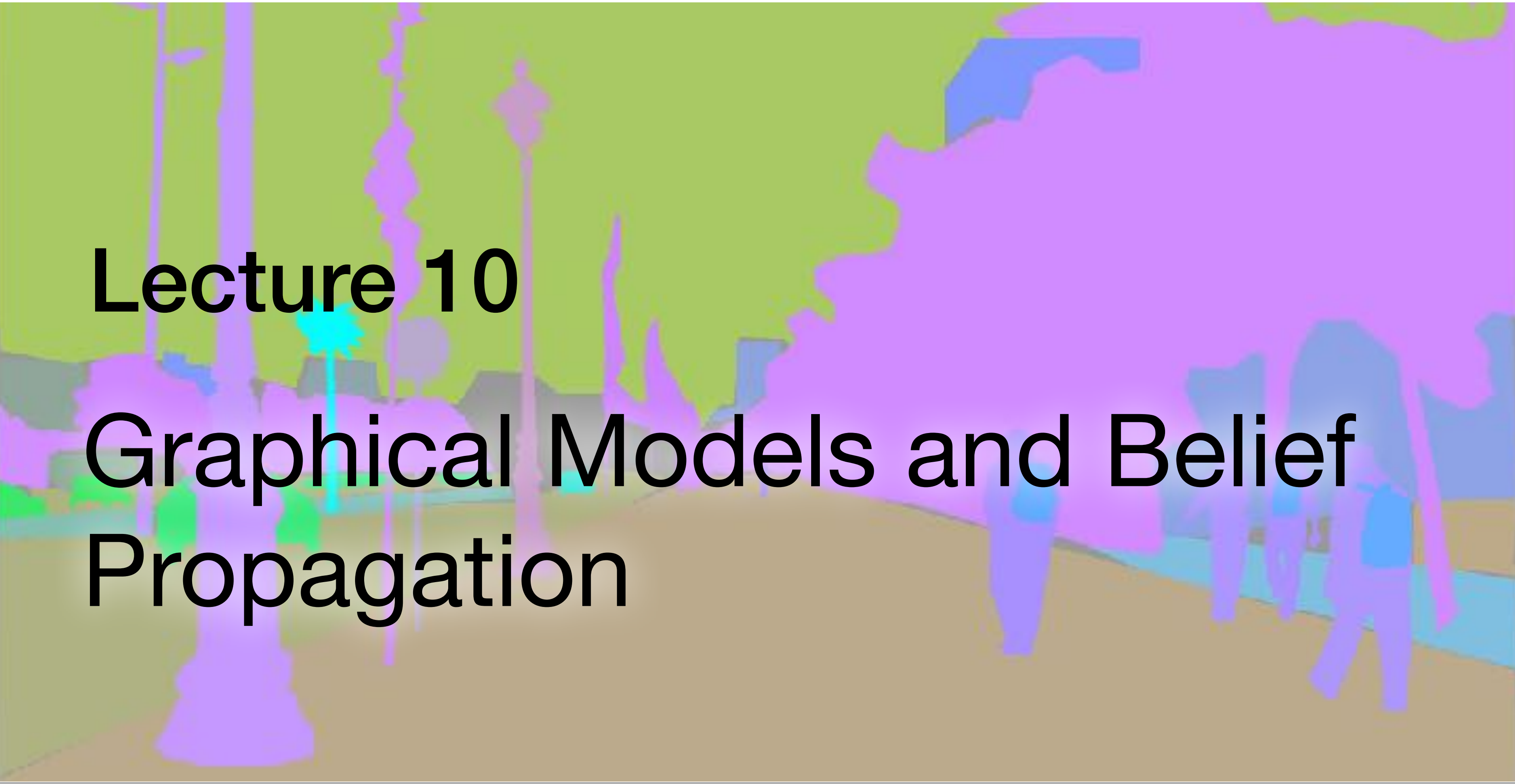


Lecture 10

Graphical Models and Belief Propagation

6.869/6.819 Advances in Computer Vision

Fall 2019

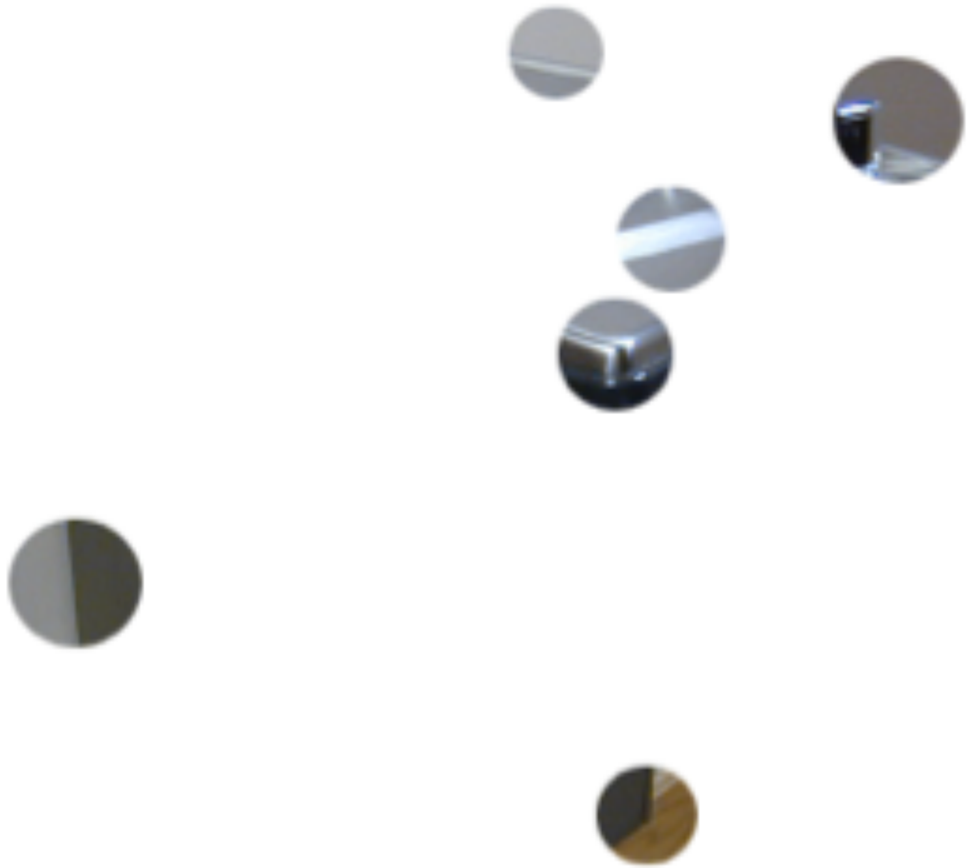
Bill Freeman, Antonio Torralba, Phillip Isola

October 8, 2019

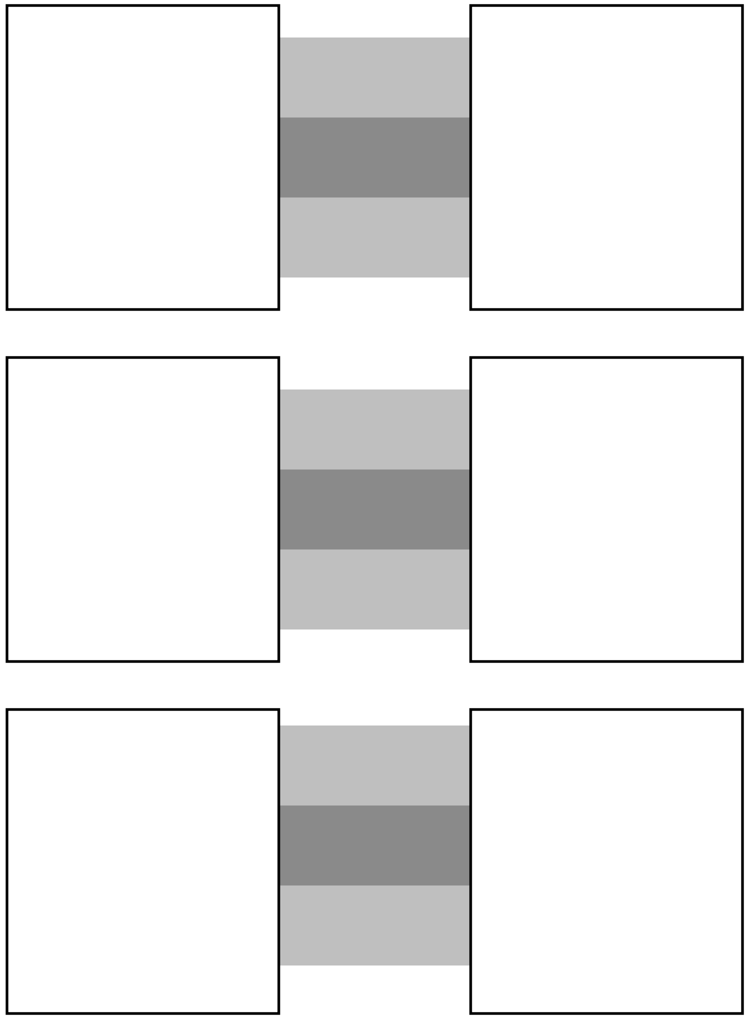
Lecture 10 Oct. 8, 2019

Belief Propagation and Graphical Models

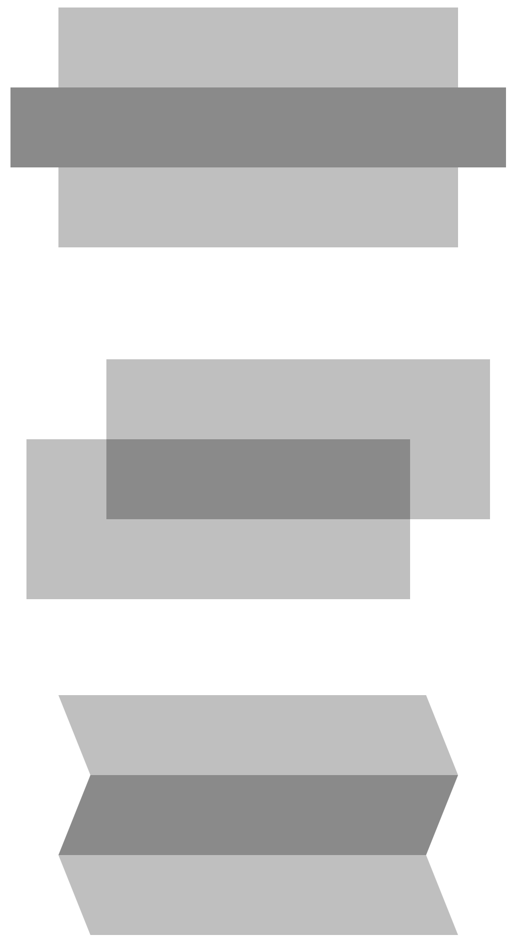
Only the first 10 slides will be presented in class; the rest are just included for reference. Most of the class will be on the blackboard.



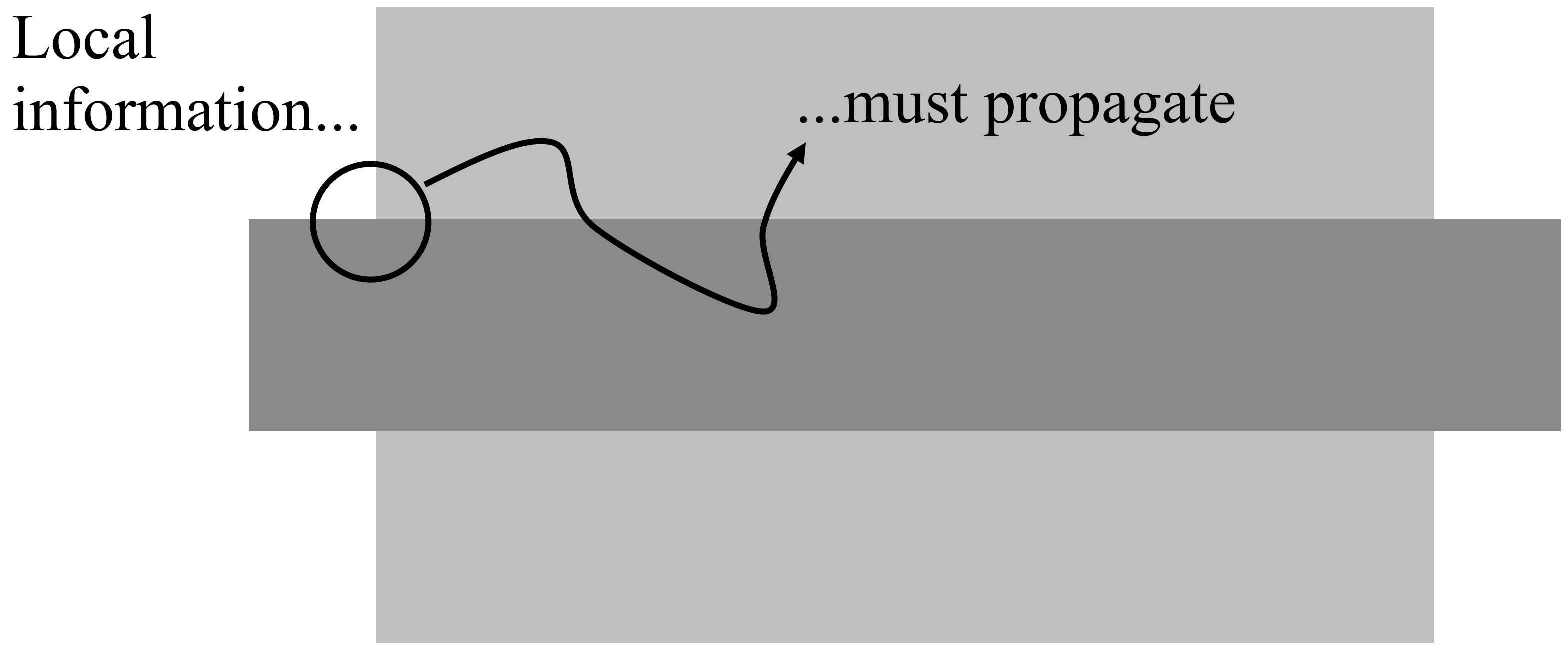
Identical local evidence...



…different interpretations



Information must propagate over the image.



Probabilistic graphical models are a powerful tool for propagating information within an image. And these tools are used everywhere within computer vision now.

http://www.cvpapers.com/cvpr2014.html

From a random sample of 6 papers from CVPR 2014, half had figures that look like this...

Partial Optimality by Pruning for MAP-inference with General Graphical Models, Swoboda et al

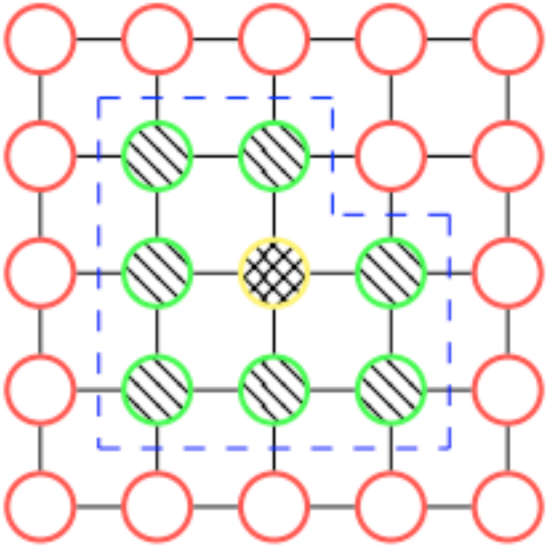


Figure 1. An exemplary graph containing inside nodes (yellow with crosshatch pattern) and boundary nodes (green with diagonal pattern). The blue dashed line encloses the set A. Boundary edges are those crossed by the dashed line.

http://hci.iwr.uni-heidelberg.de/Staff/bsavchyn/papers/swoboda-GraphicalModelsPersistency-with-Supplement-cvpr2014.pdf

Active flattening of curved document images via two structured beams, Meng et al.

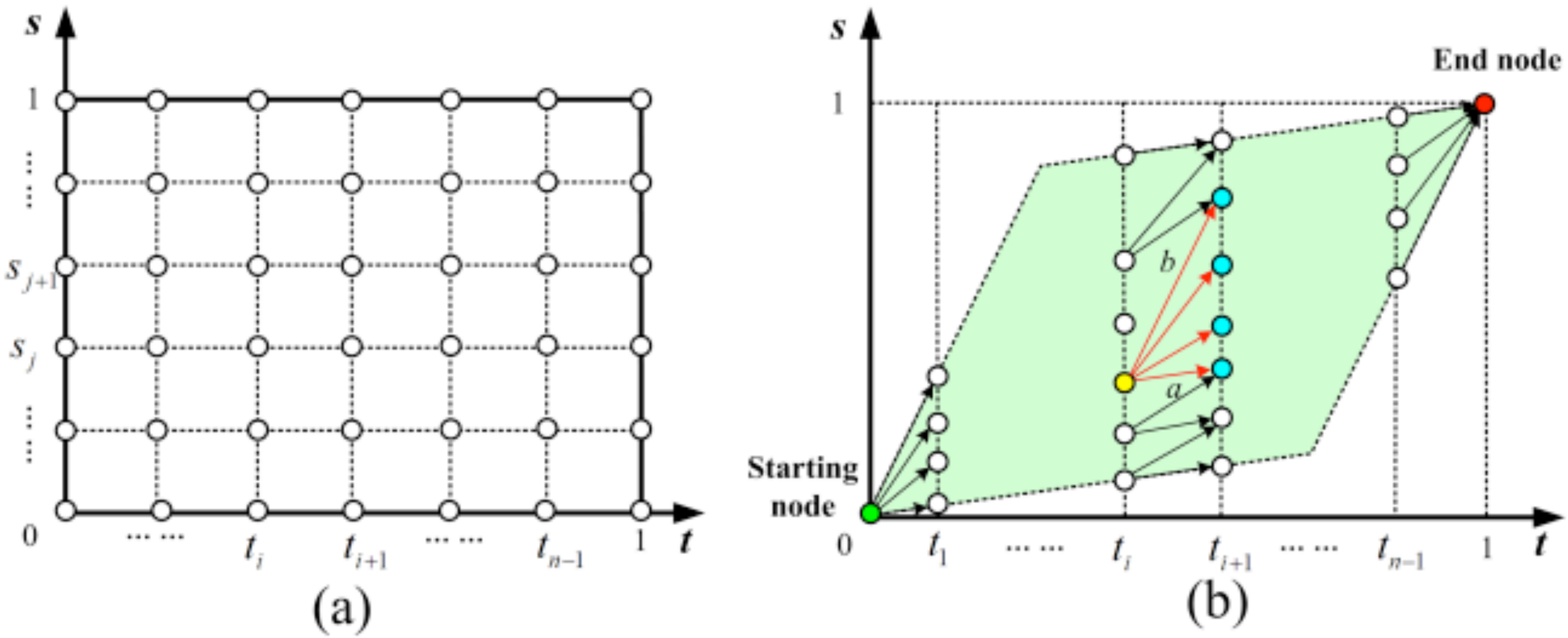


Figure 5. The directed graph G for computing the correspondence function. (a) discretization of the $t-s$ plane, (b) the constructed graph. All the vertices of the graph locate in a parallelogram. The slops of its edges are a and b, respectively.

file:///Users/billf/Downloads/dewarp_high.pdf

A Mixture of Manhattan Frames: Beyond the Manhattan World, Straub et al

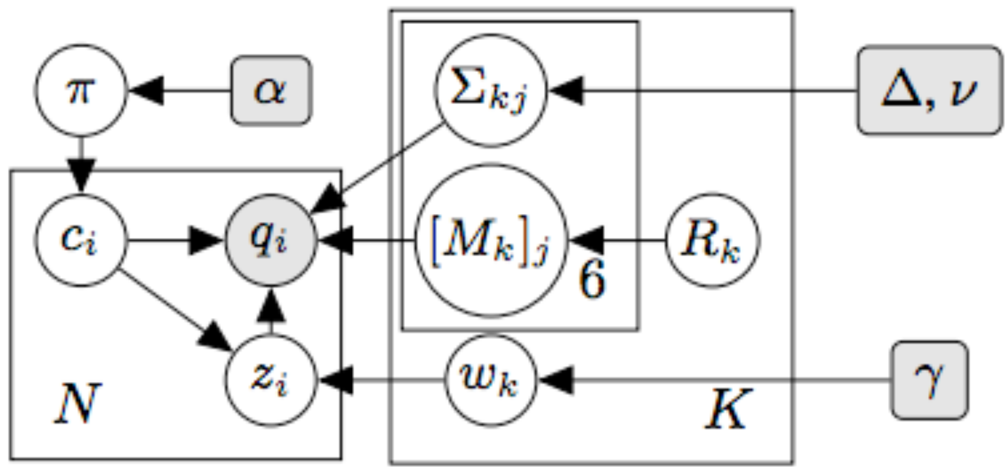
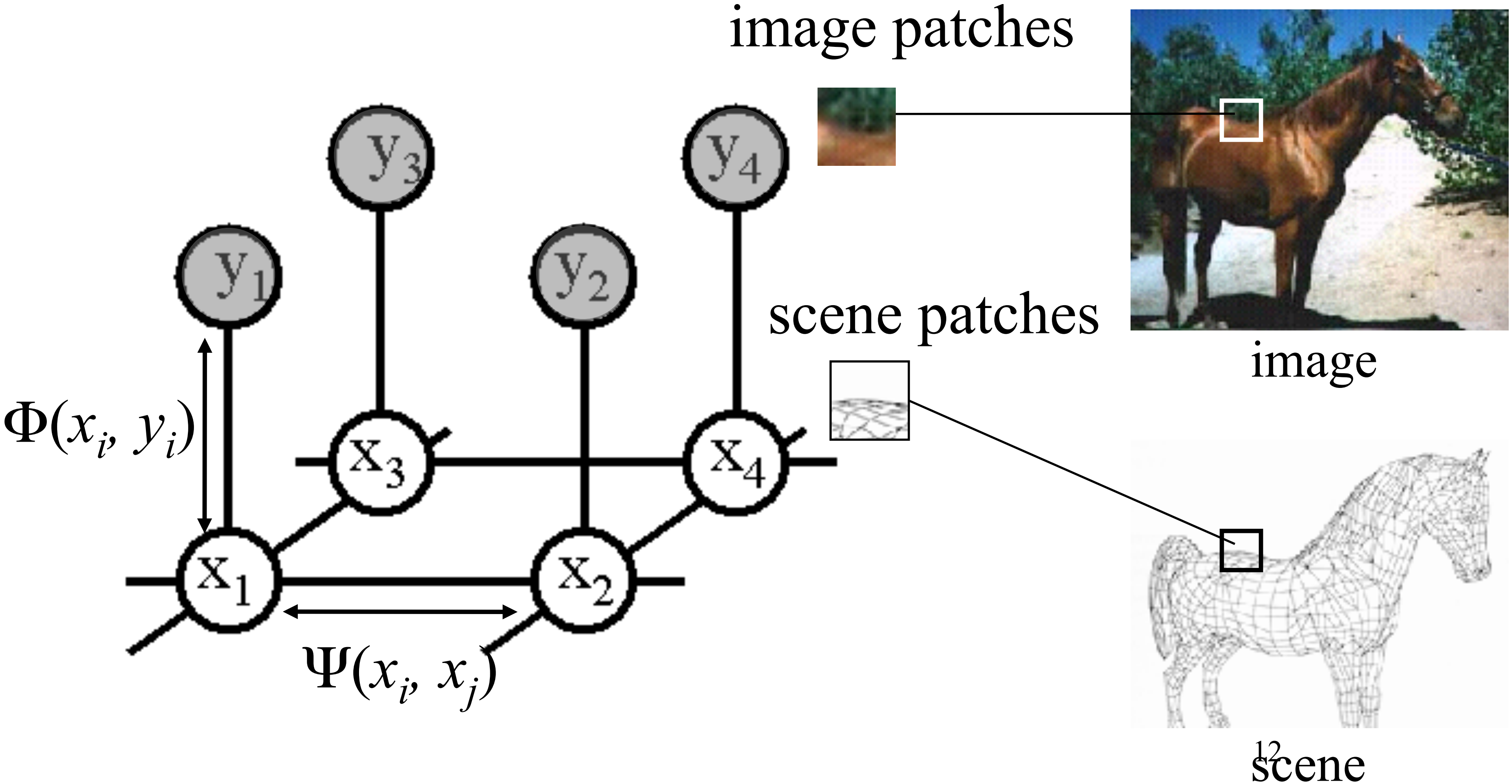


Figure 3: Graphical model for a mixture of K MFs.

http://www.jstraub.de/download/straub2014mmf.pdf

MRF nodes as patches



Network joint probability

$$P(x, y) = \frac{1}{Z} \prod_{i,j} \Psi(\underbrace{x_i, x_j}_{\text{neighboring scene nodes}}) \prod_i \Phi(x_i, y_i)$$

scene

image

Scene-scene
compatibility
function

Image-scene
compatibility
function

local
observations

Energy formulation

$$\mathrm{E}(x,y) = k + \sum_{(i,j)} \beta\underbrace{(x_i, x_j)}_{\text{neighboring scene nodes}} + \sum \alpha(x_i, y_i)$$

scene

image

Scene-scene compatibility function

neighboring scene nodes

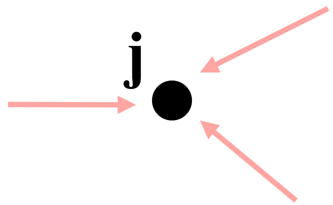
Image-scene compatibility function

local observations

Outline of MRF section

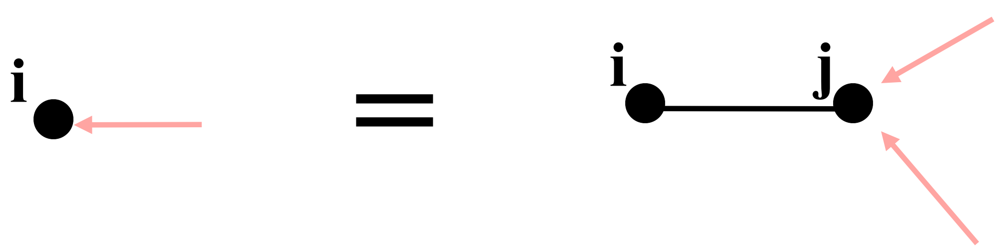
- Inference in MRF's.
  - Gibbs sampling, simulated annealing
  - Iterated conditional modes (ICM)
  - Loopy belief propagation
    - Application example—super-resolution
  - Graph cuts
  - Variational methods
- Learning MRF parameters.
  - Iterative proportional fitting (IPF)

Belief, and message update rules are just local operations, and can be run whether or not the network has loops



$$b_j(x_j) = \prod_{k \in N(j)} M_j^k(x_j)$$

$$M_i^j(x_i) = \sum_{x_j} \psi_{\mathrm{ij}}(x_i, x_j) \prod_{k \in N(j) \backslash i} M_j^k(x_j)$$



Justification for running belief propagation in networks with loops

- Experimental results:
  - Comparison of methods Szeliski et al. 2008 http://vision.middlebury.edu/MRF/
  - Error-correcting codes Kschischang and Frey, 1998; McEliece et al., 1998
  - Vision applications Freeman and Pasztor, 1999; Frey, 2000
- Theoretical results:
  - For Gaussian processes, means are correct. Weiss and Freeman, 1999
  - Large neighborhood local maximum for MAP. Weiss and Freeman, 2000
  - Equivalent to Bethe approx. in statistical physics. Yedidia, Freeman, and Weiss, 2000
  - Tree-weighted reparameterization Wainwright, Willsky, Jaakkola, 2001

Show program comparing some methods on a simple MRF

testMRF.m

Outline of MRF section

- Inference in MRF's.
  - Gibbs sampling, simulated annealing
  - Iterated conditional modes (ICM)
  - Belief propagation
    - Application example—super-resolution
  - Graph cuts
  - Variational methods
- Learning MRF parameters.
  - Iterative proportional fitting (IPF)

Super-resolution

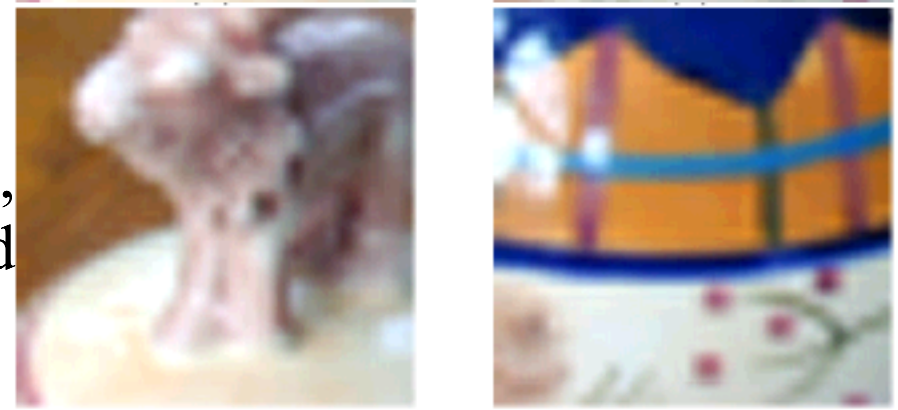
- Image: low resolution image
- Scene: high resolution image

ultimate goal...

Pixel-based images are not resolution independent

Pixel replication

Cubic spline, sharpened

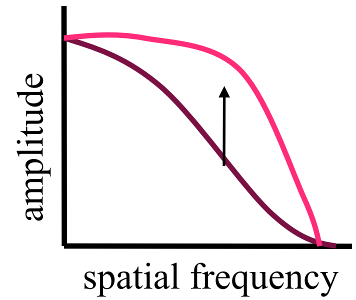


Training-based super-resolution

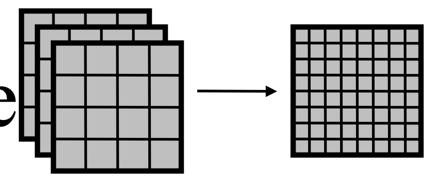
Polygon-based graphics images are resolution independent

3 approaches to perceptual sharpening

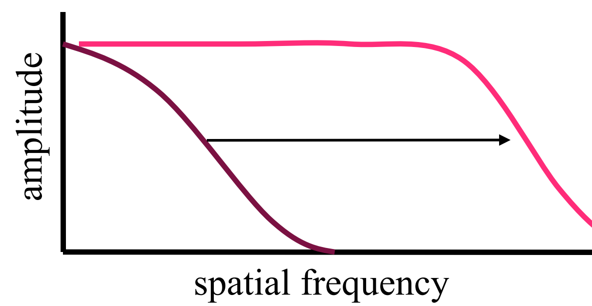
(1) Sharpening; boost existing high frequencies.



(2) Use multiple frames to obtain higher sampling rate in a still frame



(3) Estimate high frequencies not present in image, although implicitly defined.



In this talk, we focus on (3), which we'll call "super-resolution".

Super-resolution: other approaches

- Schultz and Stevenson, 1994
- Pentland and Horowitz, 1993
- fractal image compression (Polvere, 1998; Iterated Systems)
- astronomical image processing (eg. Gull and Daniell, 1978; “pixons” http://casswww.ucsd.edu/puetter.html)
- Follow-on: Jianchao Yang, John Wright, Thomas S. Huang, Yi Ma: Image super-resolution as sparse representation of raw image patches. CVPR 2008

Training images, **~100,000 image/scene patch pairs**

Images from two Corel database categories:
"giraffes" and "urban skyline".

Do a first interpolation

Zoomed low-resolution

Low-resolution

Zoomed low-resolution

Full frequency original

Low-resolution

Representation

Zoomed low-freq.

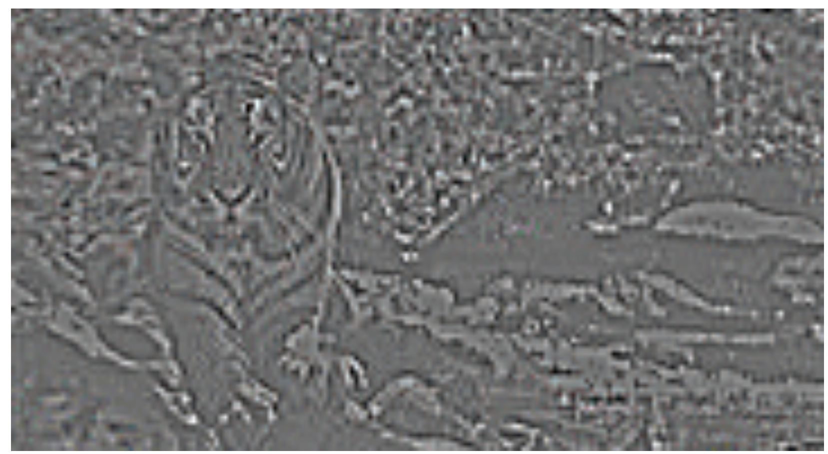
Full freq. original

Representation

Zoomed low-freq.

Full freq. original

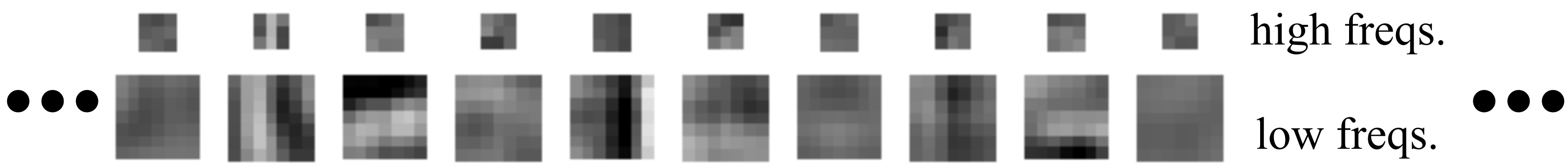
Low-band input
(contrast normalized,
PCA fitted)



True high freqs

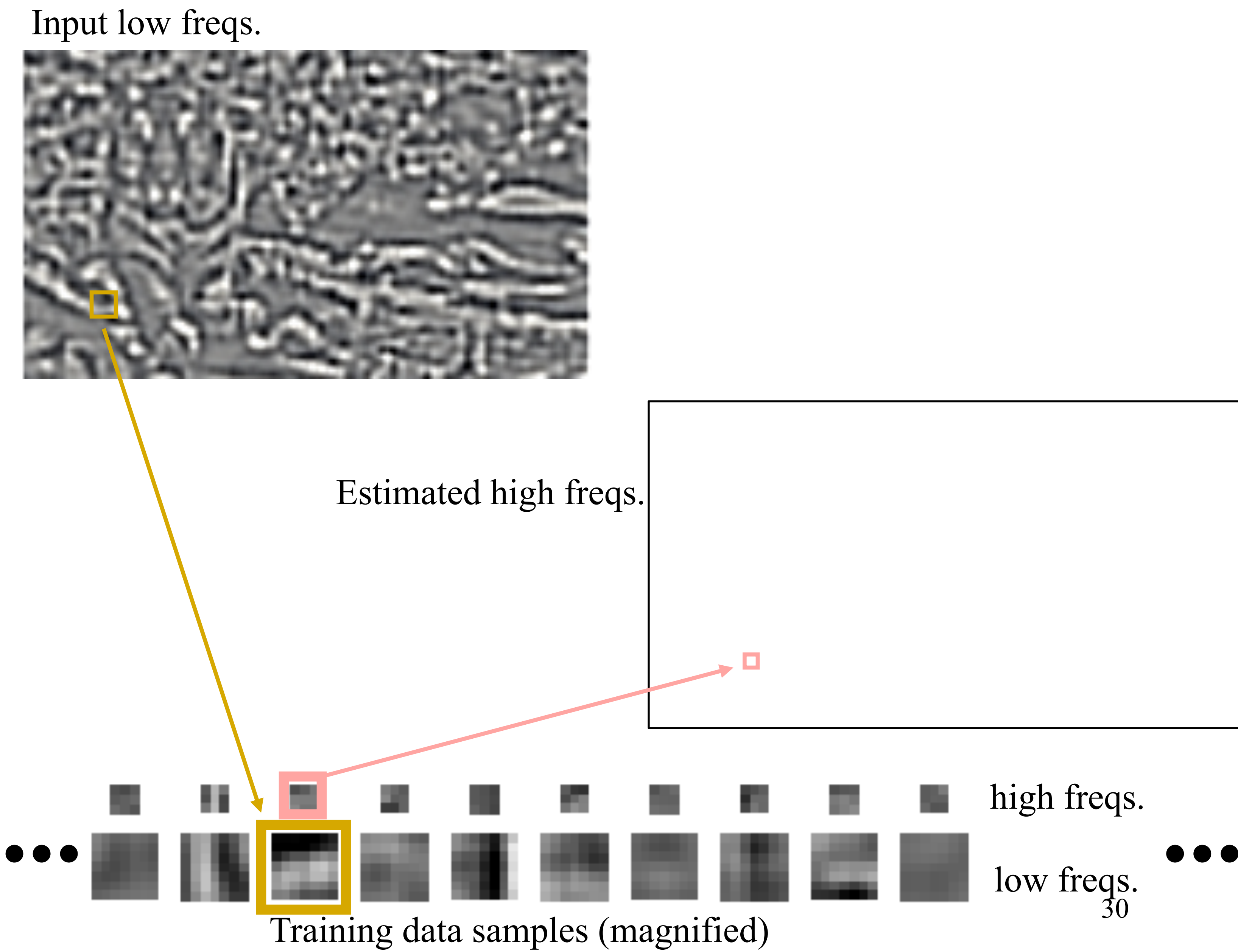
(to minimize the complexity of the relationships we have to learn,
we remove the lowest frequencies from the input image,
and normalize the local contrast level).

Gather ~100,000 patches

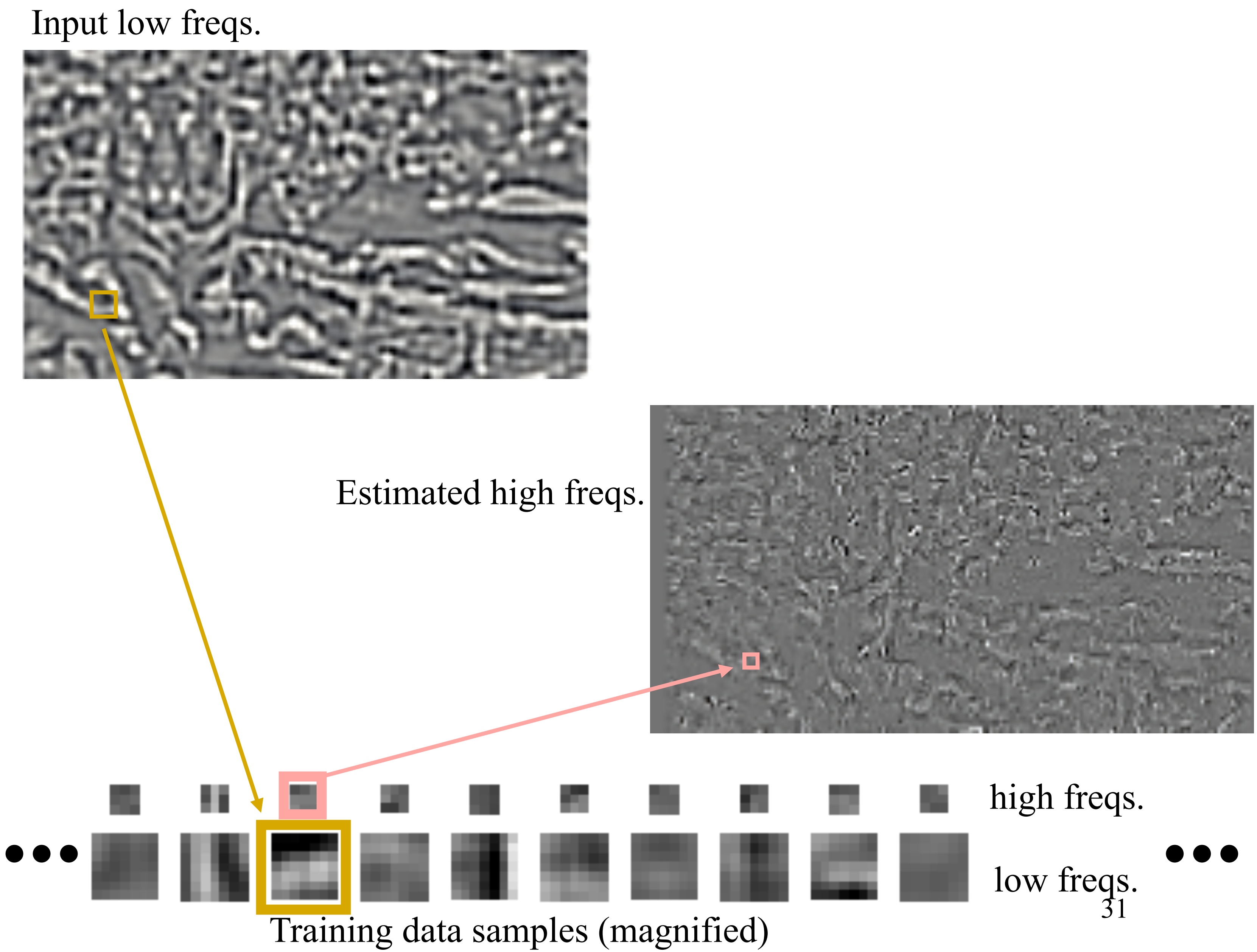


Training data samples (magnified)

Nearest neighbor estimate

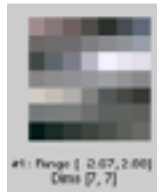


Nearest neighbor estimate

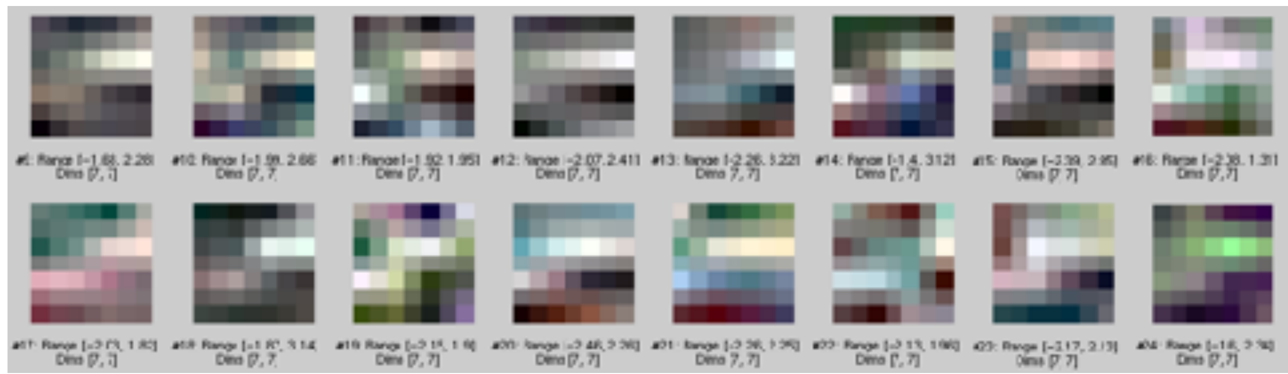


Example: input image patch, and closest matches from database

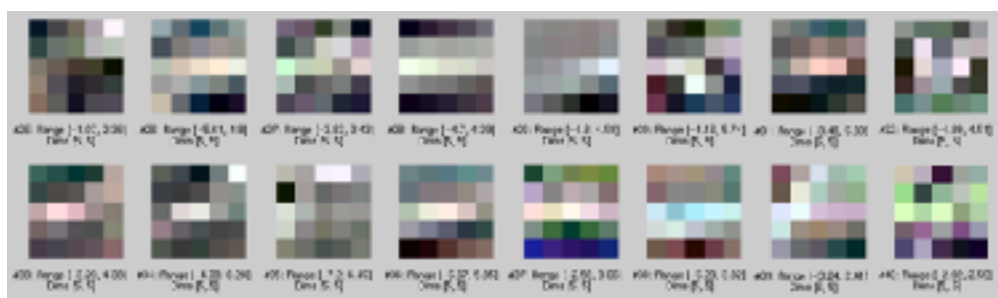
Input patch



Closest image patches from database



Corresponding high-resolution patches from database



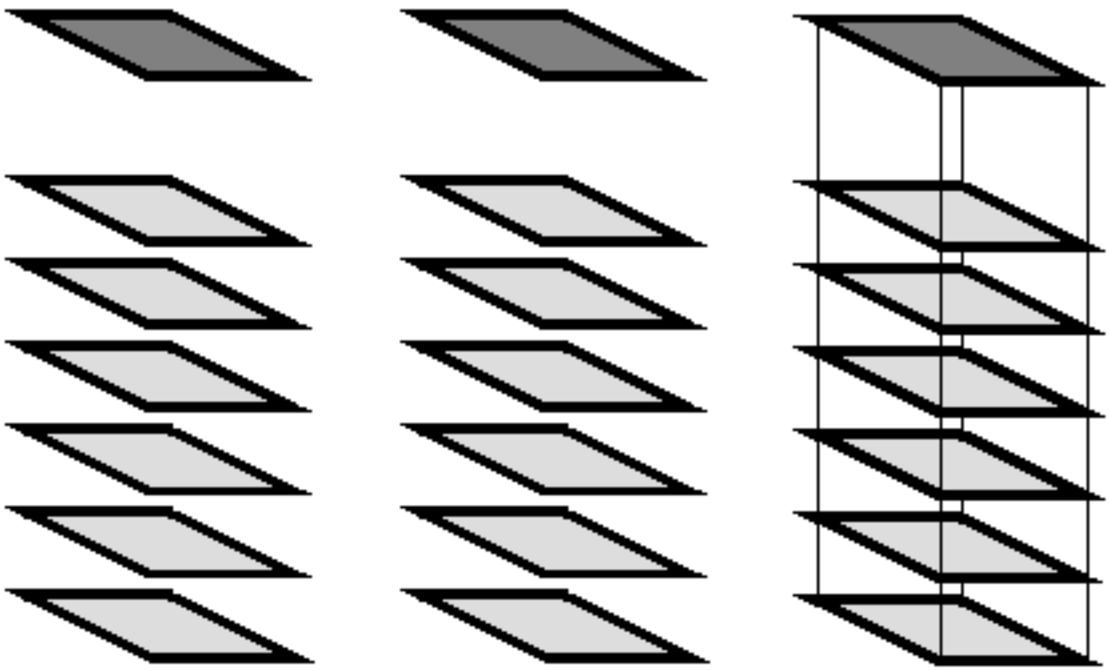
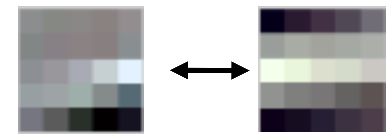


Image patch

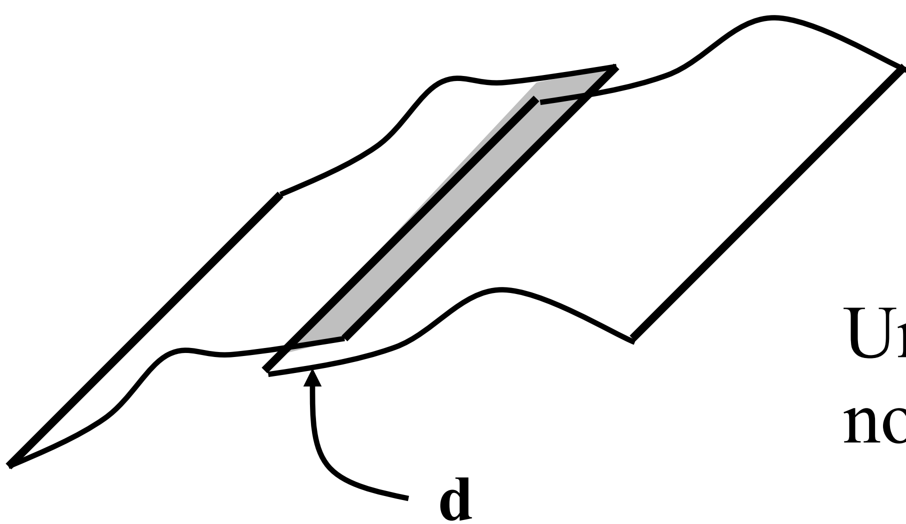
Underlying candidate scene patches. Each renders to the image patch.

Scene-scene compatibility function, $\Psi(x_i, x_j)$



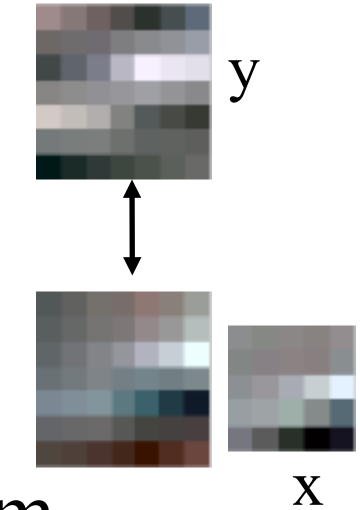
Assume overlapped regions, d, of hi-res. patches differ by Gaussian observation noise:

$$\Psi(x_i, x_j) = \exp^{-|d_i - d_j|^2 / 2\sigma^2}$$



Uniqueness constraint, not smoothness.

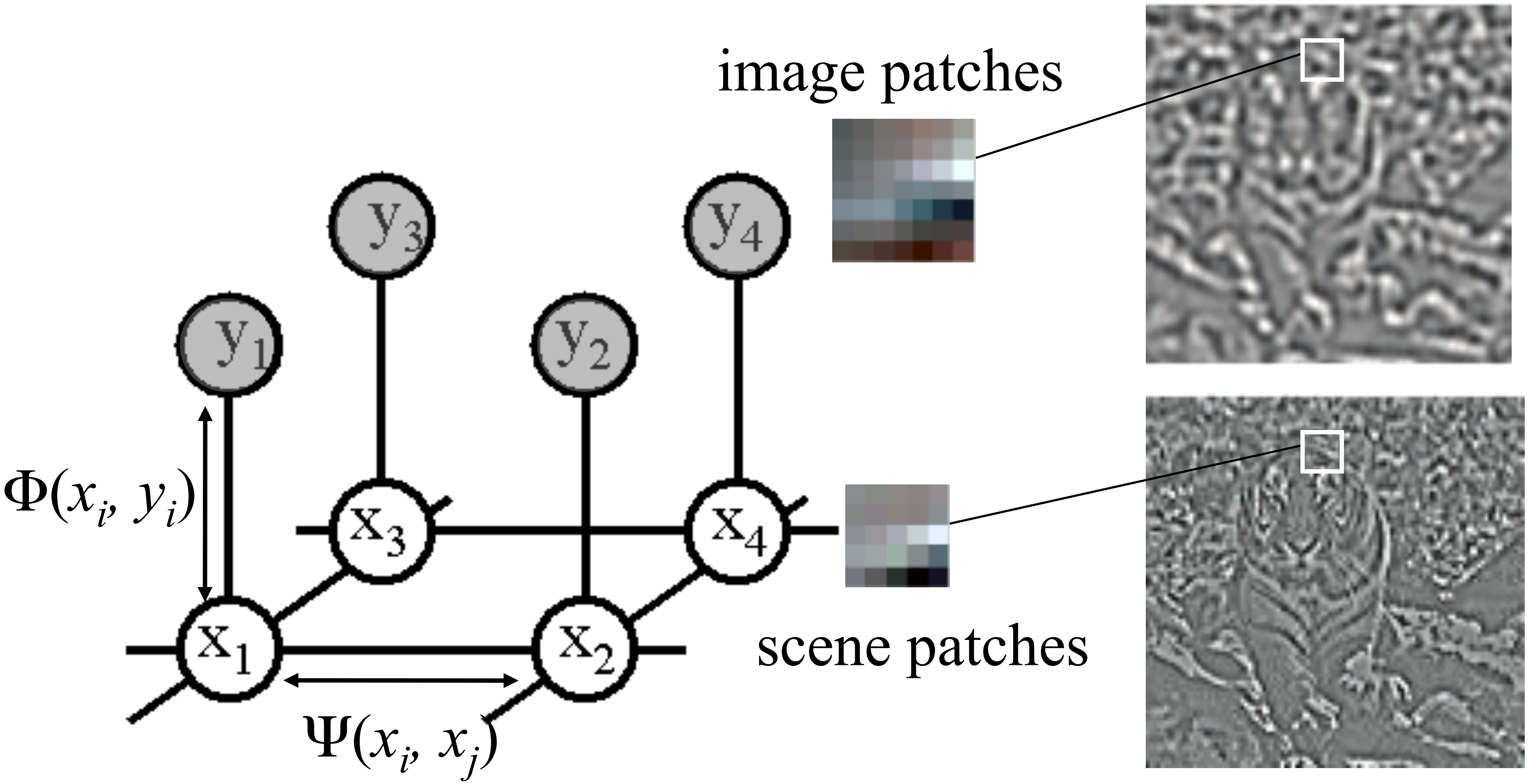
Image-scene compatibility function, $\Phi(x_i, y_i)$



Assume Gaussian noise takes you from observed image patch to synthetic sample:

$$\Phi(x_i, y_i) = \exp^{-|y_i - y(x_i)|^2 / 2\sigma^2}$$

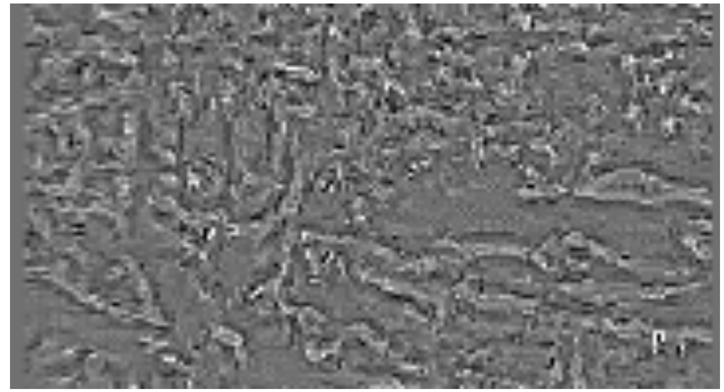
Markov network



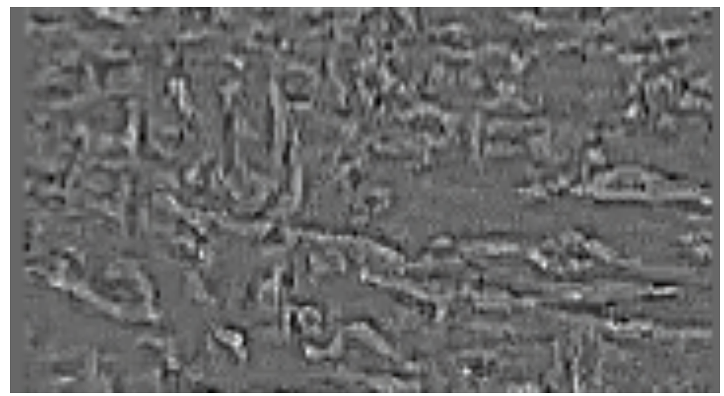
Belief Propagation

After a few iterations of belief propagation, the algorithm selects spatially consistent high resolution interpretations for each low-resolution patch of the input image.

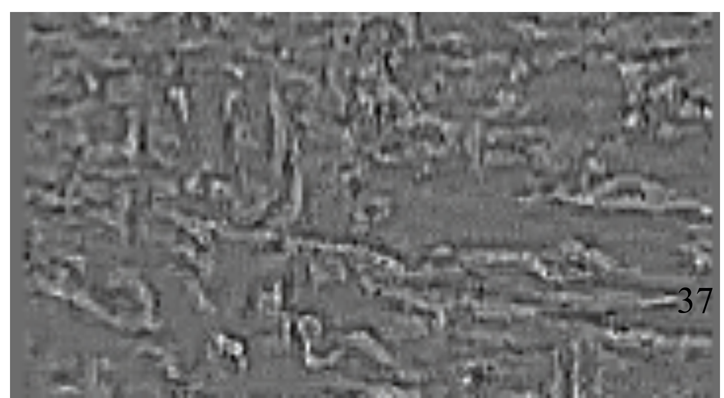
Input



Iter. 0



Iter. 1



Iter. 3

Zooming 2 octaves

We apply the super-resolution algorithm recursively, zooming up 2 powers of 2, or a factor of 4 in each dimension.

85 x 51 input

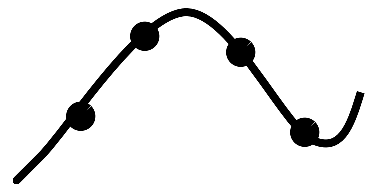
Cubic spline zoom to 340x204

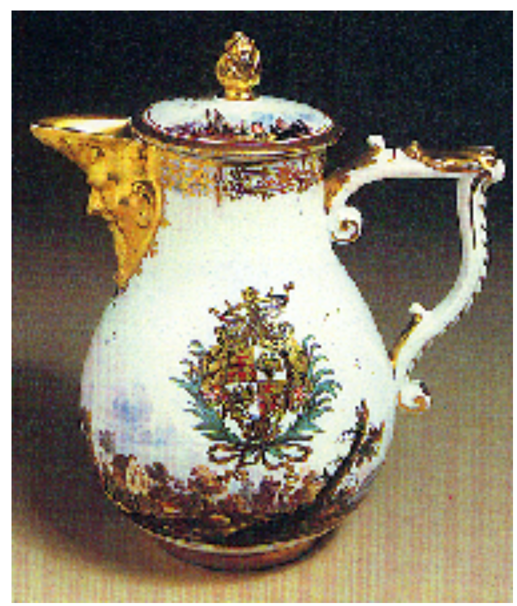
Max. likelihood zoom to 340x204

Now we examine the effect of the prior assumptions made about images on the high resolution reconstruction.
First, cubic spline interpolation.

Original
50x58

(cubic spline implies thin plate prior)

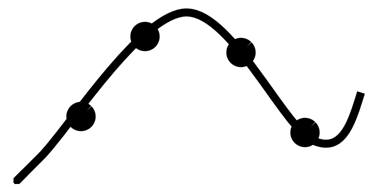




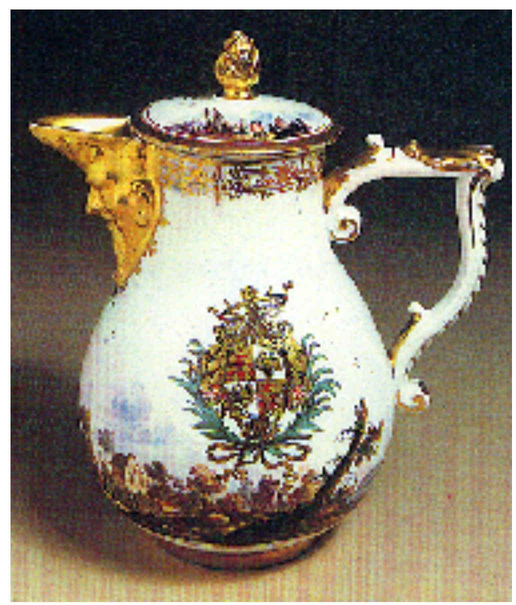
True
200x232

Original
50x58

(cubic spline implies
thin plate prior)



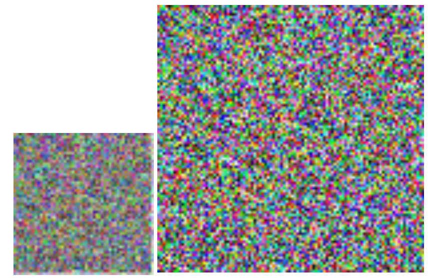
Cubic spline



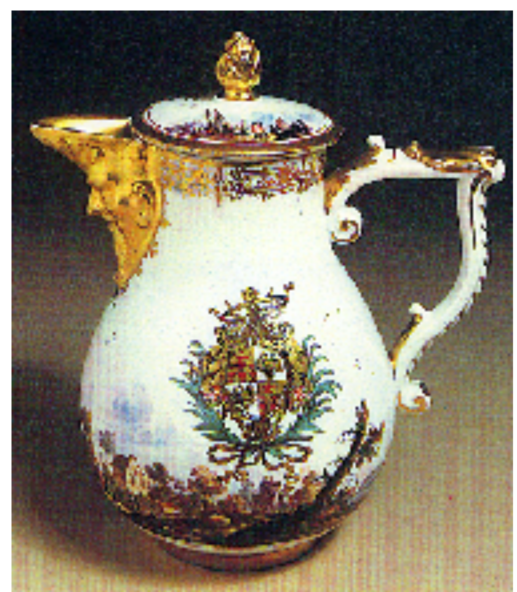
True
200x232

Next, train the Markov network algorithm on a world of random noise images.

Original 50x58



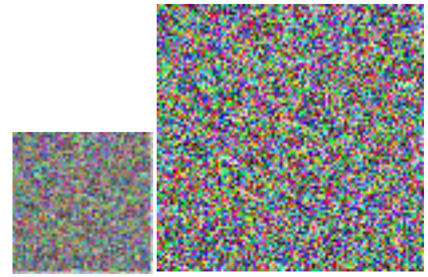
Training images



True

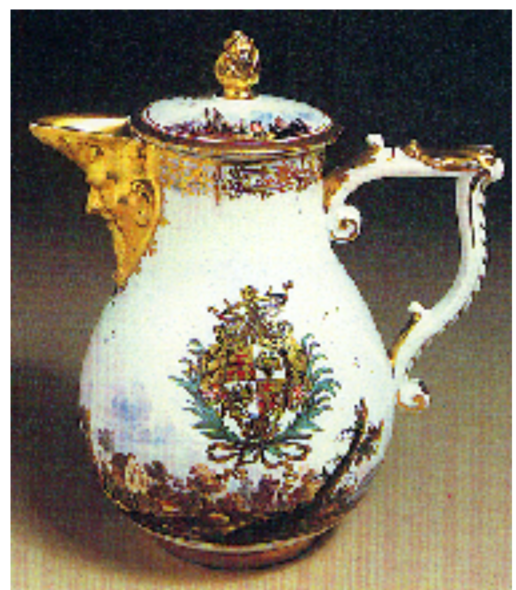
The algorithm learns that, in such a world, we add random noise when zoom to a higher resolution.

Original
50x58



Training images

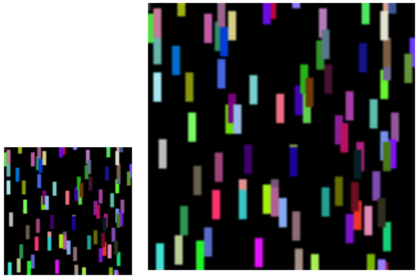
Markov
network



True

Next, train on a world of vertically oriented rectangles.

Original
50x58

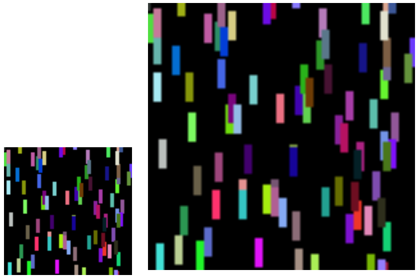


Training images

True

The Markov network algorithm hallucinates those vertical rectangles that it was trained on.

Original
50x58



Training images

Markov
network

True

Now train on a generic collection of images.

Original
50x58

Training images

True

The algorithm makes a reasonable guess at the high resolution image, based on its training images.

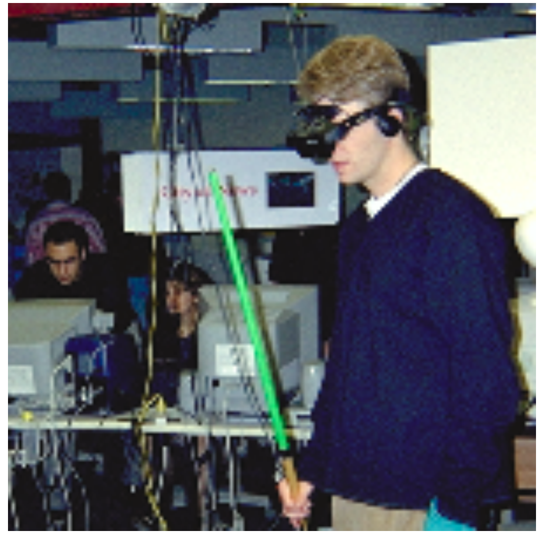
Original 50x58

Training images

Markov network

True

Generic training images



Next, train on a generic set of training images. Using the same camera as for the test image, but a random collection of photographs.

Original
70x70

Cubic
Spline

Markov
net,
training:
generic

True
280x280

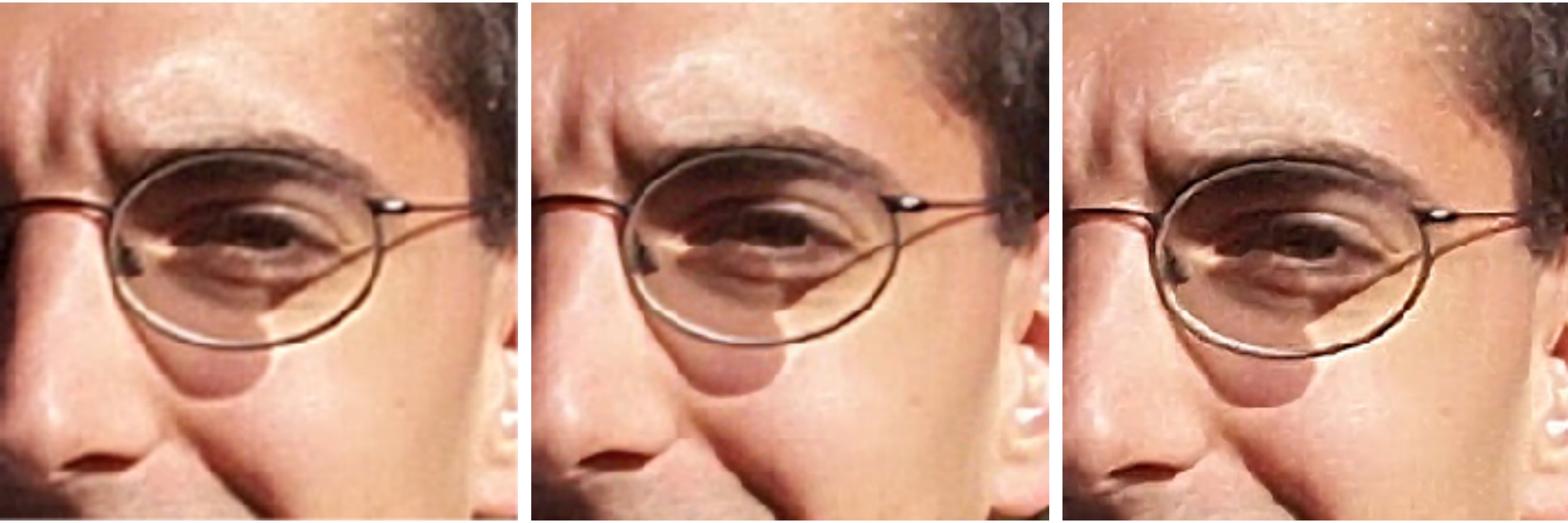
Kodak Imaging Science Technology Lab test.

3 test images, 640x480, to be zoomed up by 4 in each dimension.

8 judges, making 2-alternative, forced-choice comparisons.

Algorithms compared

- Bicubic Interpolation
- Mitra's Directional Filter
- Fuzzy Logic Filter
- Vector Quantization
- VISTA



Bicubic spline Altamira VISTA

Bicubic spline Altamira VISTA

User preference test results

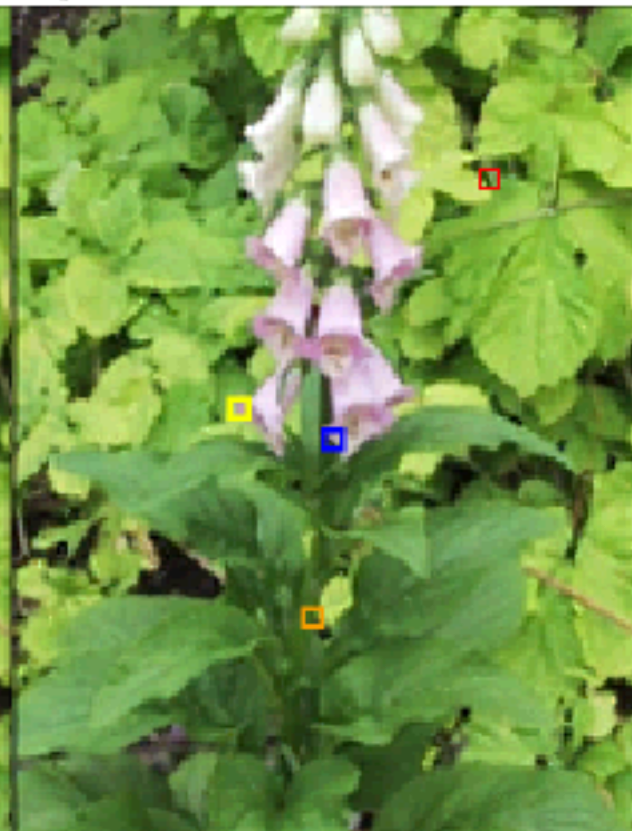
“The observer data indicates that six of the observers ranked Freeman’s algorithm as the most preferred of the five tested algorithms. However the other two observers rank Freeman’s algorithm as the least preferred of all the algorithms….

Freeman’s algorithm produces prints which are by far the sharpest out of the five algorithms. However, this sharpness comes at a price of artifacts (spurious detail that is not present in the original scene). Apparently the two observers who did not prefer Freeman’s algorithm had strong objections to the artifacts. The other observers apparently placed high priority on the high level of sharpness in the images created by Freeman’s algorithm.”

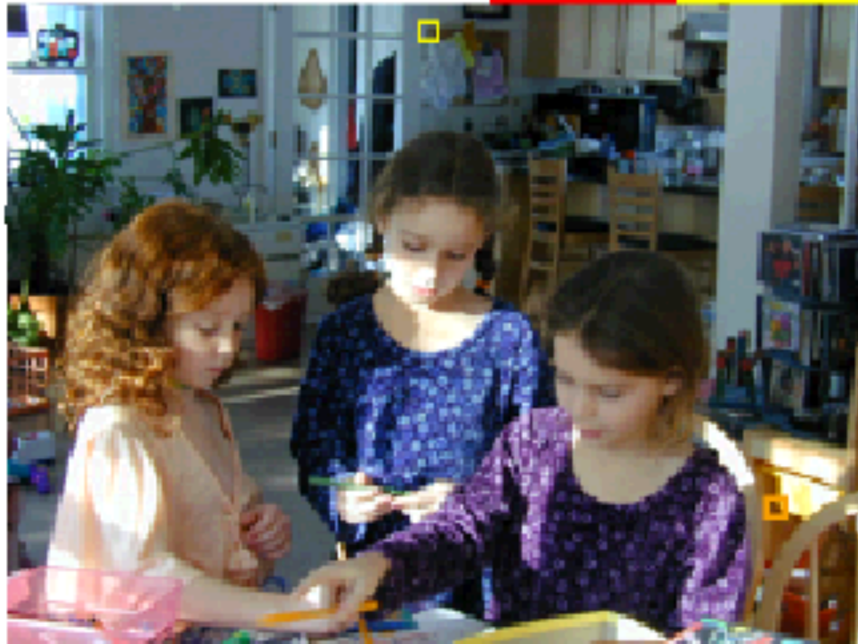
Input

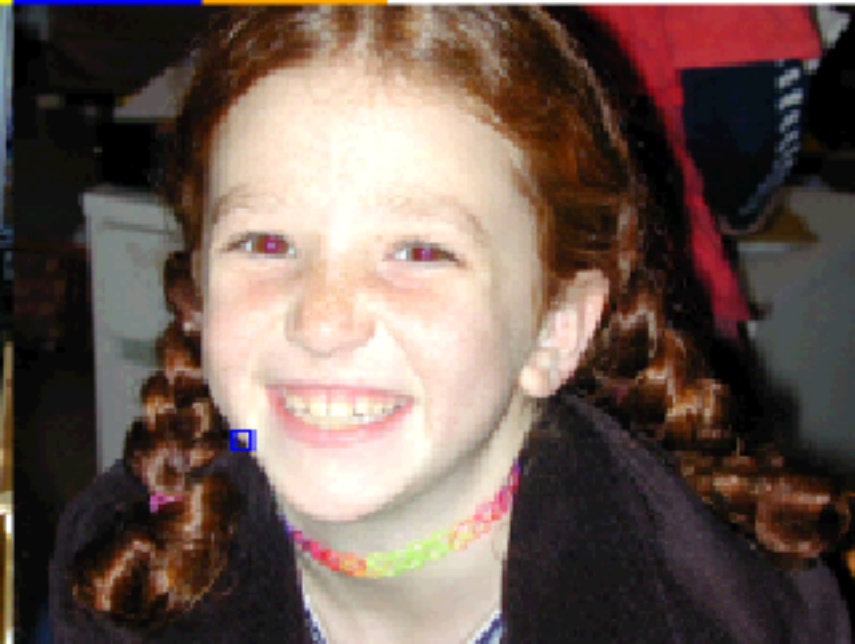
Cubic spline zoom

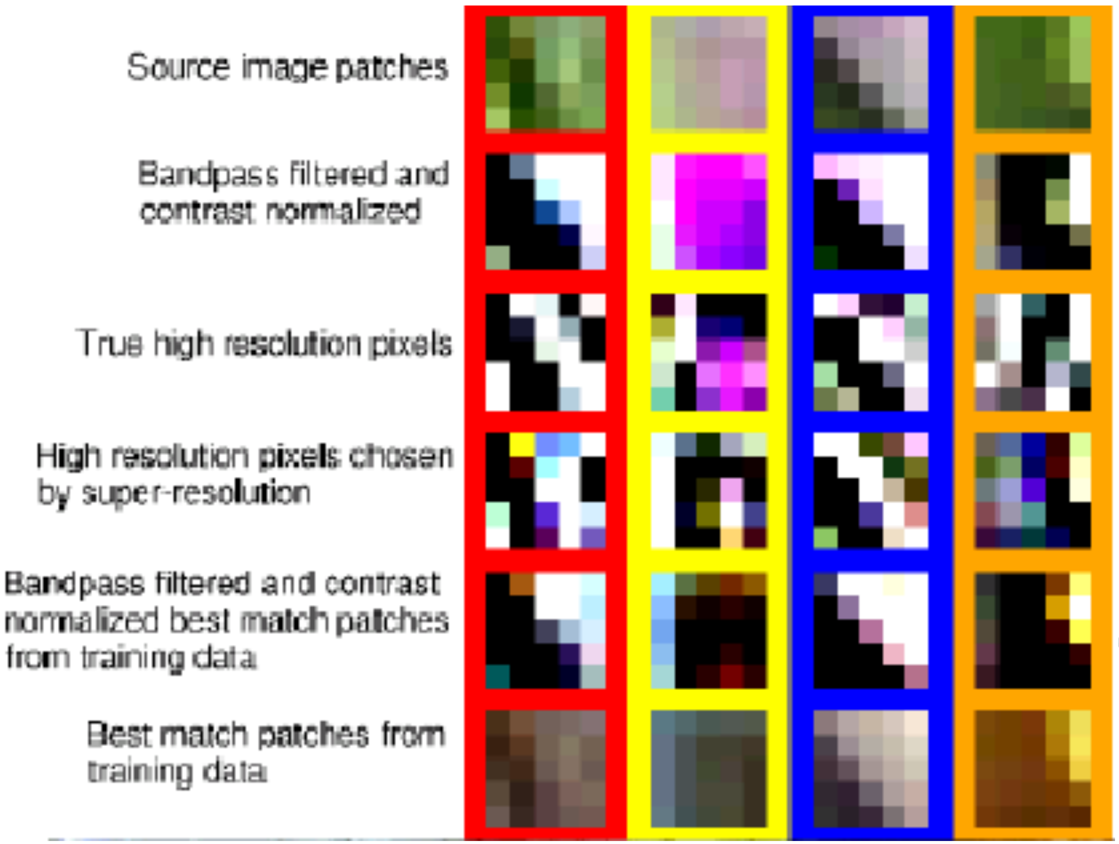
Super-resolution zoom

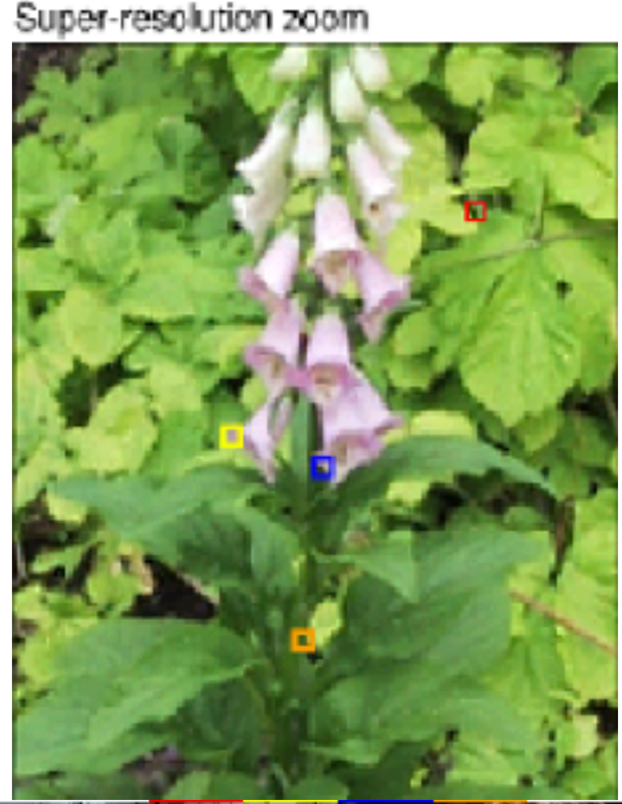


True high-resolution image



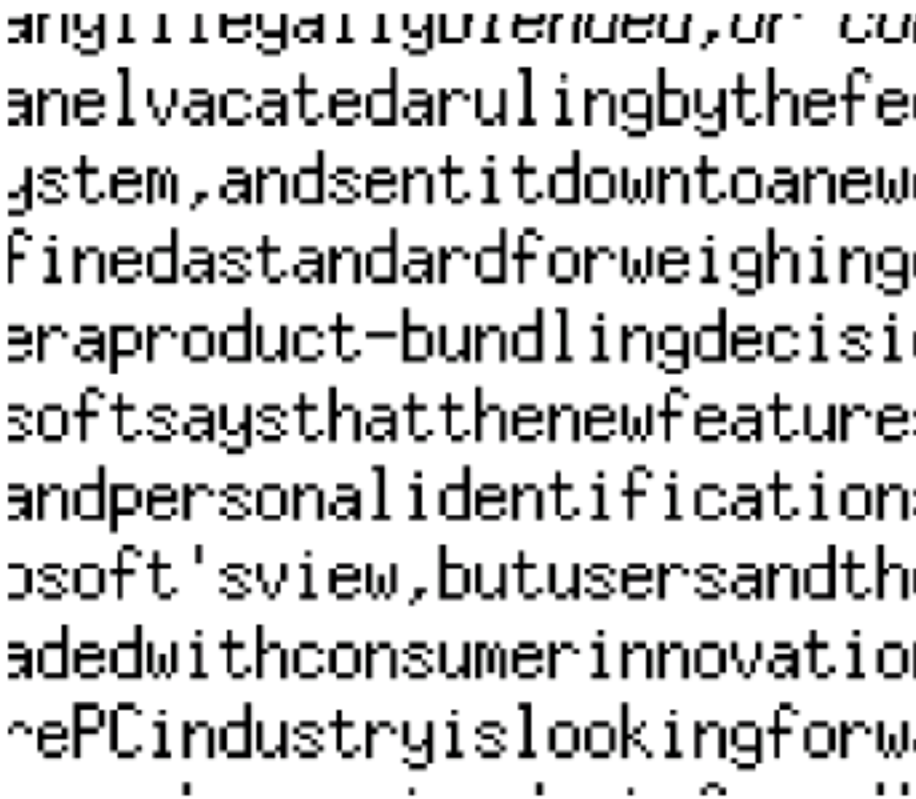






Training images

Training image



Processed image

code available online

http://people.csail.mit.edu/billf/project%20pages/sresCode/Markov%20Random%20Fields%20for%20Super-Resolution.html

Markov Random Fields for Super-Resolution

William T. Freeman	Ce Liu
Massachusetts Institute of Technology	Microsoft Research New England

[Download the package]

This is an implementation of the example-based super-resolution algorithm of [1]. Although the applications of MRFs have now extended beyond example-based super resolution and texture synthesis, it is still of great value to revisit this problem, especially to share the source code and examplar images with the research community. We hope that this software package can help to understand Markov random fields for low-level vision, and to create benchmark for super-resolution algorithms.

When you refer to this code in your paper, please cite the following book chapter:

W. T Freeman and C. Liu. **Markov Random Fields for Super-resolution and Texture Synthesis.** In A. Blake, P. Kohli, and C. Rother, eds., *Advances in Markov Random Fields for Vision and Image Processing*, Chapter 10. MIT Press, 2011. To appear.

Algorithm

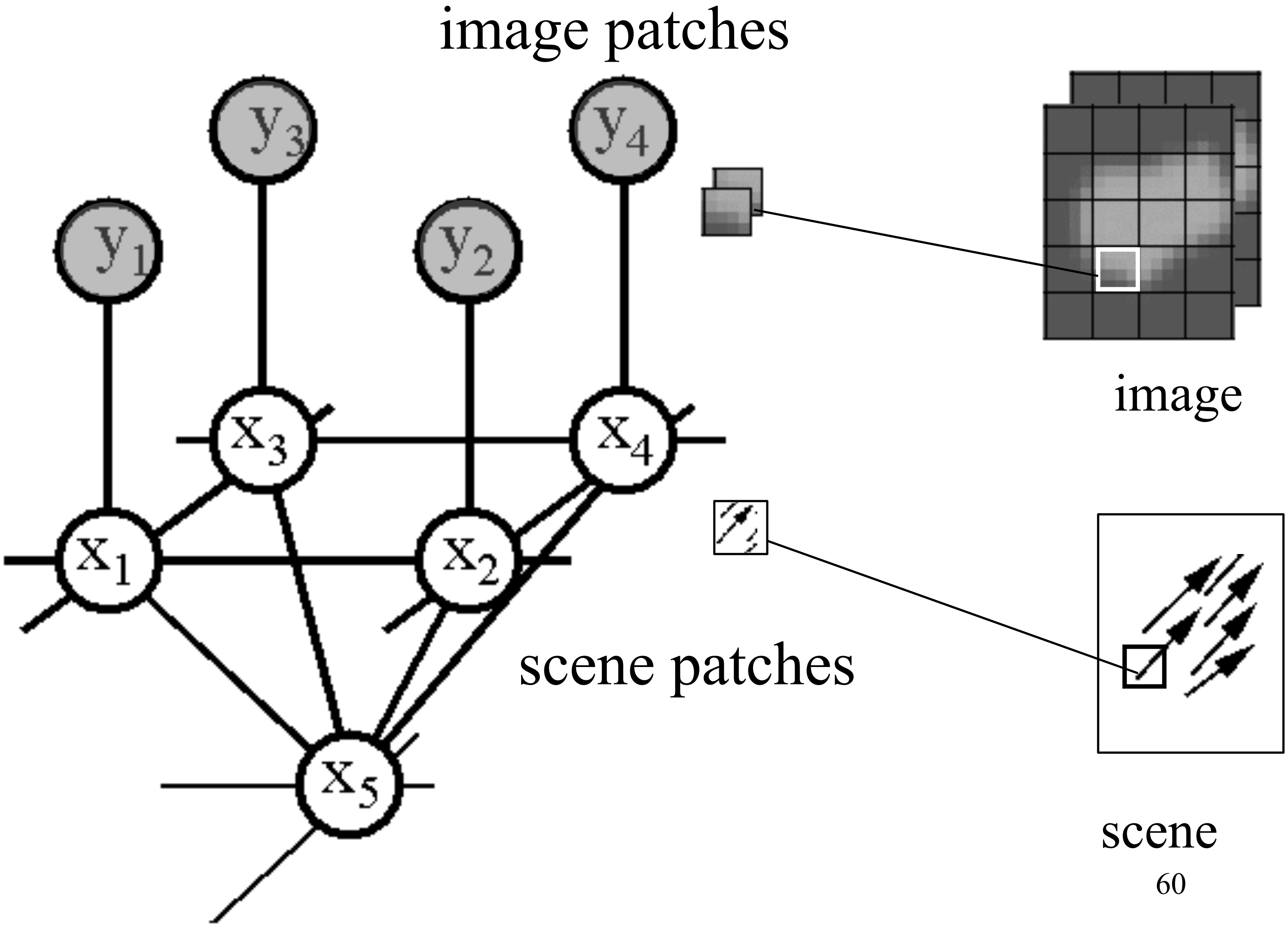
The core of the algorithm is based on [1]. We collect pairs of low-res and high-res image patches from a set of images as training. An input low-res image is decomposed to overlapping patches on a grid, and the inference problem is to find the high-res patches from the training database for each low-res patch. We use the kd-tree algorithm, which has been used for real-time texture synthesis [2], to retrieve a set of high-res, k-nearest neighbors for each low-res patch. Lastly, we run a max-product belief propagation (BP) algorithm to minimize an objective function that balances both local compatibility and spatial smoothness.

Examples

Several examples of applying the example-based super resolution code in the package are shown below. These examplar images are also included in the package. Once you run the code, it should give you the same result.

We first apply bicubic sampling to enlarge the input image (a) by a factor of 4 (b), where image details are missing. If we use the nearest neighbor for each low-res patch independently, we obtain high-res but noisy results in (c). To address this issue, we incorporating spatial smoothness into a Markov Random Fields formulation by enforcing the synthesized neighboring patches to agree on the overlapped areas. Max-product belief propagation is used to obtain high-res images in (d). The inferred high-frequency images are shown in (e), and the original high-res are shown in (f).

Motion application

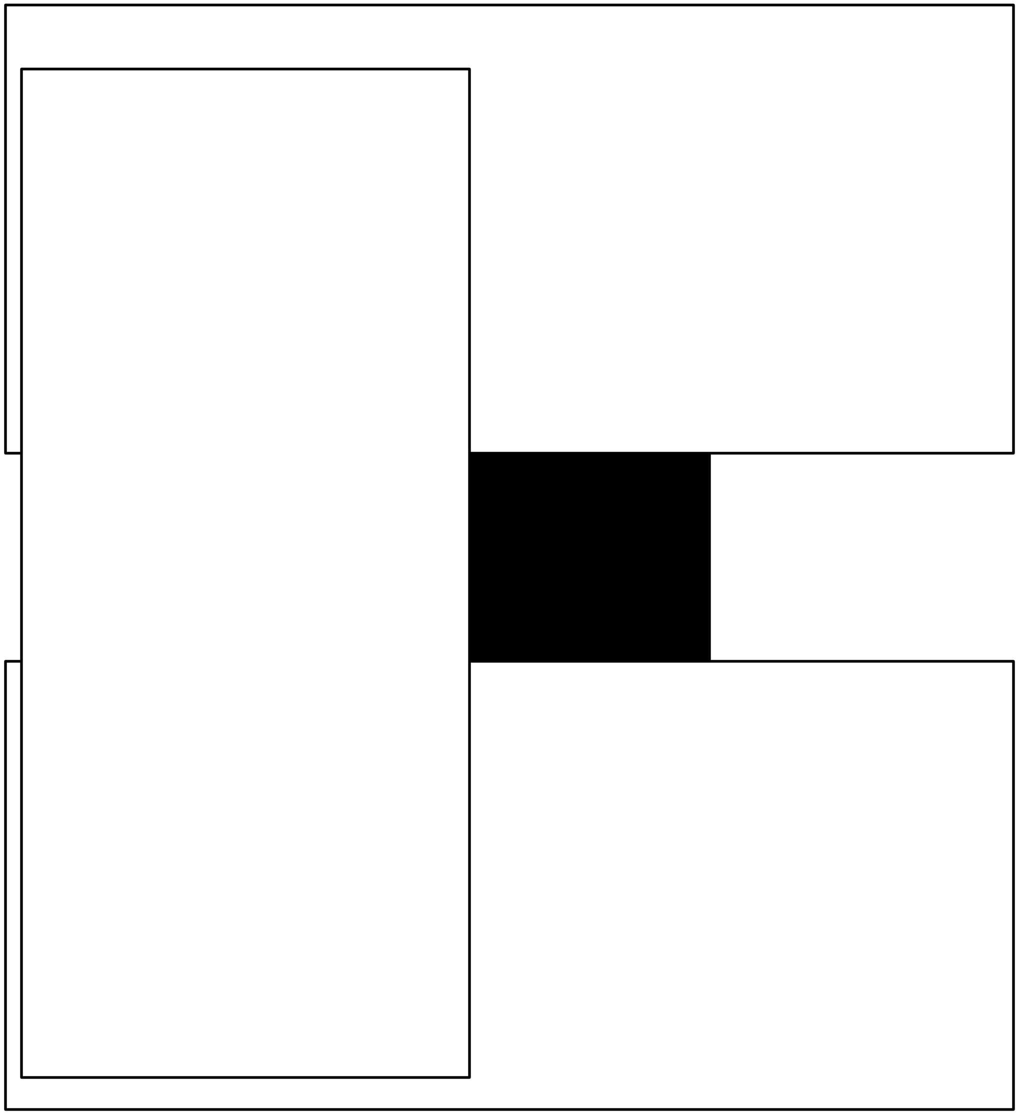


What behavior should we see in a motion algorithm?

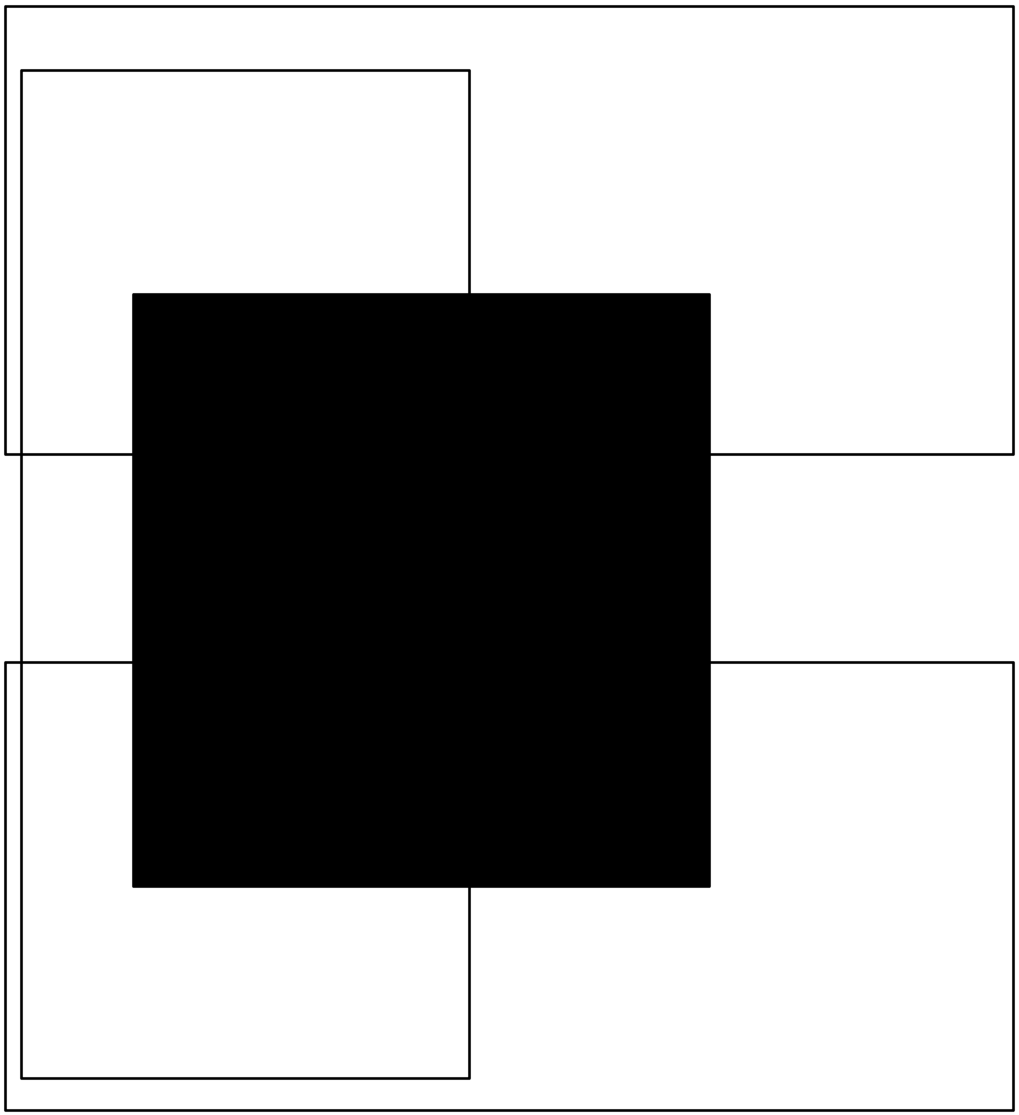
- Aperture problem
- Resolution through propagation of information
- Figure/ground discrimination

The aperture problem

http://web.mit.edu/persci/demos/Motion&Form/demos/one-square/one-square.html



The aperture problem



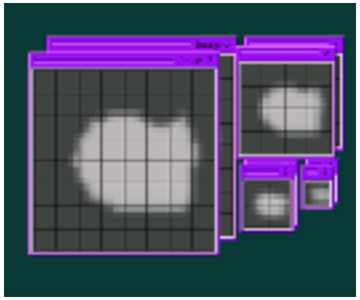
motion program demo

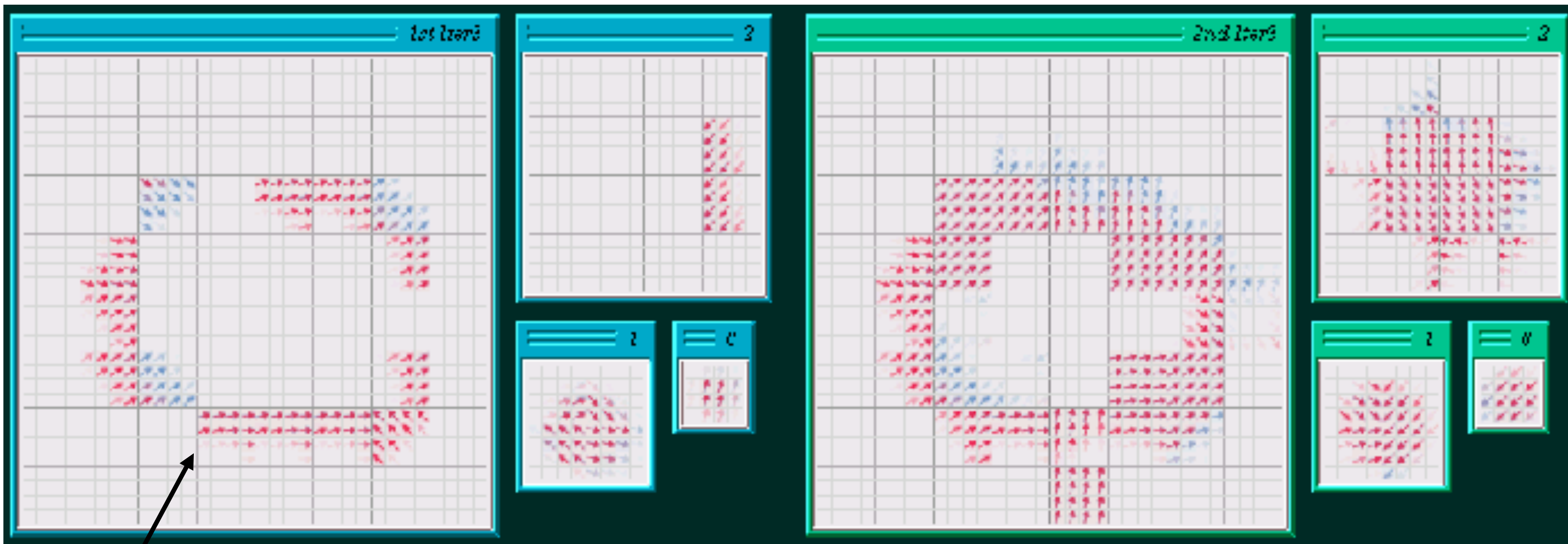
Inference:

Motion estimation results

(maxima of scene probability distributions displayed)

Image data

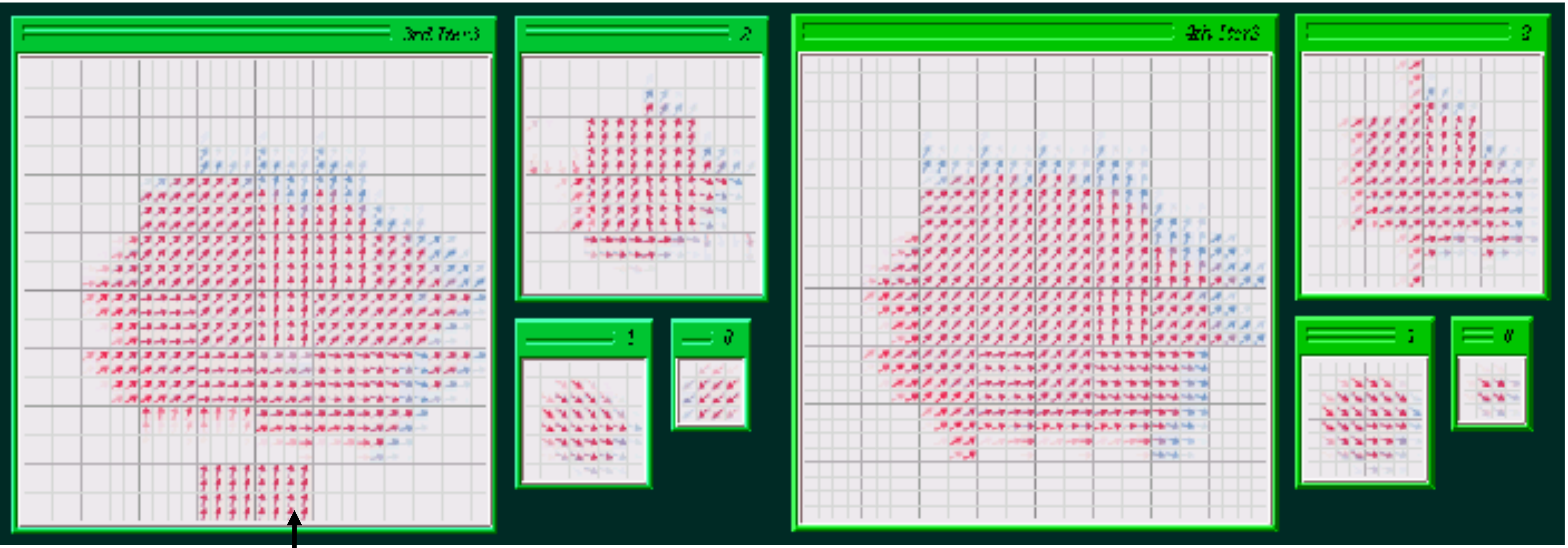




Iterations 0 and 1

Initial guesses only
show motion at edges.

Motion estimation results
(maxima of scene probability distributions displayed)

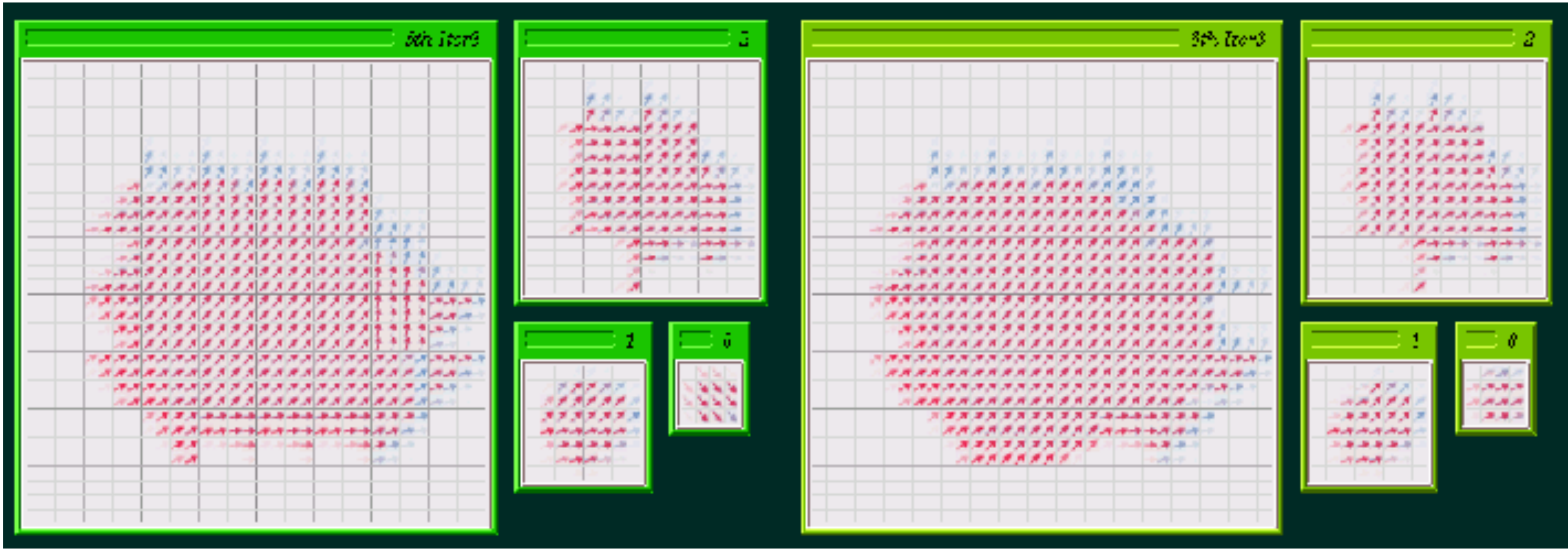


Iterations 2 and 3

Figure/ground still
unresolved here.

Motion estimation results

(maxima of scene probability distributions displayed)



Iterations 4 and 5

Final result compares well with vector quantized true (uniform) velocities.