MIT CSAIL

6.869 Advances in Computer Vision

Spring 2011

Problem Set 10: Kalman Filter and Structure-from-Motion

Posted: Wednesday, April 14, 2011 **Due:** Wednesday, April 27, 2011

You should submit a hard copy of your work in class, and upload your code (and all files needed to run it, images, etc) to stellar.
Your report should include images and plots showing your results, as well as pieces of your code that you find relevant.

Problem 10.1 *Kalman filter*

In this problem you will implement a discrete Kalman filter for a simple linear dynamical system.
Consider a particle moving in 1-dimensional space under random forces and damping (This could be a model for the output of some algorithm tracking a point in a video). Specifically, the 2-dimensional state $\vec{x}$ of the particle at a given time step is

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{1}$$

where x_1 represents the particle's location, and x_2 is the particle's velocity. The system equations are

$$\begin{aligned} \vec{x}_{t+1} &= \begin{pmatrix} 1 & 1 \\ 0 & 0.98 \end{pmatrix} \vec{x}_t + \vec{w}_t, \\ y_t &= \begin{pmatrix} 1 & 0 \end{pmatrix} \vec{x}_t + v_t \end{aligned} \tag{2}$$

where $w_t \sim \mathcal{N}(\vec{0}, Q)$ and $v_t \sim \mathcal{N}(0, R)$, with covariance and variance

$$Q = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad R = \epsilon \tag{3}$$

For initial condition assume that $\vec{x}_0 \sim \mathcal{N}(\vec{0}, I)$.

(a) Let $\epsilon = 100$, and $T = 200$. Simulate the above system to generate state and measurement sequences for $t = 1, 2, \ldots, T$. Plot the evolution of the particle's true position, along with the (noisy) measurements.

(b) Implement the Kalman filter and find $\vec{x}_{t|t}$ for each time t. Plot the resulting estimate of the particle positions across time, on top of the true positions.

(c) Restricting attention again to the position component x_1 of the particle state, compute:

$$\frac{1}{T}\sum_t (y_t - x_t)^2, \qquad \frac{1}{T}\sum_t (x_{t|t} - x_t)^2 \tag{4}$$

Comment on the relative qualities of using the measurements y_t directly as estimates of the particle position, and the Kalman filter estimates.

(d) Repeat parts (a-c) for $\epsilon = 5$. How does this parameter affect the system model? How does it affect the estimates?

(e) **[Optional]** Implement the Kalman smoother (Rauch–Tung–Striebel) and find $\vec{x}_{t|T}$ for each time t. Although we did not cover RTS in class, it is a rather simple extension of the Kalman filter, and you should be able to read and understand its derivation. Similar to belief propagation on Markov chains, the algorithm uses two sweeps: the forward sweep, running the standard Kalman filter, followed by a backward sweep for incorporating the future measurements into past estimates. You can find the update equations for the backward sweep in the literature. Plot the resulting estimate of the particle positions across time, on top of the true positions, and compare the quantity $\frac{1}{T}\sum_t (x_{t|T} - x_t)^2$ with the ones in part (c).

Add your answers and plots to your report, and submit all your code online.

Problem 10.2 *SFM*

In this problem you have to implement the Tomasi and Kanade algorithm, described in class for structure from motion. The script `sfm.m` contains code to generate the rotating cylinder from class. Given the 2D trajectories in that sequence, our goal is to reconstruct the 3D coordinates of the points.

(a) Build the registered measurement matrix

(b) Apply SVD, and find the matrix Q to recover the rotation and structure matrices. Save your 3D reconstruction as a MATLAB `fig` file, and submit it online.

(c) Try to vary the number of frames and the number of points. Produce new sequences with (`Npoints = 80, Nframes = 5`), and (`Npoints = 40, Nframes = 20`). Do these settings affect your reconstruction? Why?
Submit your reconstructions online, as done in part (b).