Lecture 6

Motion Filtering and Sampling

6.869/6.819 Advances in Computer Vision

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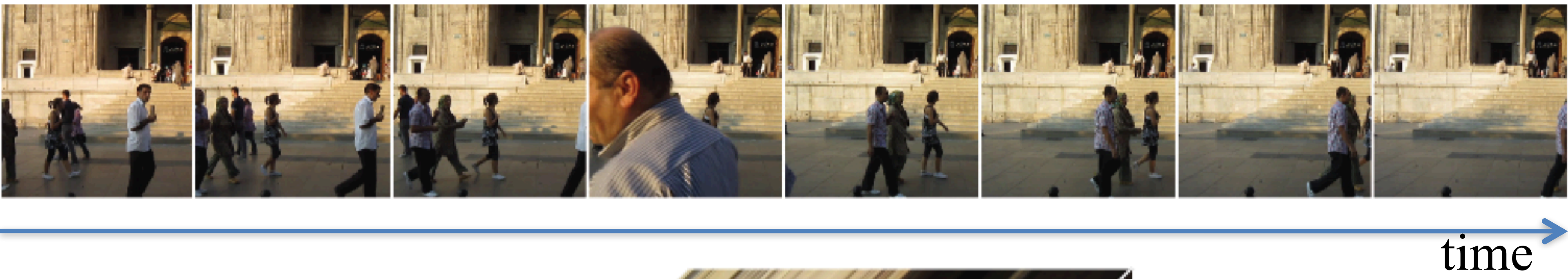
Today’s content

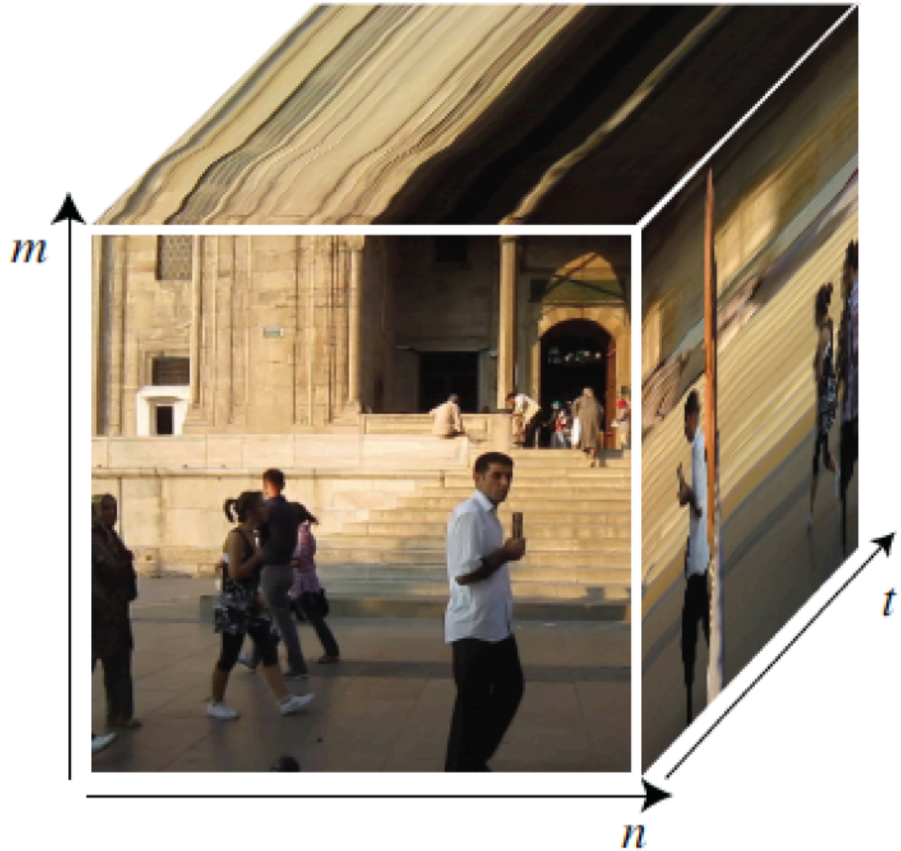
- **Temporal filtering**—what sorts of things can you do with it?
  - picking out objects moving with a certain velocity.
- **Gabor filters** and quadrature pairs.
- Measuring or synthesizing motion.
- **Aliasing**
- **Motion illusion**, involving aliasing, addressing whether humans match spatial patterns, or use temporal filters, to measure motion.
- If there’s time: using temporal filtering to remove objects moving with a certain velocity.

Temporal filtering

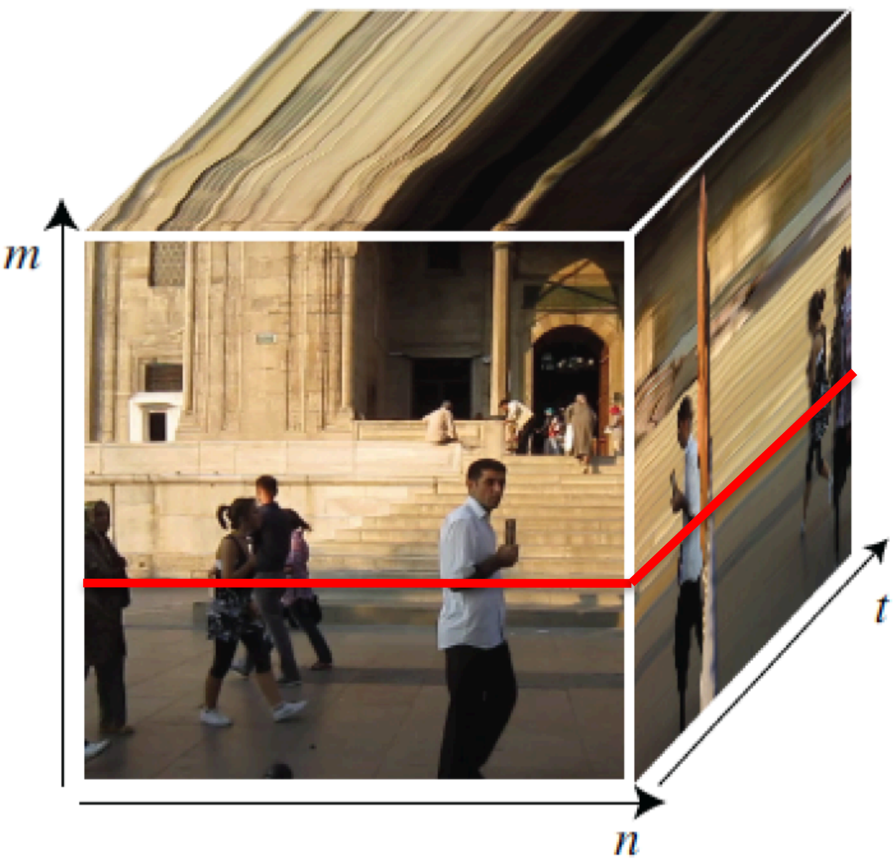
why filter videos over time?

Sequences

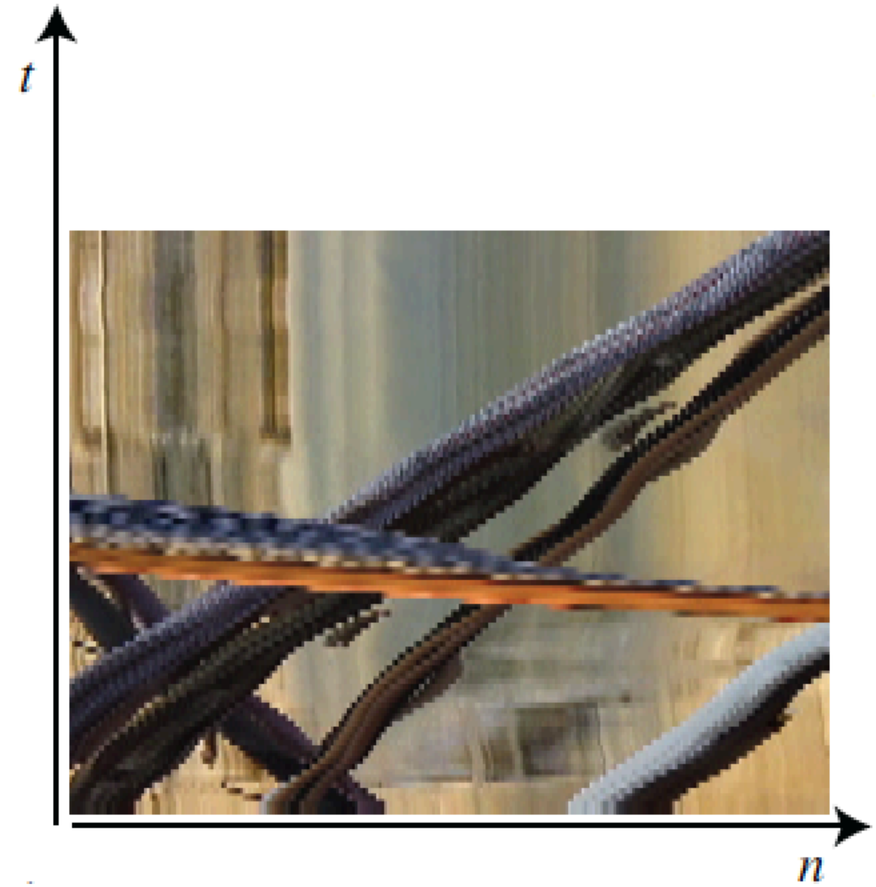




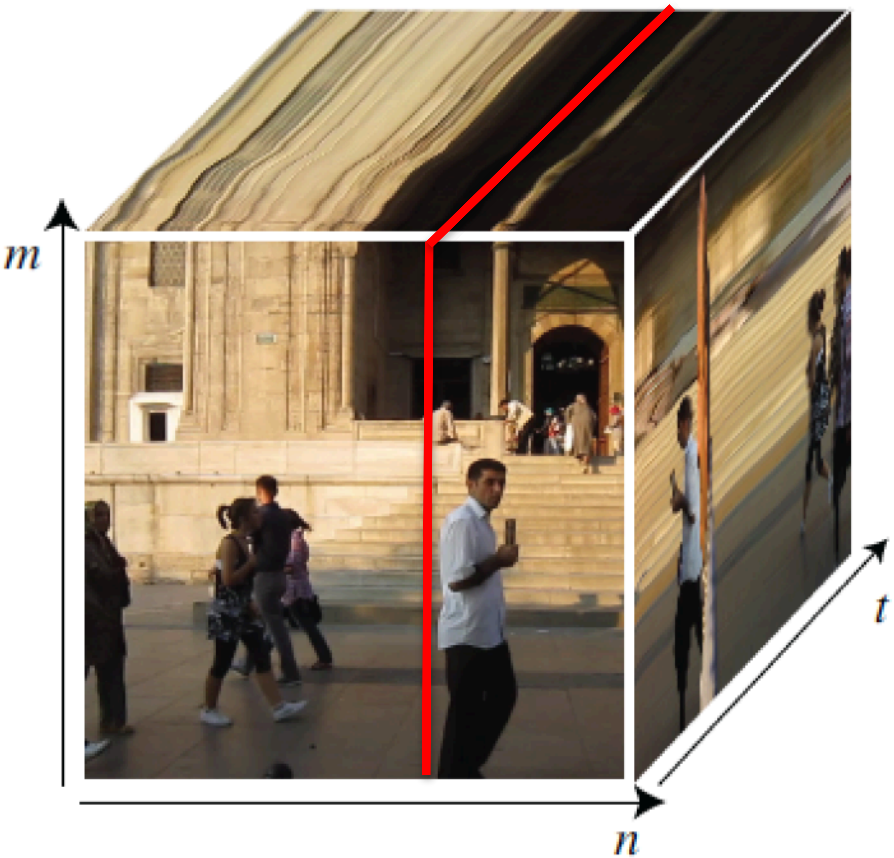
Sequences



Cube size = 128x128x90

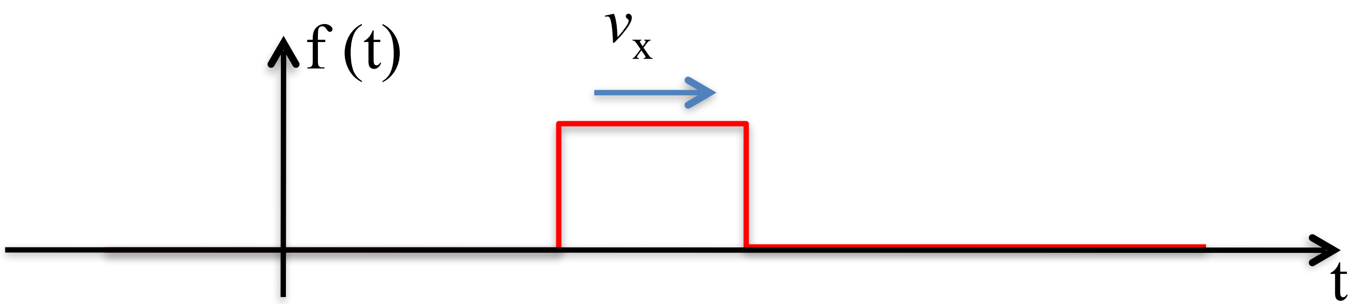


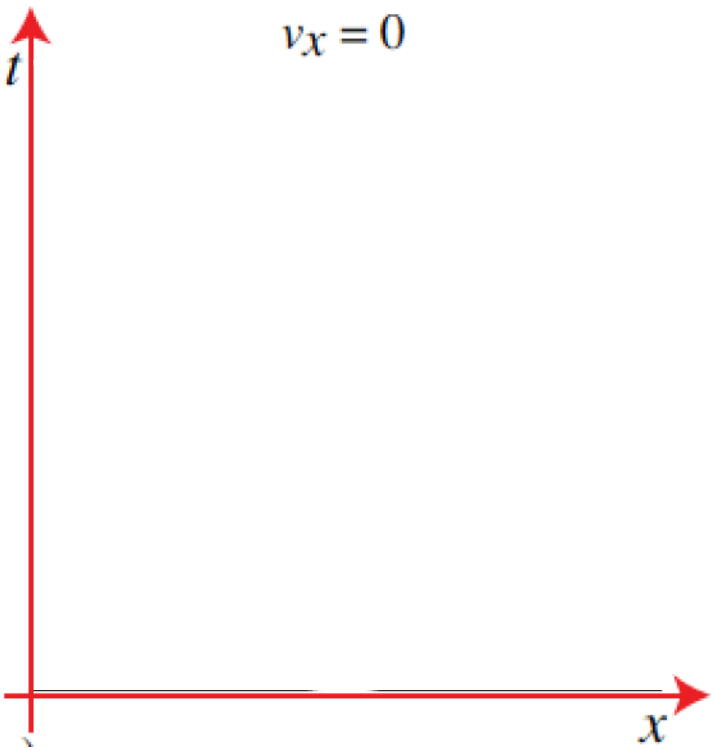
Sequences

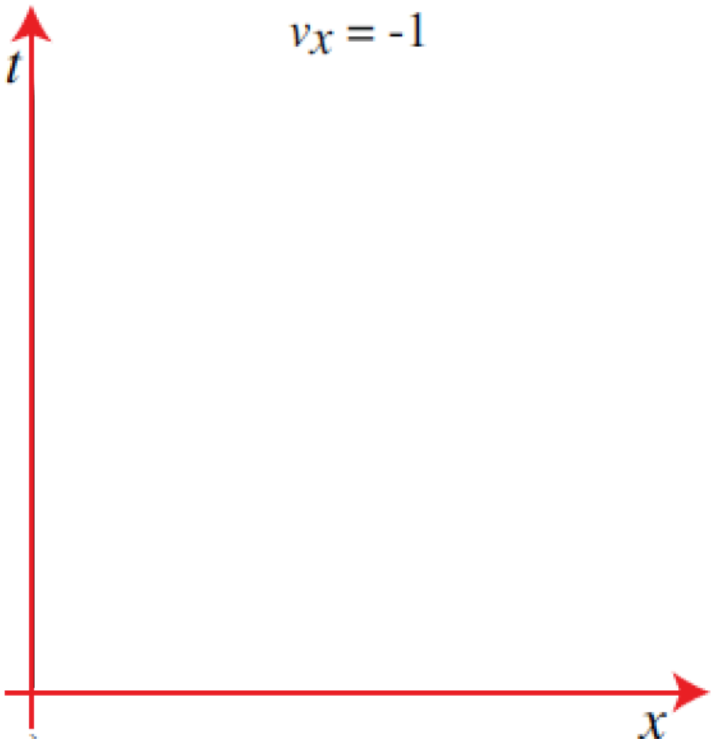


Cube size = 128x128x90

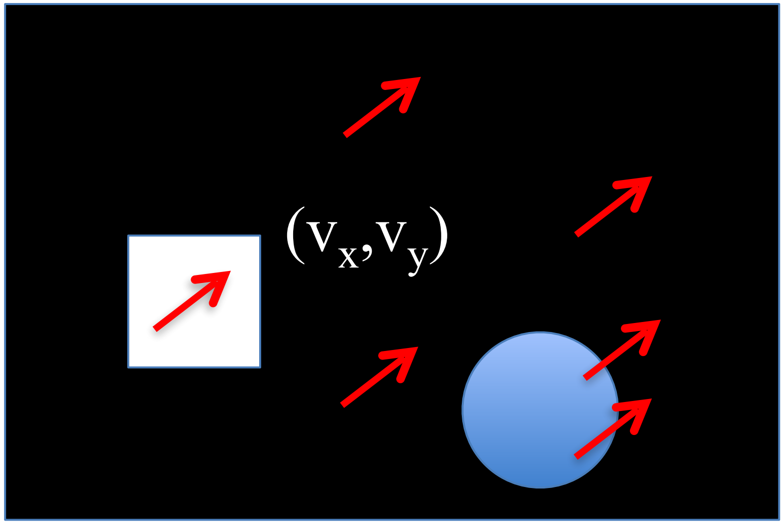
A box moving with speed v_x







Global constant motion



A global motion of the image can be written as:

$$f(x,y,t) = f_0(x - v_x t, y - v_y t)$$

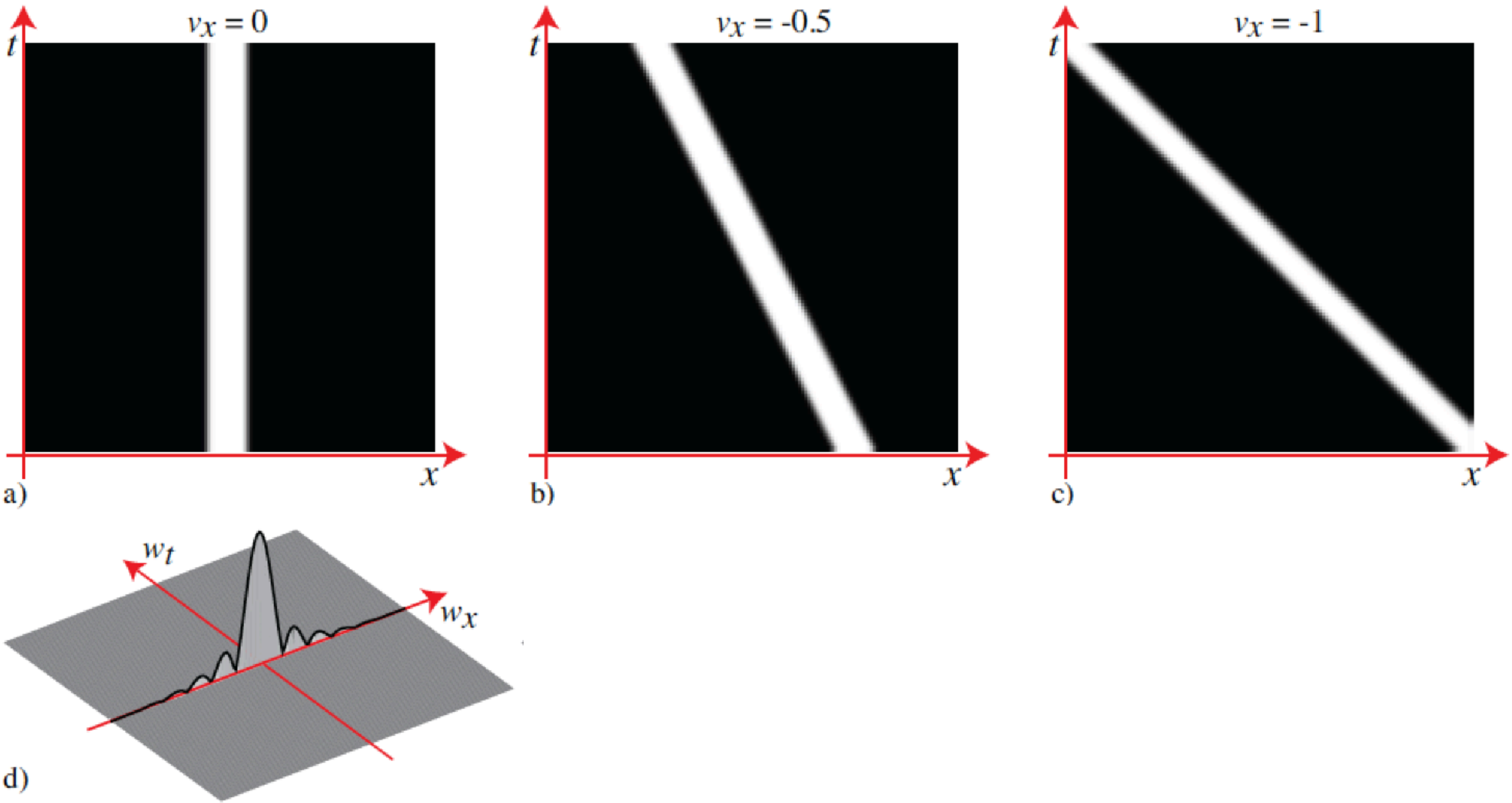
Where:

$$f_0(x,y) = f(x,y,0)$$

$$f(x, y, t) = f_0(x - v_x t, y - v_y t)$$

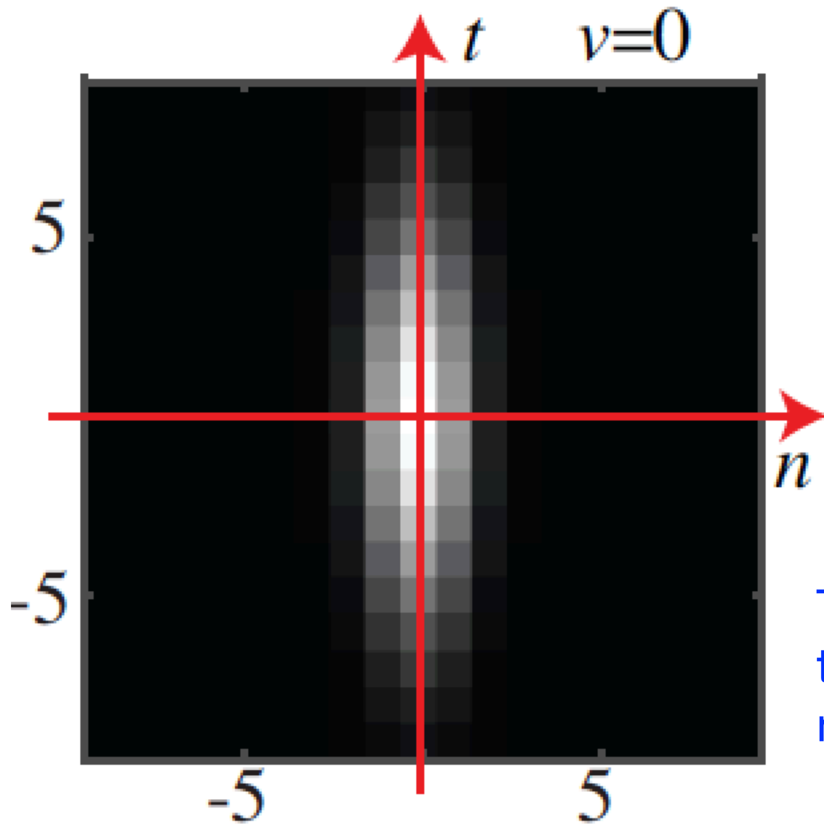
$$\updownarrow$$

$$F(w_x, w_y, w_t) = F_0(w_x, w_y)\delta(w_t + v_x w_x + v_y w_y)$$



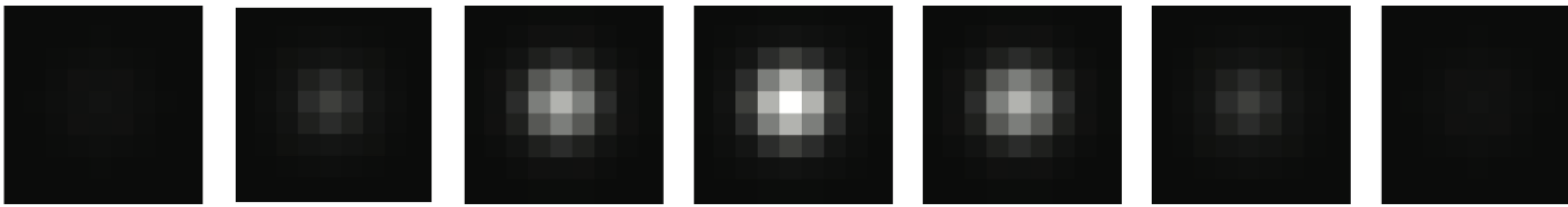
Temporal Gaussian

$$g(x,y,t;\sigma_x,\sigma_t) = \frac{1}{(2\pi)^{3/2}\sigma_x^2\sigma_t} \exp - \frac{x^2+y^2}{2\sigma_x^2} \exp - \frac{t^2}{2\sigma_t^2}$$

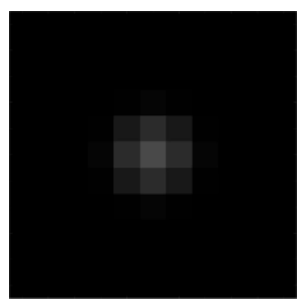


This filter keeps stationary things sharp, and blurs moving things.

Spatio-temporal Gaussian



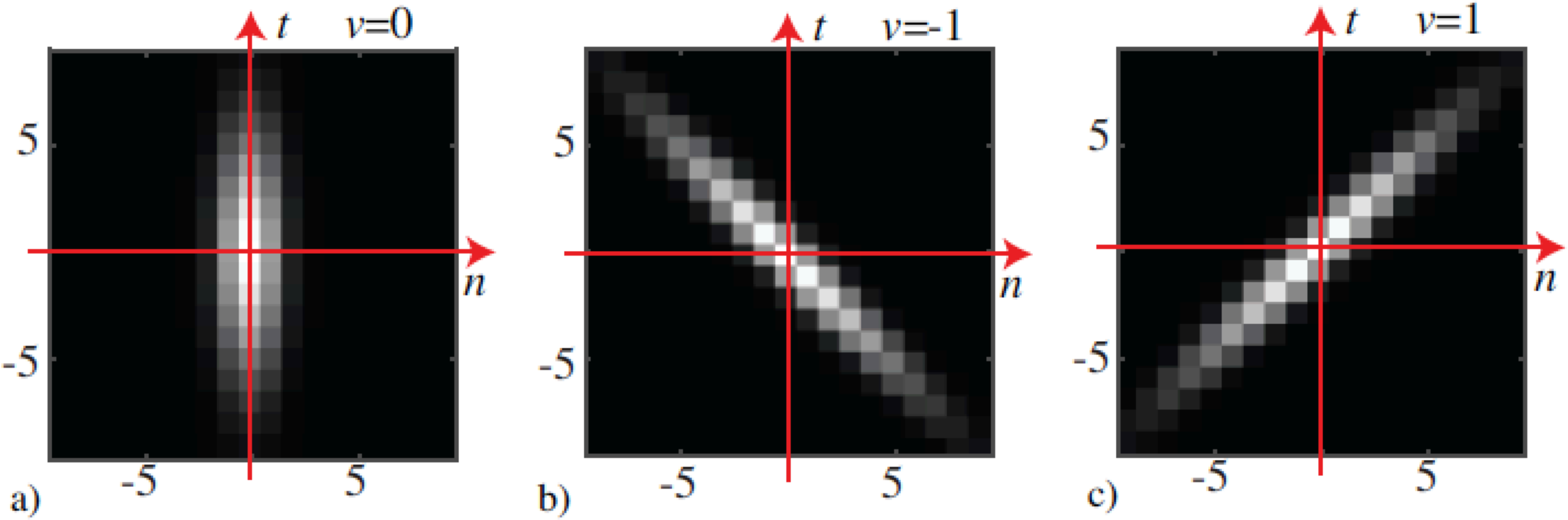
t=-3 *t*=-2 *t*=-1 *t*=0 *t*=1 *t*=2 *t*=3

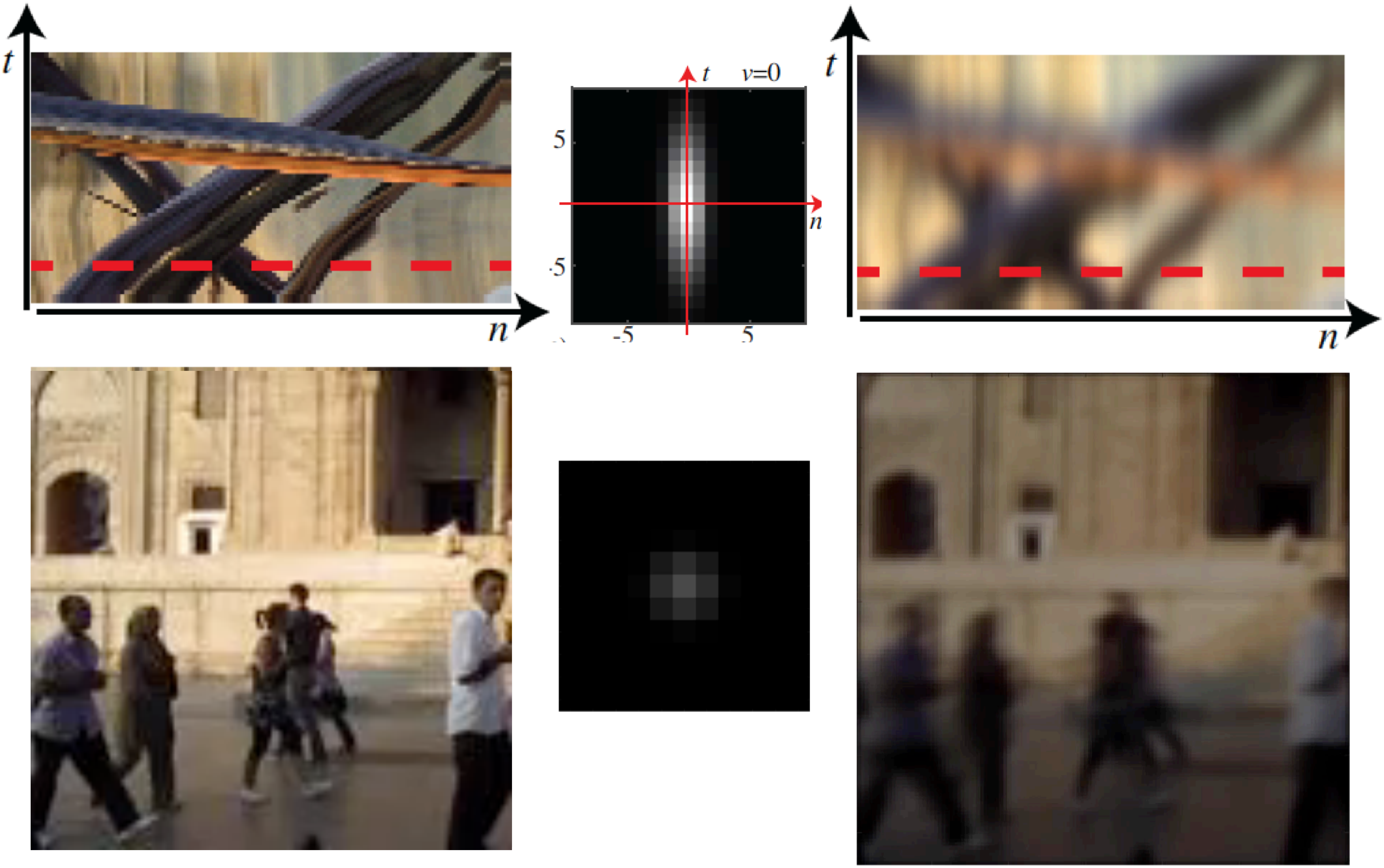


Spatio-temporal Gaussian

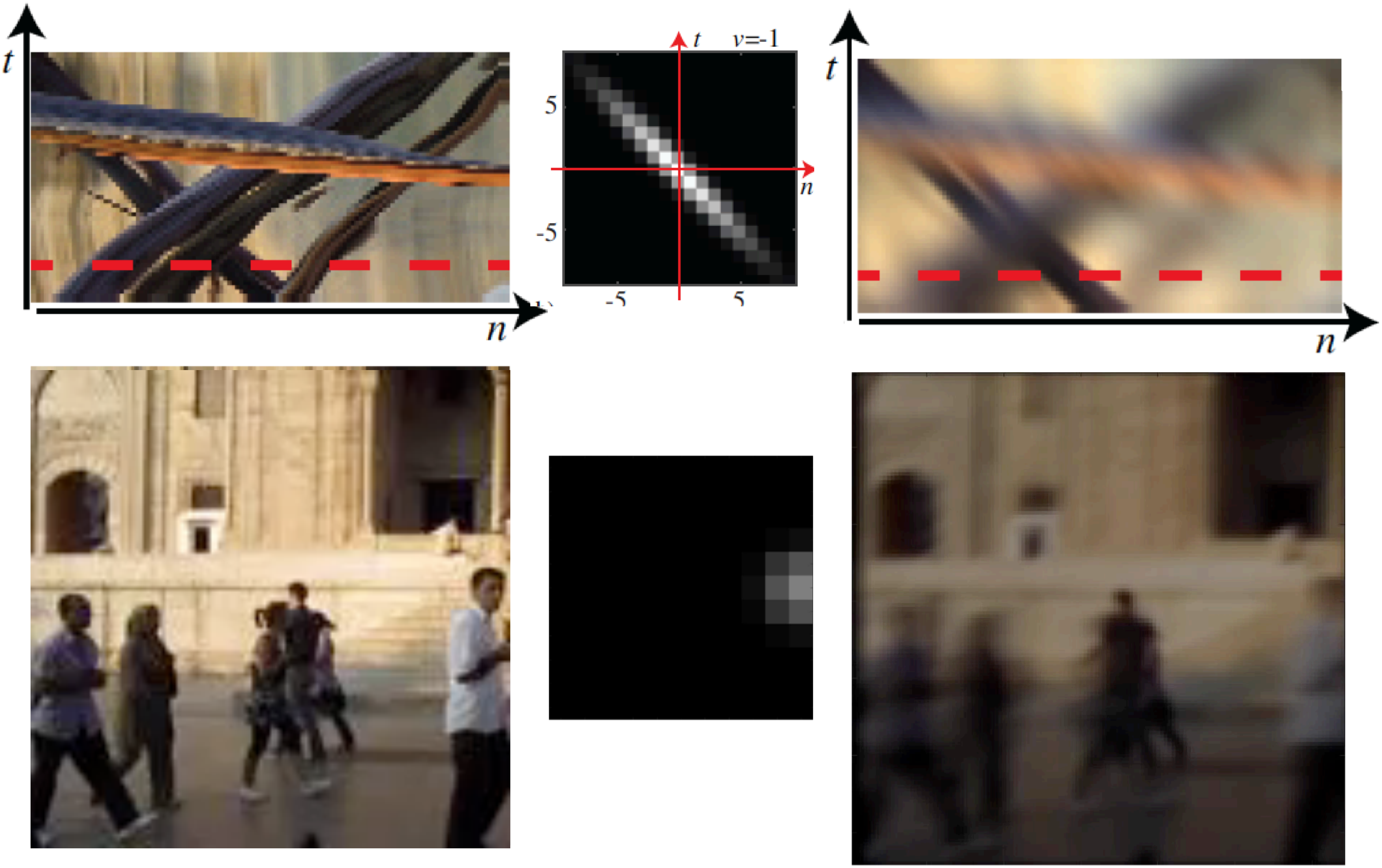
How could we create a filter that keeps sharp objects that move at some velocity **(vx , vy)** while blurring the rest?

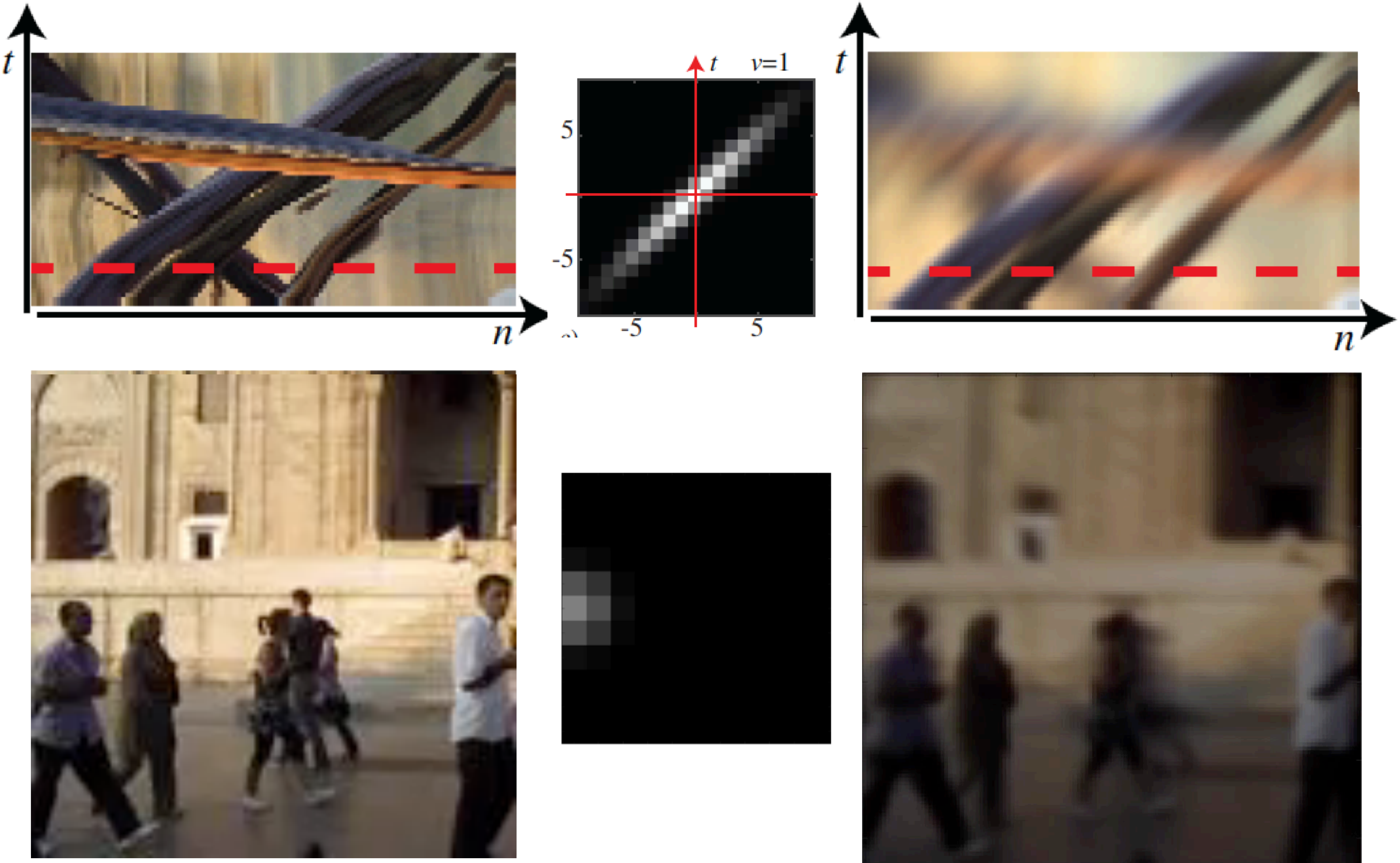
$$g_{v_x,v_y}(x,y,t) = g(x - v_x t, y - v_y t, t)$$





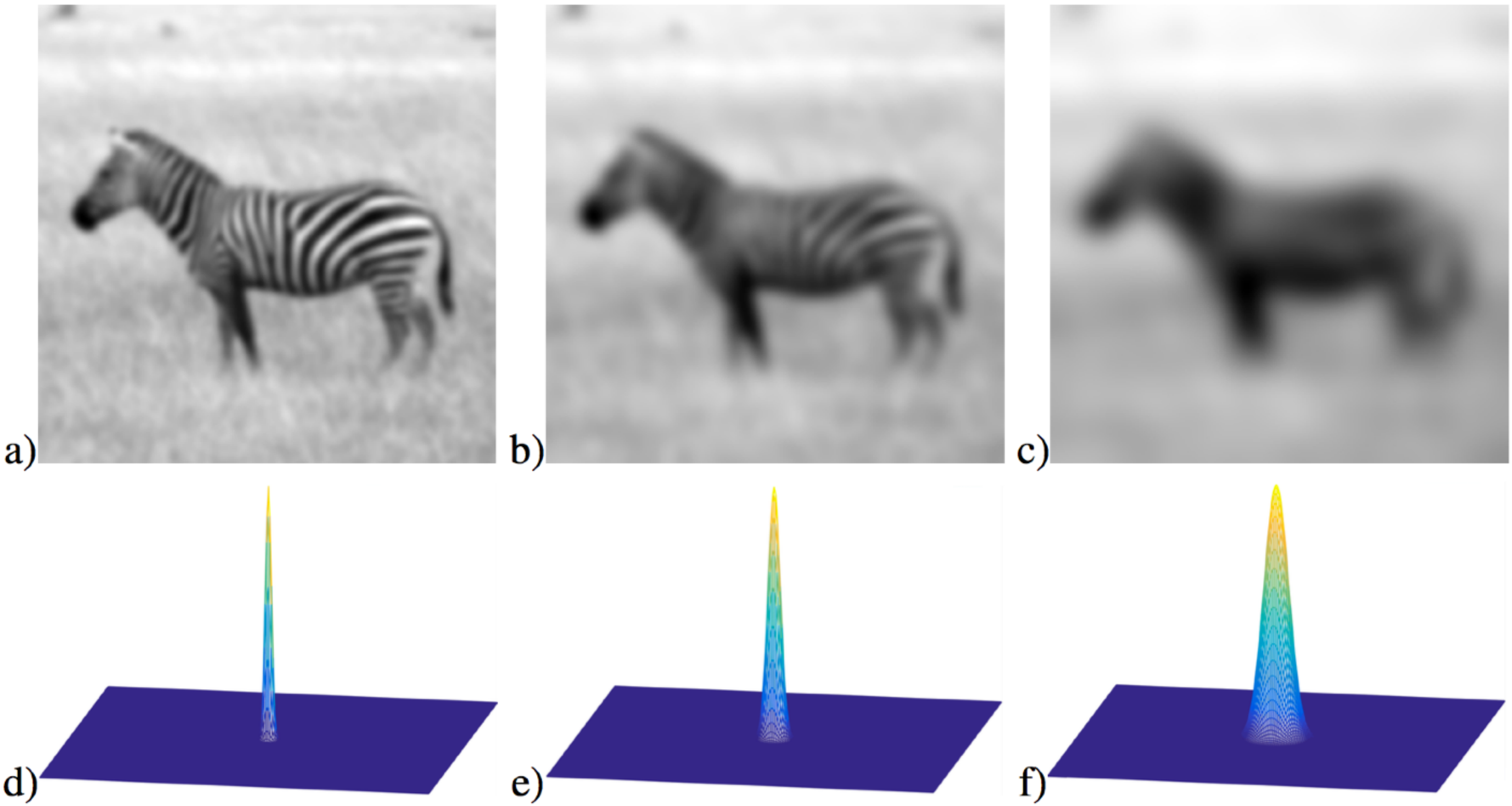
t
n
t
v=0
5
-5
-5
5
n
t
n


t
n
t
v=-1
5
-5
n
-5
5
t
n



t
n
t
v=1
5
-5
-5
5
t
n

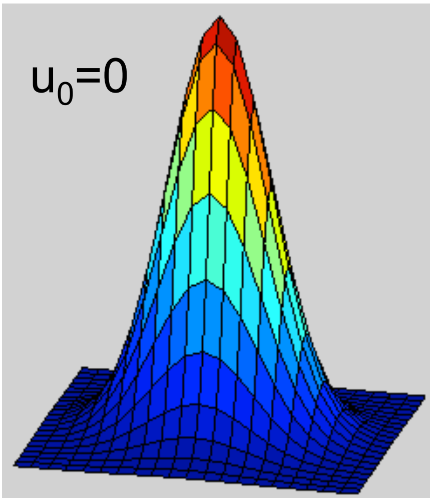
(last class) Gaussians set scale

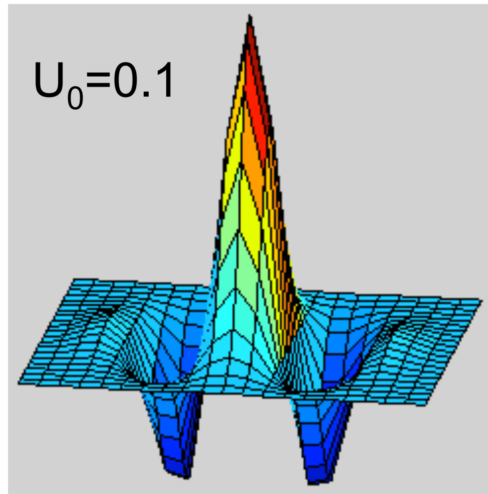


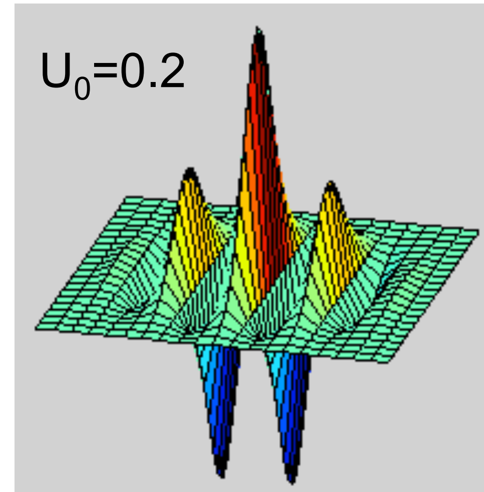
Gabor wavelets

Good for both temporal and spatial filtering

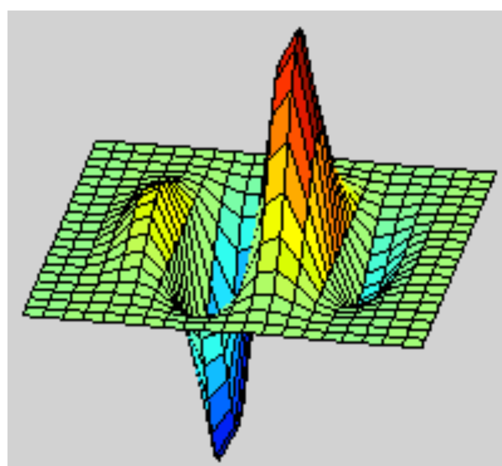
$$\psi_c(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \cos(2\pi u_0 x)$$

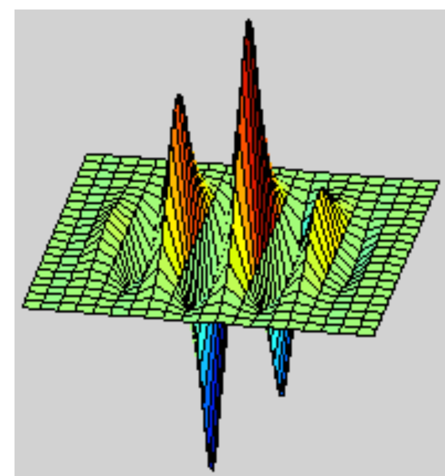




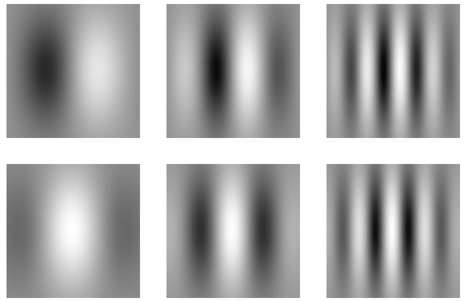


$$\psi_s(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \sin(2\pi u_0 x)$$

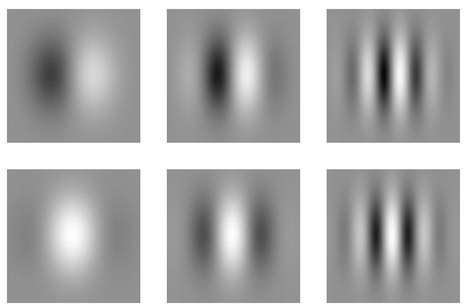




Gabor wavelets are like sinusoids, only they are localized, as enforced by the Gaussian multiplicative window.

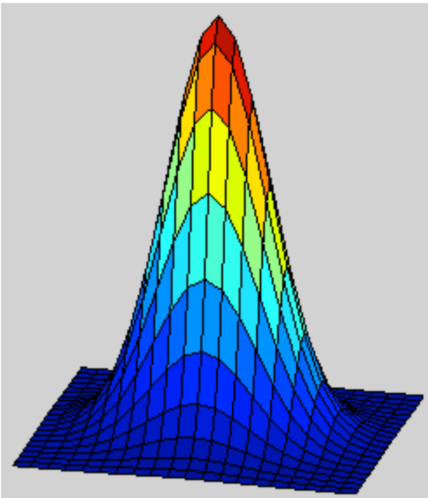


Gabor filters at different scales and spatial frequencies

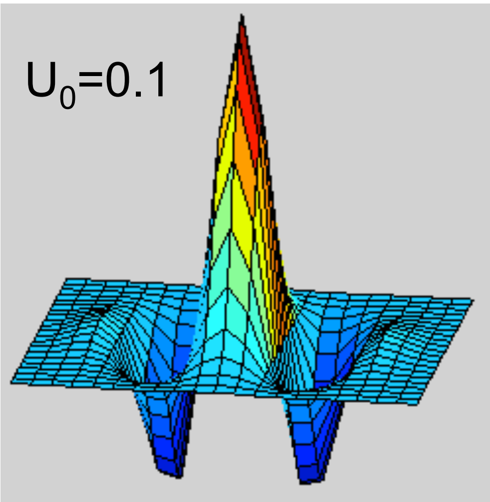


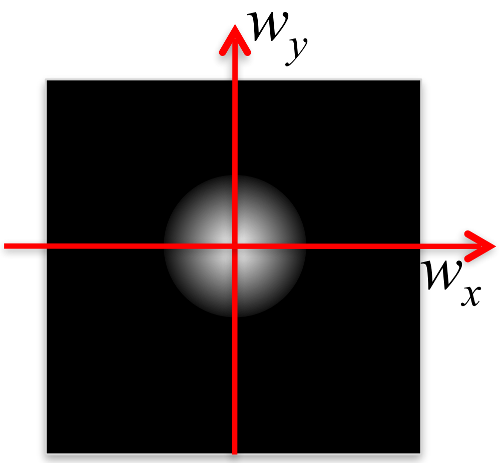
<u>Top row</u> shows anti-symmetric (or odd) filters; these are good for detecting odd-phase structures like edges.
<u>Bottom row</u> shows the symmetric (or even) filters, good for detecting line phase contours.

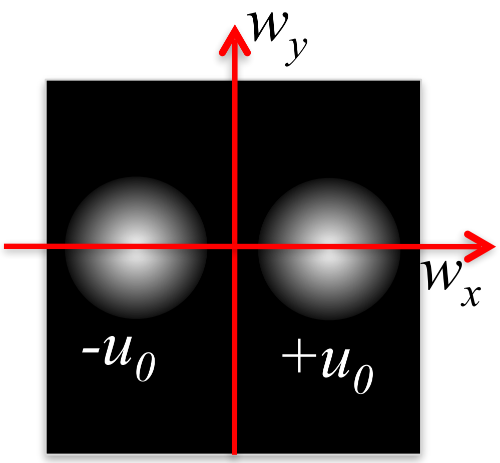
Fourier transform of a Gabor wavelet

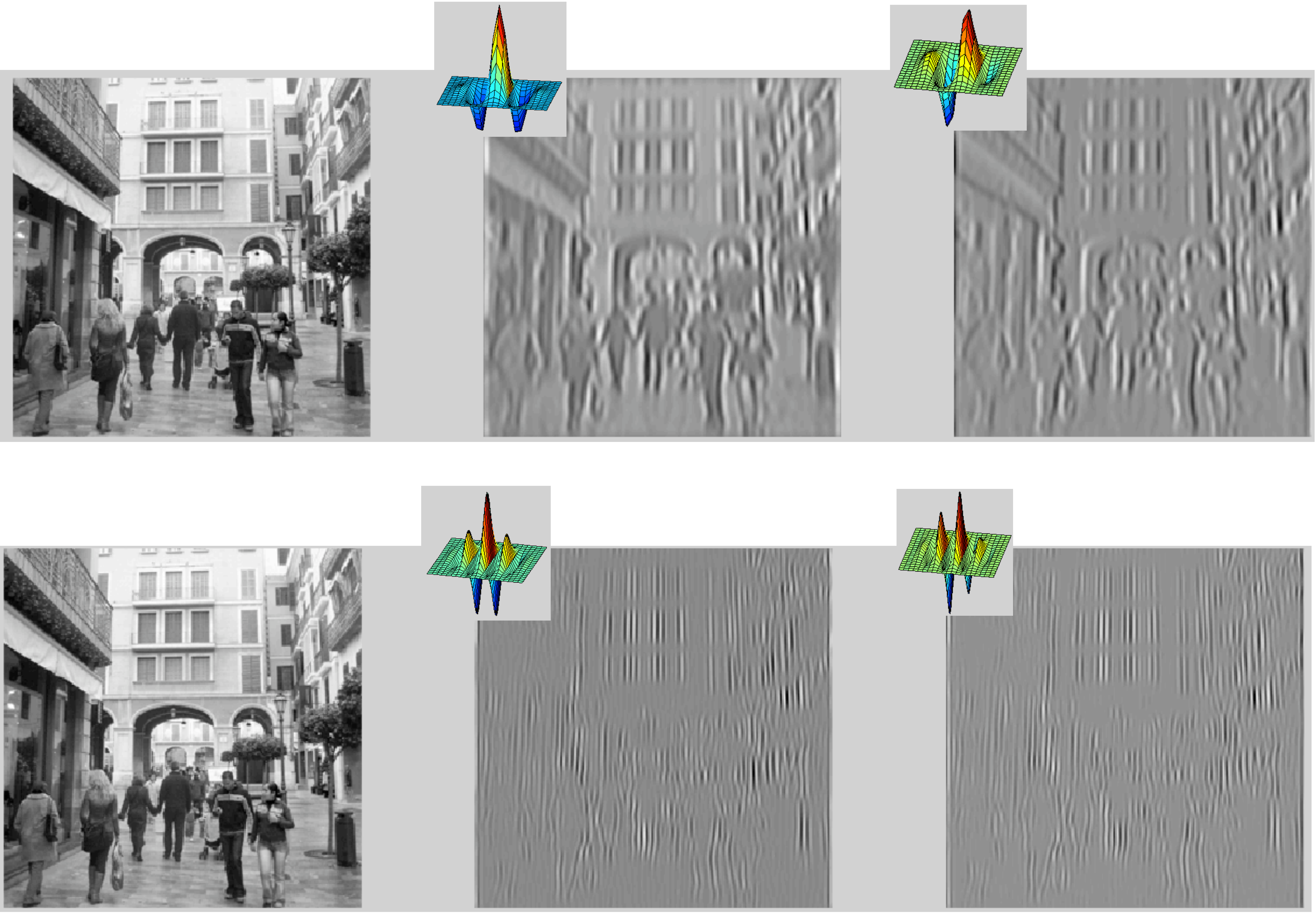


$$\psi_c(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \cos(2\pi u_0 x)$$



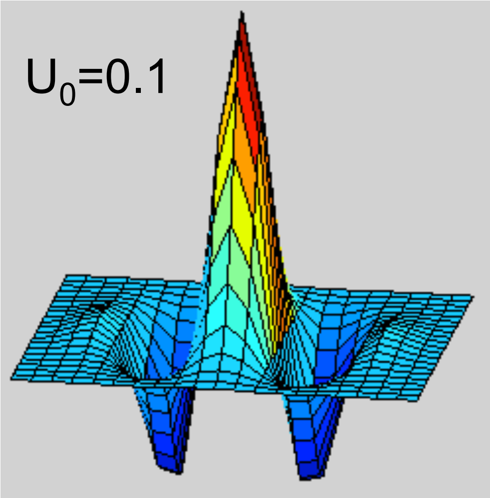




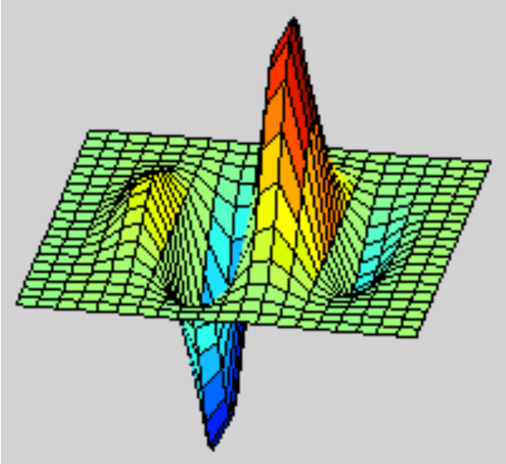


Quadrature pair

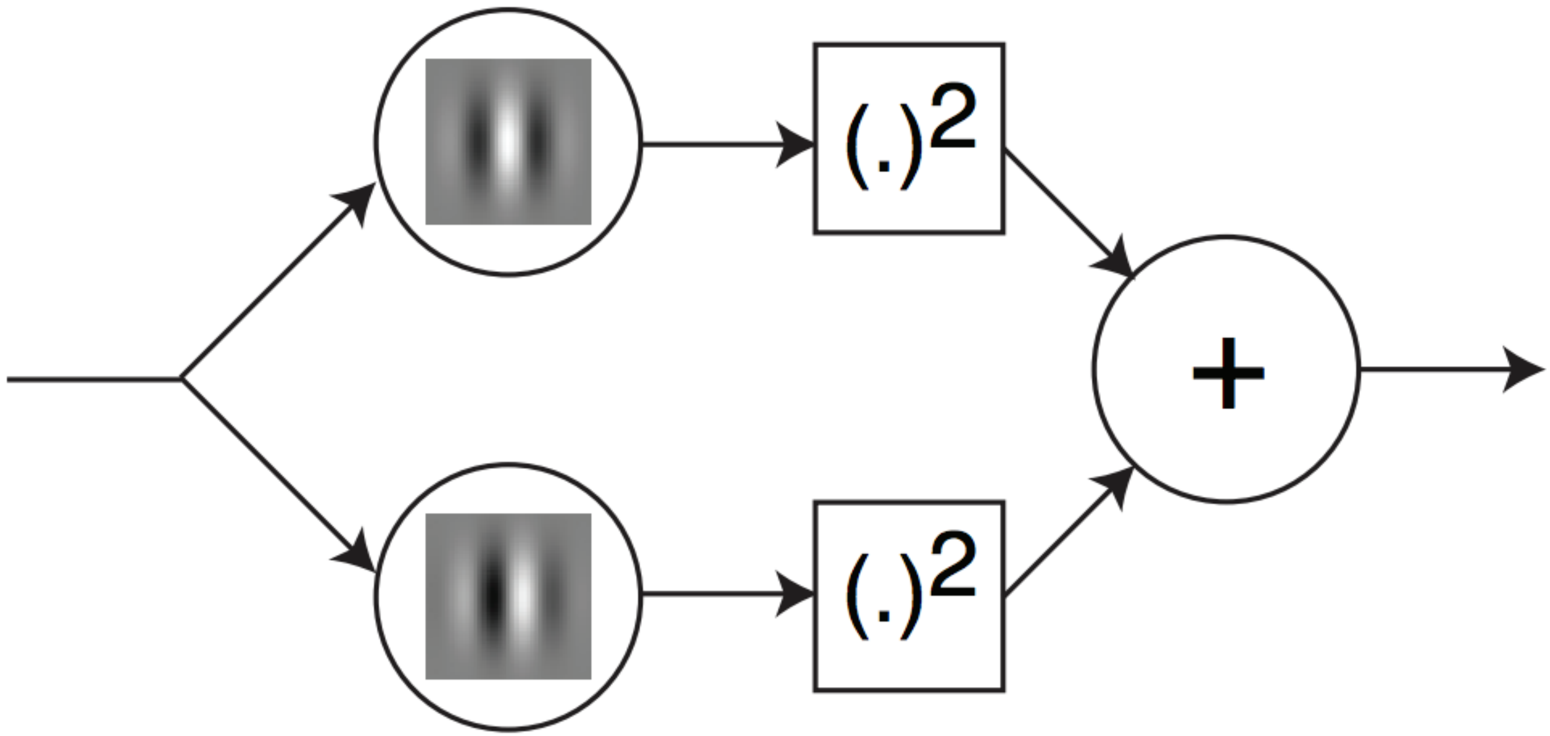
$$\psi_c(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \cos(2\pi u_0 x)$$



$$\psi_s(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \sin(2\pi u_0 x)$$

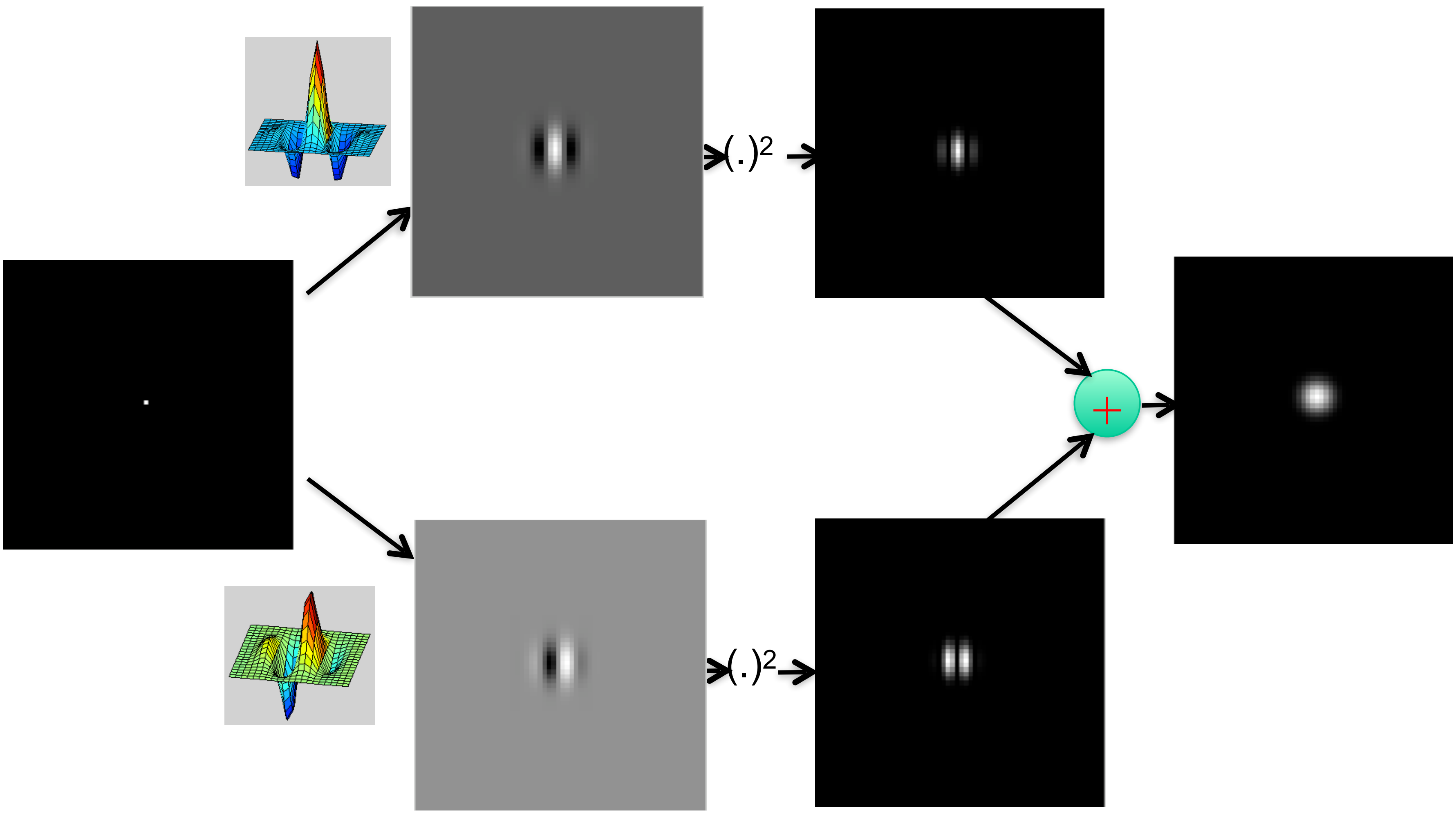


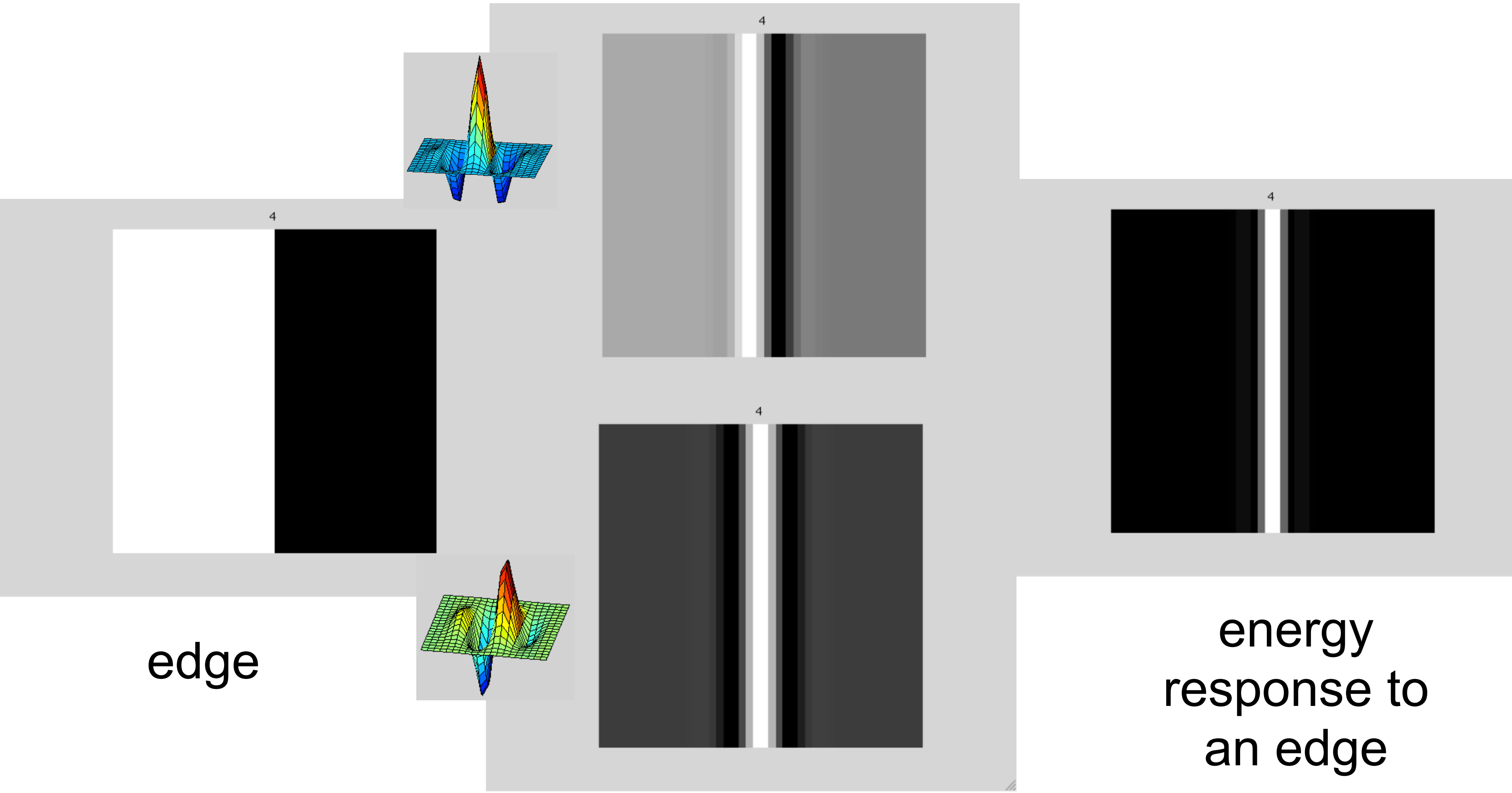
“oriented energy” from a quadrature pair



Quadrature filter pairs

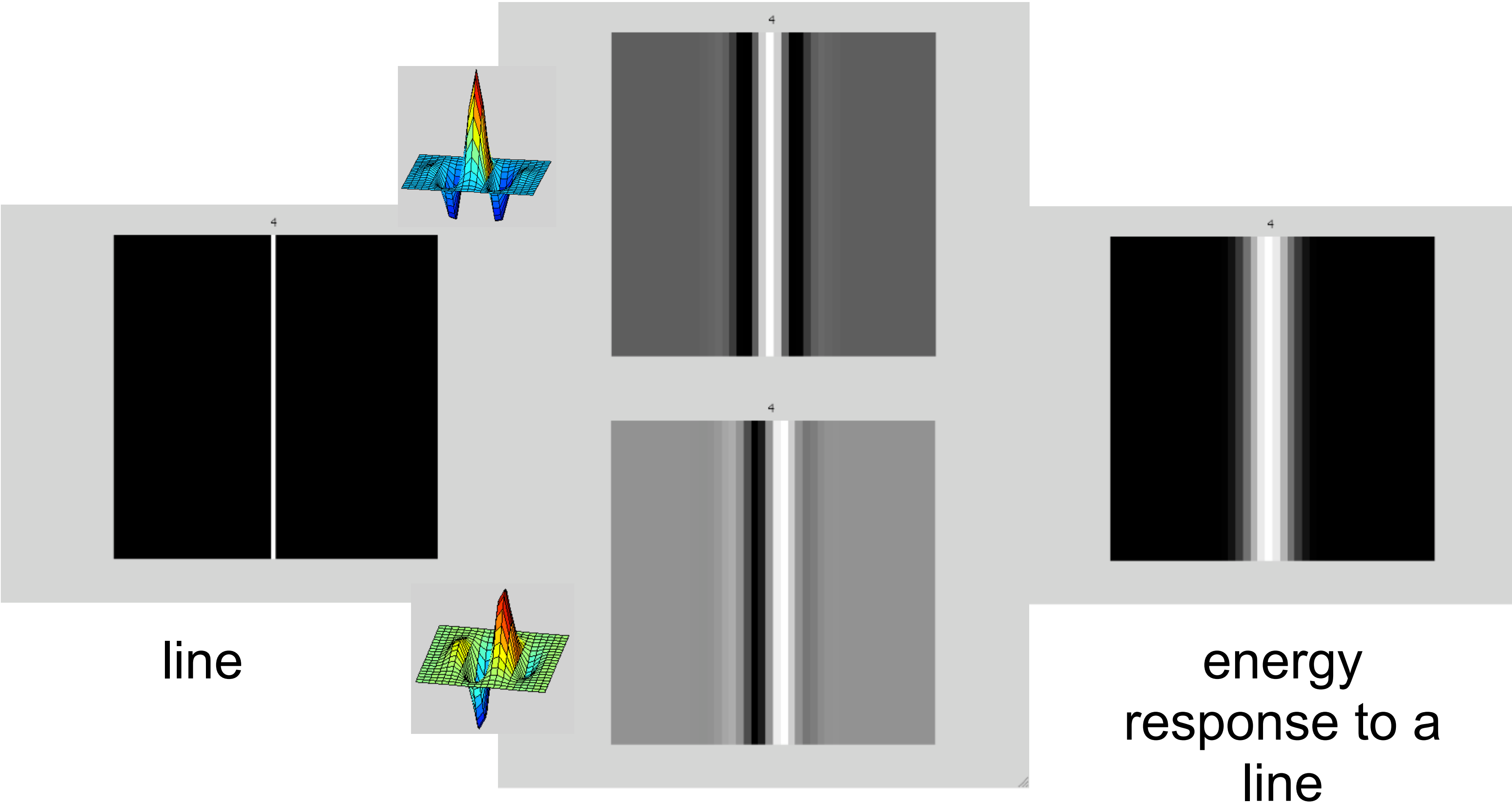
A quadrature filter is a complex filter whose real part is related to its imaginary part via a Hilbert transform along a particular axis through origin of the frequency domain.





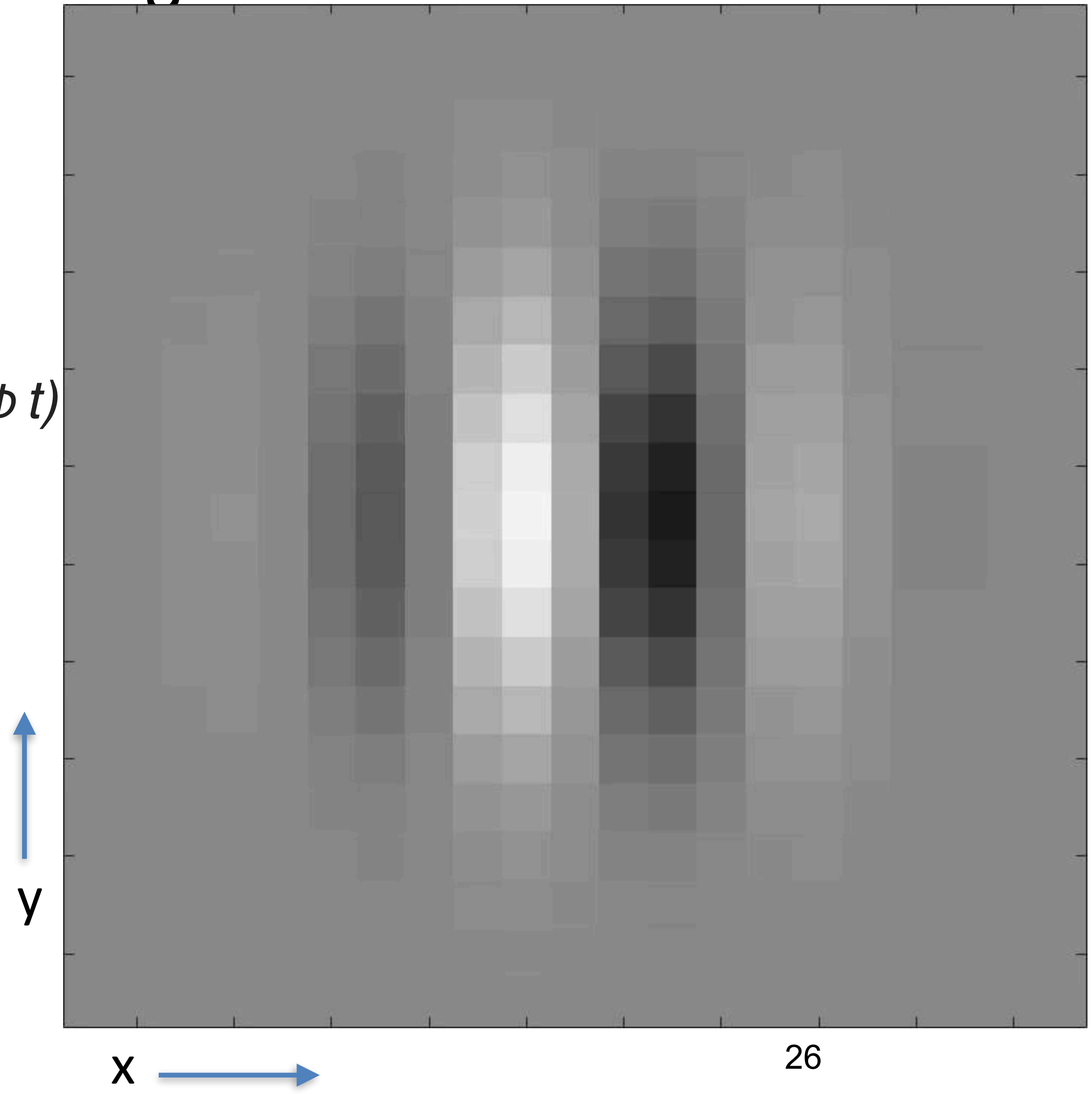
edge

energy response to an edge


line
energy response to a line

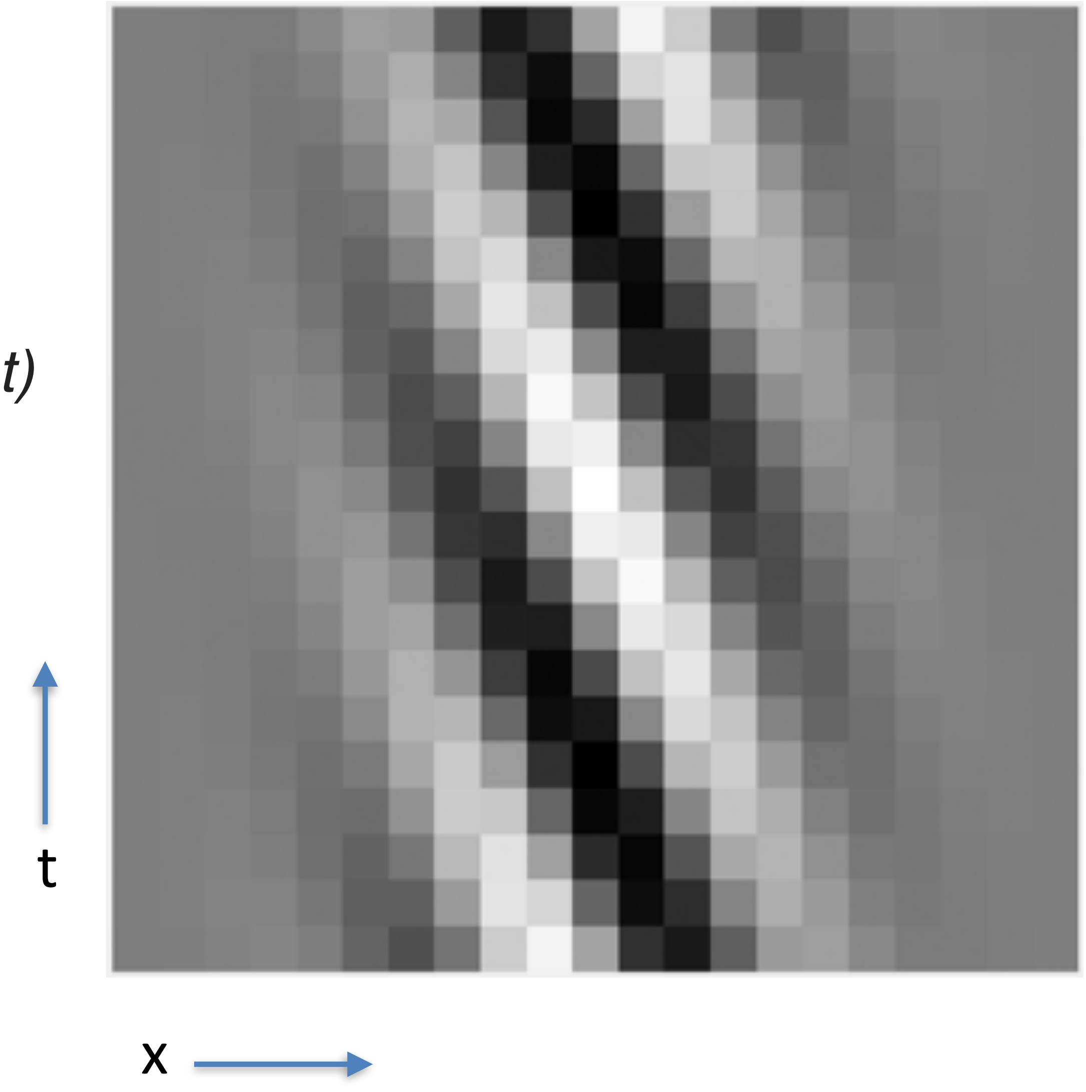
Using phase changes of local Gabor filters to analyze or generate motion

$$\psi_c(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \cos(2\pi u_0 x + \phi t)$$

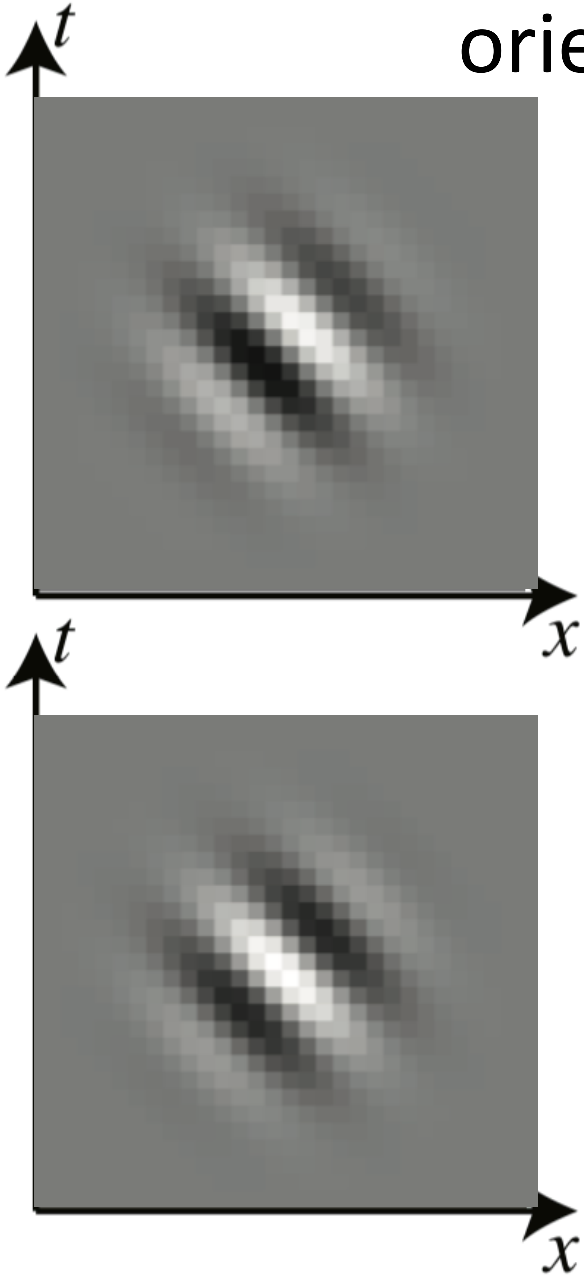


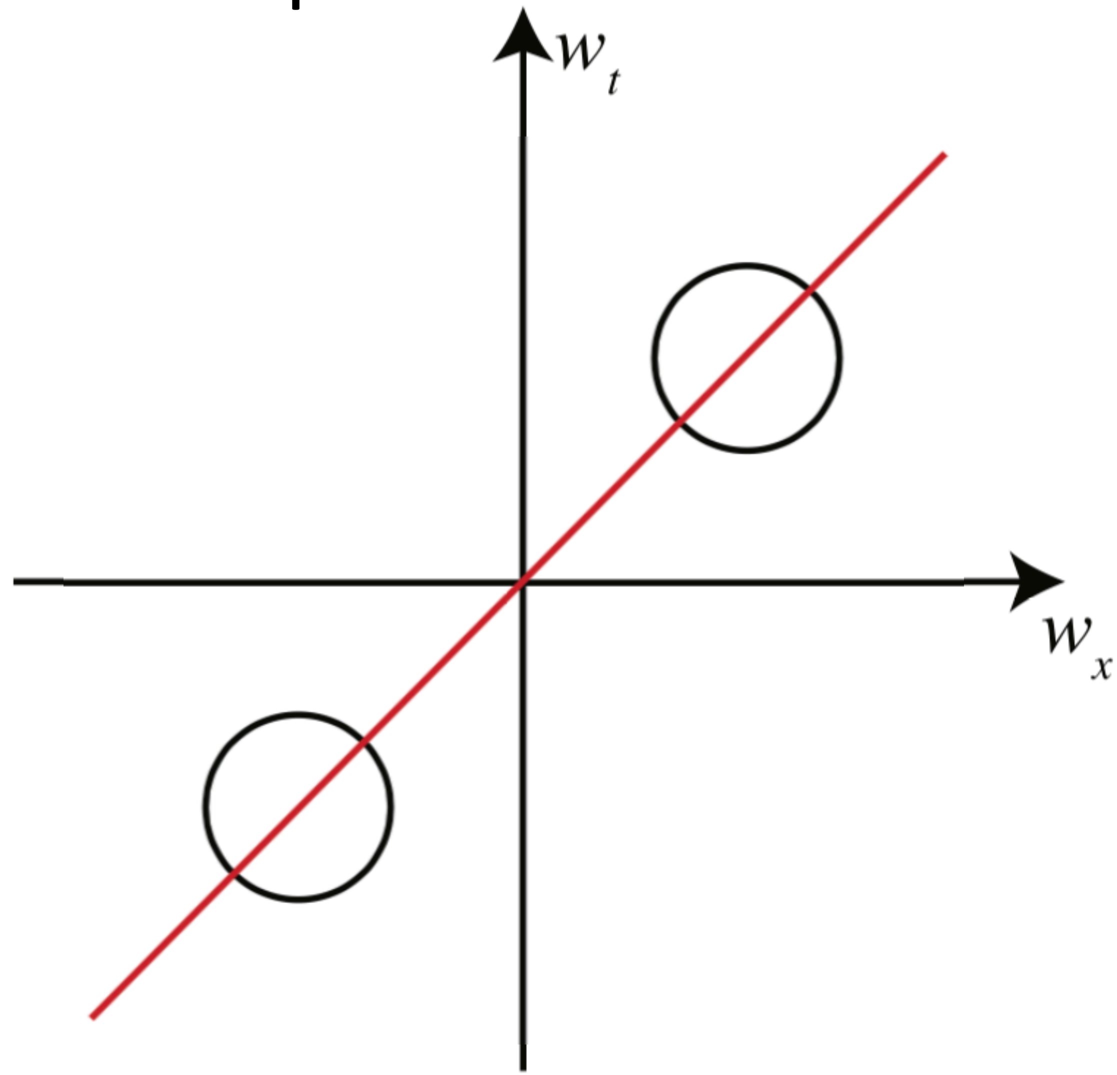
Space-time plot of the a slice through the patio-temporal filter of the previous slide

$$\psi_c(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \cos(2\pi u_0 x + \phi\, t)$$



Gabor filters for analyzing motion as orientation in space-time





Remember: uniform motion at a particular speed and direction means the spatio-temporal Fourier transform of the local patch is non-zero only along a particular plane in the frequency domain.

Gabor filters for analyzing motion

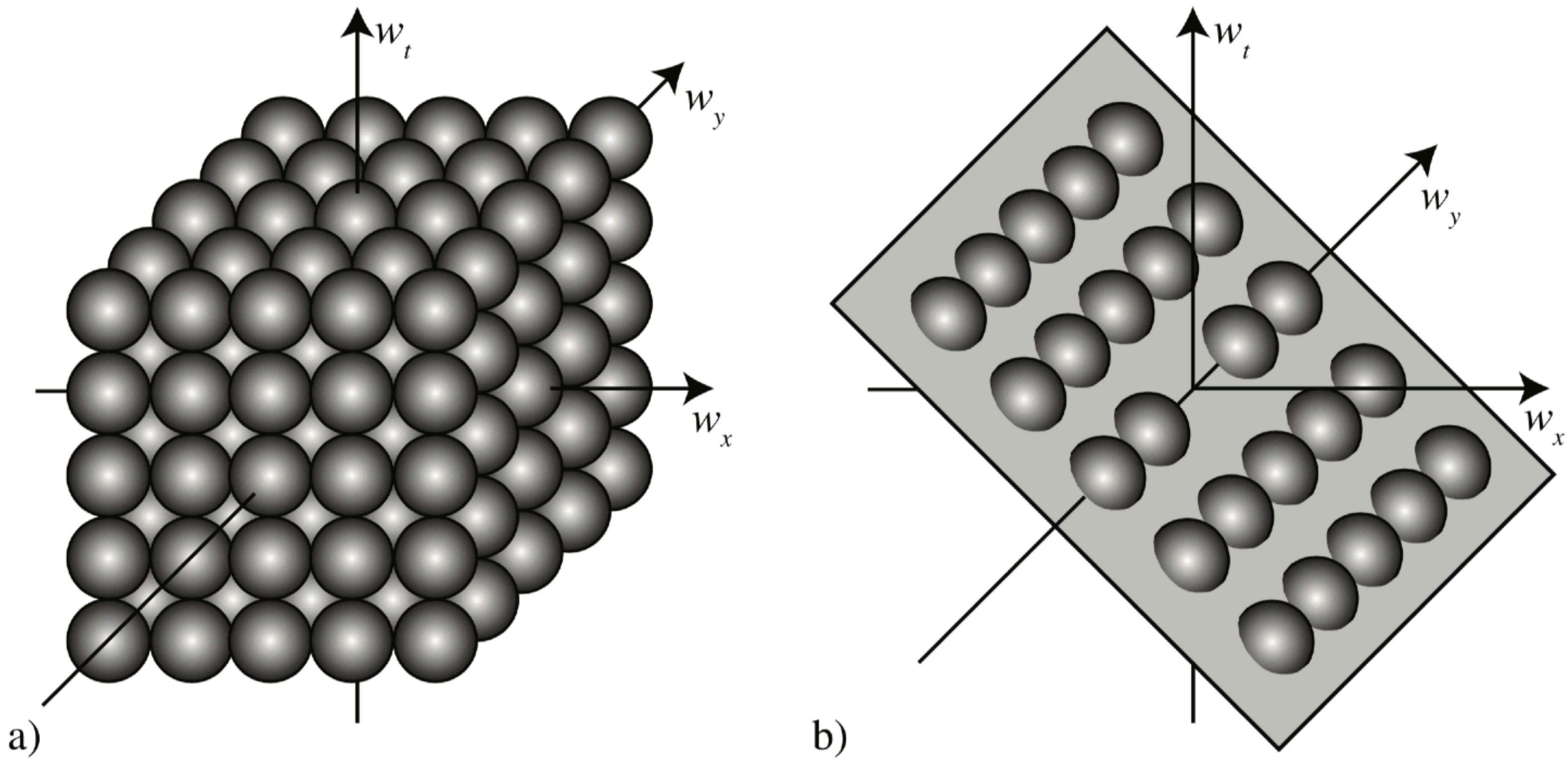


Figure 1.9: a) Space-time Gabor filters tiles. b) Set of Gabor filters selective to a particular velocity.

Motion without movement

SIGGRAPH '91 Las Vegas, 28 July-2 August 1991

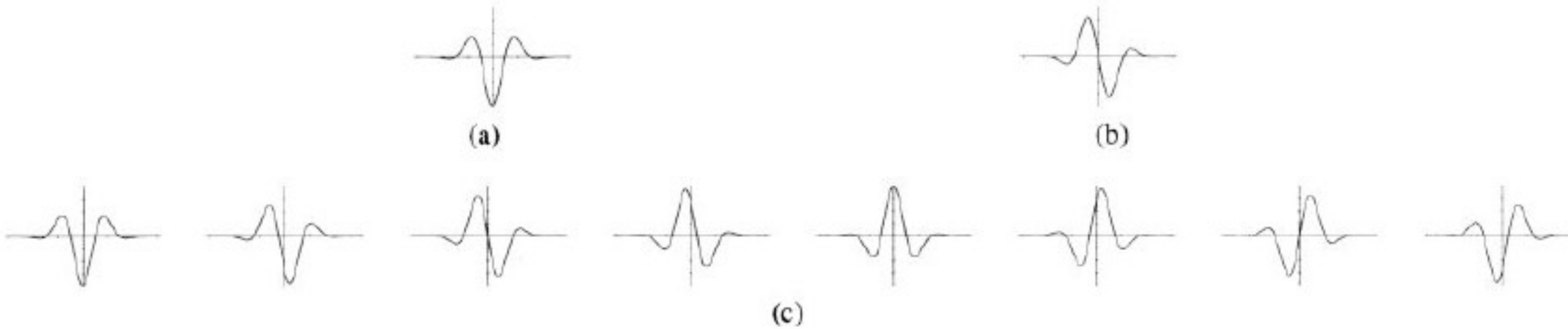


Figure 1: 1-d cross-sections of filters. (a) Even phase (G_2). (b) Odd phase (H_2). (c) Filters modulated in phase according to Eq. (1). Note the apparent rightward motion of the filter ripples.

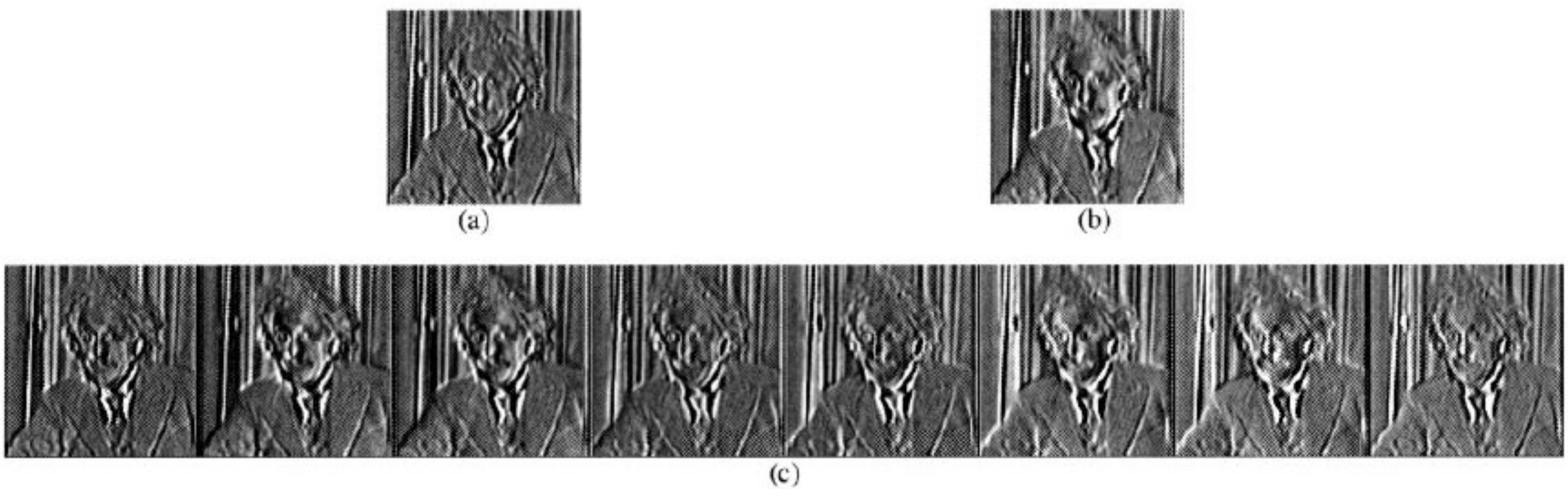
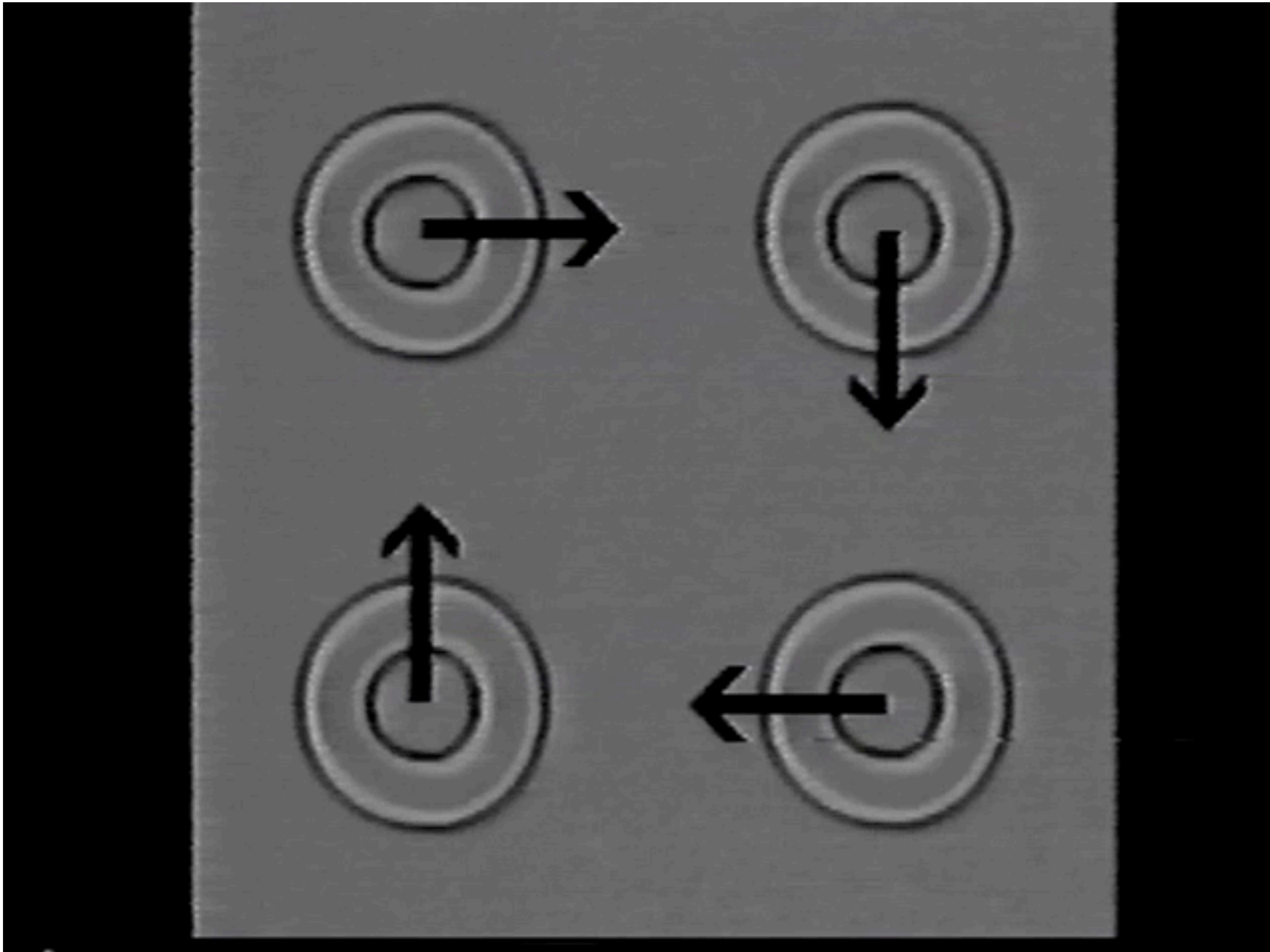


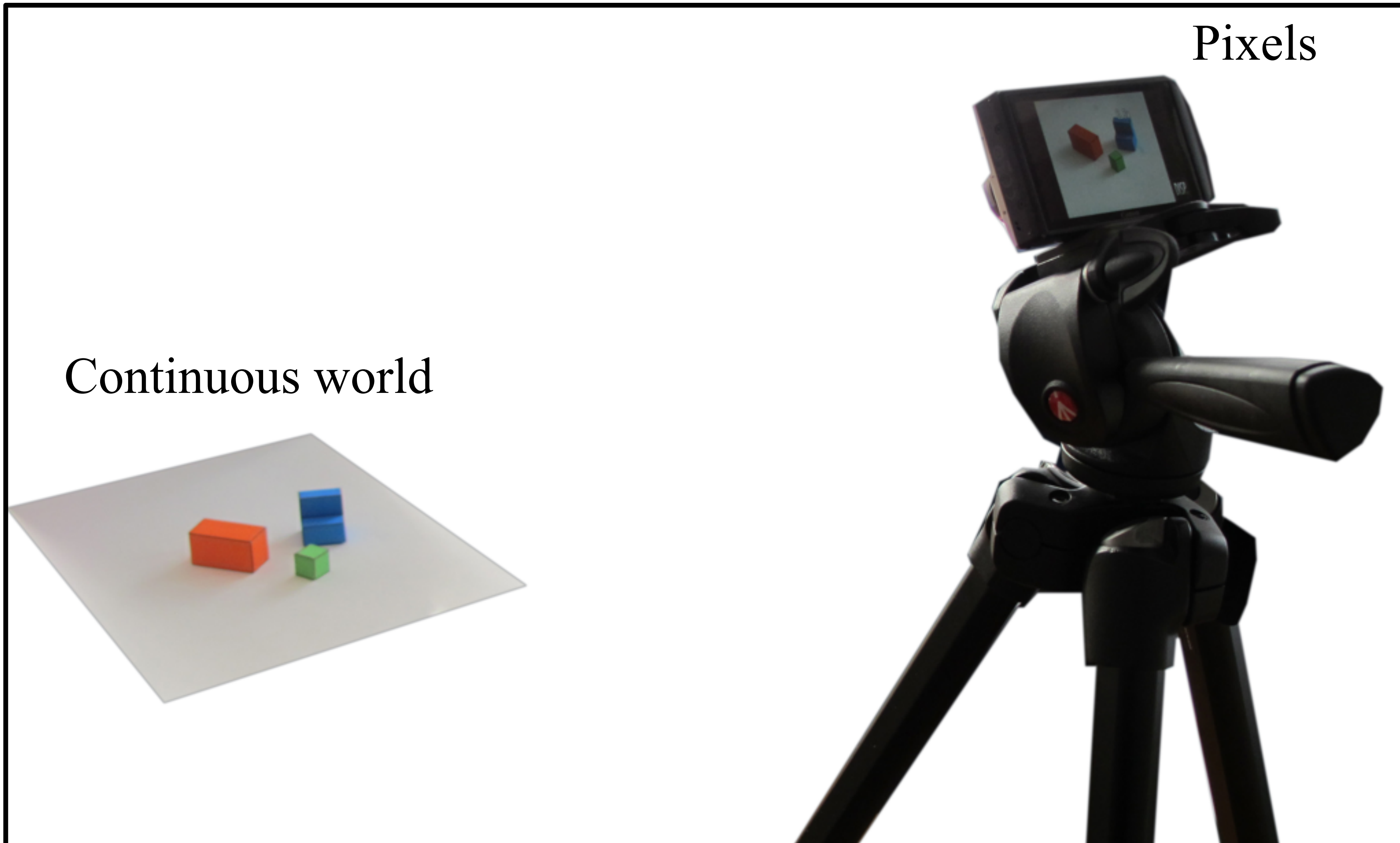
Figure 2: (a) and (b): G_2 and H_2 filters were applied to an image of Einstein. (c) Images modulated as in Eq. (1). When viewed as a temporal sequence, this generates the perception of rightward motion, yet image remains stationary.

Motion without movement

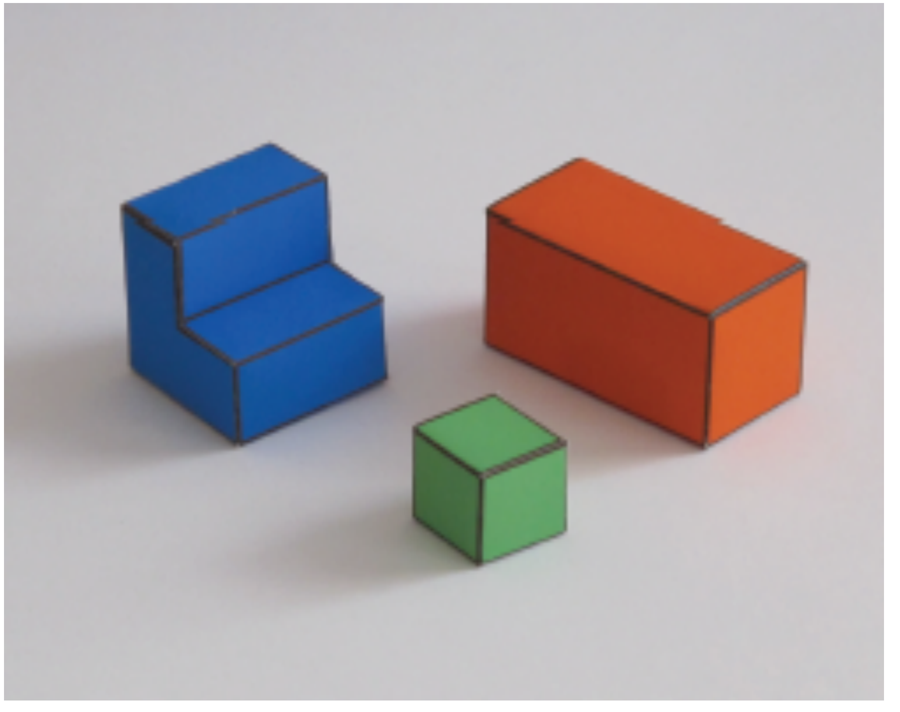


Sampling

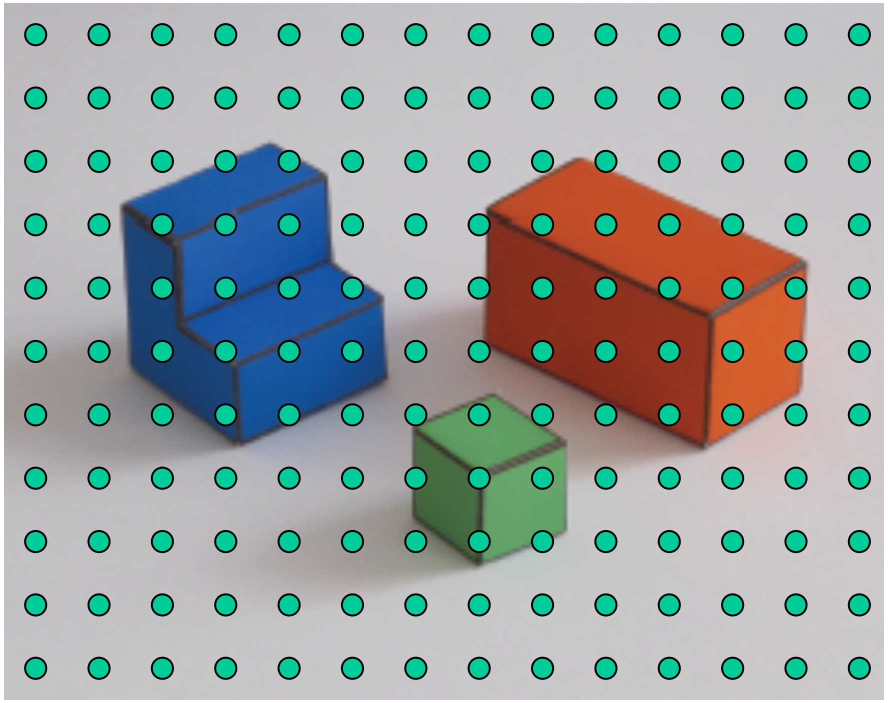
Sampling



Sampling



Sampling

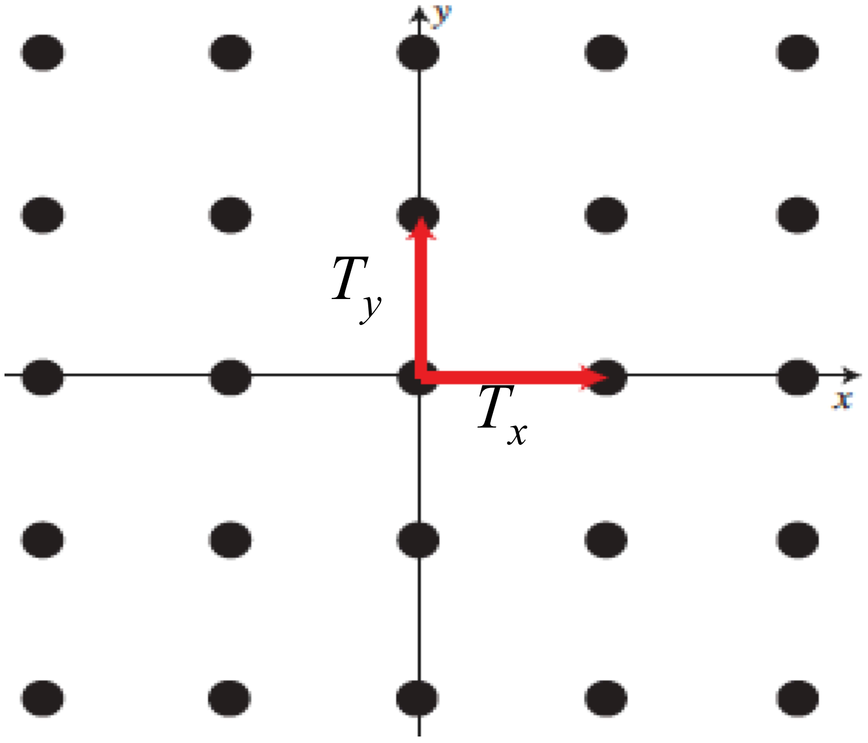


Sampling

Continuous image $f(x, y)$

We can sample it using a rectangular grid as

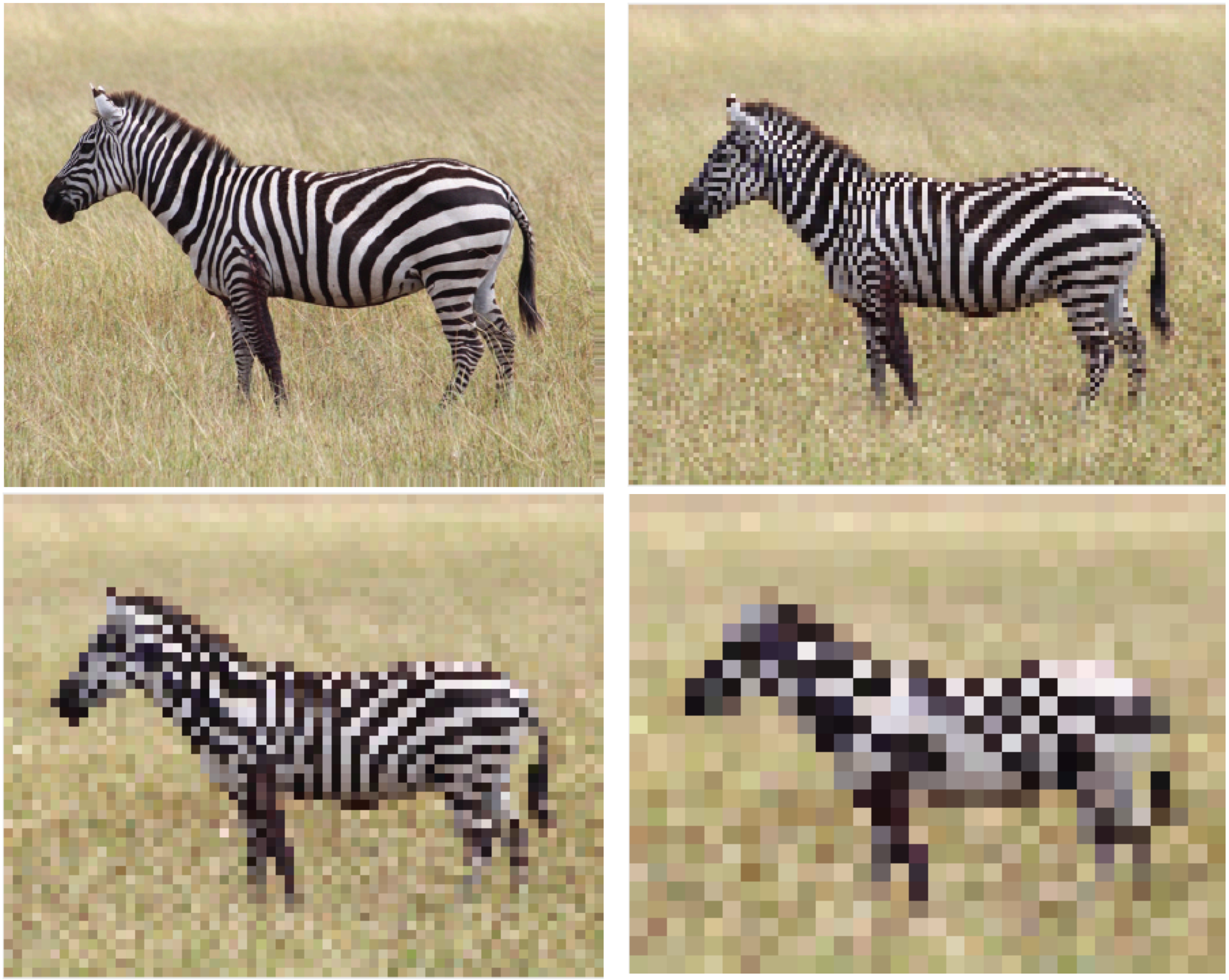
$$f[n,m] = f\left(nT_x, mT_y\right)$$



Aliasing

Let's start with this continuous image (it is not really continuous…)

Aliasing



103×128

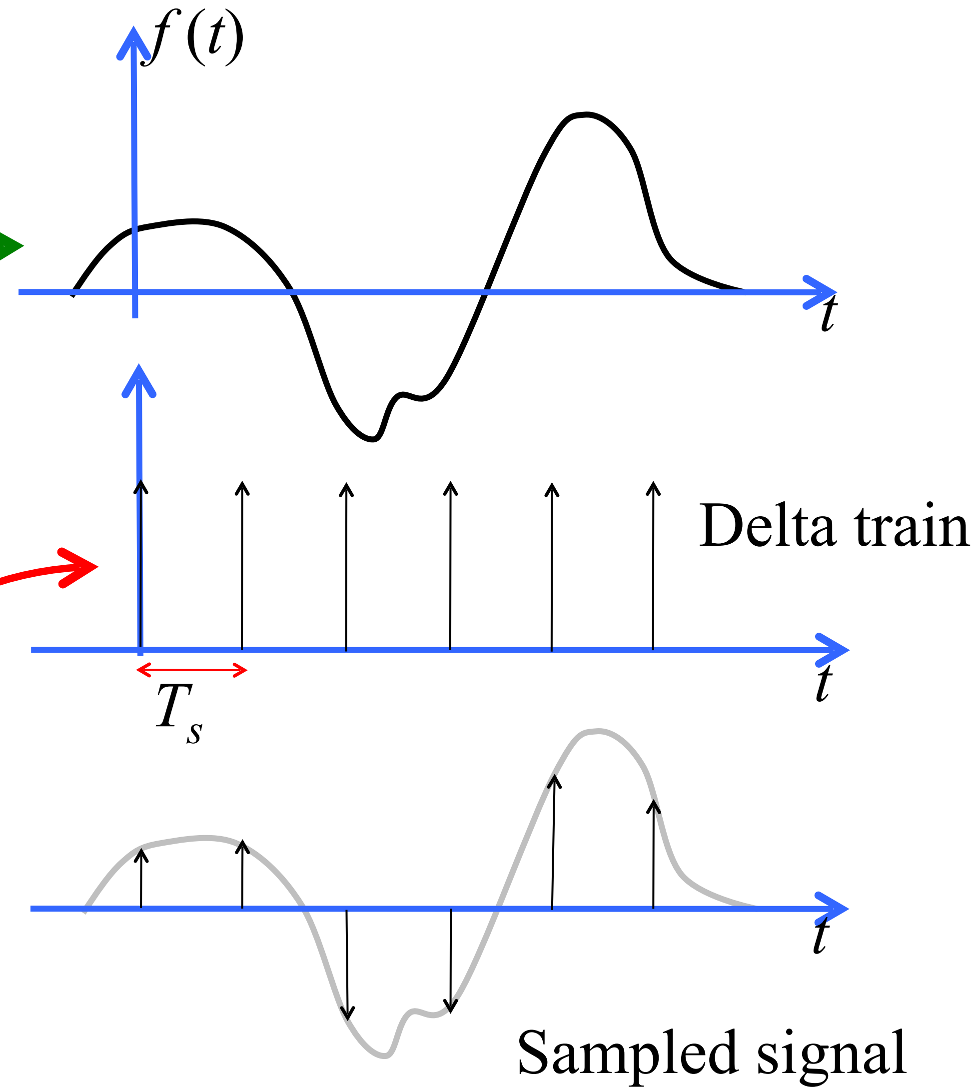
52×64

26×32

Modeling the sampling process

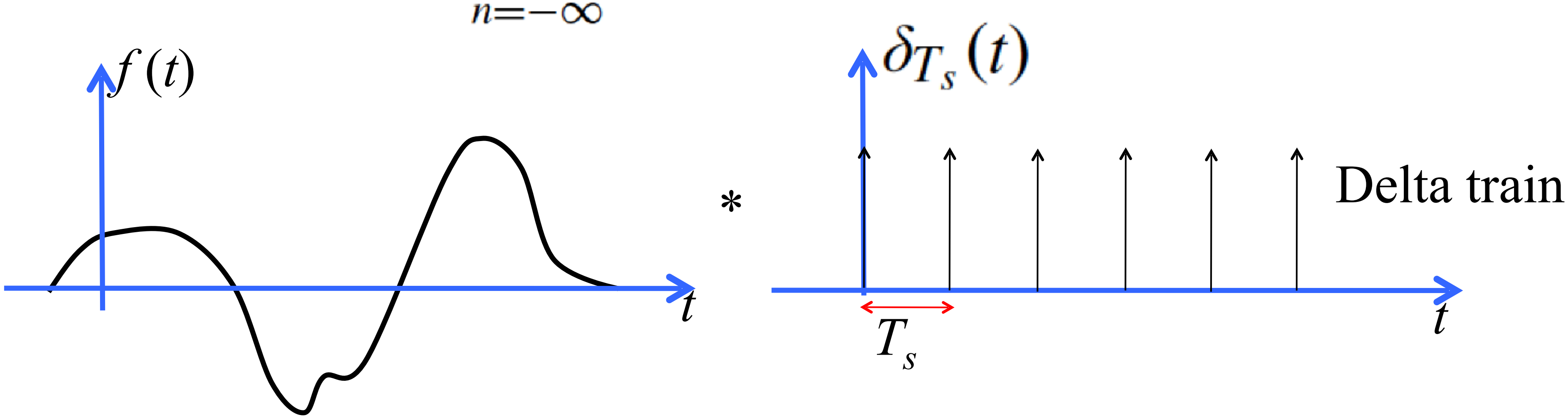
$$f[n] = f(nT_s)$$

$$\hat{f}(t) = f(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$



Modeling the sampling process

$$\hat{f}(t) = f(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = f(t)\delta_{T_s}(t)$$

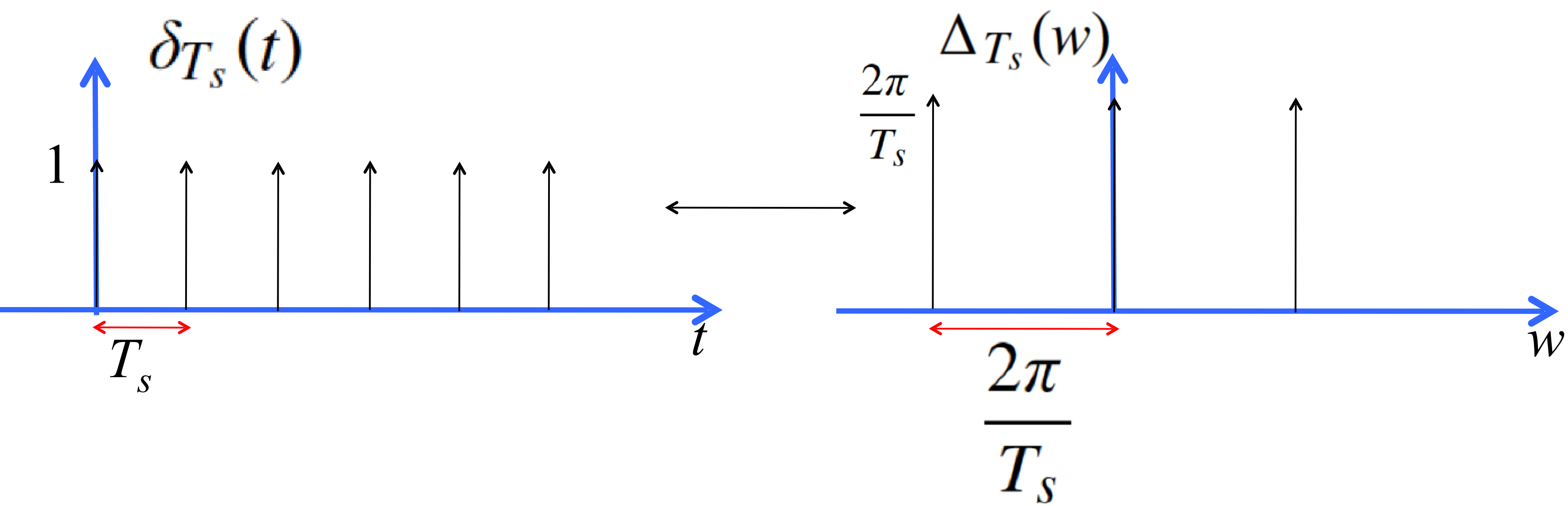


The Fourier transform is a convolution…

Interesting property of the delta train: the Fourier transform of a delta train of period T is another delta train with period 2π/T

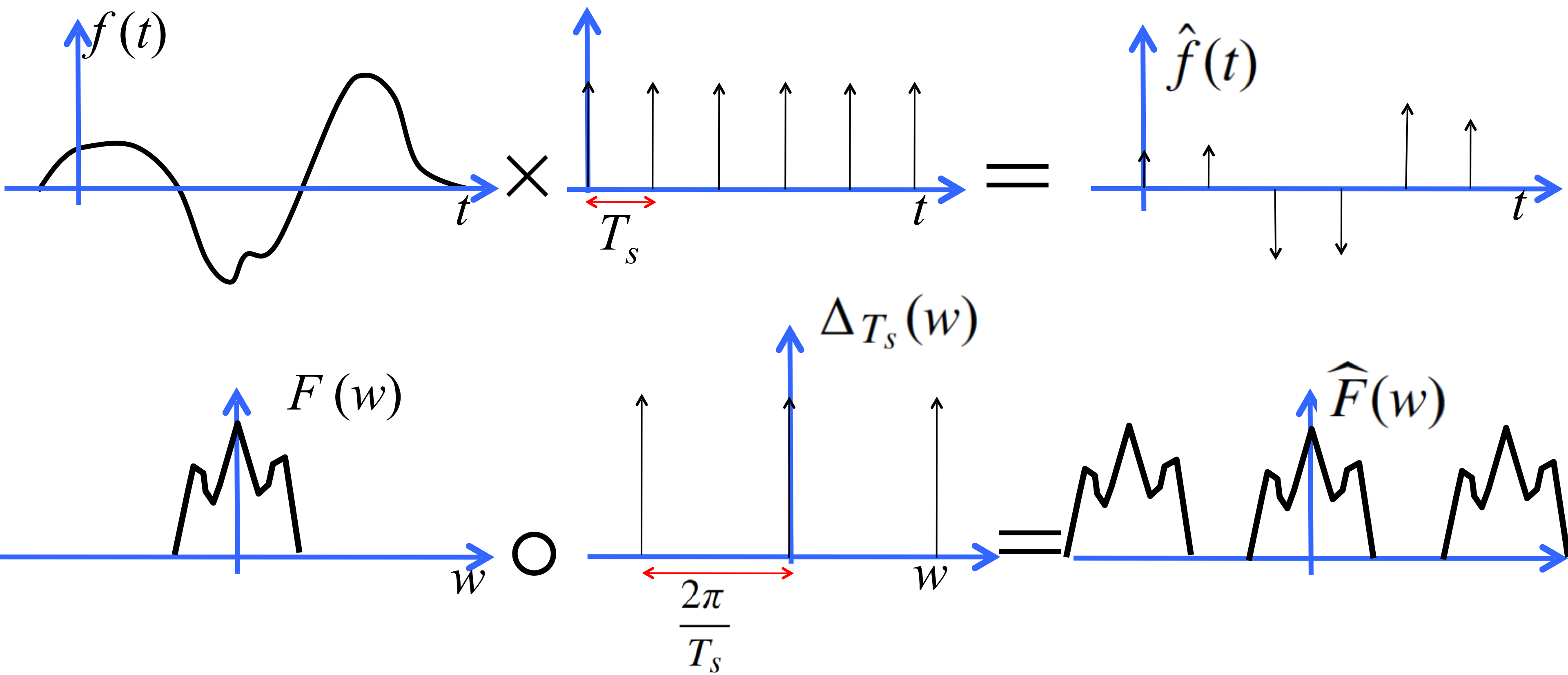
Modeling the sampling process

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \longleftrightarrow \Delta_{T_s}(w) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta\left(w - k\frac{2\pi}{T_s}\right)$$

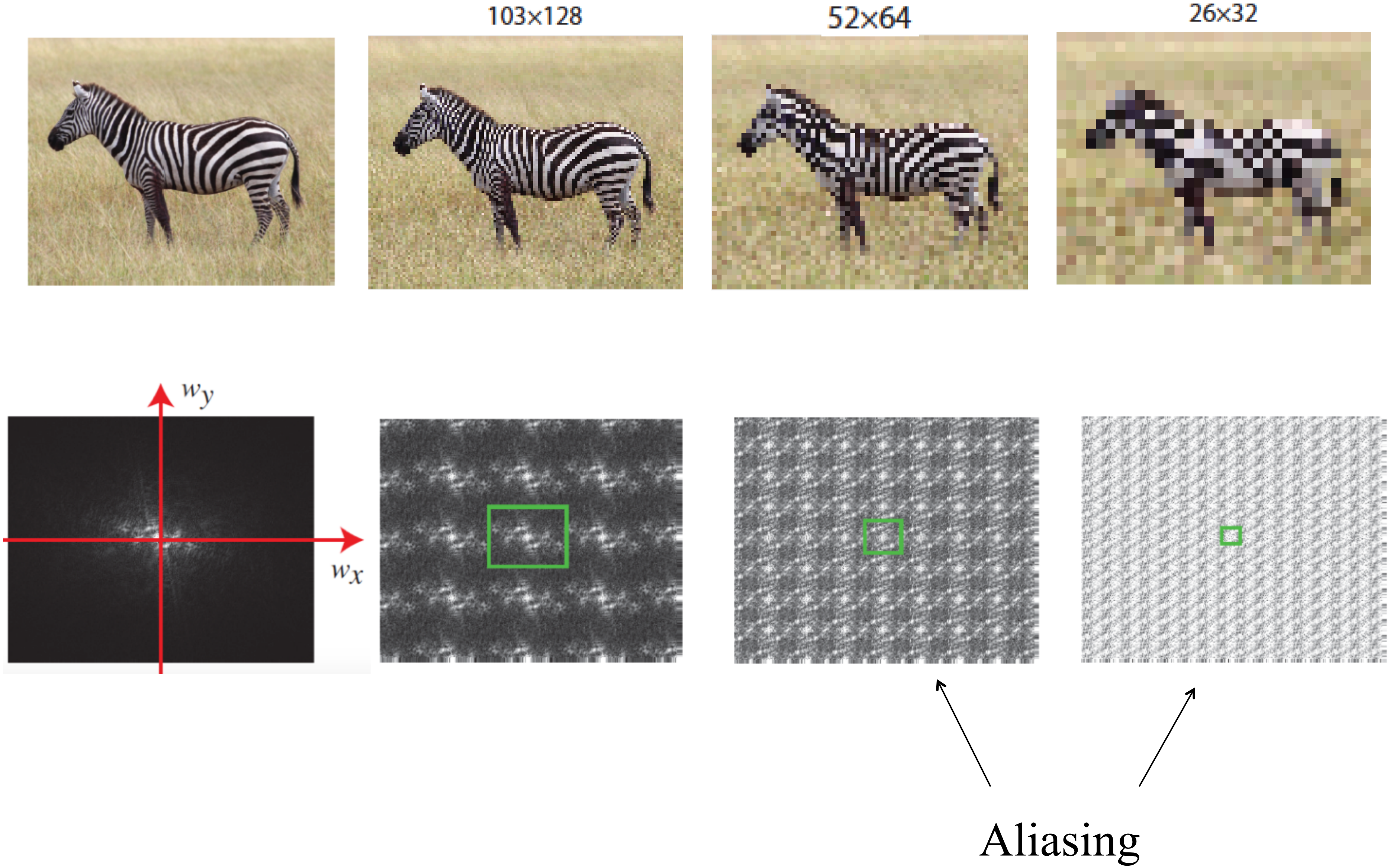


Interesting property of the delta train: the Fourier transform of a delta train of period T is another delta train with period $2\pi/T$.

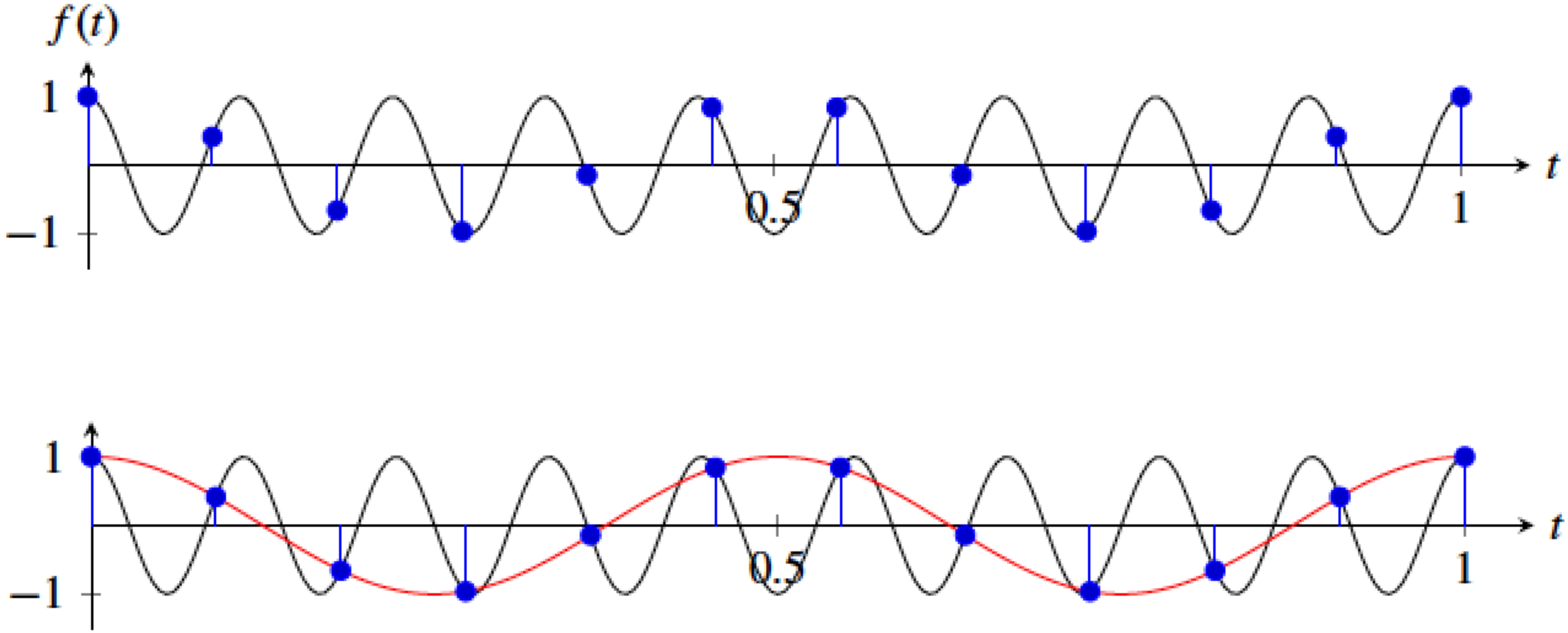
Modeling the sampling process



What happens when the repetitions overlap?

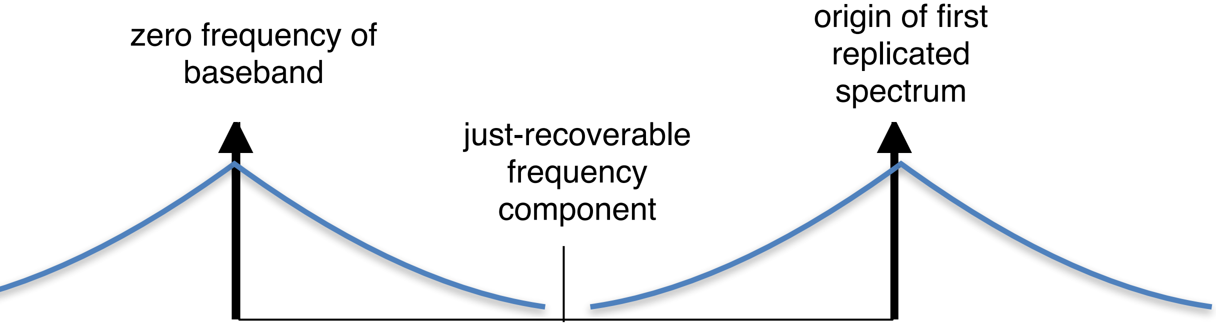

103×128
52×64
26×32
w_y
w_x
Aliasing

Aliasing

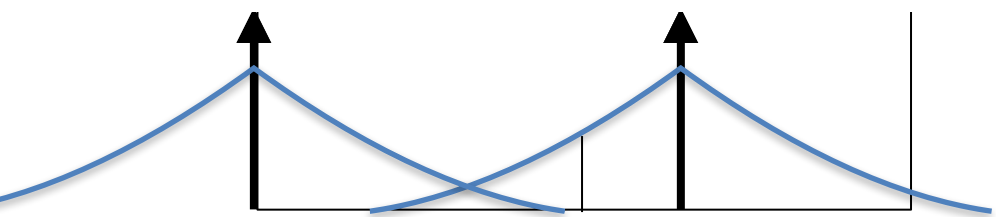


Both waves fit the same samples. Aliasing consists in "perceiving" the red wave when the actual input was the blue wave.

aliasing



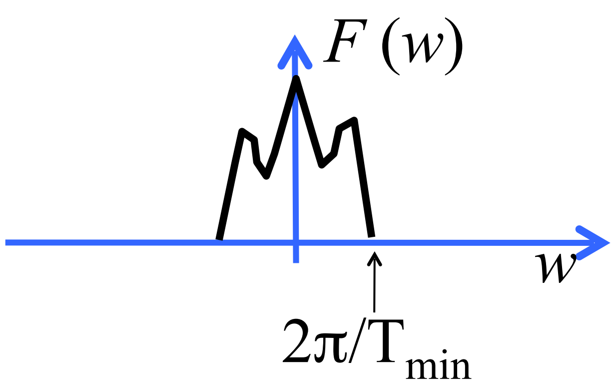
no aliasing



aliasing

Sampling theorem

The sampling theorem (also known as Nyquist theorem) states that for a signal to be perfectly reconstructed from it samples, the sampling period T_s has to be $T_s > T_{min}/2$ where T_{min} is the period of the highest frequency present in the input signal.



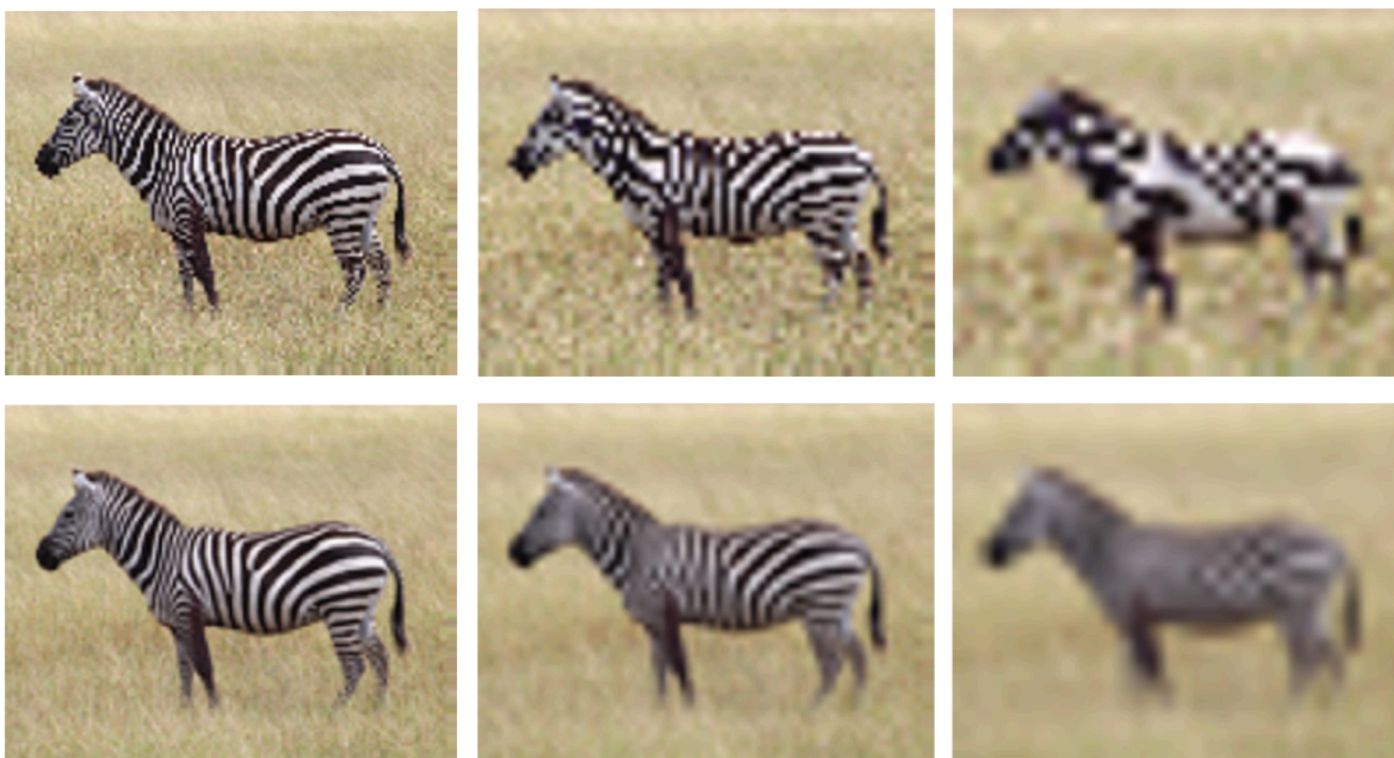
Antialising filtering

Before sampling, apply a low pass-filter to remove all the frequencies that will produce aliasing.

103×128 52×64 26×32

Without antialising filter.

With antialising filter.



Spatio-temporal sampling illusion

Evidence for filter-based analysis of motion in the human visual system shown via spatio-temporal visual illusion based on sampling

Two potential theories for how humans compute our motion perceptions:

(a) We match the pattern in the image that we see at one moment and compare it with what we see at subsequent times.
(b) We use spatio-temporal filters to measure spatio-temporal energy in order to measure local motion.

This illusion favors one theory over the other.

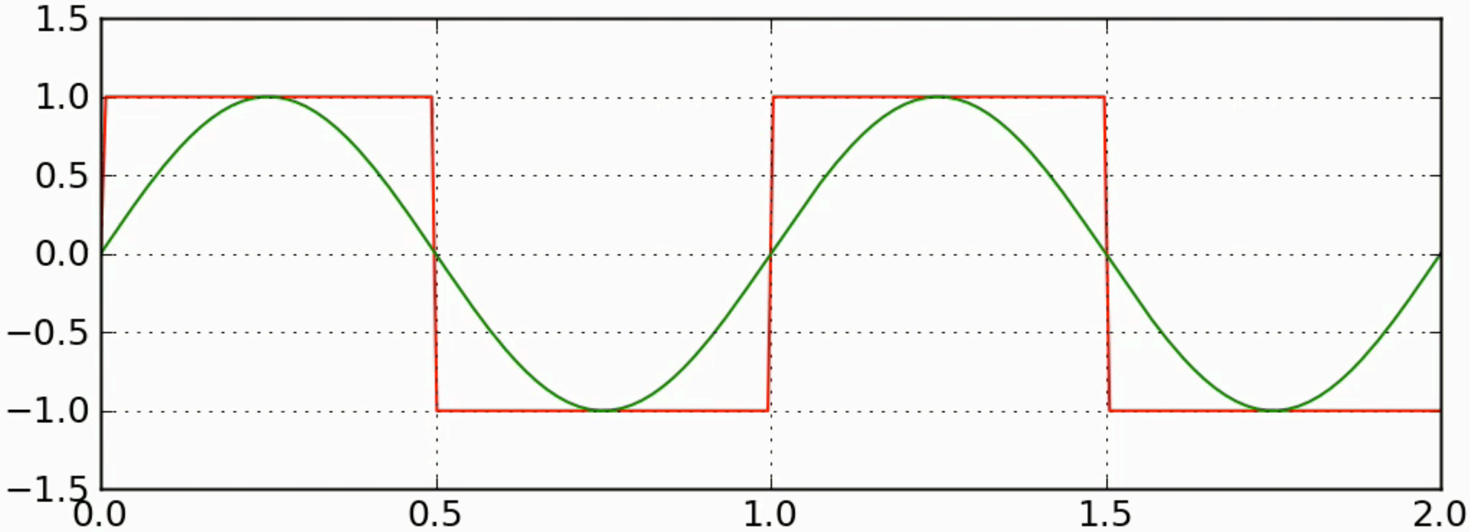
Square wave Fourier components

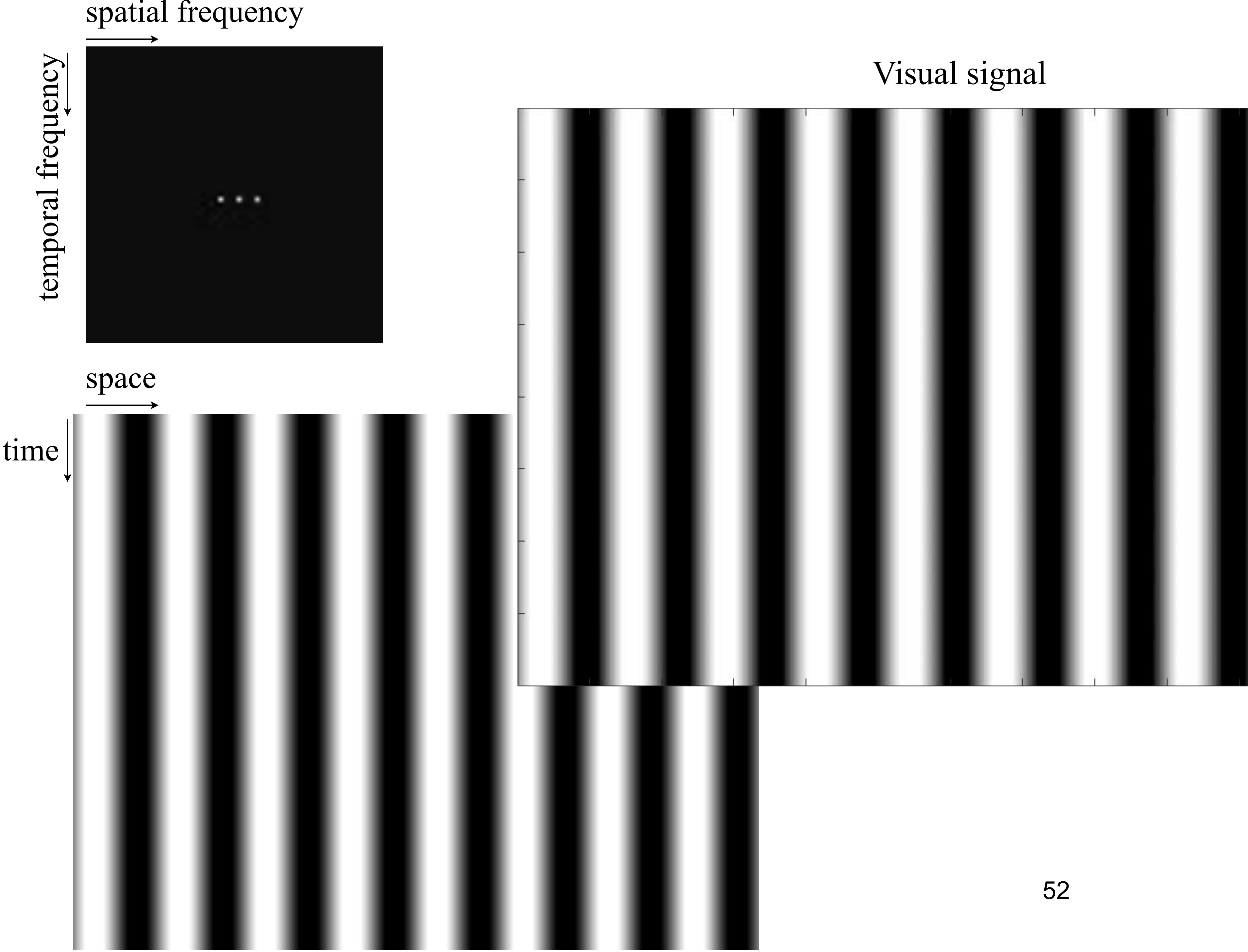
Using Fourier series we can write an ideal square wave as an infinite series of the form

$$x_{\mathrm{square}}(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left((2k-1)2\pi f t\right)}{(2k-1)}$$

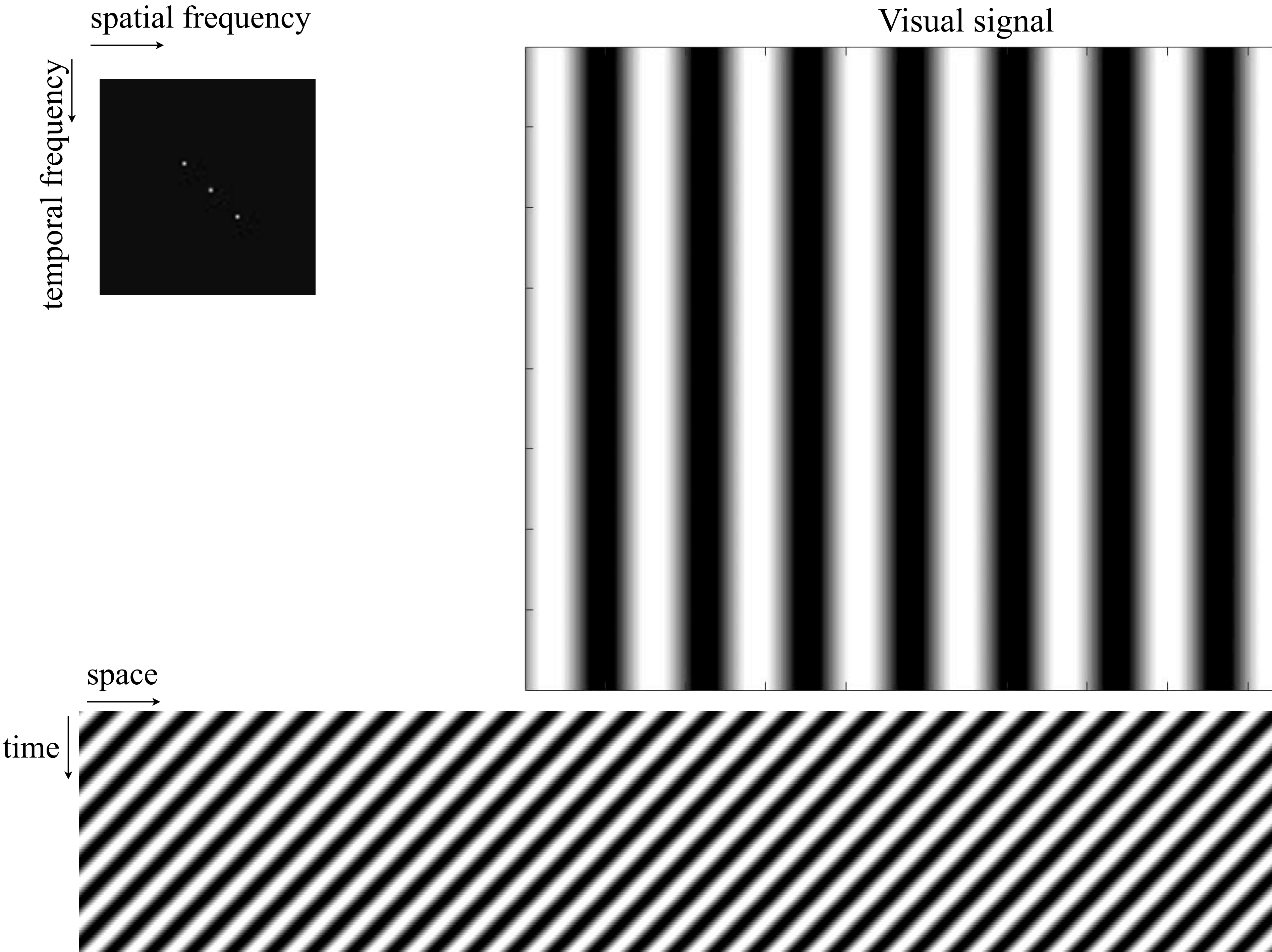
$$= \frac{4}{\pi}\left(\sin(2\pi f t) + \frac{1}{3}\sin(6\pi f t) + \frac{1}{5}\sin(10\pi f t) + \cdots\right).$$

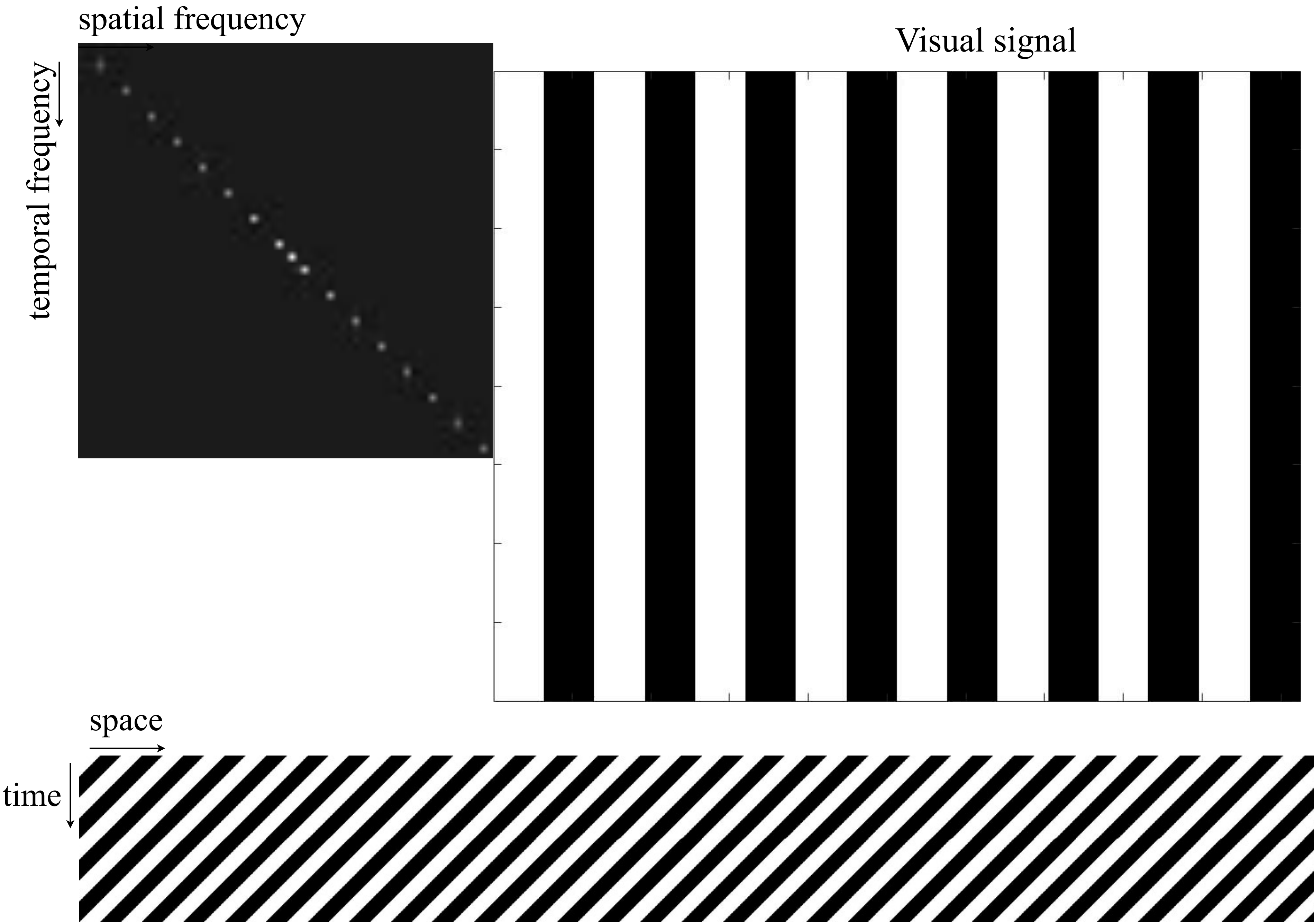
A square wave is an infinite sum of sinusoids





spatial frequency
temporal frequency
space
time
Visual signal


spatial frequency
temporal frequency
Visual signal
space
time



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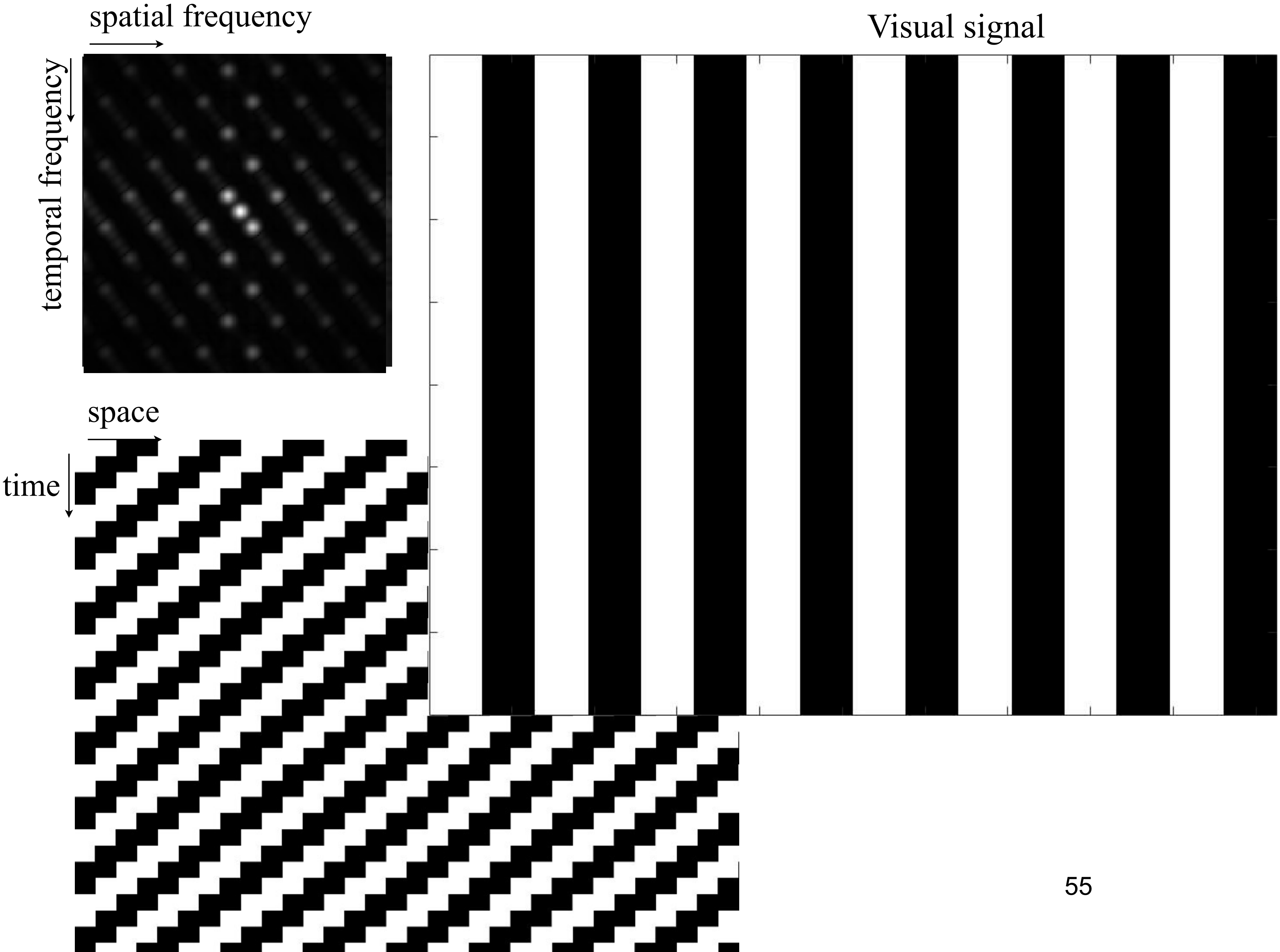
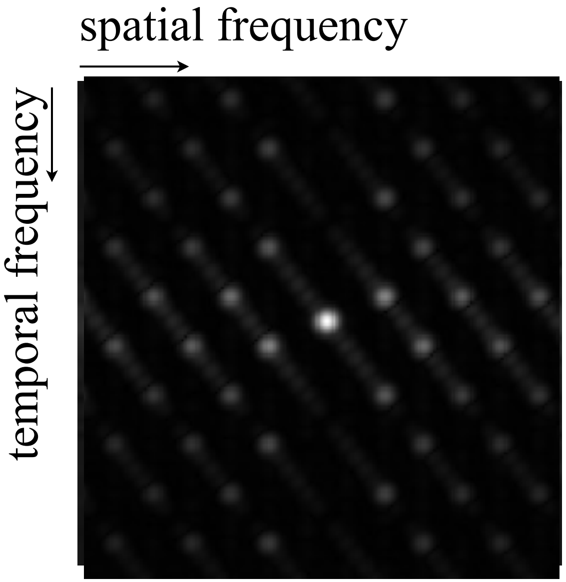
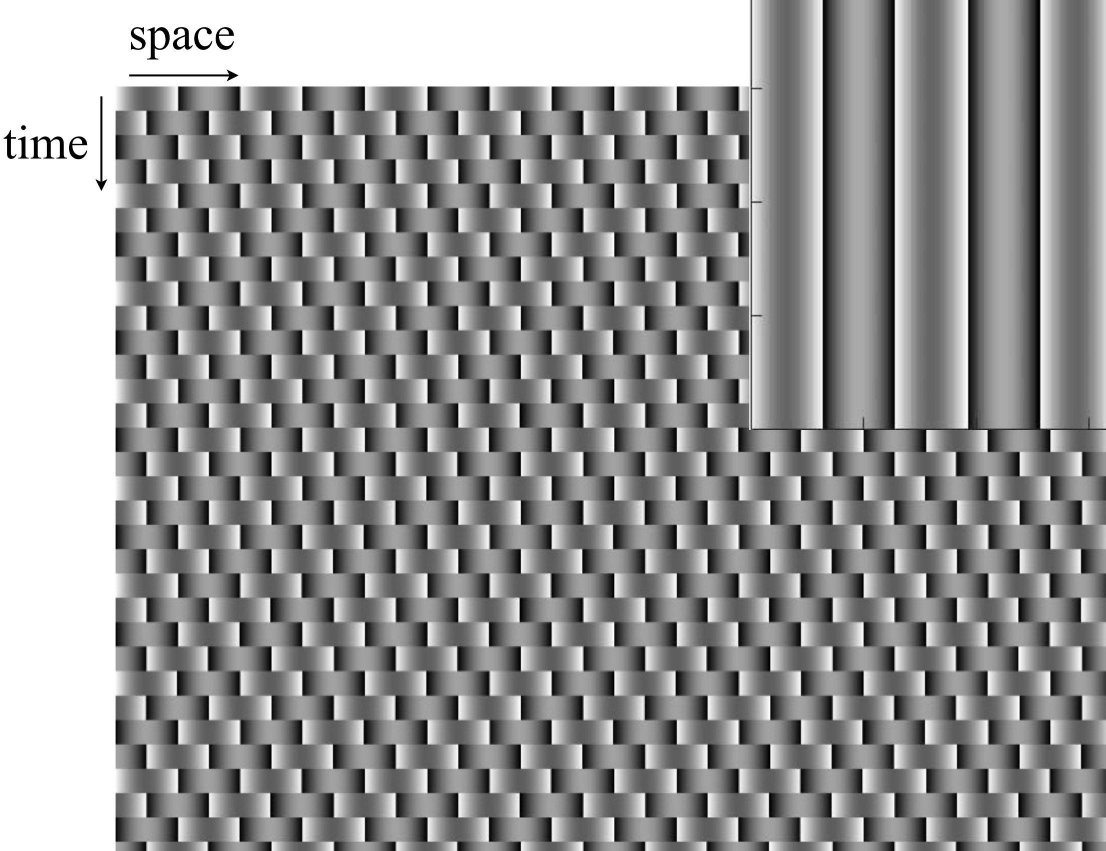
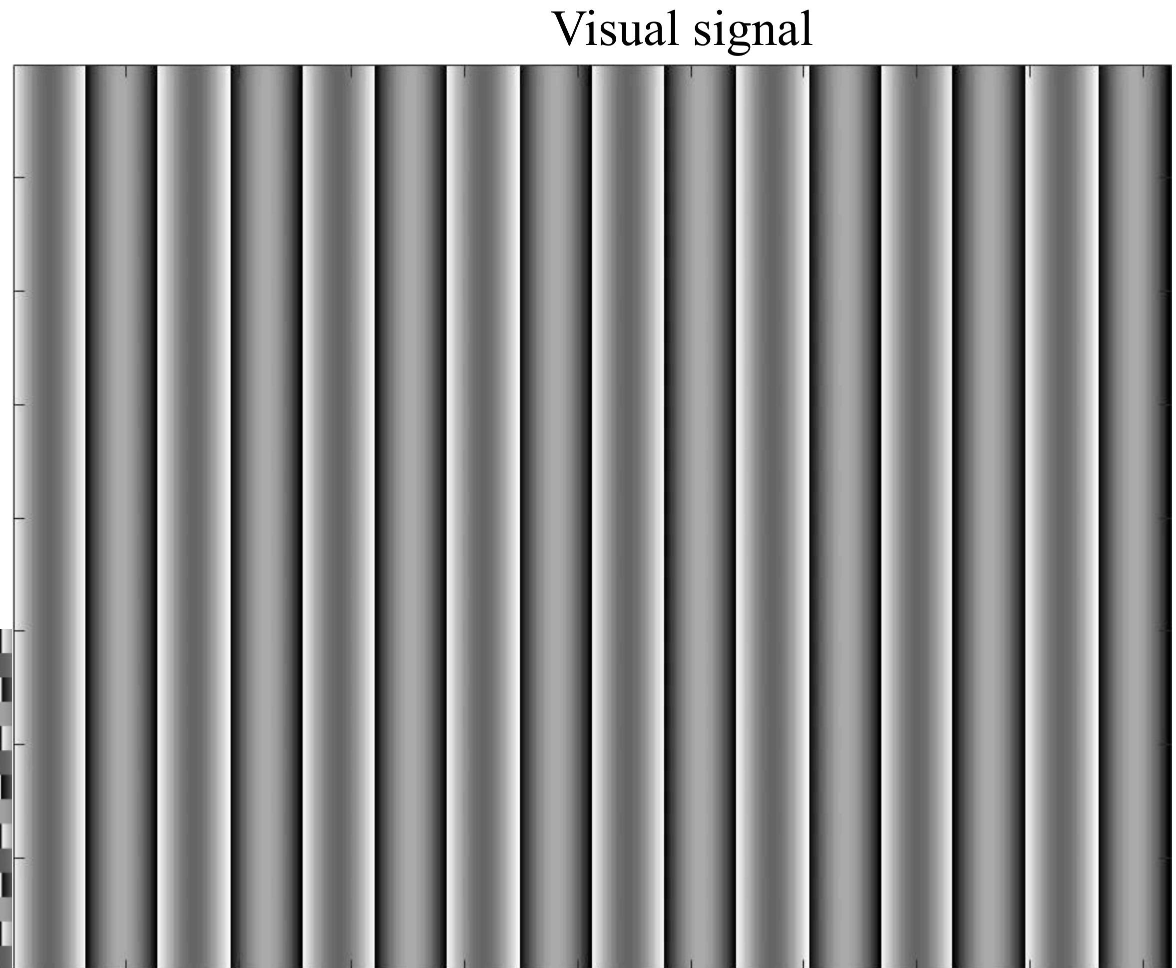
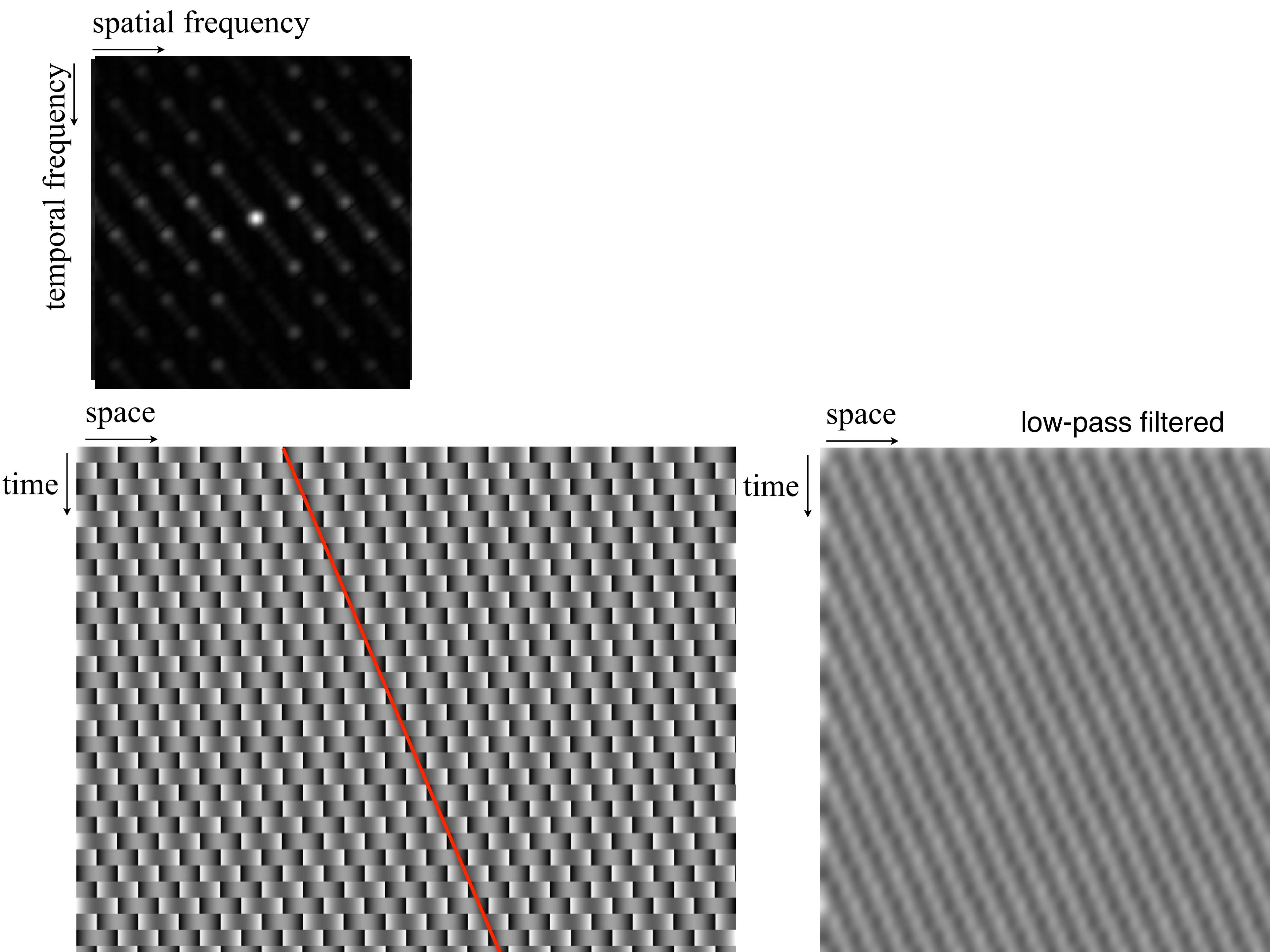

spatial frequency
temporal frequency
Visual signal
space
time

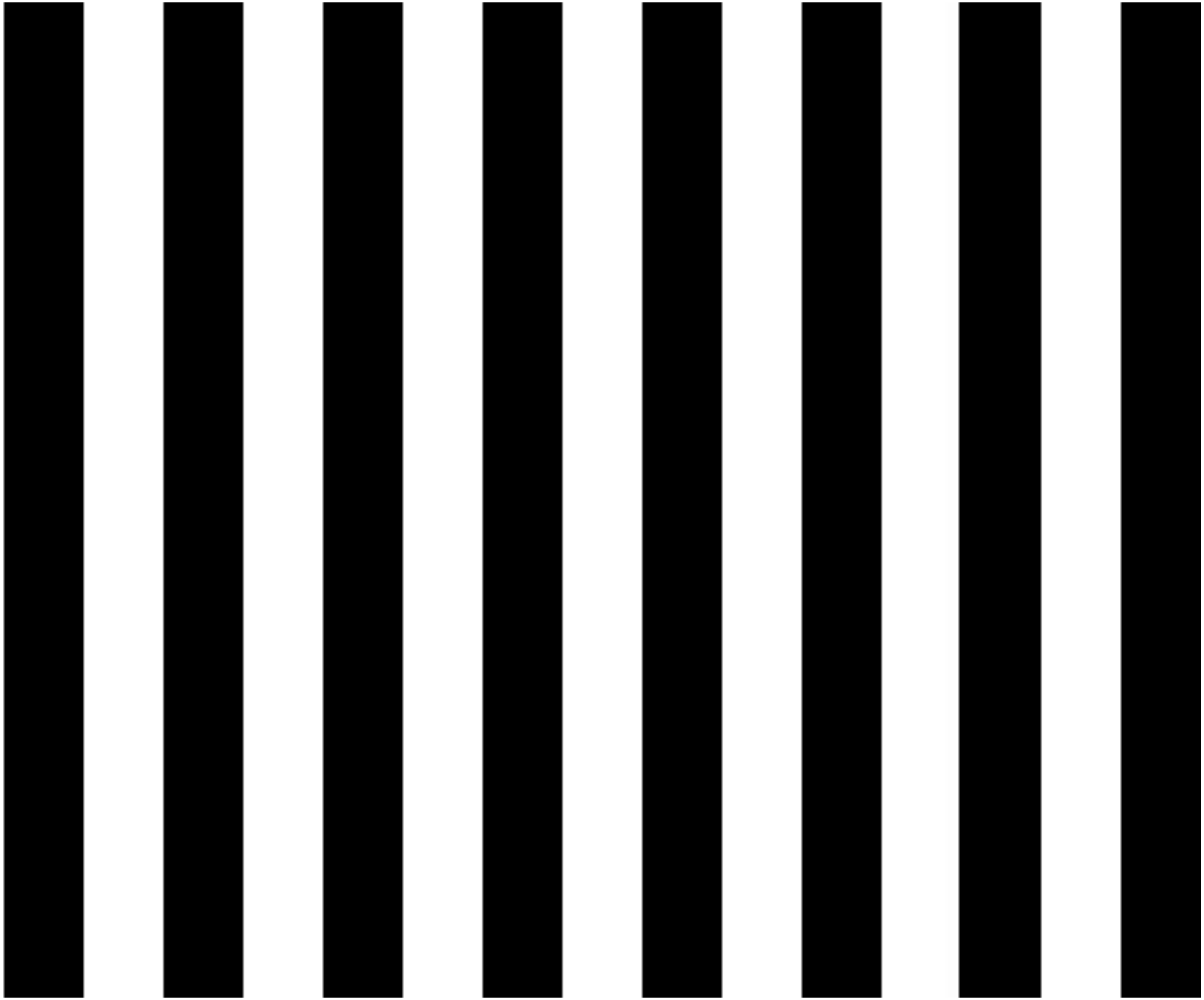

spatial frequency
temporal frequency

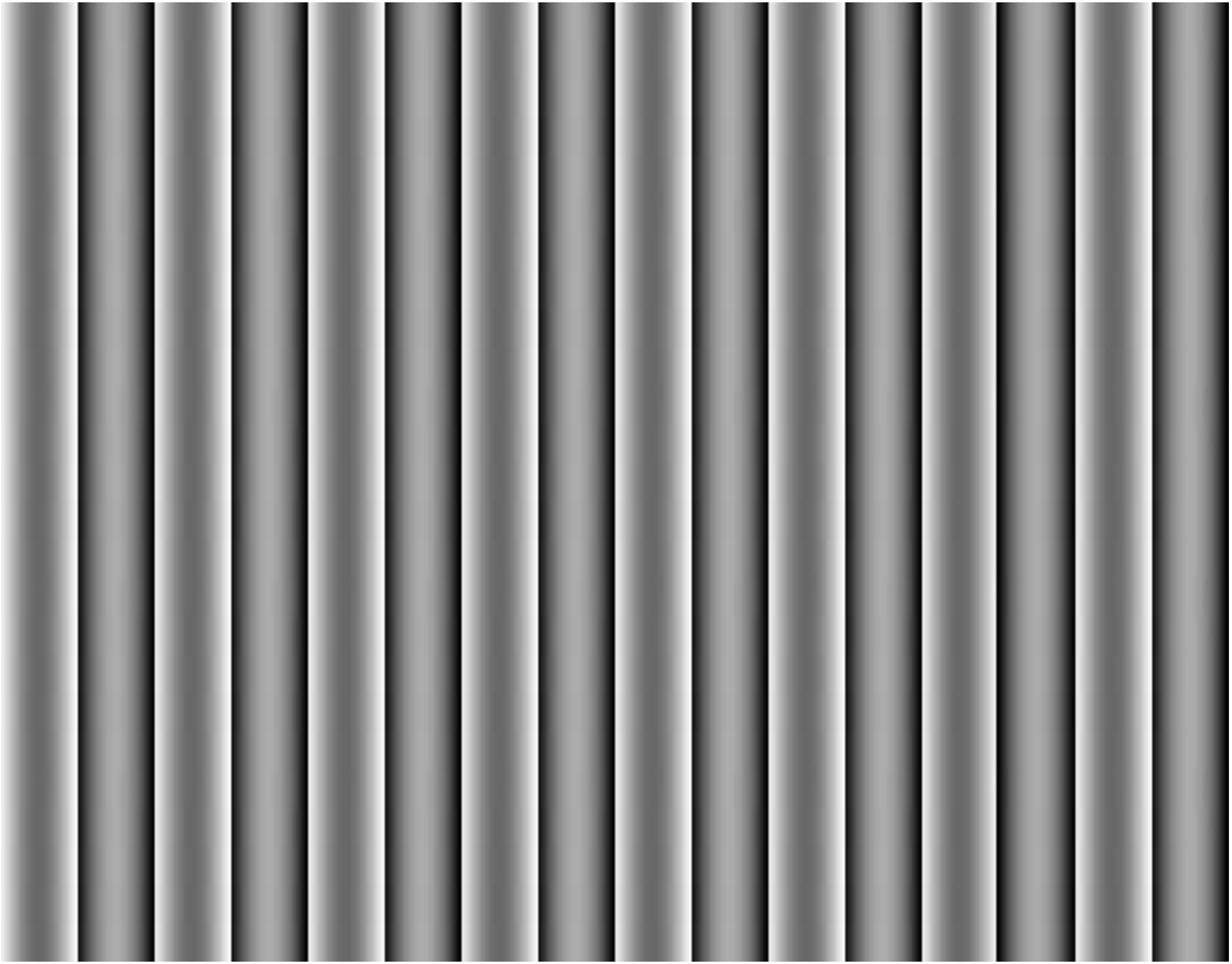

space
time


Visual signal

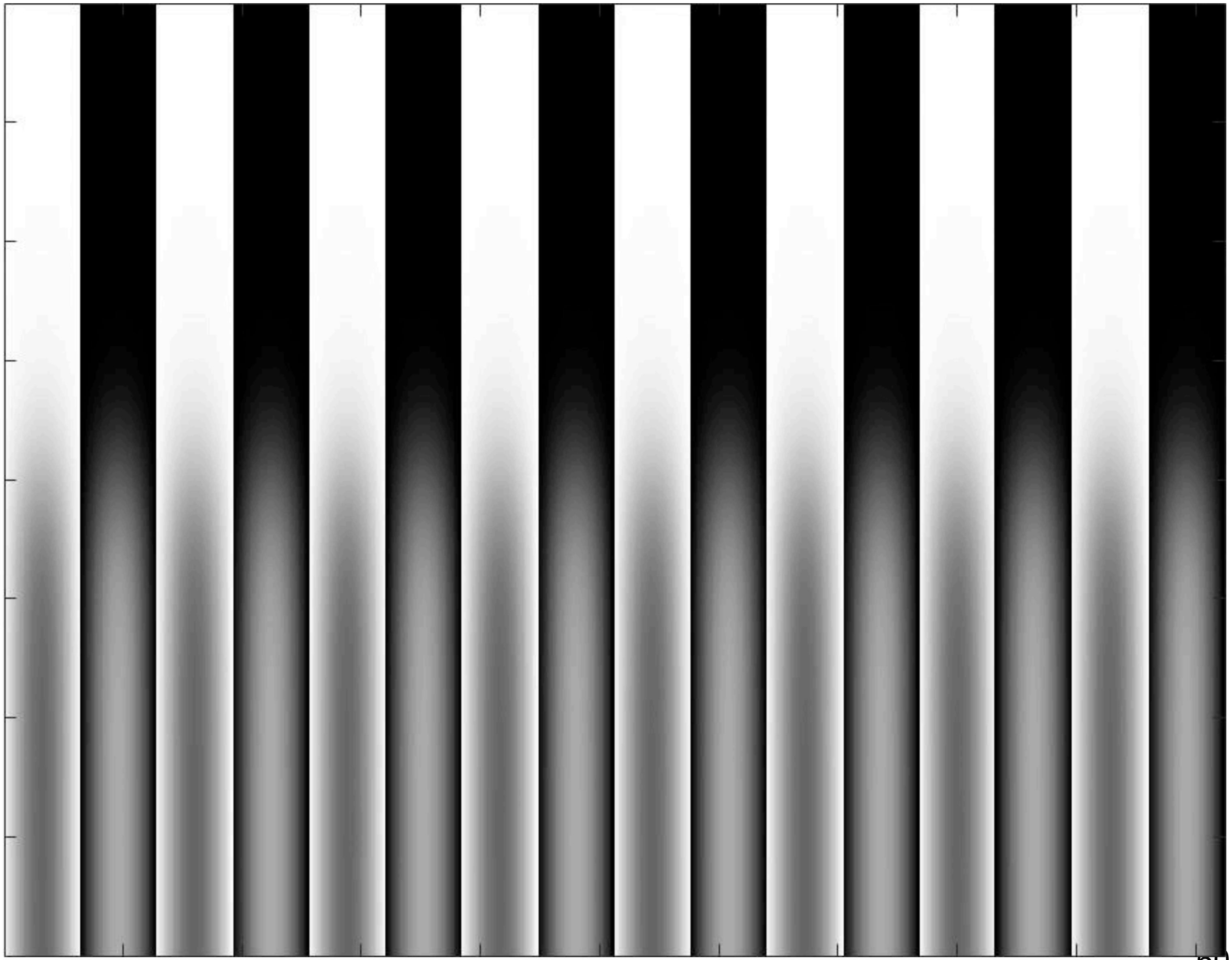


spatial frequency
temporal frequency
space
time
space
low-pass filtered
time

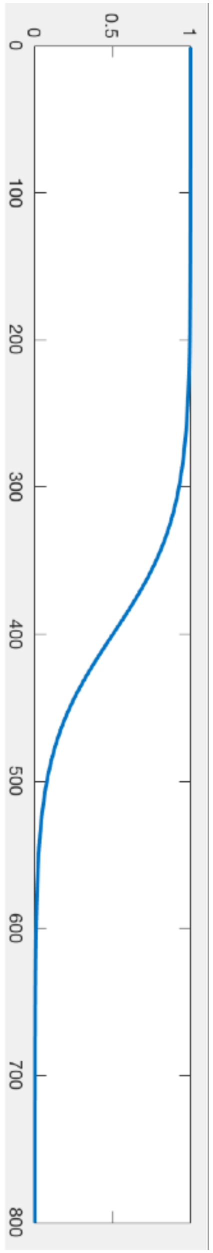




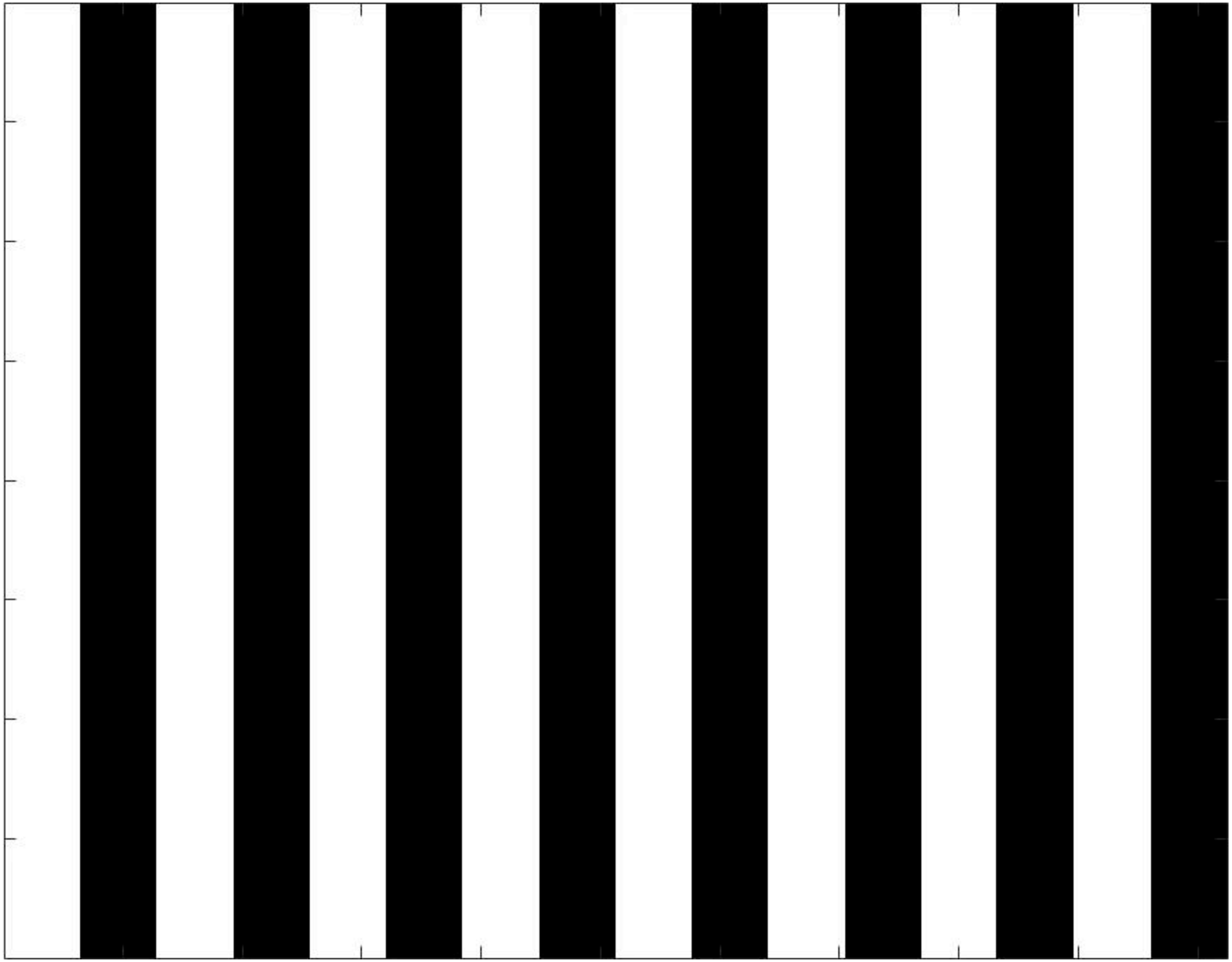
blend over the two conditions



fraction of square wave
fundamental frequency

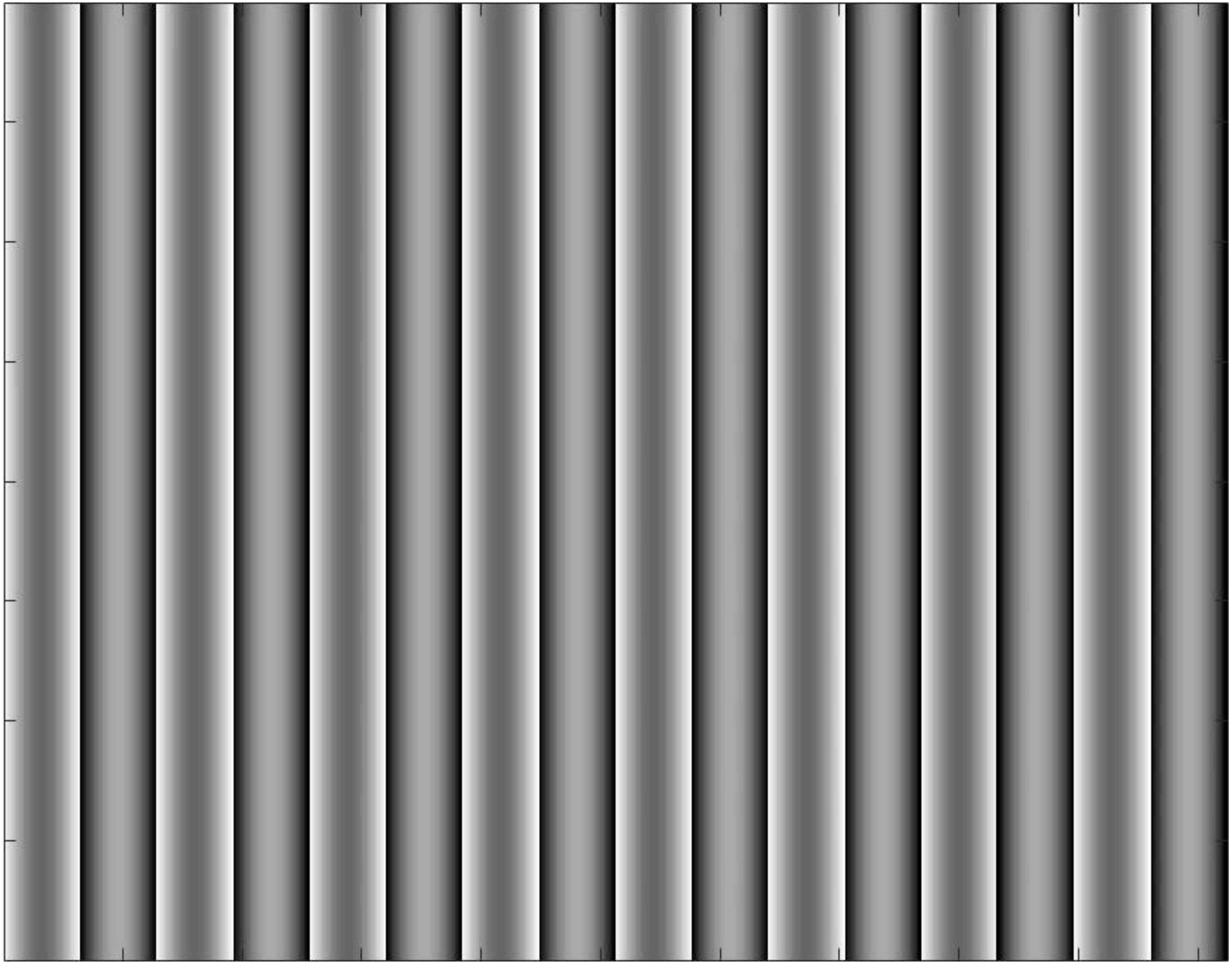


faster display speed



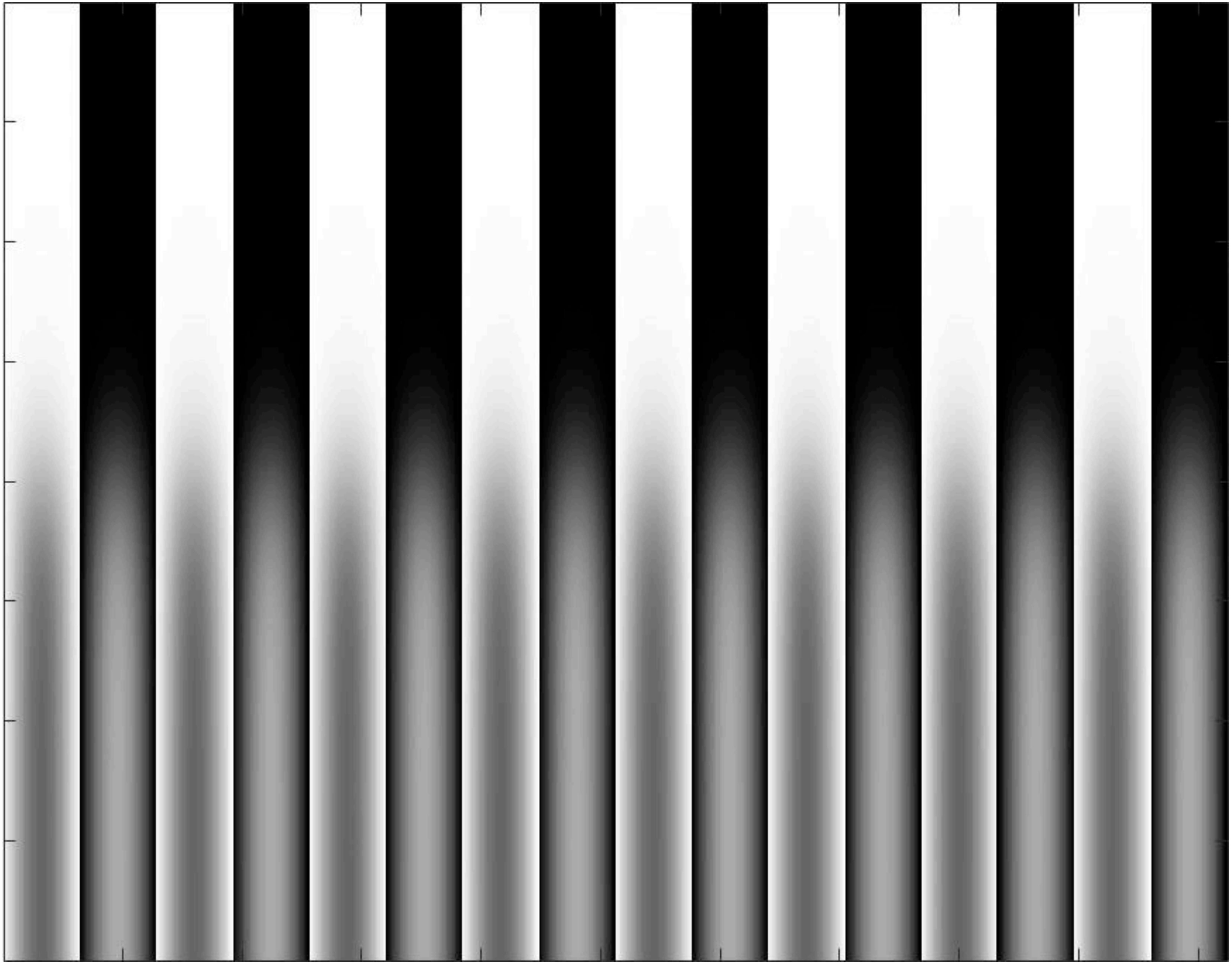
alpha: 1 squareFlag: 1 offset: period/4

faster display speed

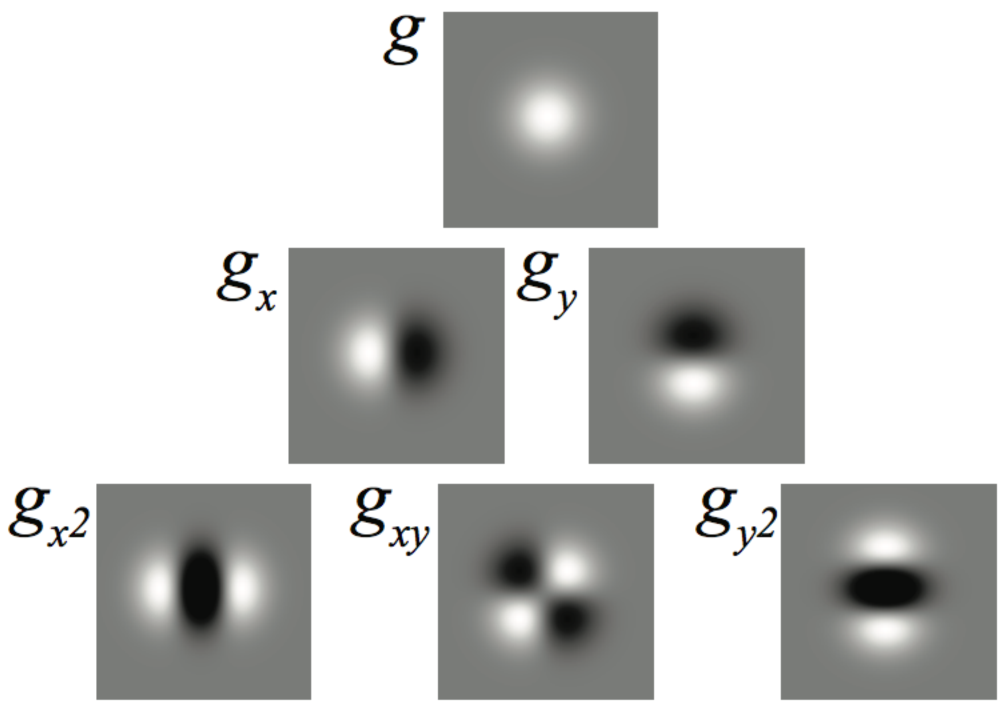


alpha: 0 squareFlag: 1 offset: period/4

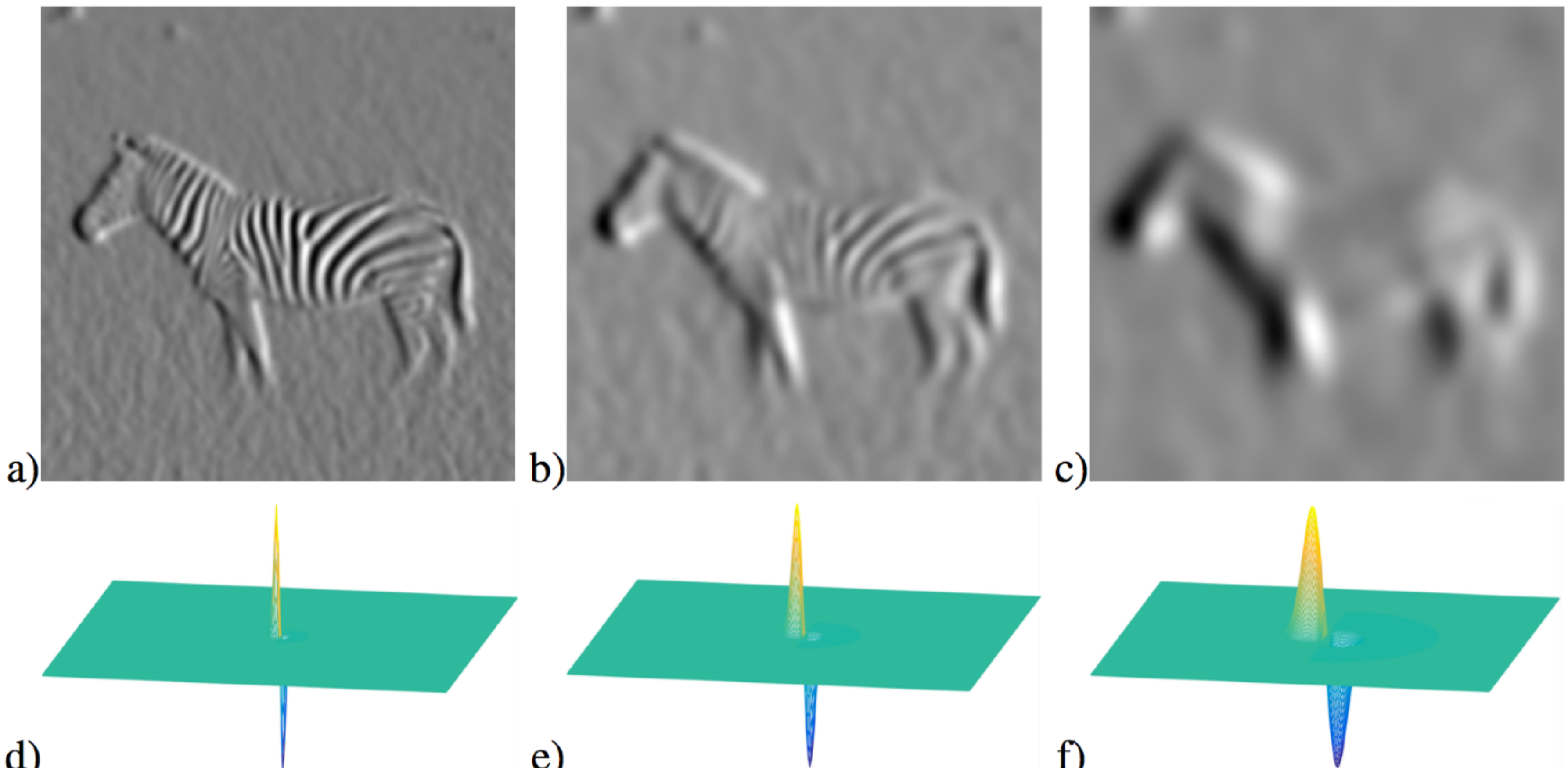
fast blended…



derivatives of Gaussians



derivatives of Gaussians



Space-time Gaussian derivatives

$$\frac{\partial g}{\partial t} = \frac{-t}{\sigma_t^2} g(x, y, t)$$

$$\nabla g = \left(g_x(x,y,t), g_y(x,y,t), g_t(x,y,t)\right) =$$
$$= \left(-x/\sigma^2, -y/\sigma^2, -t/\sigma_t^2\right) g(x,y,t)$$

Note: we can discretize time derivatives in the same way we discretized spatial derivatives. For instance:

$$f[m,n,t] - f[m,n,t-1]$$

Cancelling moving objects

Can we create a filter that *removes* objects that move at some velocity **(vx , vy)** while keeping the rest?

Space-time Gaussian derivatives

For a global translation, we can write:

$$f(x,y,t) = f_0(x - v_x t, y - v_y t)$$

Therefore, we can write the temporal derivative of f as a function of the spatial derivatives of f_0 :

$$\frac{\partial f}{\partial t} = \frac{\partial f_0}{\partial t} = -v_x \frac{\partial f_0}{\partial x} - v_y \frac{\partial f_0}{\partial y}$$

And from here (using derivatives of f, which will be the same):

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} = 0$$

This relation is known as the "Brightness change constraint equation", introduced by Horn & Schunck in 1981

Space-time Gaussian derivatives

Can we create a filter that removes objects that move at some velocity **(vx , vy)** while keeping the rest?

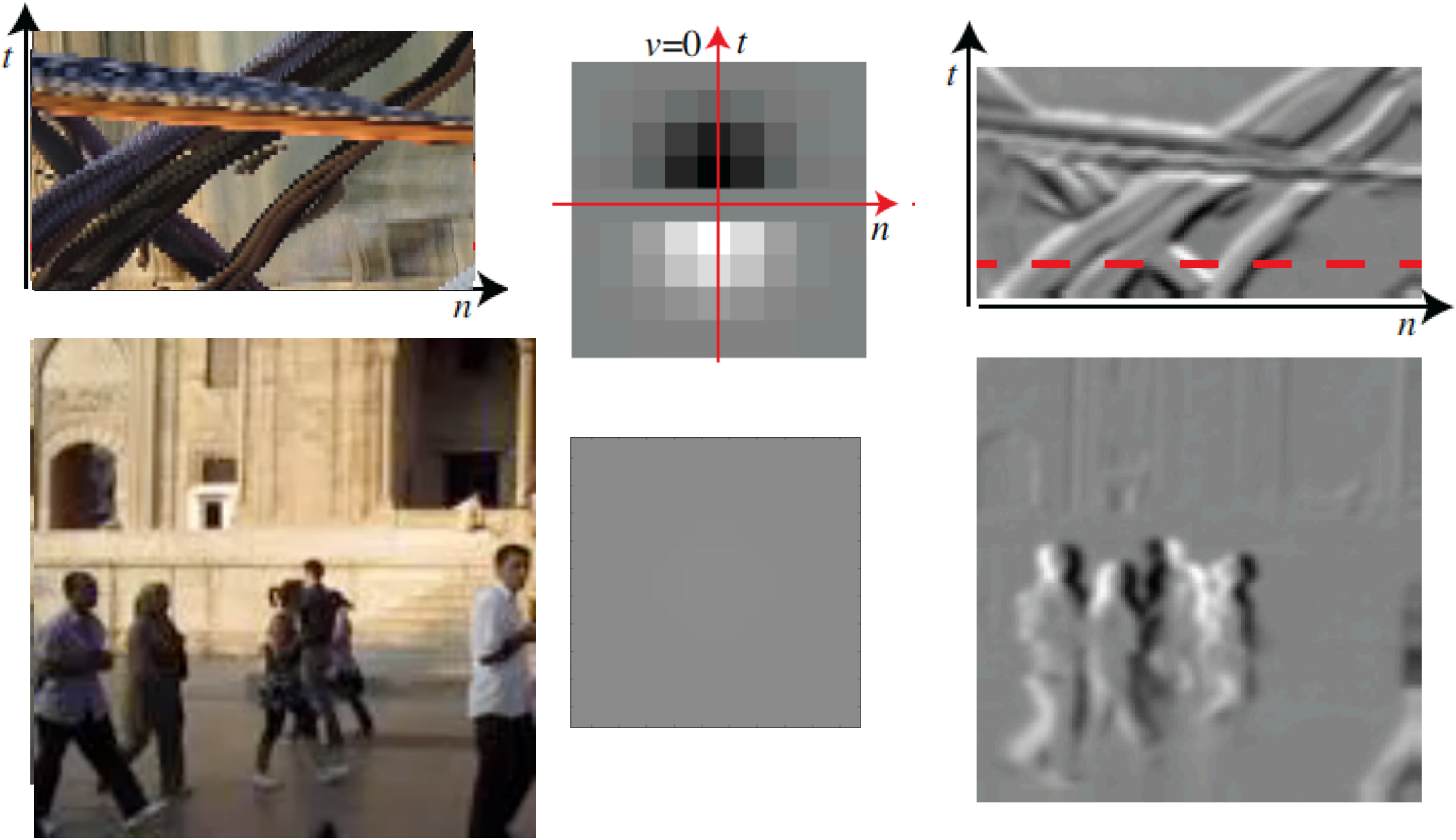
Yes, we could create a filter that implements this constraint:

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} = 0$$

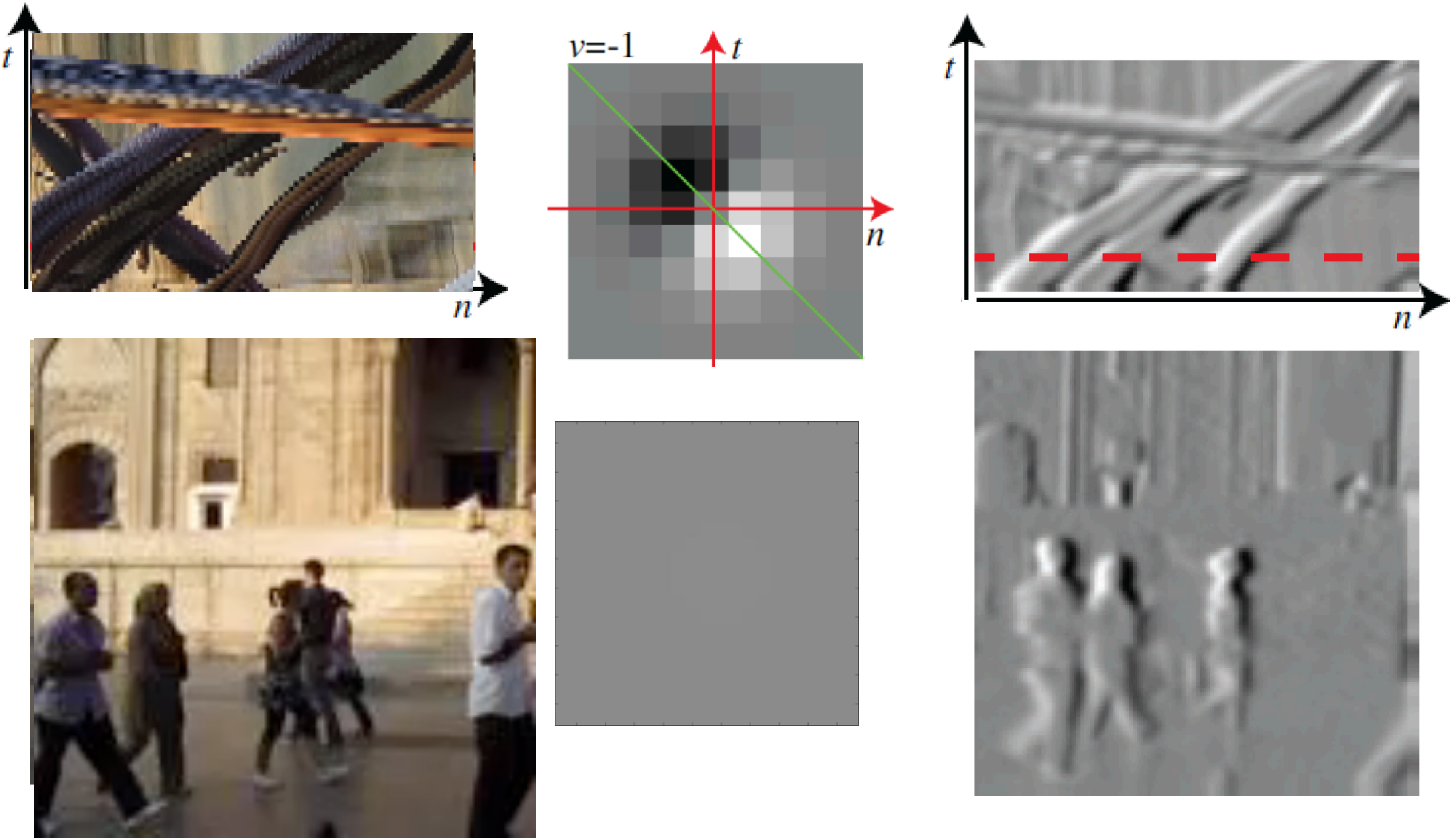
We can create this filter as a combination of Gaussian derivatives:

$$\begin{aligned} h(x,y,t;v_x,v_y) &= g_t + v_x g_x + v_y g_y \\ &= \nabla g \left(1, v_x, v_y\right)^T \end{aligned}$$

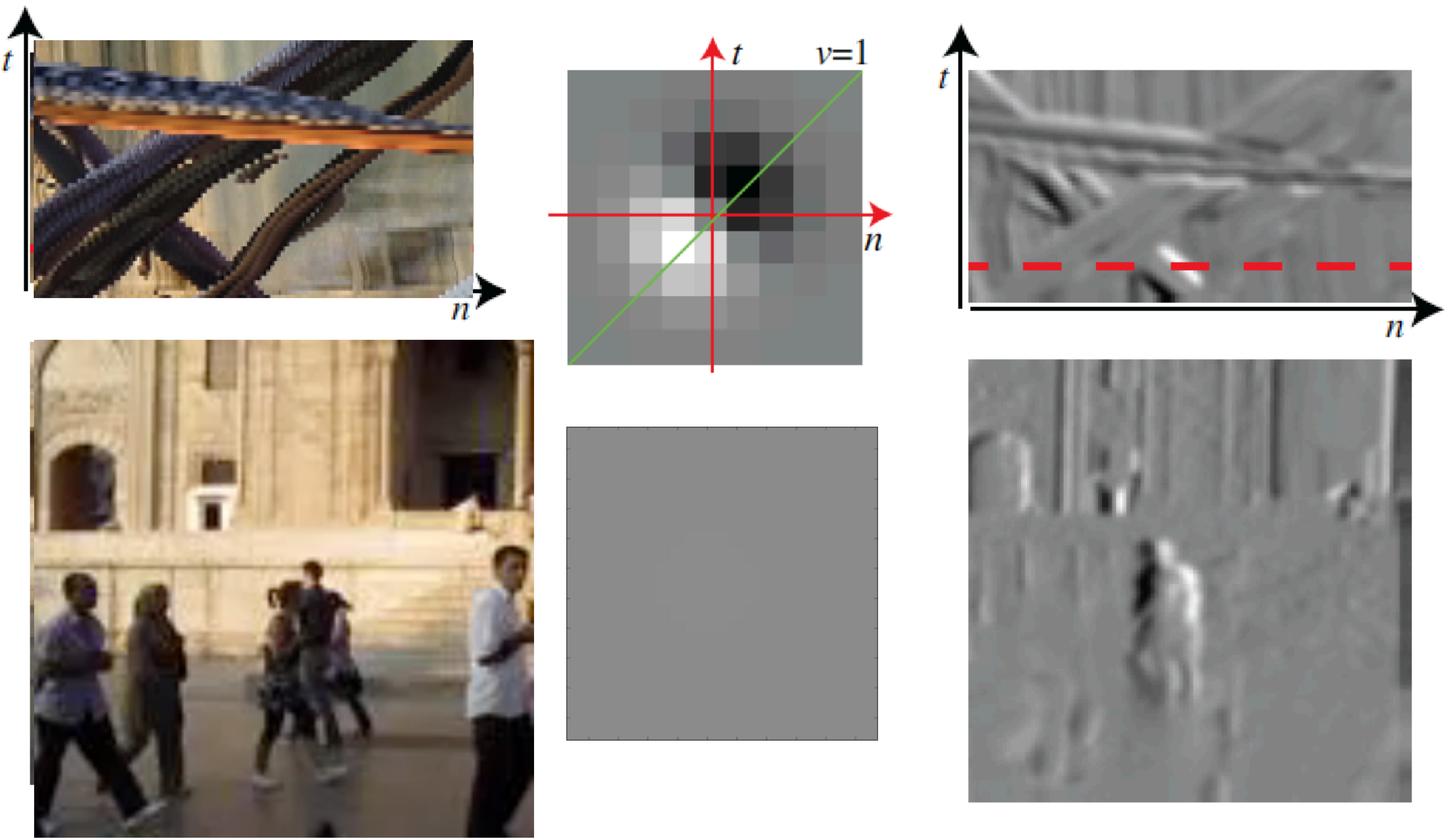
Space-time Gaussian derivatives



Nulling-out $v_x=0$, $v_y=0$ motion



Nulling-out $v_x = -1$, $v_y = 0$ motion



Nulling-out $v_x = 1$, $v_y = 0$ motion

end