Lecture 7

Spatial Pyramids

6.869/6.819 Advances in Computer Vision

Spring 2021

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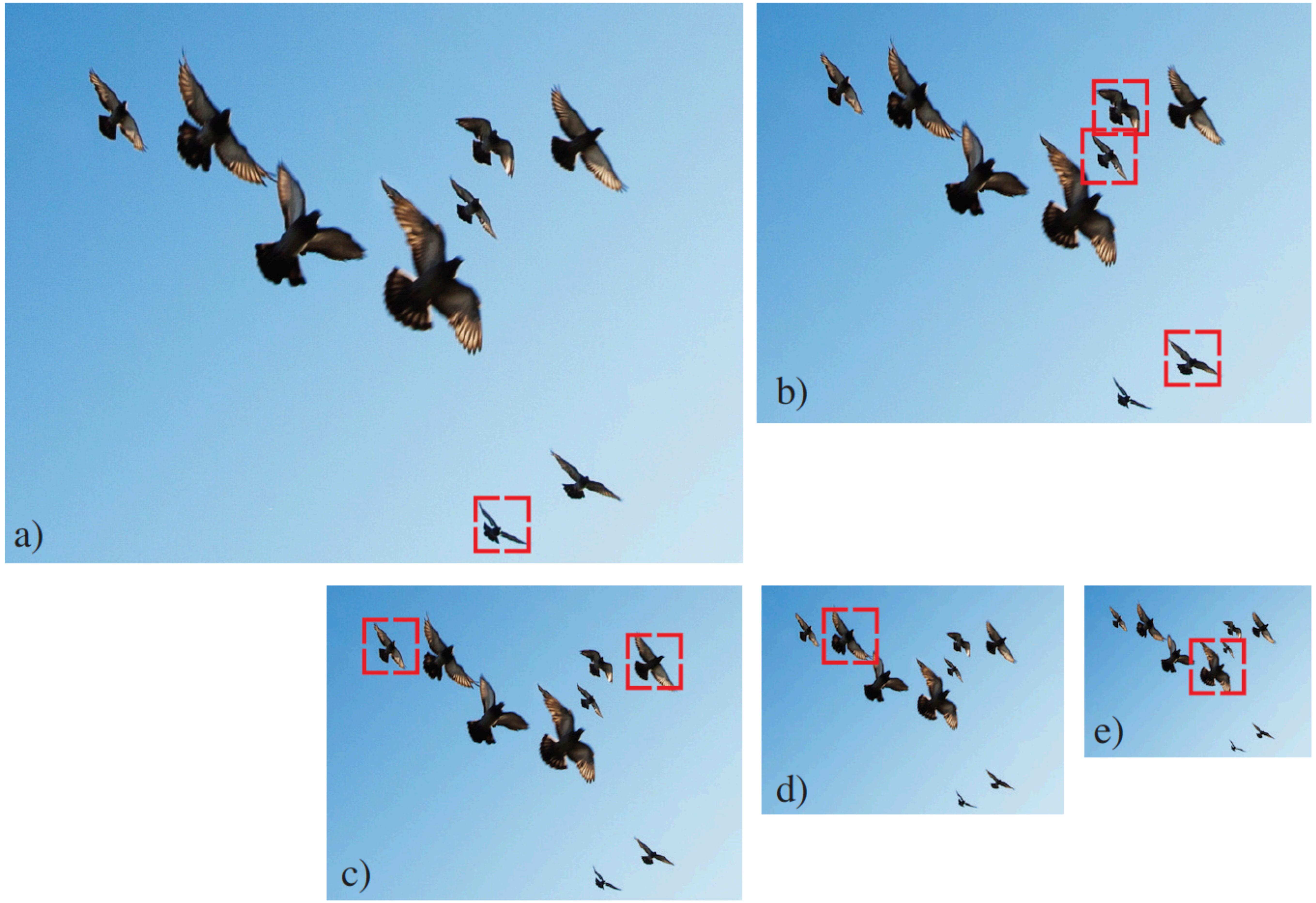
Today’s lecture

- Gaussian pyramid
  - application: recognition
- Laplacian pyramid
  - application in image blending
- Steerable pyramid
  - application in texture synthesis

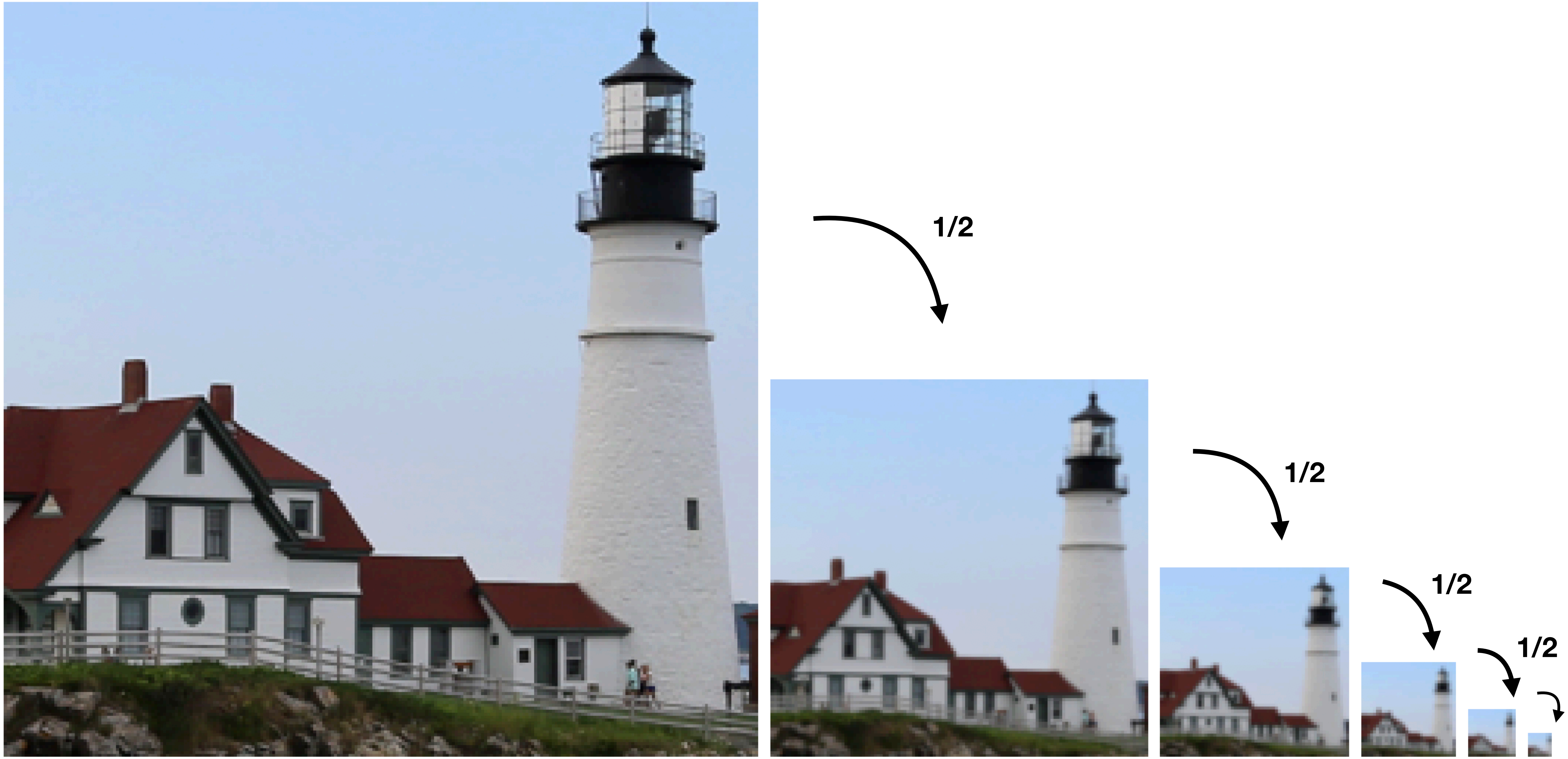
We need translation invariance

We need translation and scale invariance

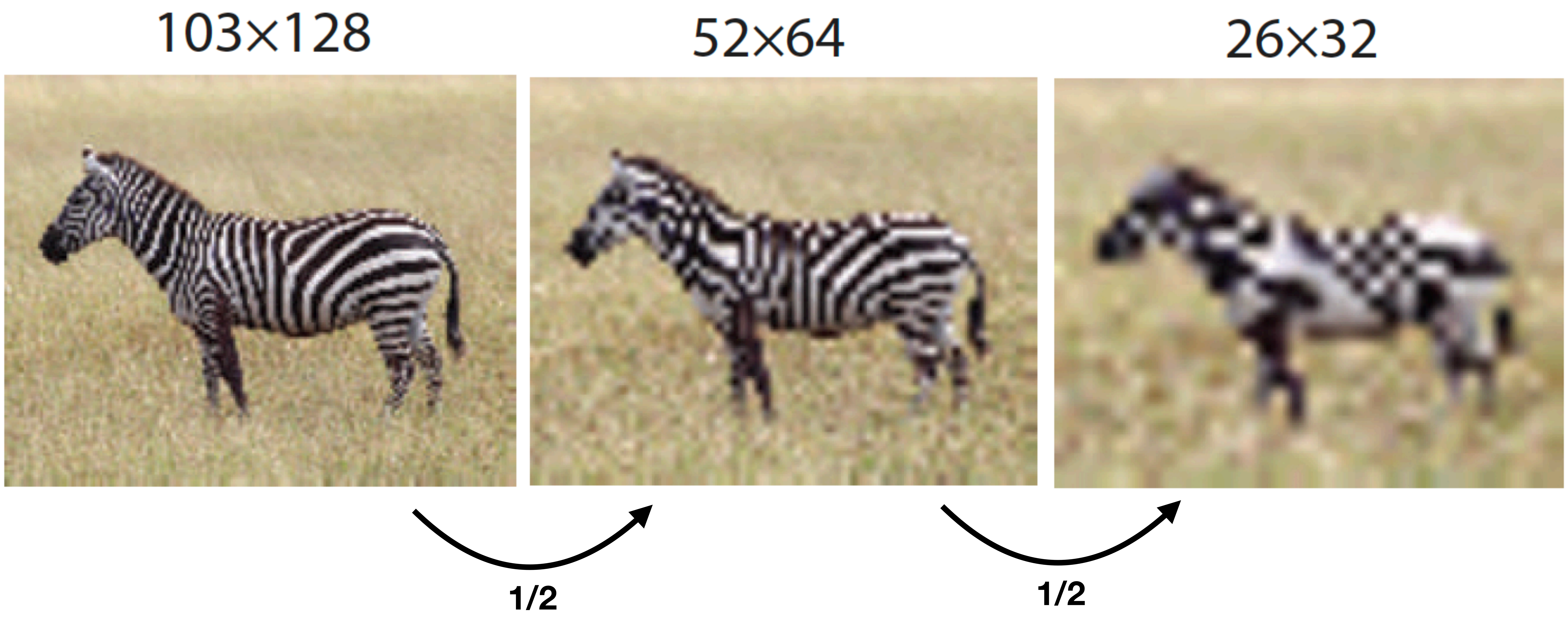
Image pyramids



Gaussian Pyramid



Subsampling and aliasing



Subsampling without blurring

The Gaussian pyramid

For each level

1. Blur input image with a Gaussian filter

[1, 4, 6, 4, 1]

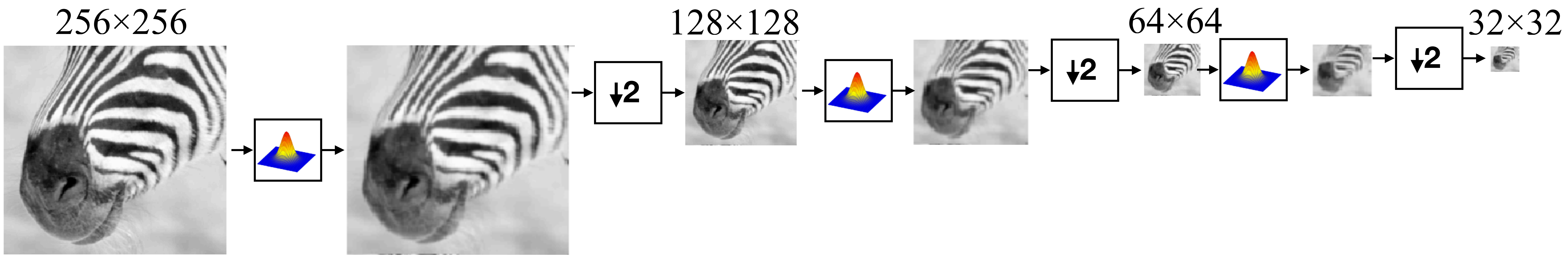
$$\bigcirc [1, 4, 6, 4, 1] \bigcirc \begin{bmatrix}1\\4\\6\\4\\1\end{bmatrix} \longrightarrow$$

The Gaussian pyramid

For each level

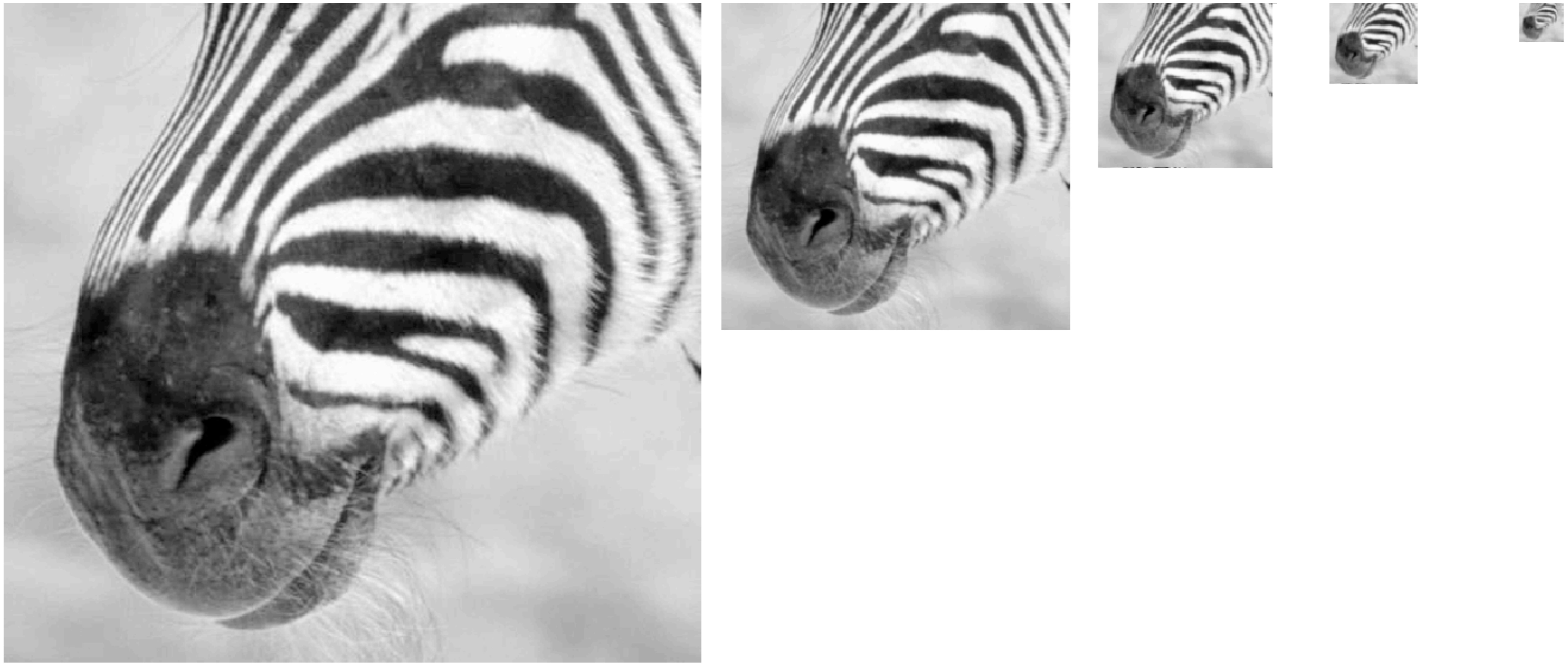
1. Blur input image with a Gaussian filter
2. Downsample image

The Gaussian pyramid



The Gaussian pyramid

512×512 256×256 128×128 64×64 32×32

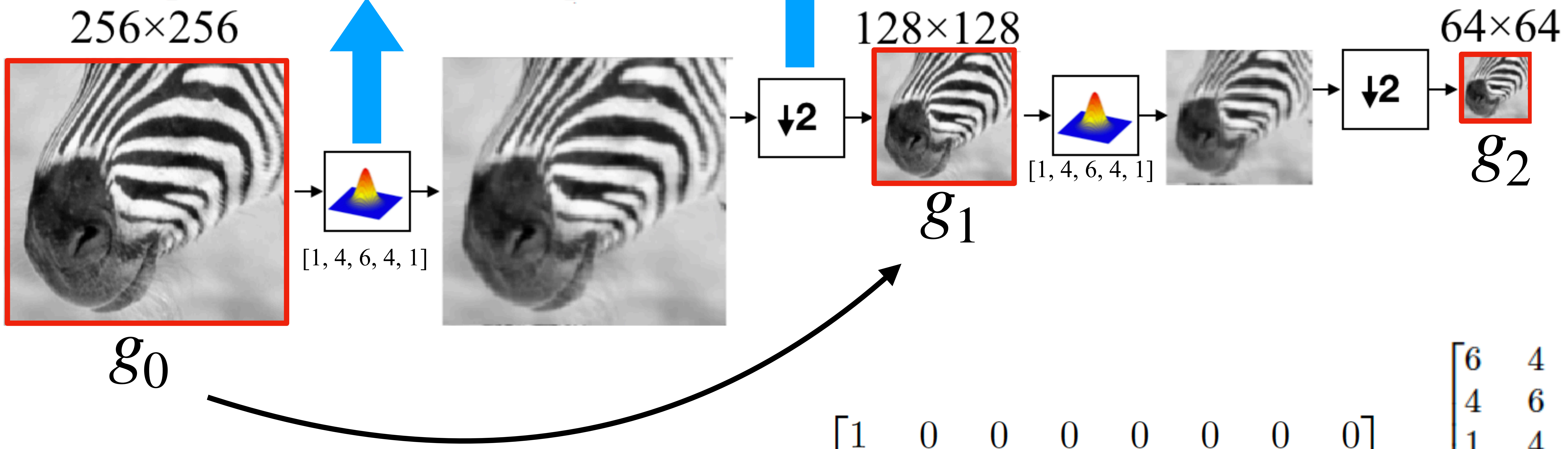


(original image)

The Gaussian pyramid

$$\frac{1}{16}\begin{bmatrix}6&4&1&0&0&0&0&0\\4&6&4&1&0&0&0&0\\1&4&6&4&1&0&0&0\\0&1&4&6&4&1&0&0\\0&0&1&4&6&4&1&0\\0&0&0&1&4&6&4&1\\0&0&0&0&1&4&6&4\\0&0&0&0&0&1&4&6\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&0&1&0\end{bmatrix}$$



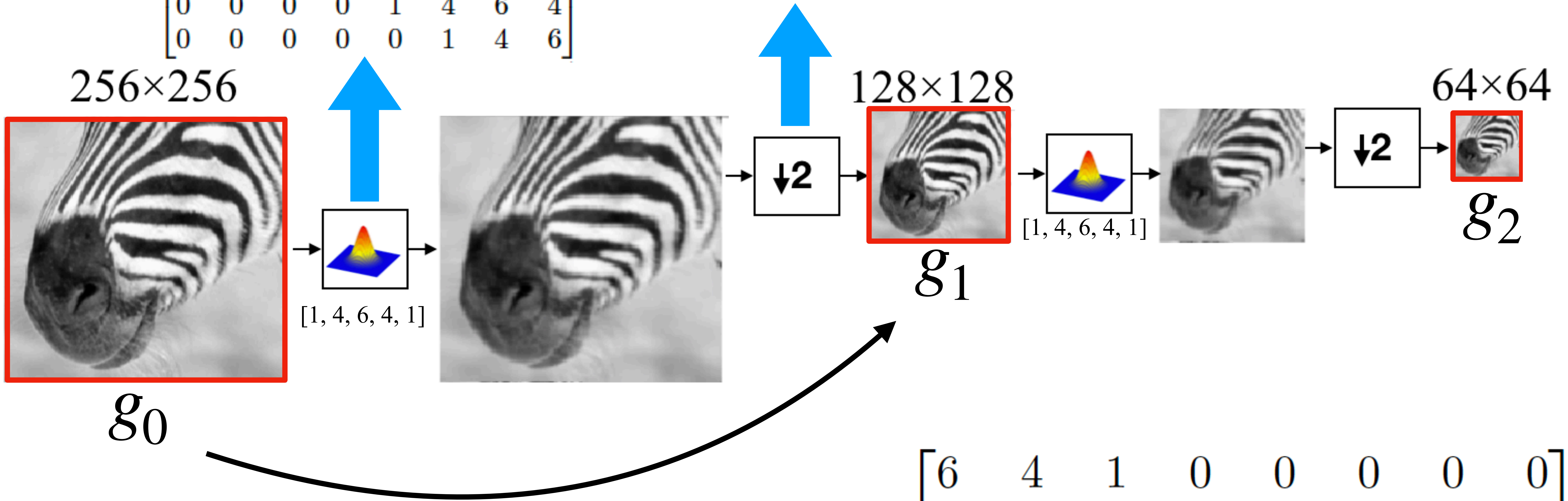
$$g_1 = G_0 g_0$$

$$G_0 = \begin{bmatrix}1&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&0&1&0\end{bmatrix}\frac{1}{16}\begin{bmatrix}6&4&1&0&0&0&0&0\\4&6&4&1&0&0&0&0\\1&4&6&4&1&0&0&0\\0&1&4&6&4&1&0&0\\0&0&1&4&6&4&1&0\\0&0&0&1&4&6&4&1\\0&0&0&0&1&4&6&4\\0&0&0&0&0&1&4&6\end{bmatrix}$$

The Gaussian pyramid

$$\frac{1}{16}\begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}$$

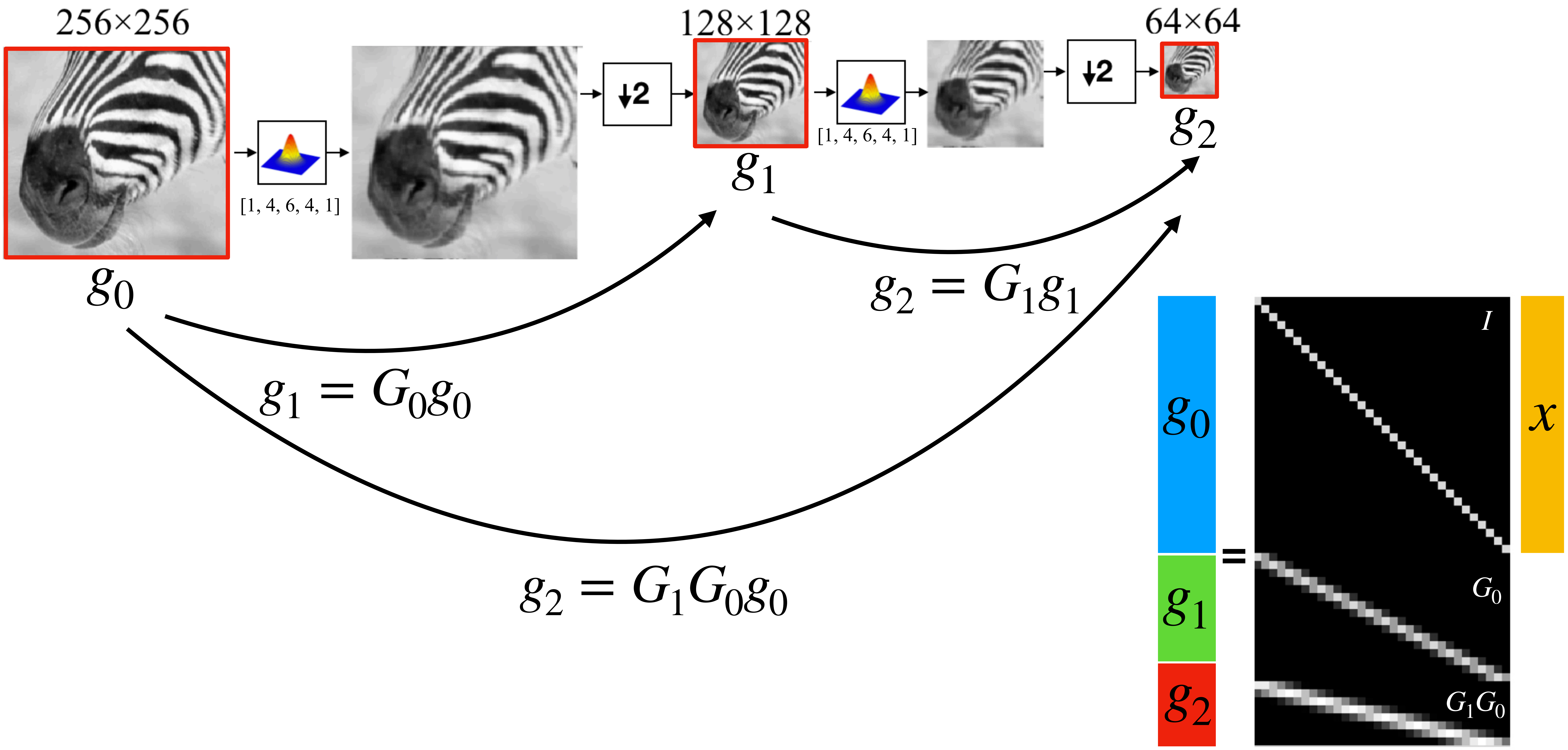
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



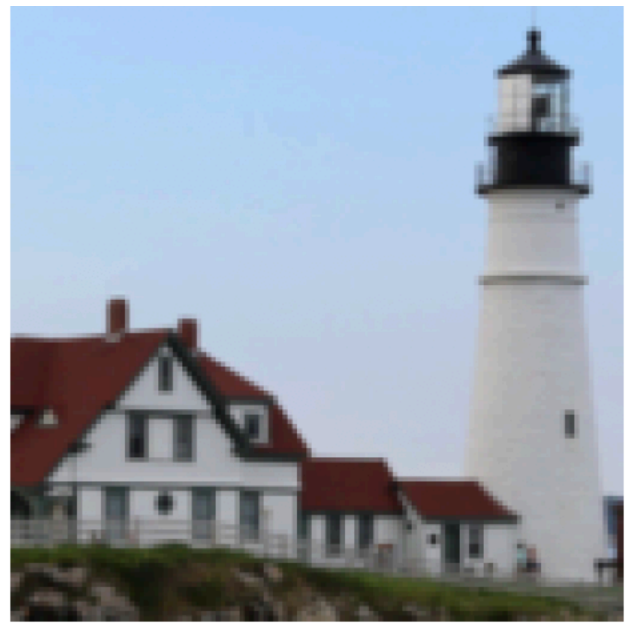
$$g_1 = G_0 g_0$$

$$G_0 = \frac{1}{16}\begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \end{bmatrix}$$

The Gaussian pyramid



The Gaussian pyramid



$$g_k \longrightarrow \boxed{G_k} \longrightarrow g_{k+1}$$

For each level

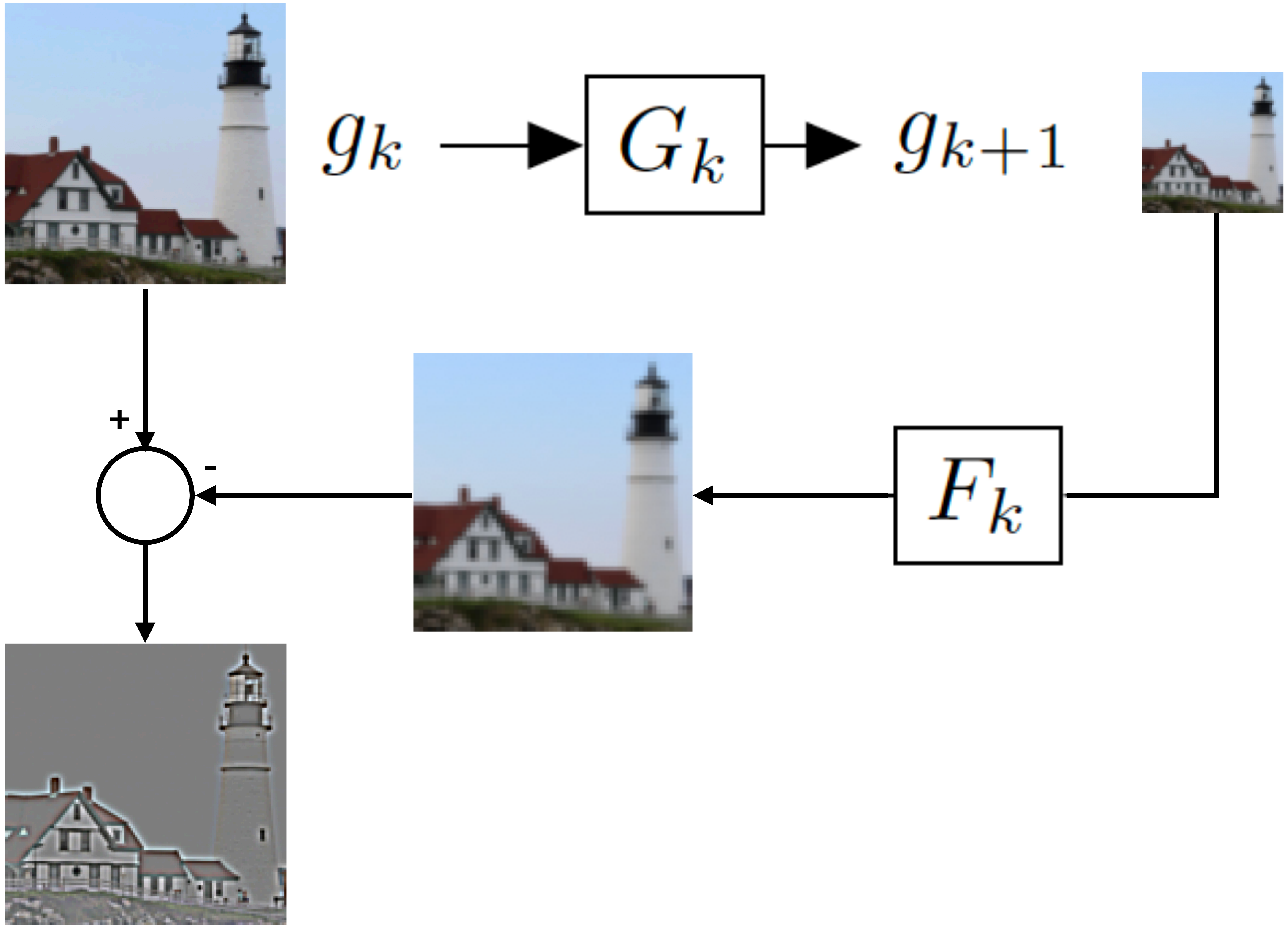
1. Blur input image with a Gaussian filter
2. Downsample image

Gaussian pyramid applications

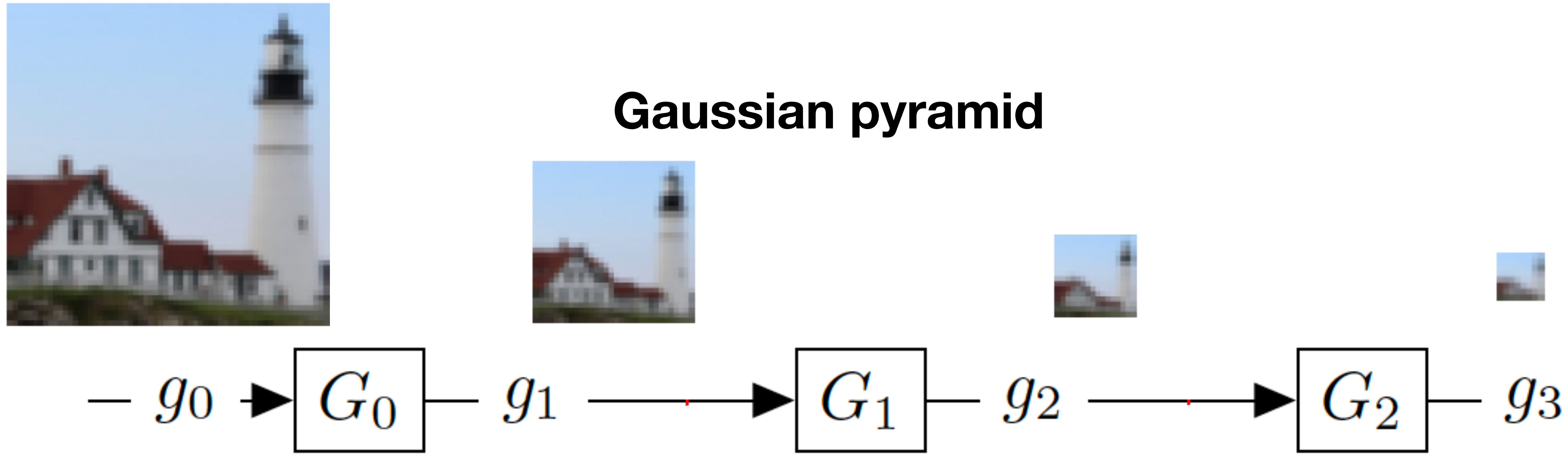
- Texture synthesis
- Object recognition
- Neural Network image synthesis

The Laplacian Pyramid

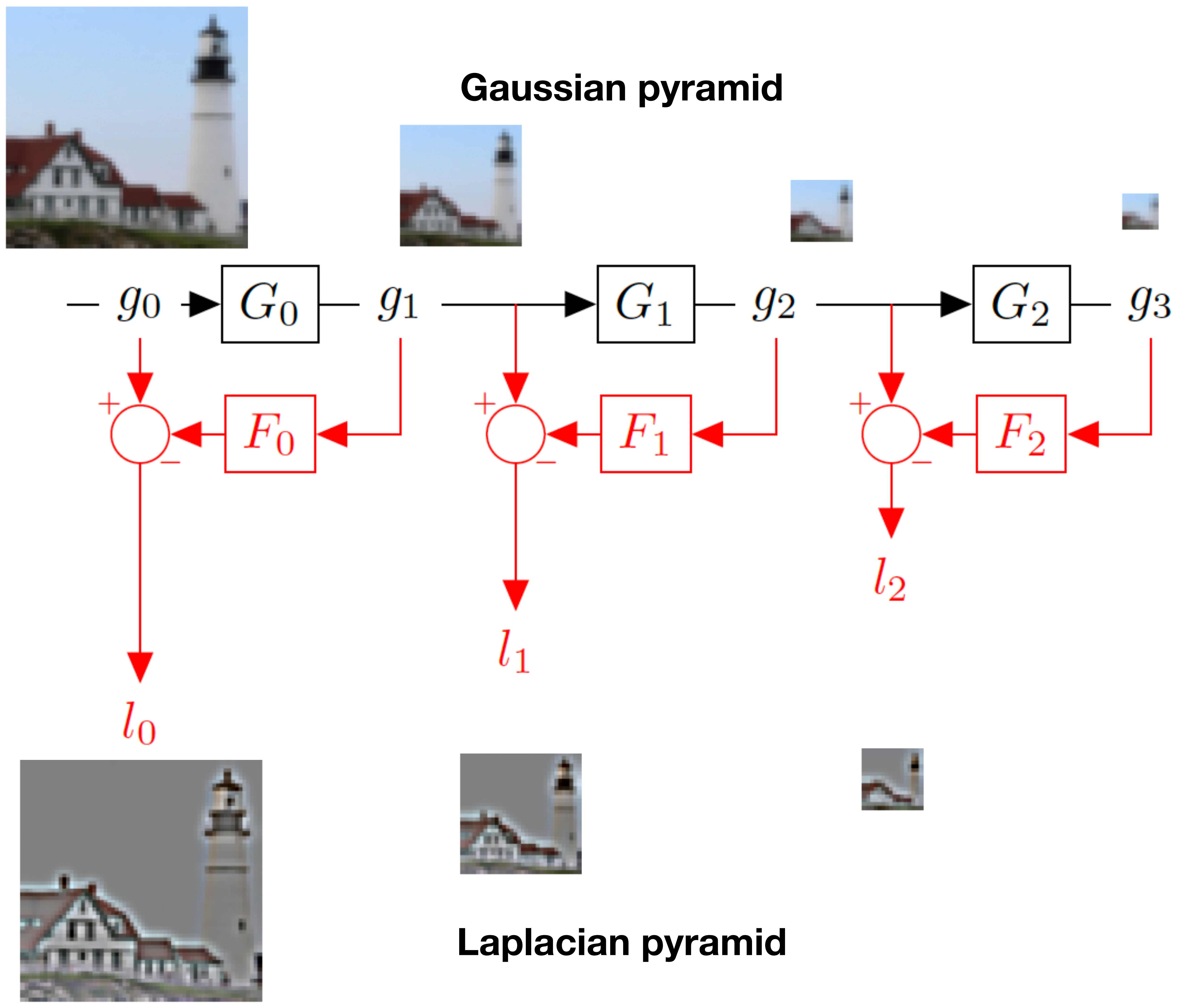
Compute the difference between upsampled Gaussian pyramid level k+1 and Gaussian pyramid level k.



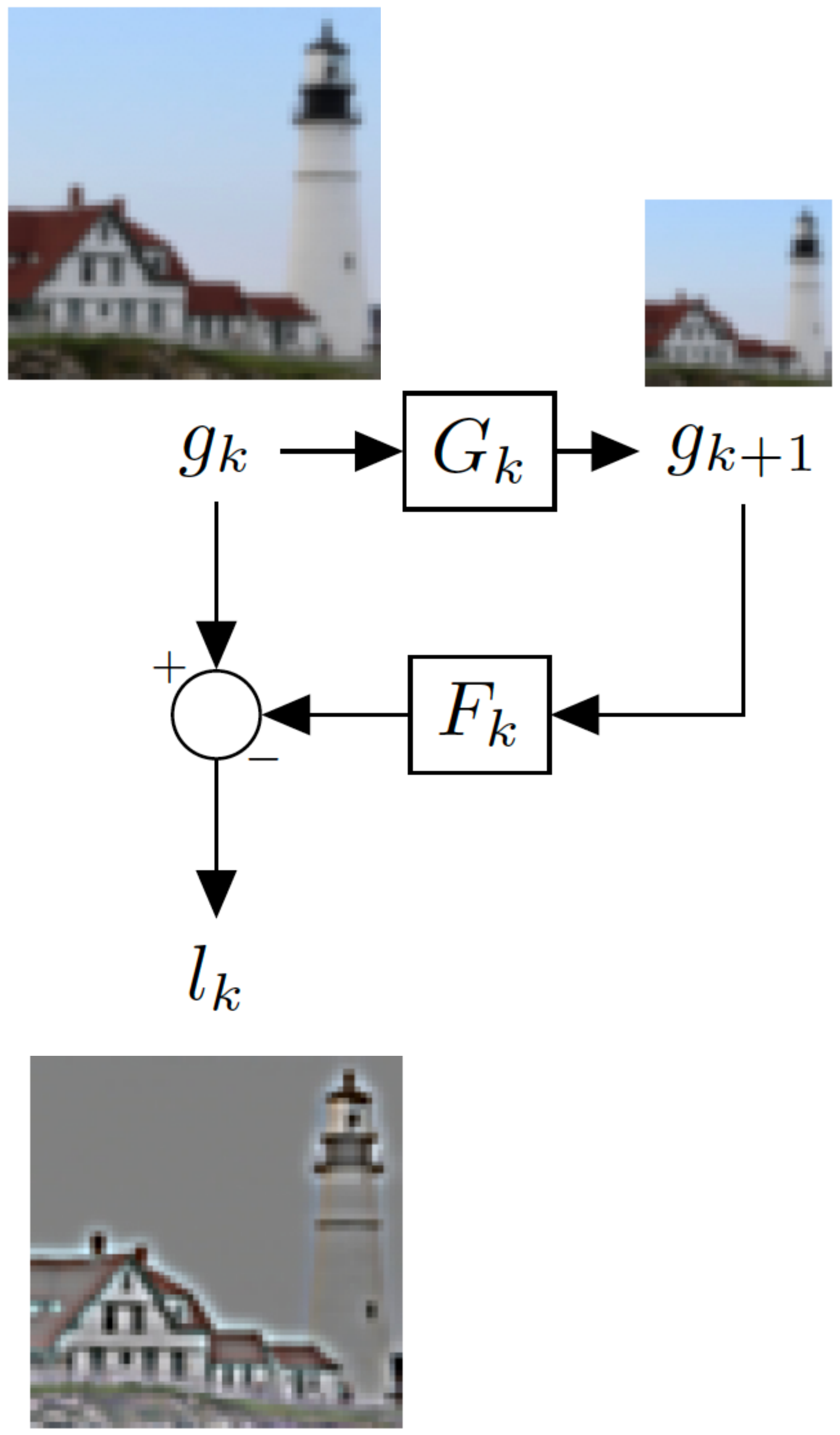
The Laplacian Pyramid



The Laplacian Pyramid



The Laplacian Pyramid



Blurring and downsampling:

$$G_0 = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{(Downsampling by 2)}} \underbrace{\frac{1}{16} \begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}}_{\text{(blur)}}$$

Upsampling and blurring:

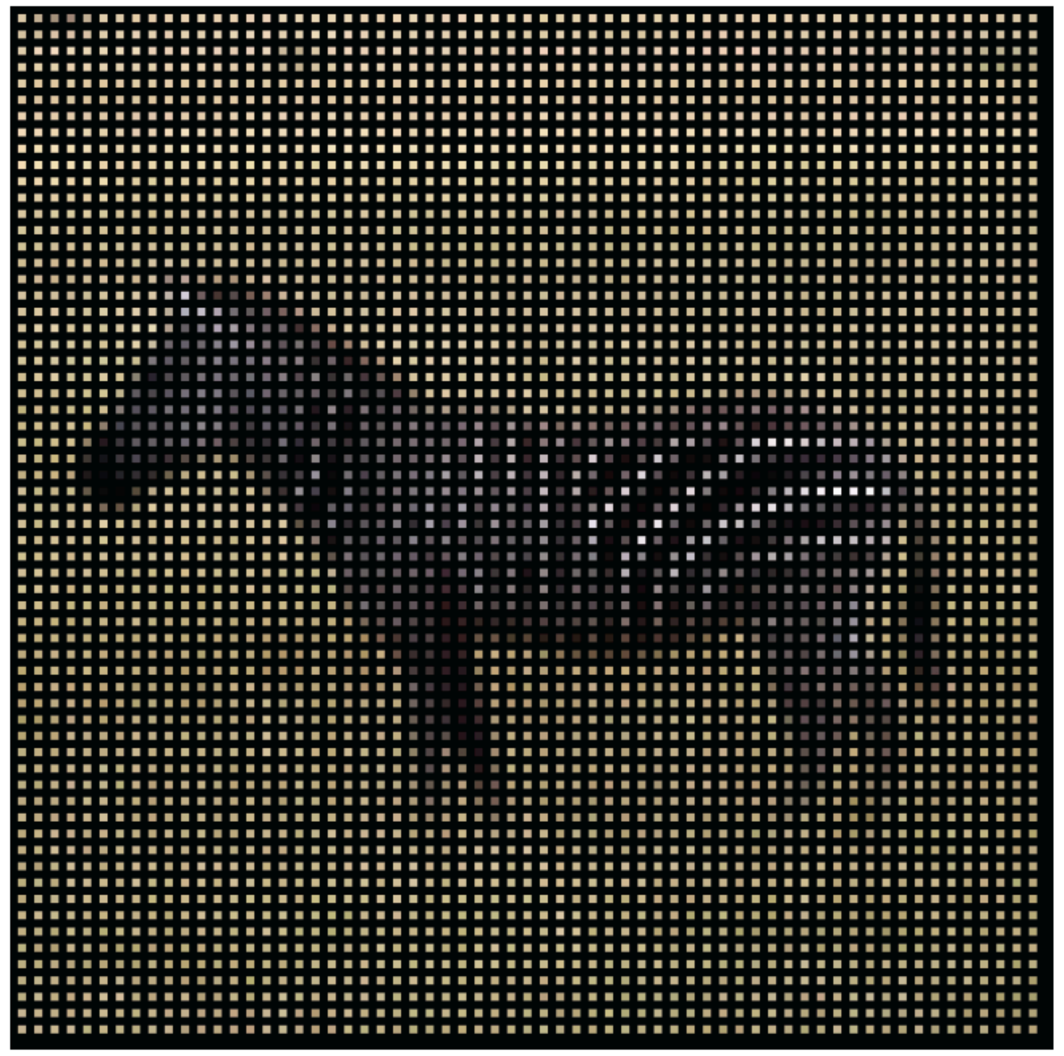
$$F_0 =$$

Upsampling

64x64

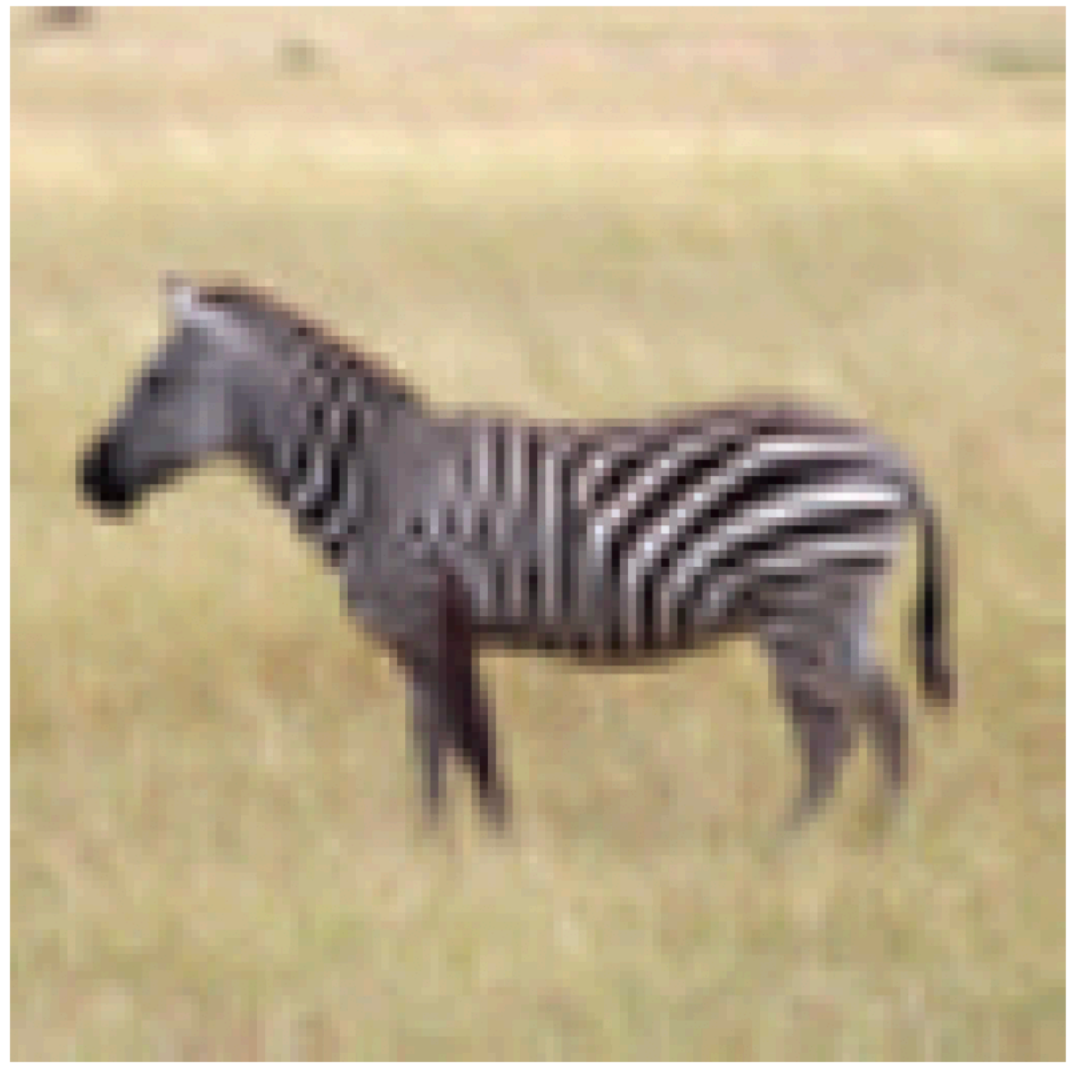
Insert zeros

128x128

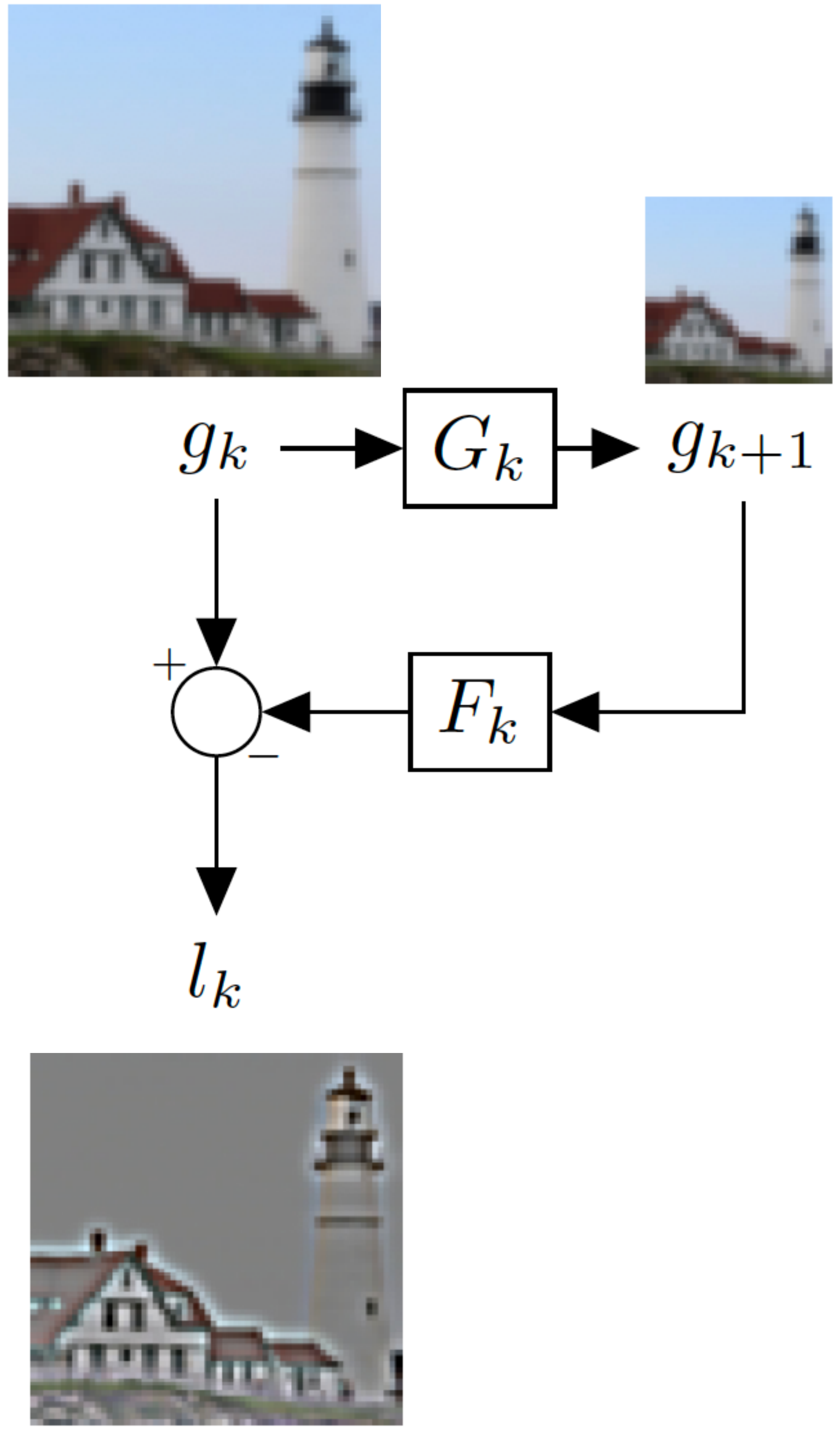


$$\circ \begin{bmatrix} 0.25 & 0.5 & 0.25 \\ 0.5 & 1 & 0.5 \\ 0.25 & 0.5 & 0.25 \end{bmatrix} =$$

128x128



The Laplacian Pyramid



Blurring and downsampling:

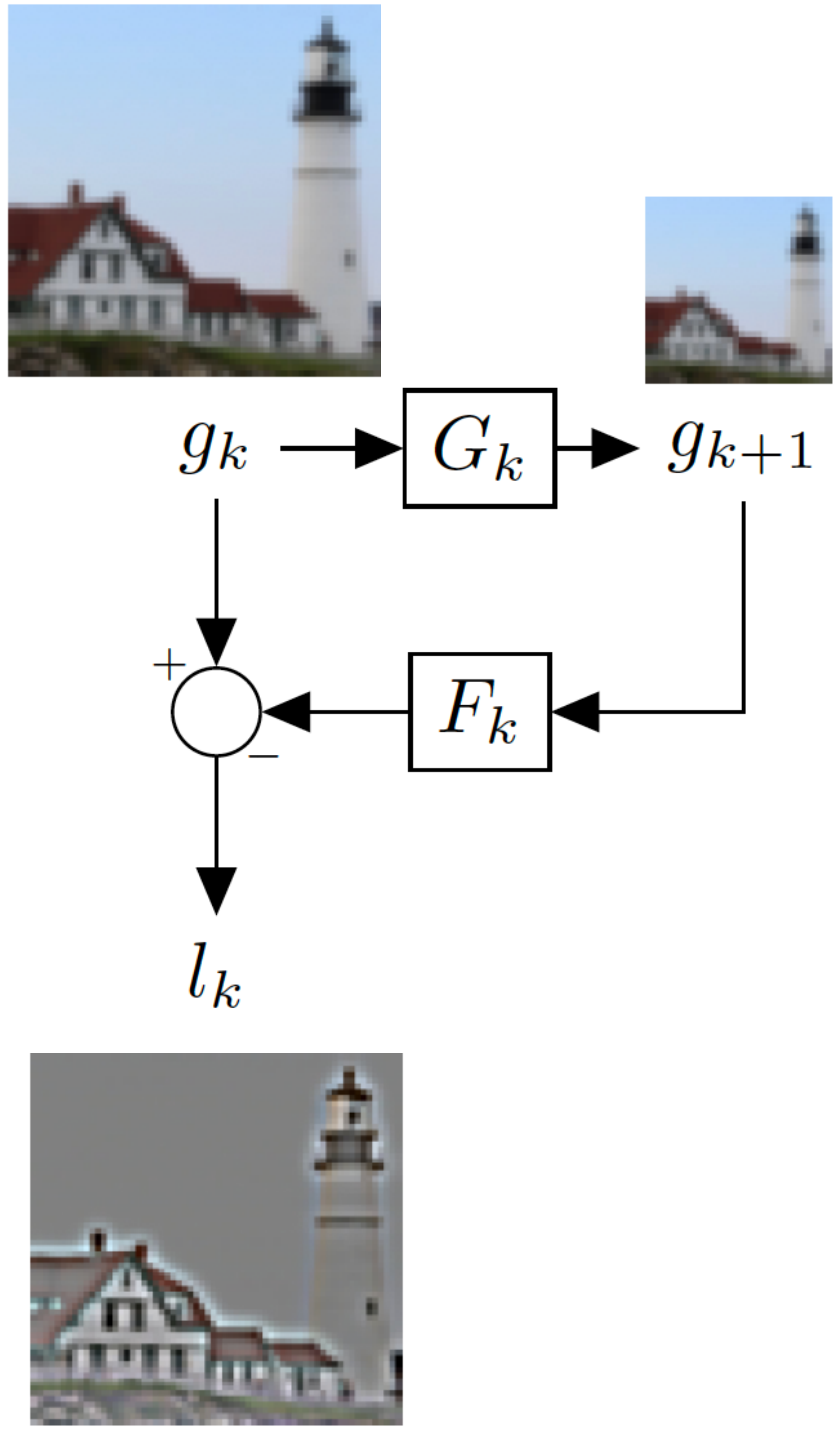
$$G_0 = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{(Downsampling by 2)}} \frac{1}{16} \underbrace{\begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}}_{\text{(blur)}}$$

Upsampling and blurring:

$$F_0 = \frac{1}{8} \underbrace{\begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}}_{\text{(blur)}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{(Upsampling by 2)}}$$

$$\boxed{l_0 = (I_0 - F_0 G_0) g_0}$$

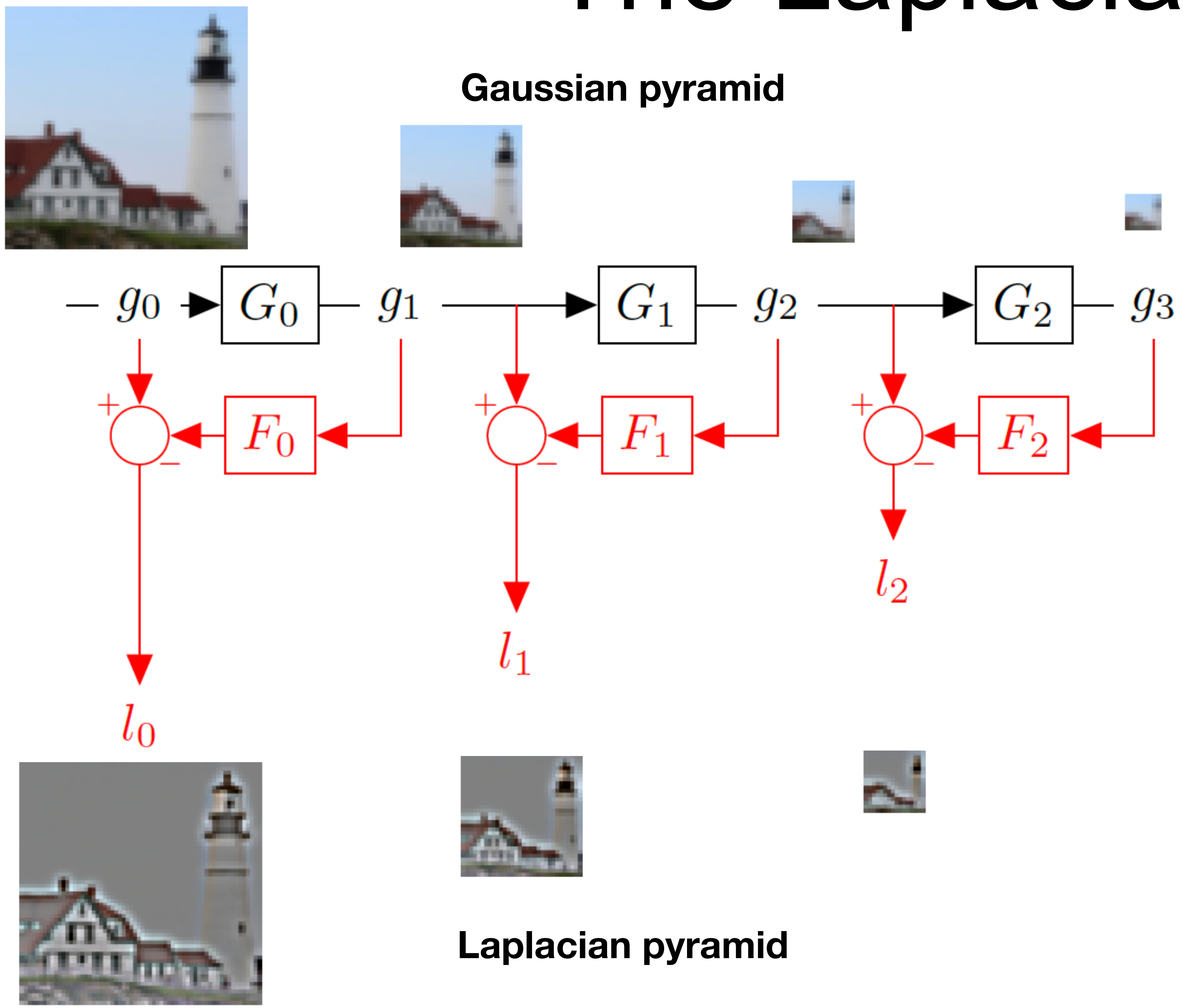
The Laplacian Pyramid

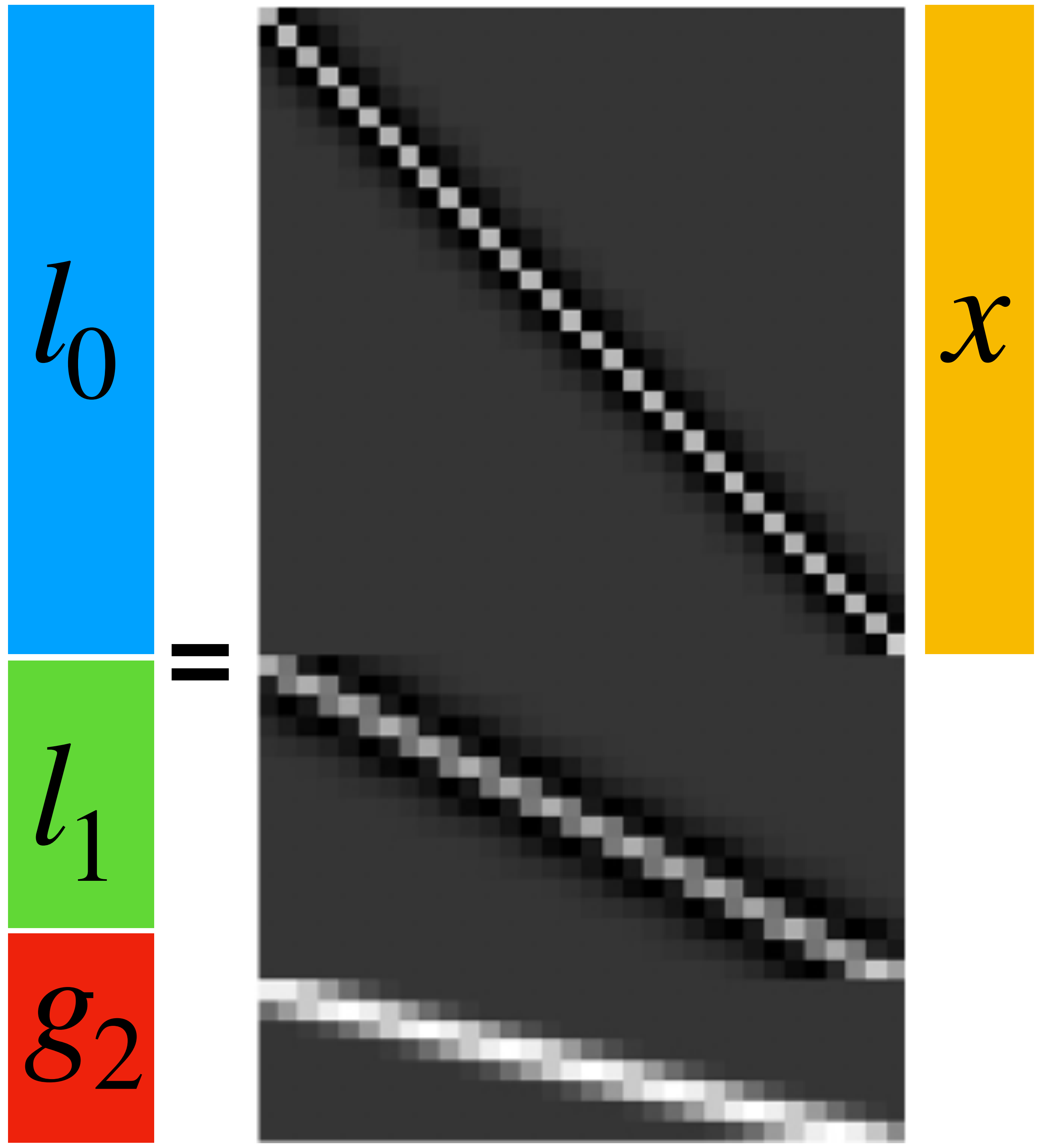


$$\boxed{l_0 = (I_0 - F_0 G_0) g_0}$$

$$= \frac{1}{256} \begin{bmatrix} 182 & -56 & -24 & -8 & -2 & 0 & 0 & 0 \\ -56 & 192 & -56 & -32 & -8 & 0 & 0 & 0 \\ -24 & -56 & 180 & -56 & -24 & -8 & -2 & 0 \\ -8 & -32 & -56 & 192 & -56 & -32 & -8 & 0 \\ -2 & -8 & -24 & -56 & 180 & -56 & -24 & -8 \\ 0 & 0 & -8 & -32 & -56 & 192 & -56 & -32 \\ 0 & 0 & -2 & -8 & -24 & -56 & 182 & -48 \\ 0 & 0 & 0 & 0 & -8 & -32 & -48 & 224 \end{bmatrix} x$$

The Laplacian Pyramid





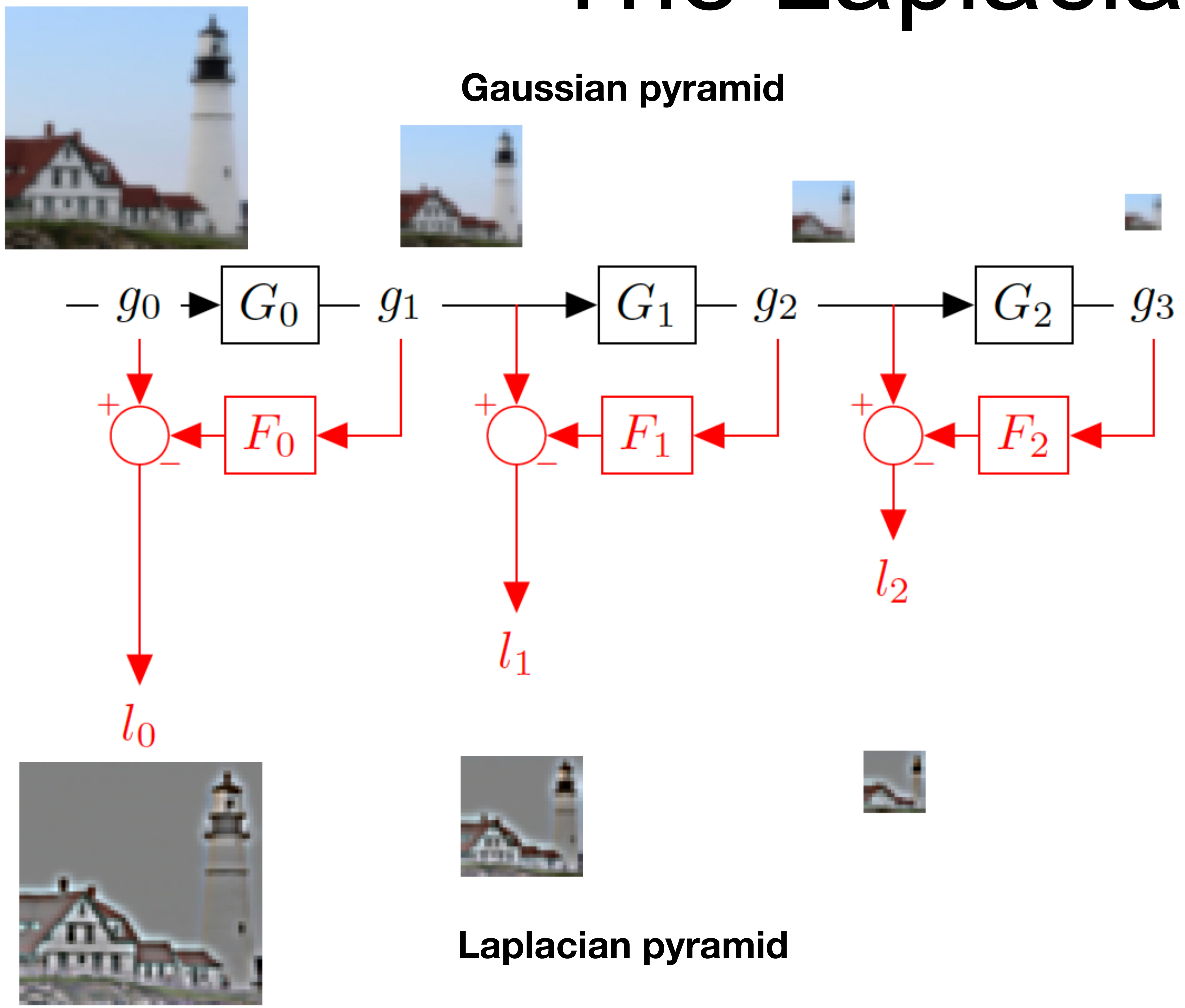
The Laplacian Pyramid

Gaussian
residual

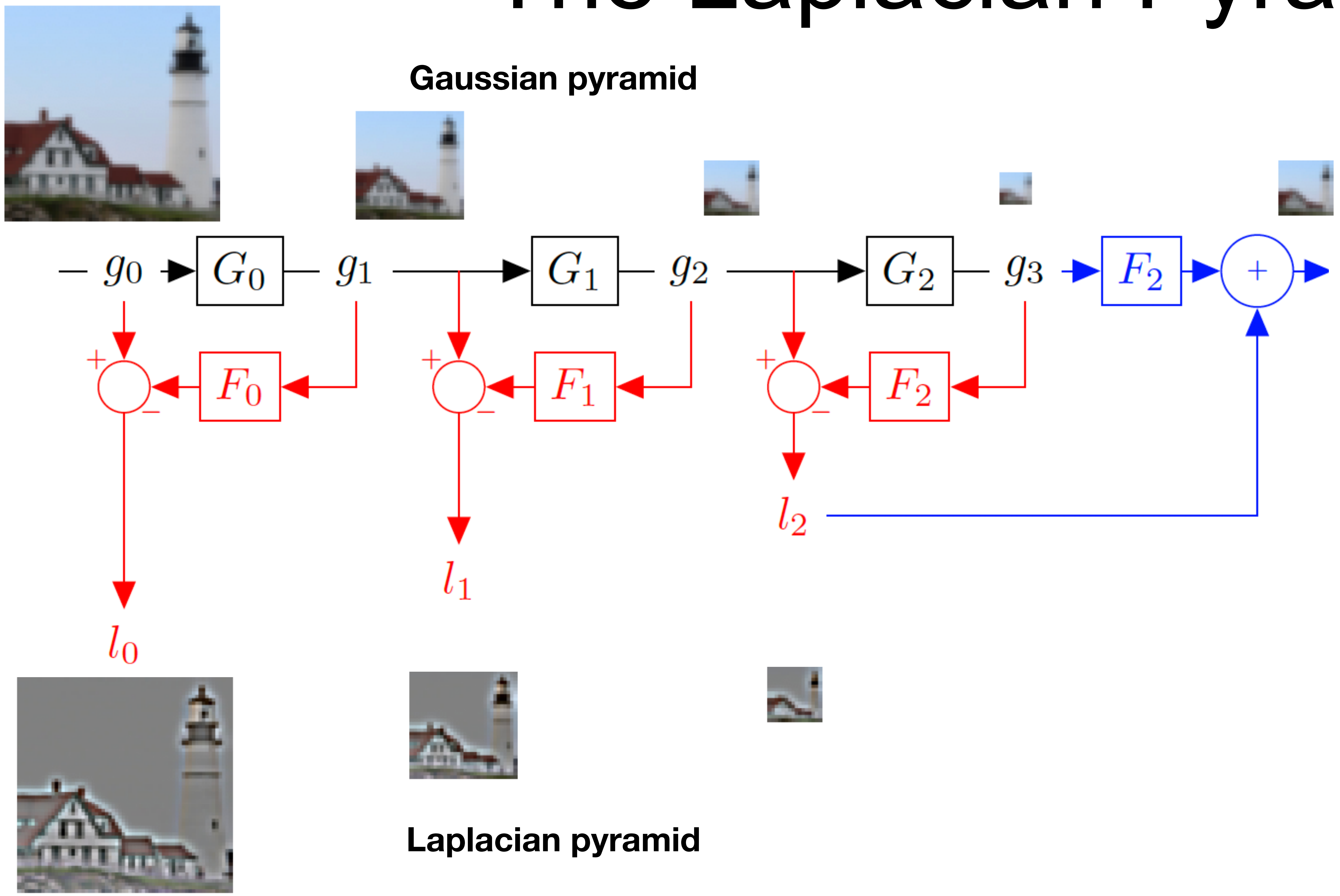
Laplacian pyramid

Can we invert the
Laplacian Pyramid?

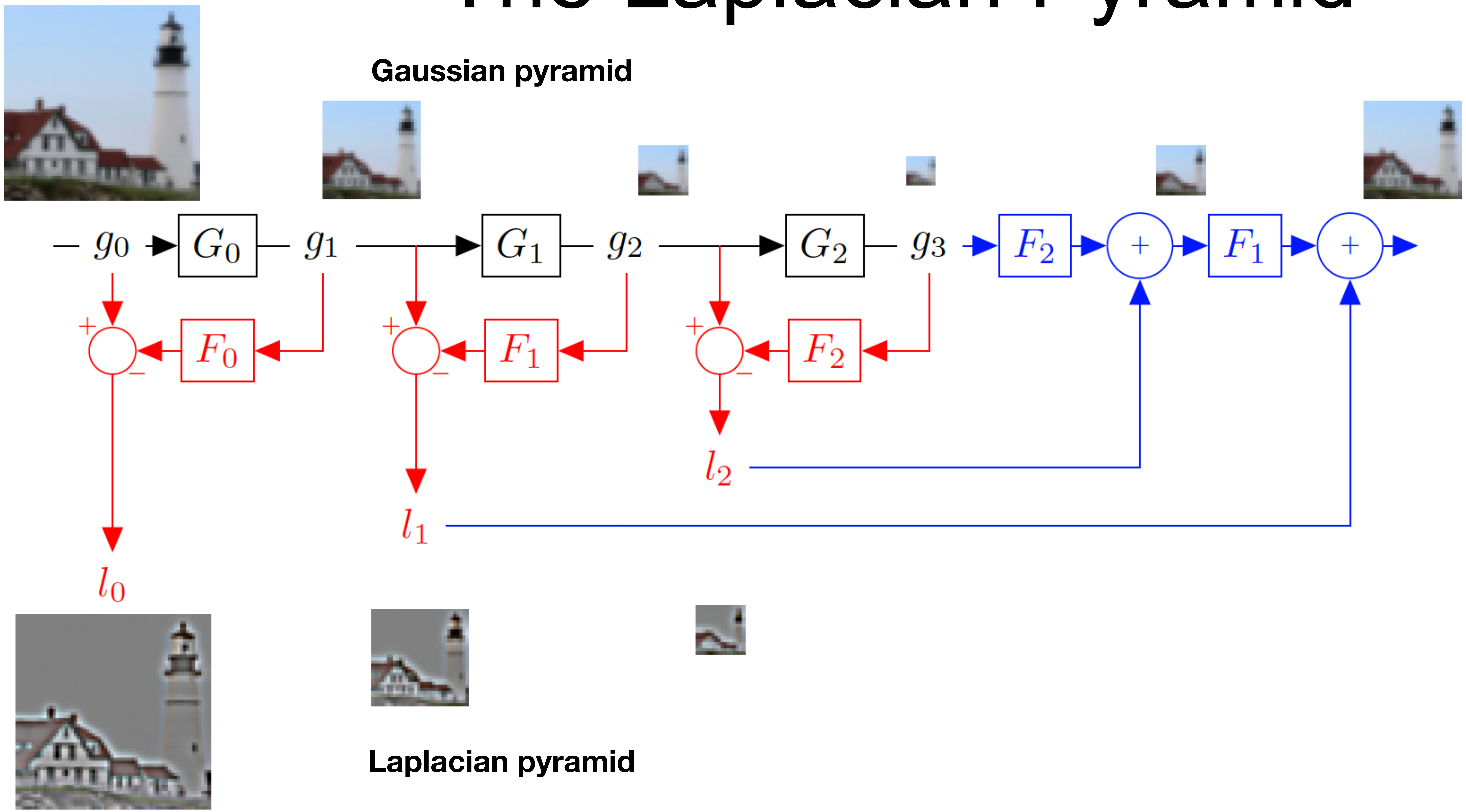
The Laplacian Pyramid



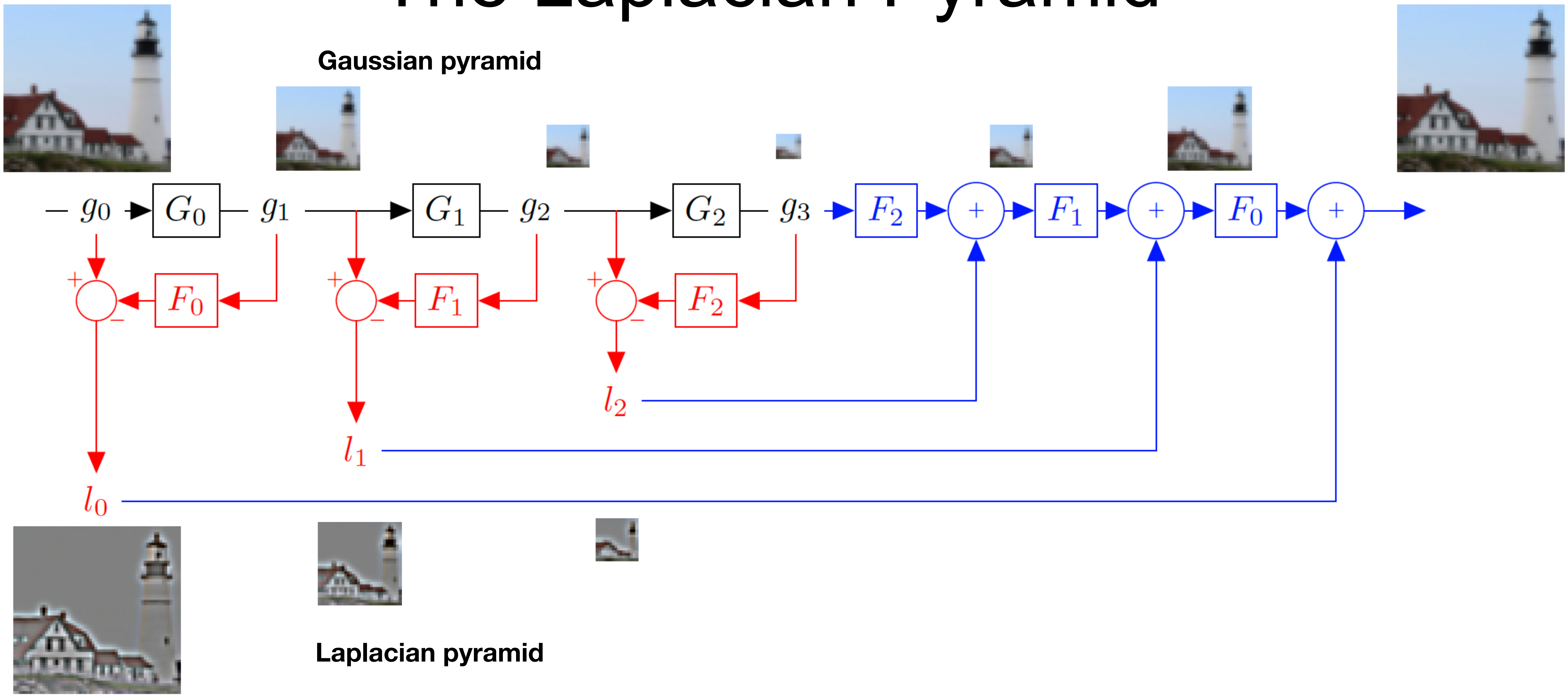
The Laplacian Pyramid



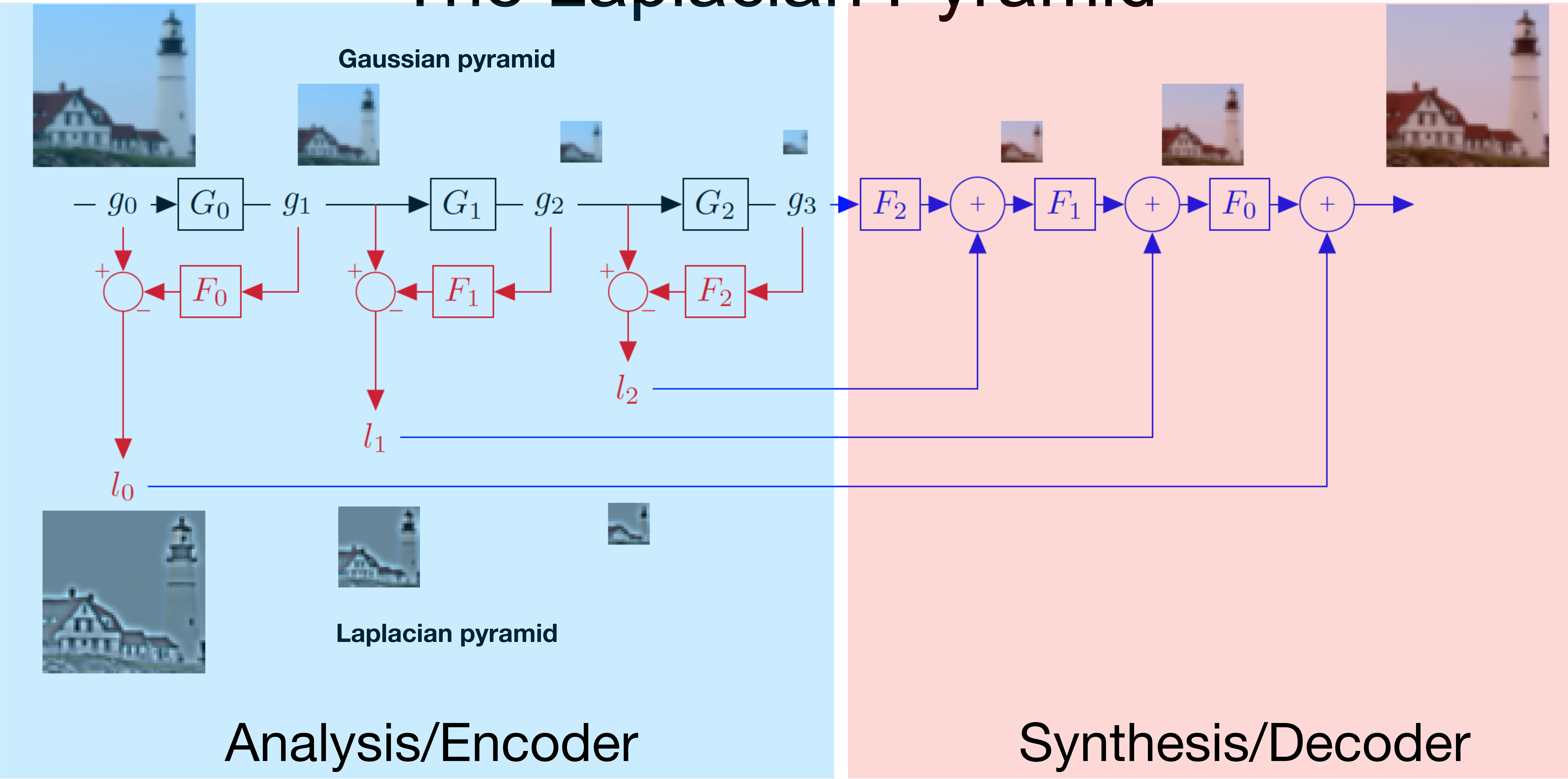
The Laplacian Pyramid



The Laplacian Pyramid



The Laplacian Pyramid



Laplacian pyramid applications

- Texture synthesis
- Image compression
- Noise removal
- Computing image features (e.g., SIFT)
- Image Blending…

Image Blending

Image Blending

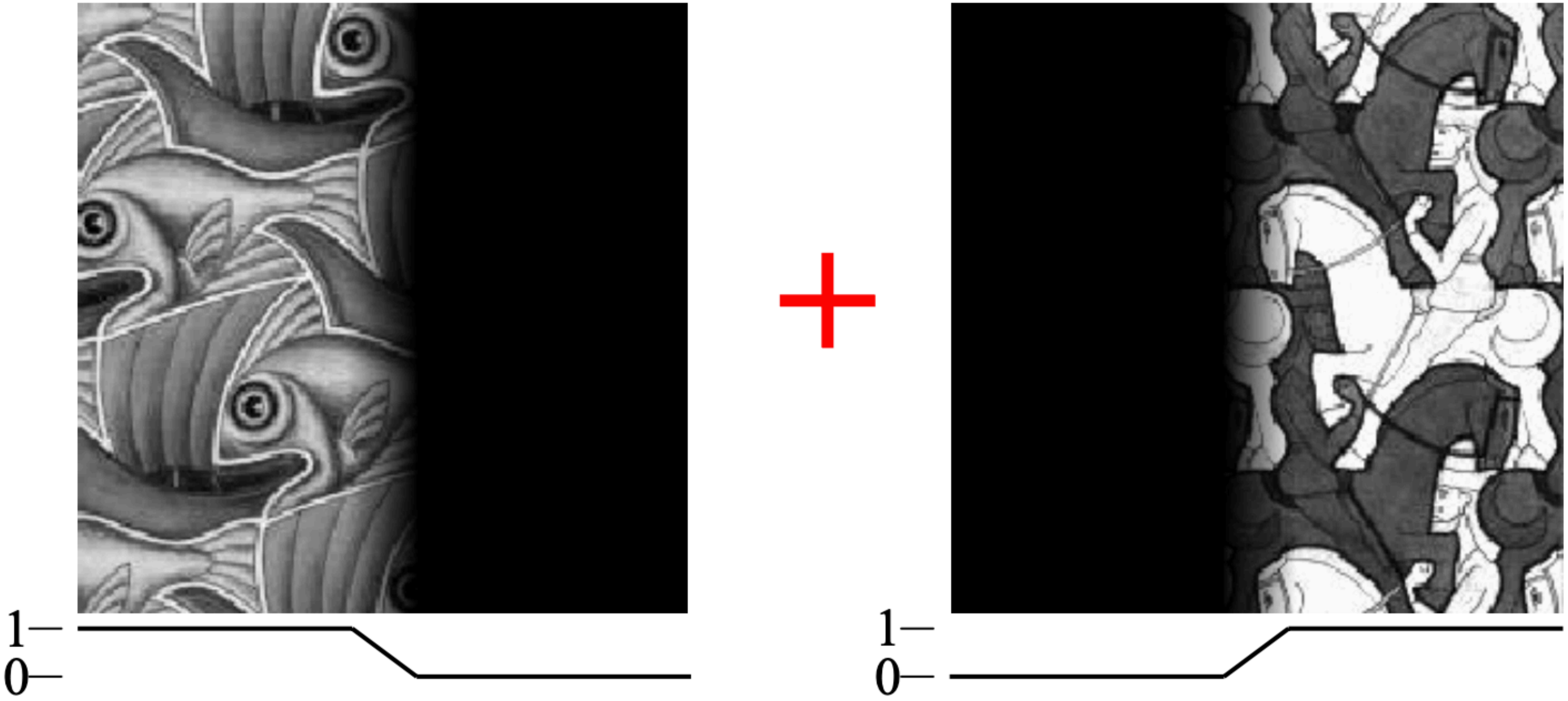
Image Blending

I^A $\qquad$ I^B $\qquad$ m $\qquad$ I

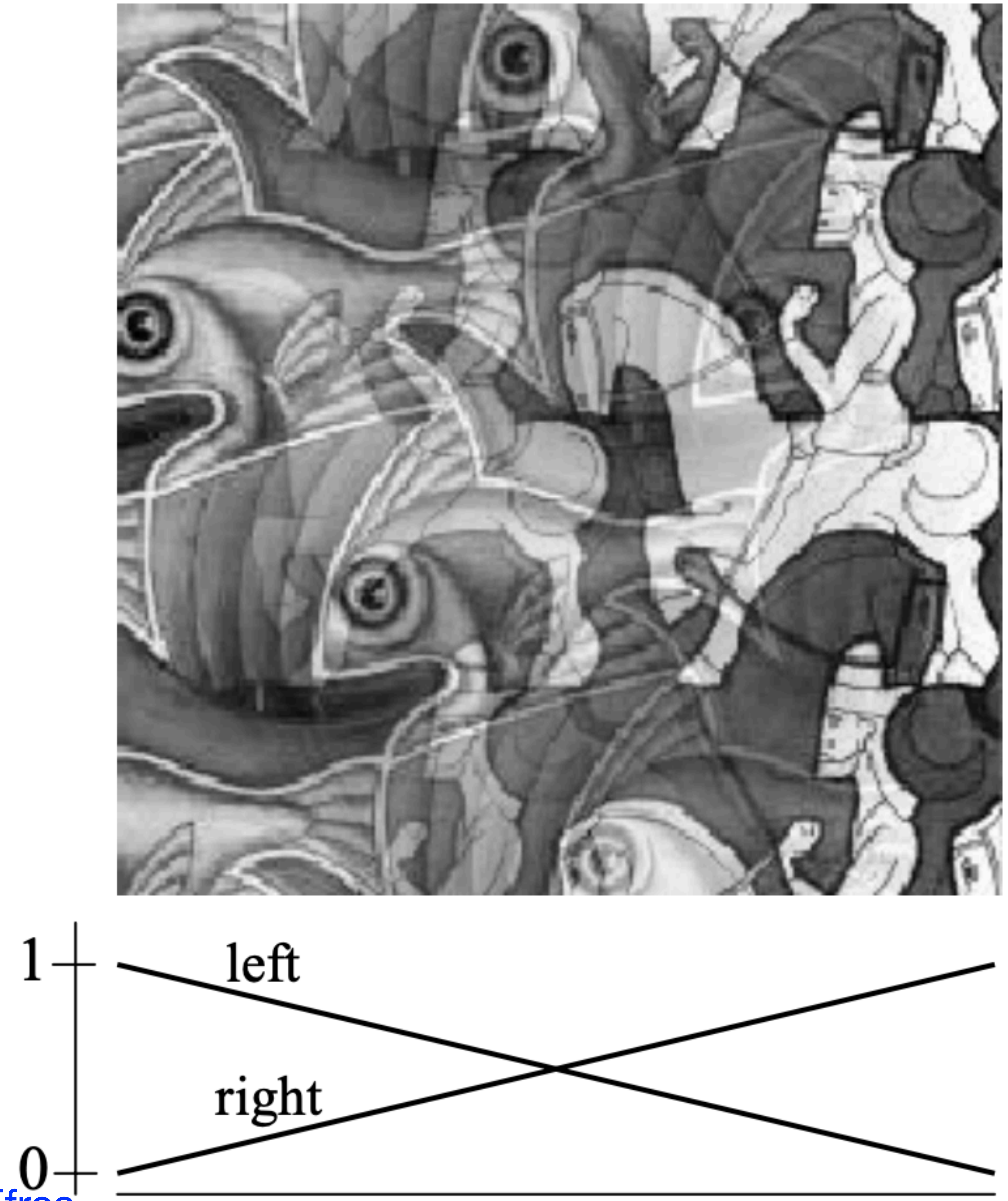
$$I = m * I^A + (1 - m) * I^B$$

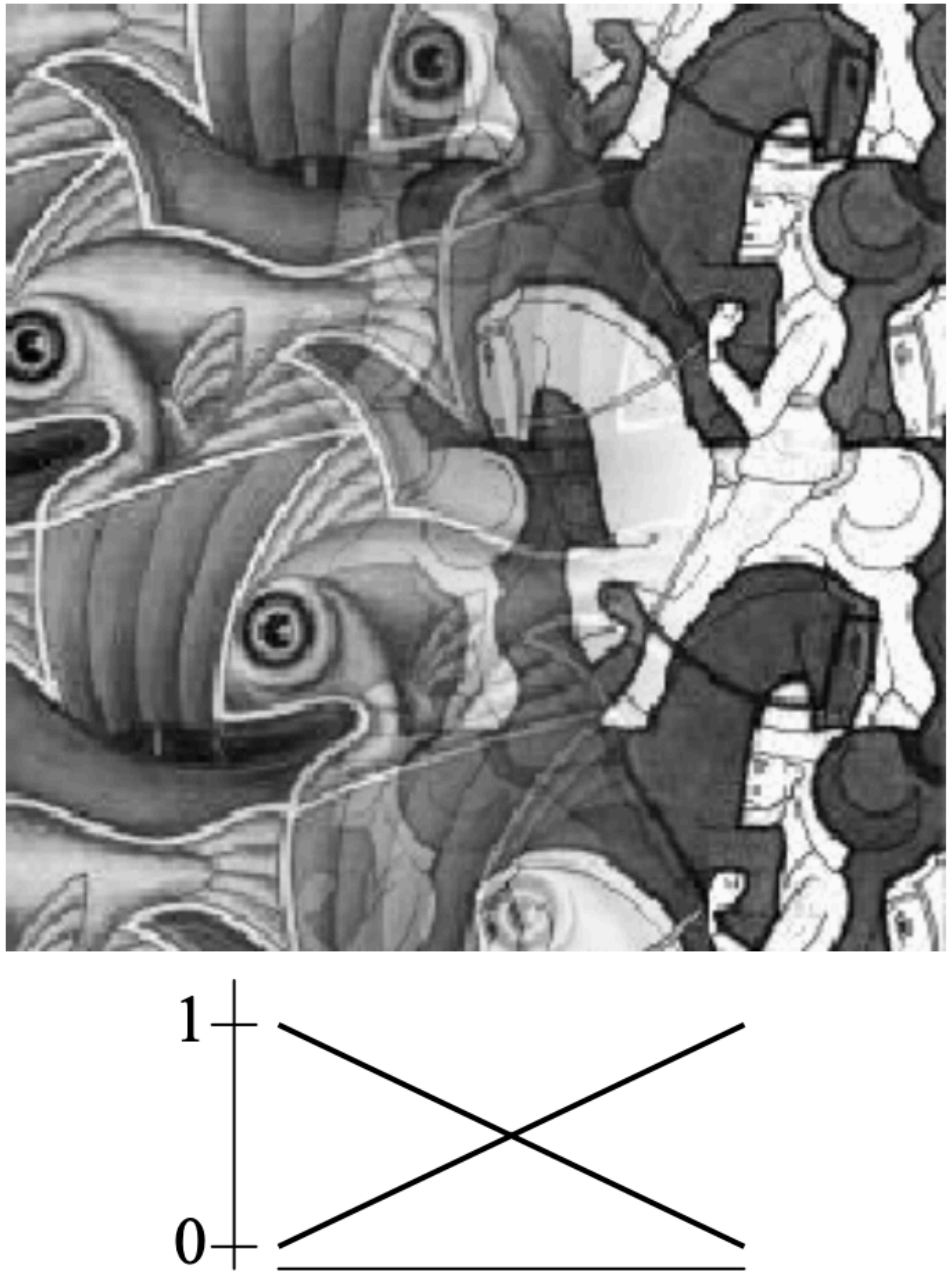
Image Blending

Feathering



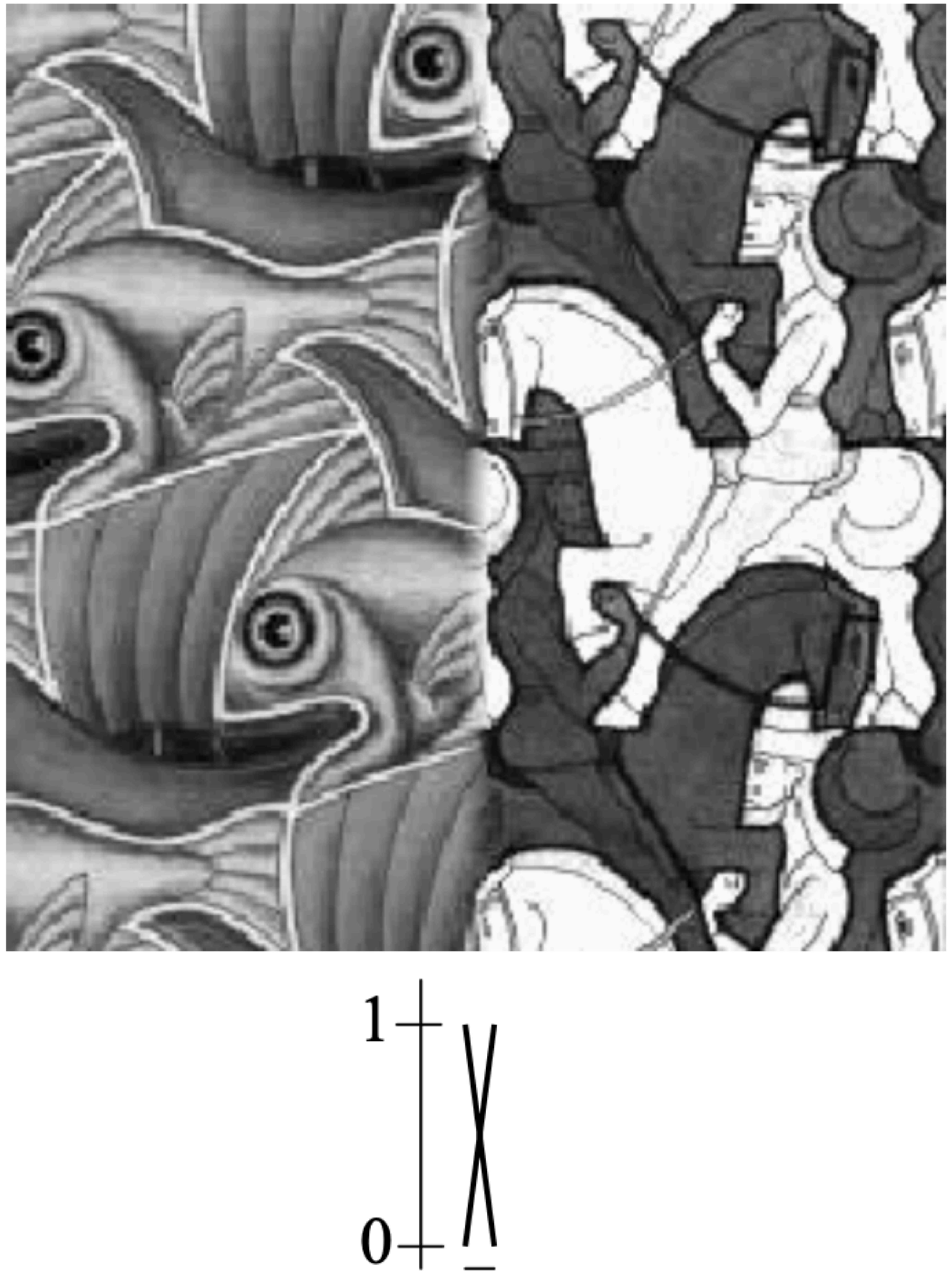
Affect of Window Size

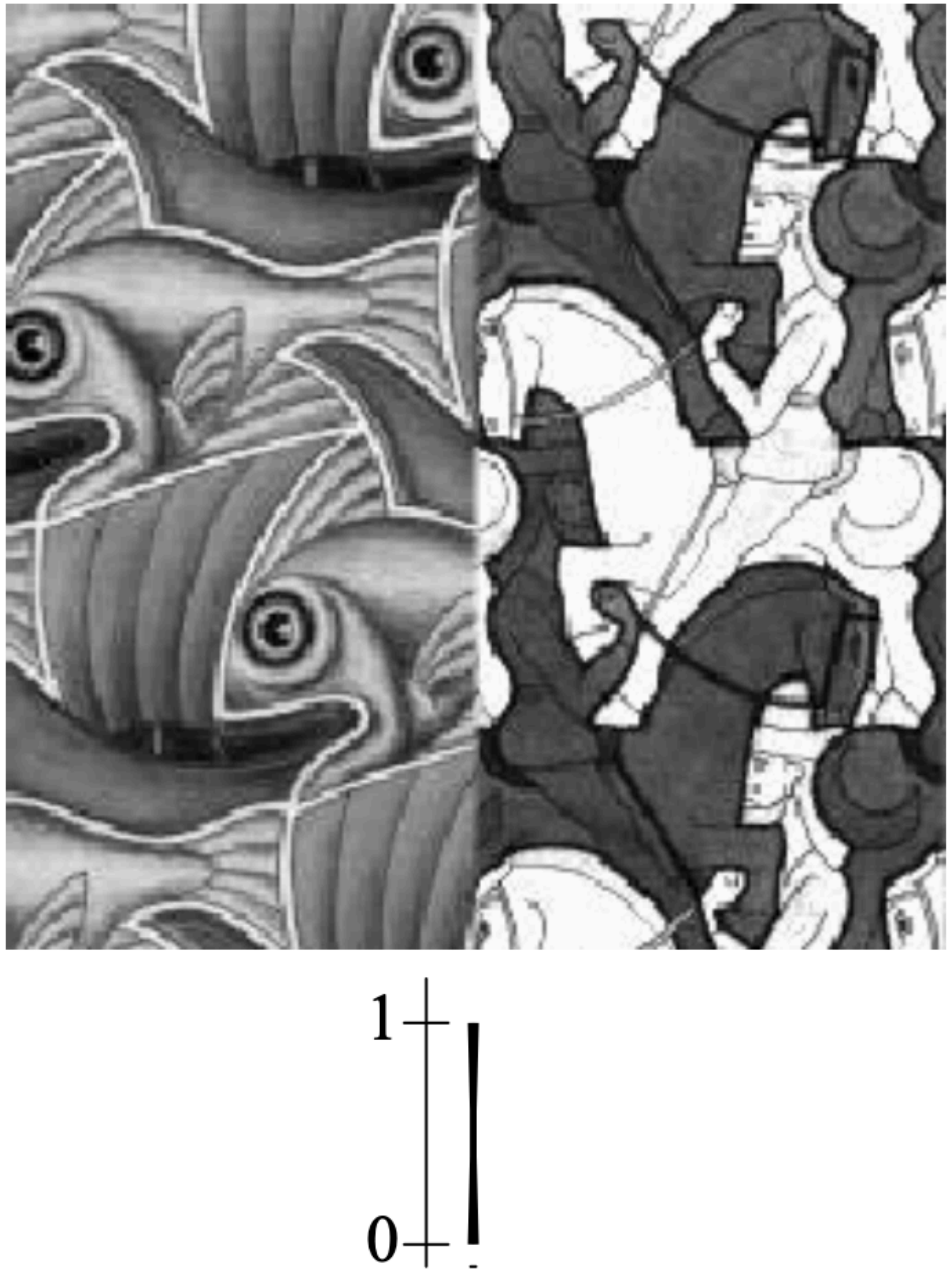




Slide by A. Efros

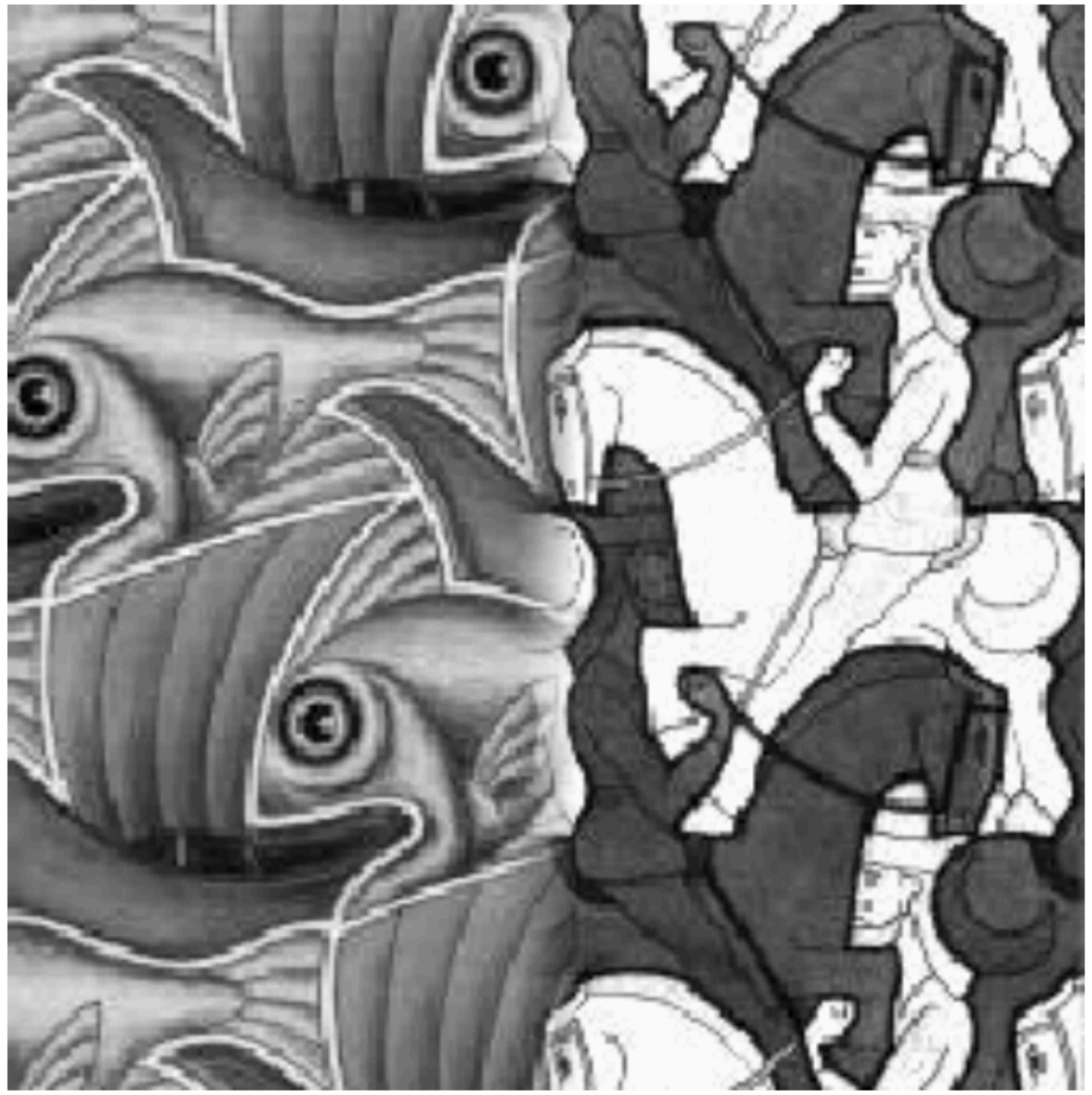
Affect of Window Size

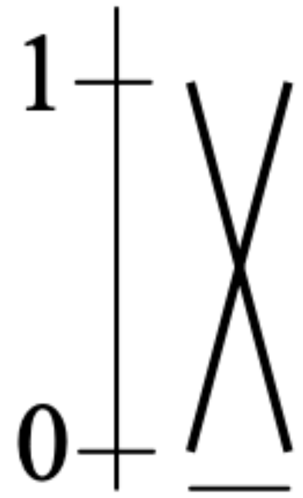




Slide by A. Efros

Good Window Size





"Optimal" Window: smooth but not ghosted

Slide by A. Efros

What is the Optimal Window?

To avoid seams

- window >= size of largest prominent feature

To avoid ghosting

- window <= 2*size of smallest prominent feature

Natural to cast this in the *Fourier domain*

- largest frequency <= 2*size of smallest frequency
- image frequency content should occupy one “octave” (power of two)

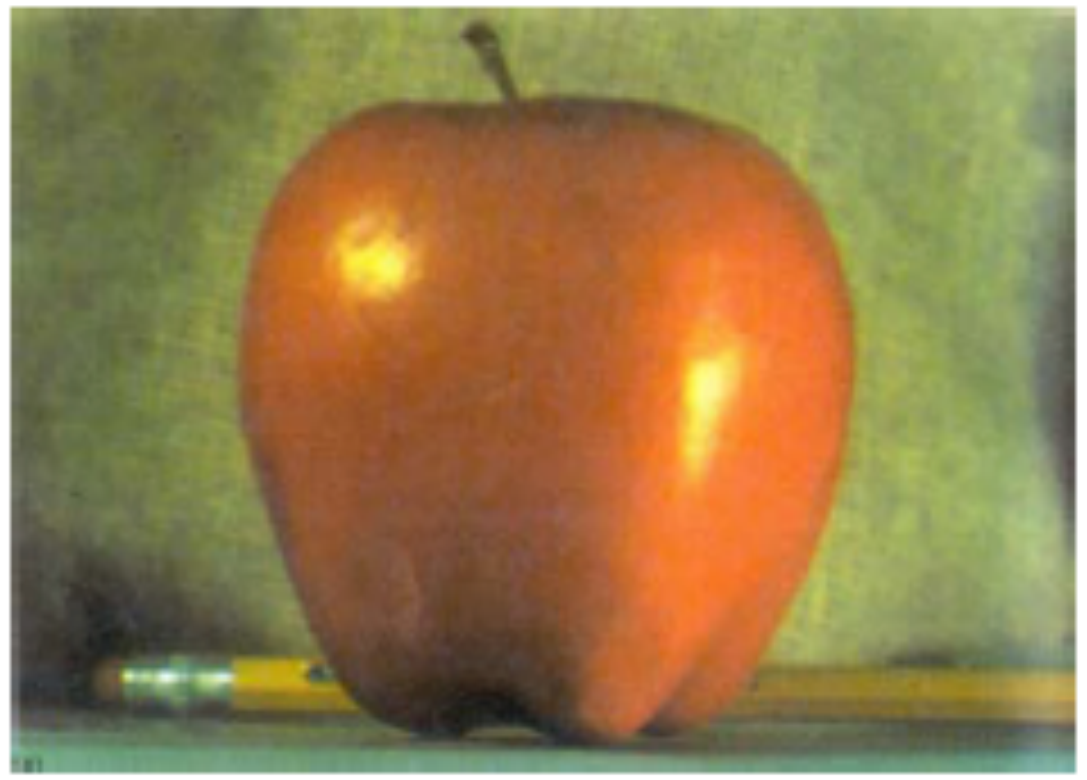
Slide by A. Efros

Image Blending with the Laplacian Pyramid

$$l_k = l_k^A * m_k + l_i^B * (1 - m_k)$$

Image Blending with the Laplacian Pyramid

Image Blending with the Laplacian Pyramid



- Build Laplacian pyramid for both images: LA, LB
- Build Gaussian pyramid for mask: G
- Build a combined Laplacian pyramid:
- Collapse L to obtain the blended image

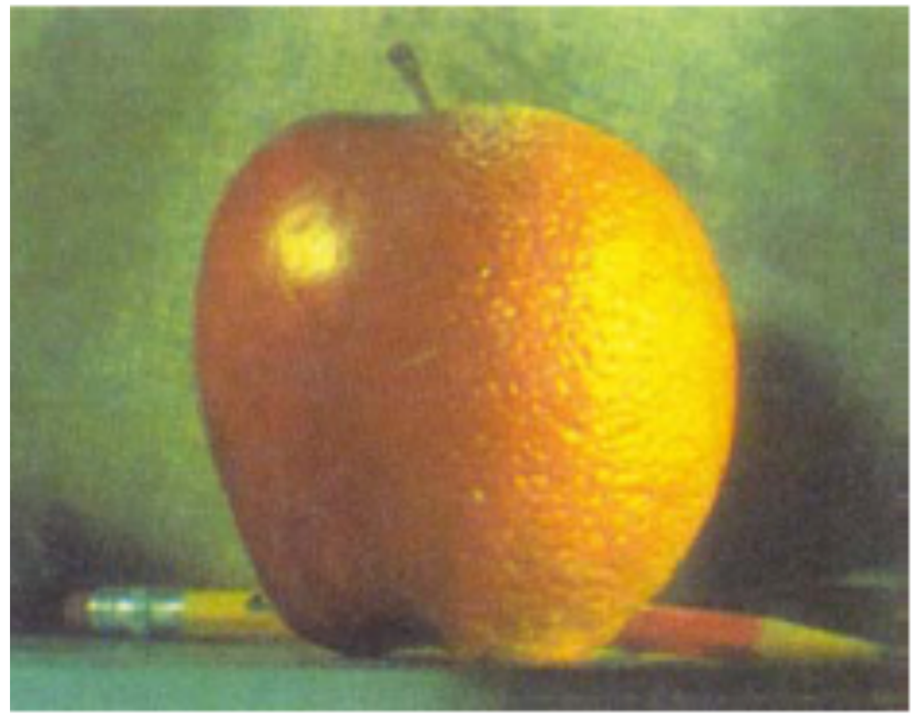
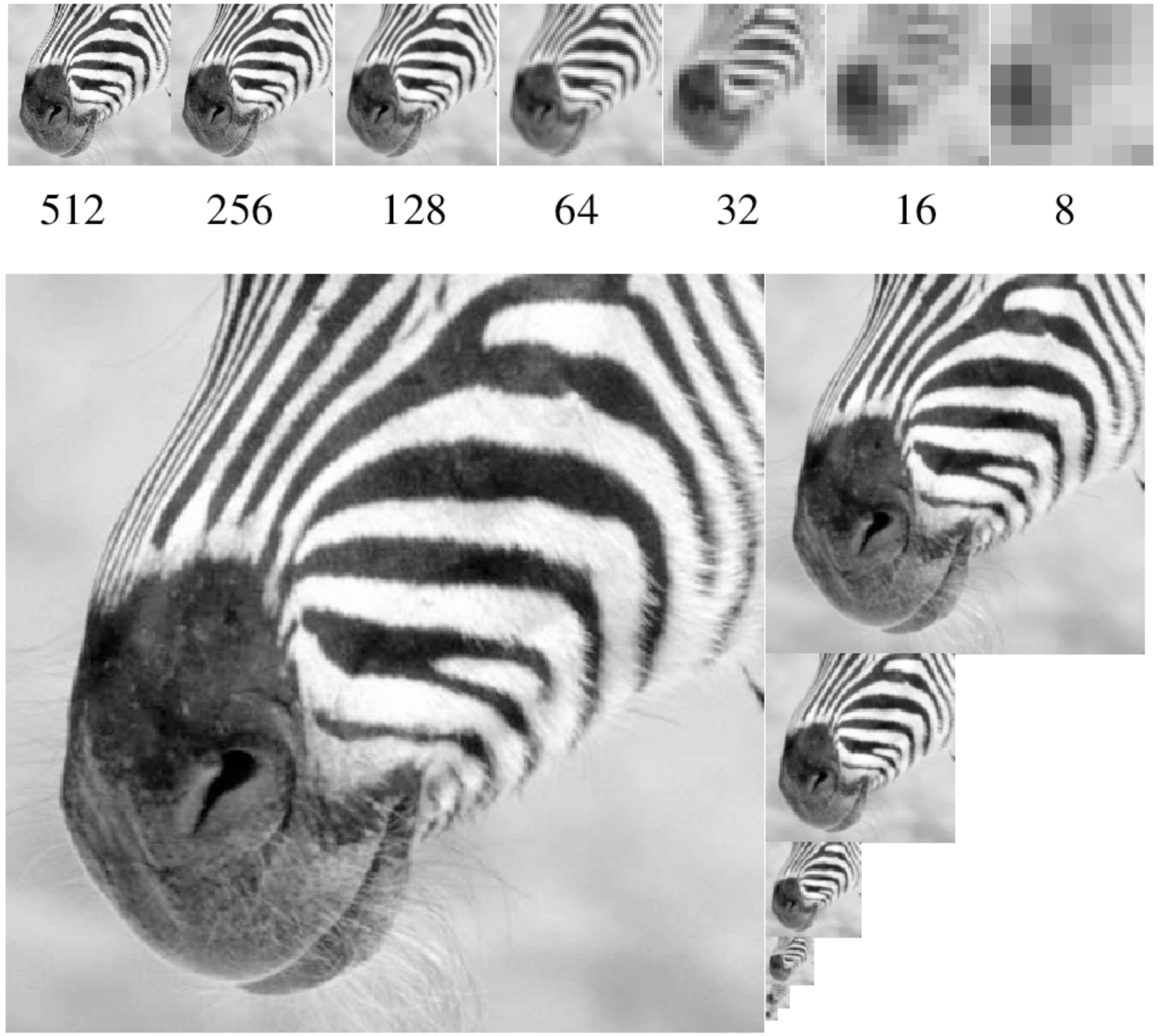
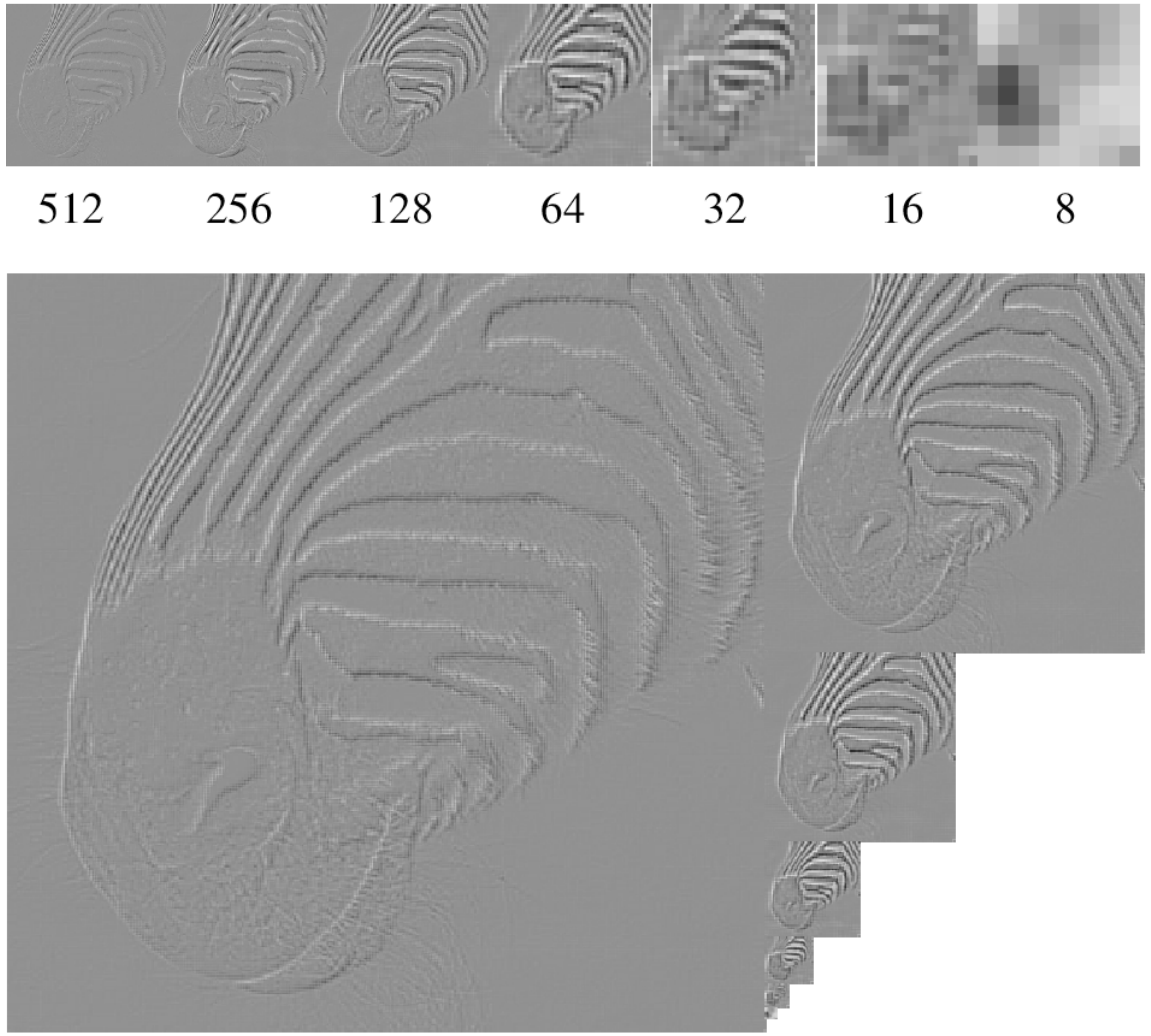


Image pyramids



Gaussian Pyr

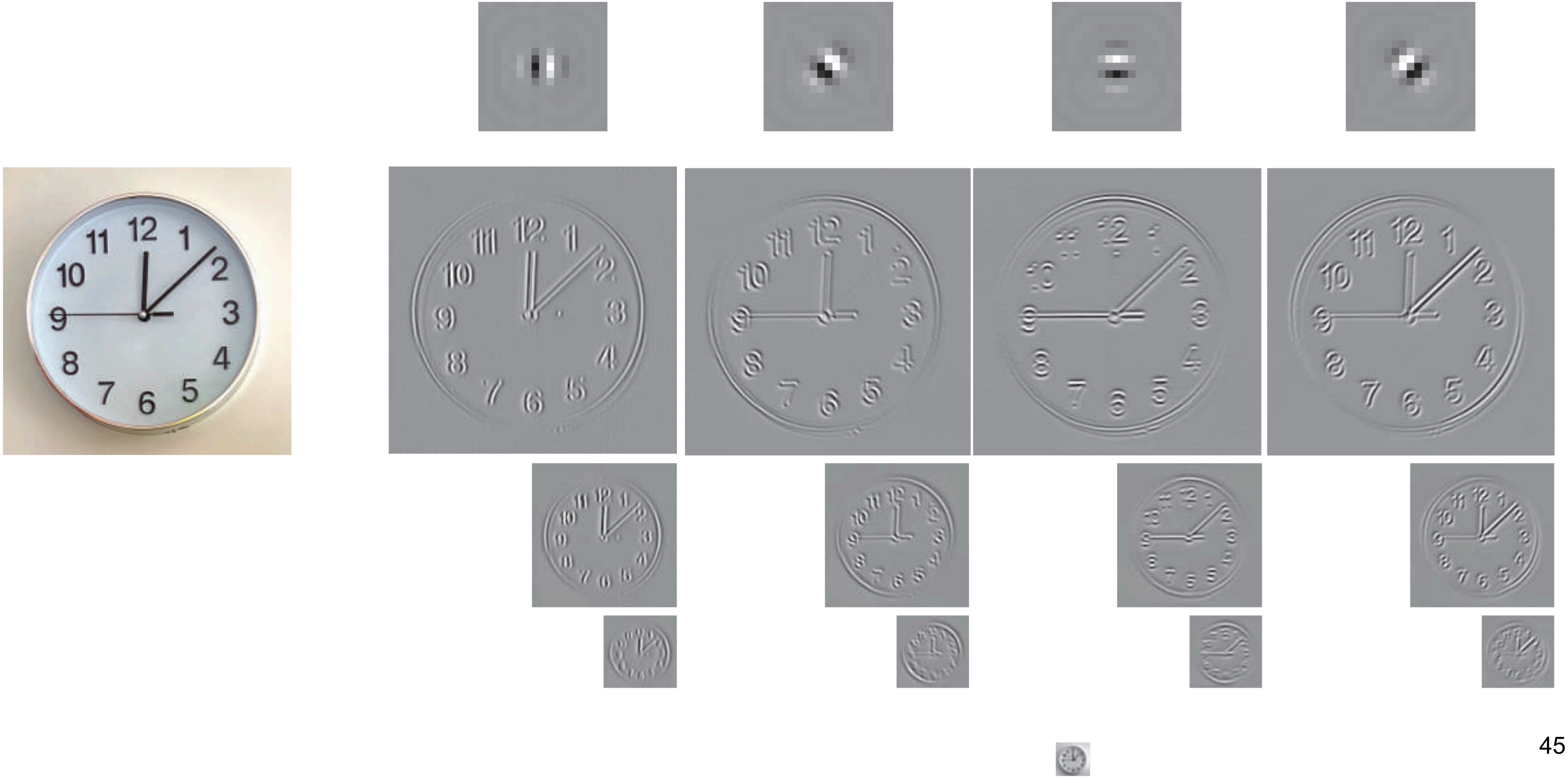


Laplacian Pyr

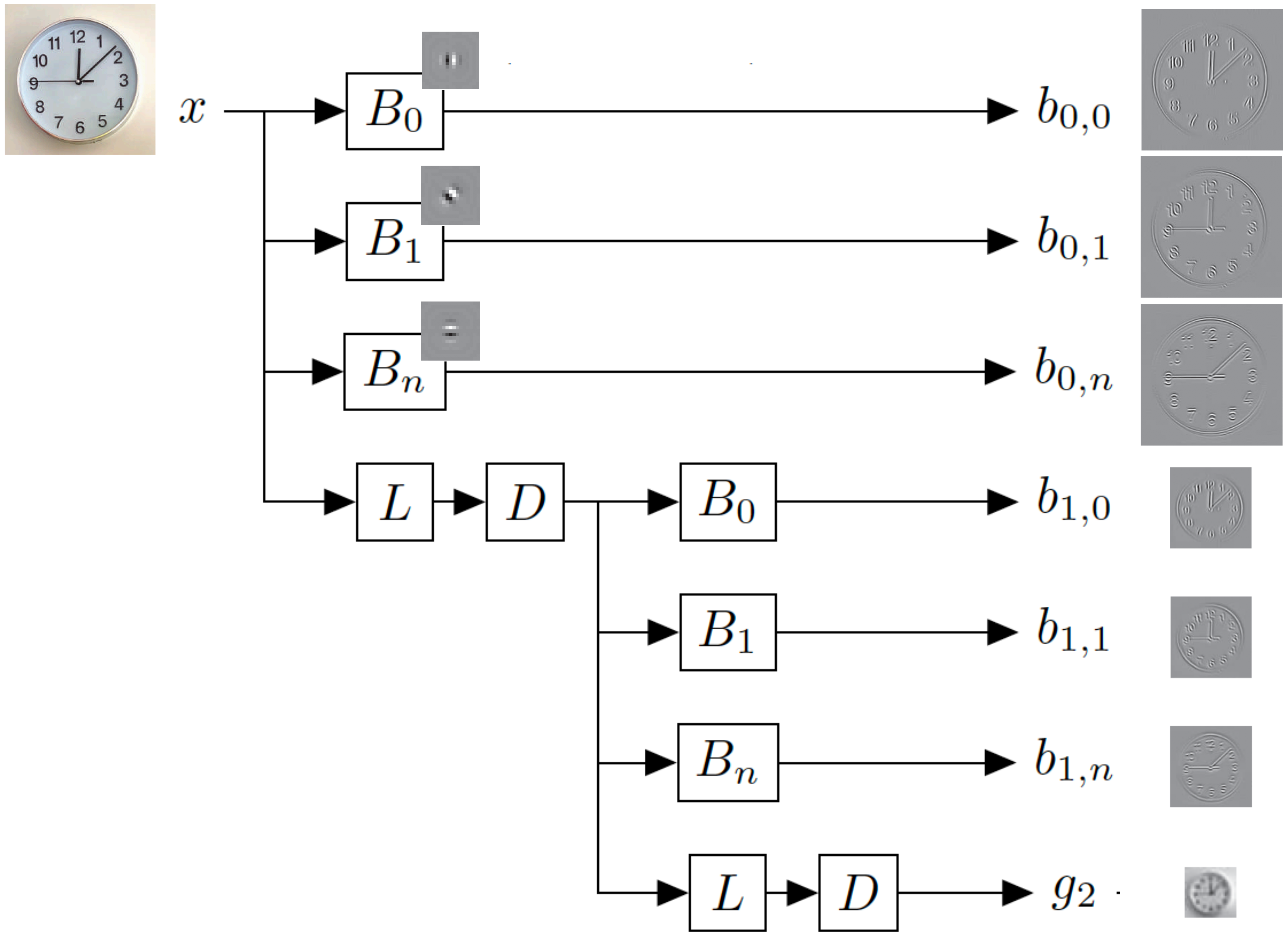
And many more: QMF, steerable, … Convnets!

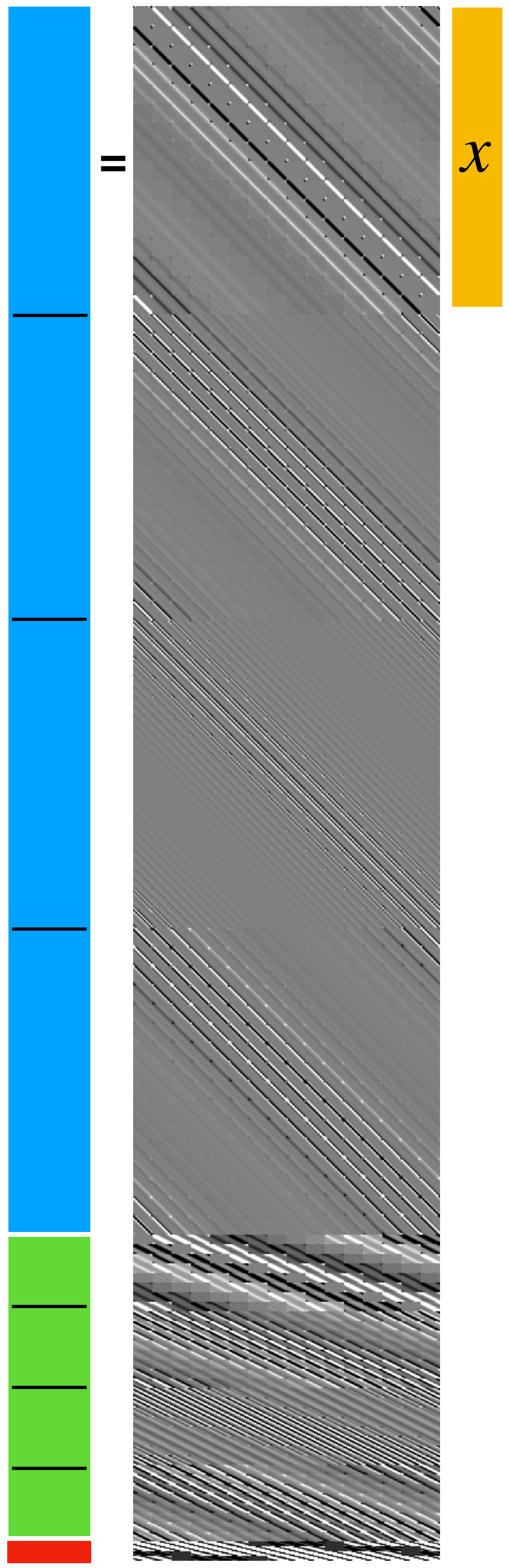
Orientations

Steerable Pyramid

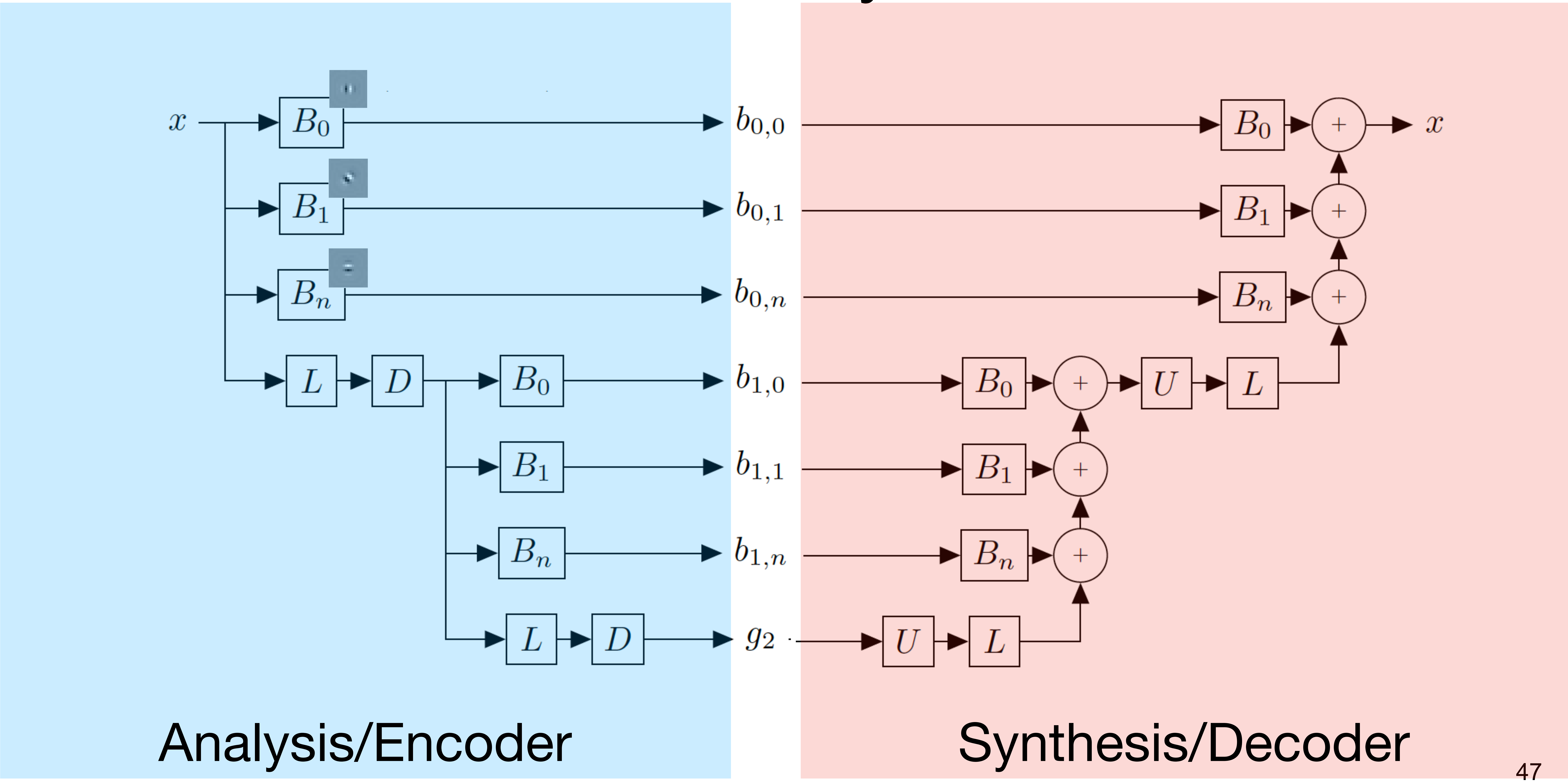


Steerable Pyramid





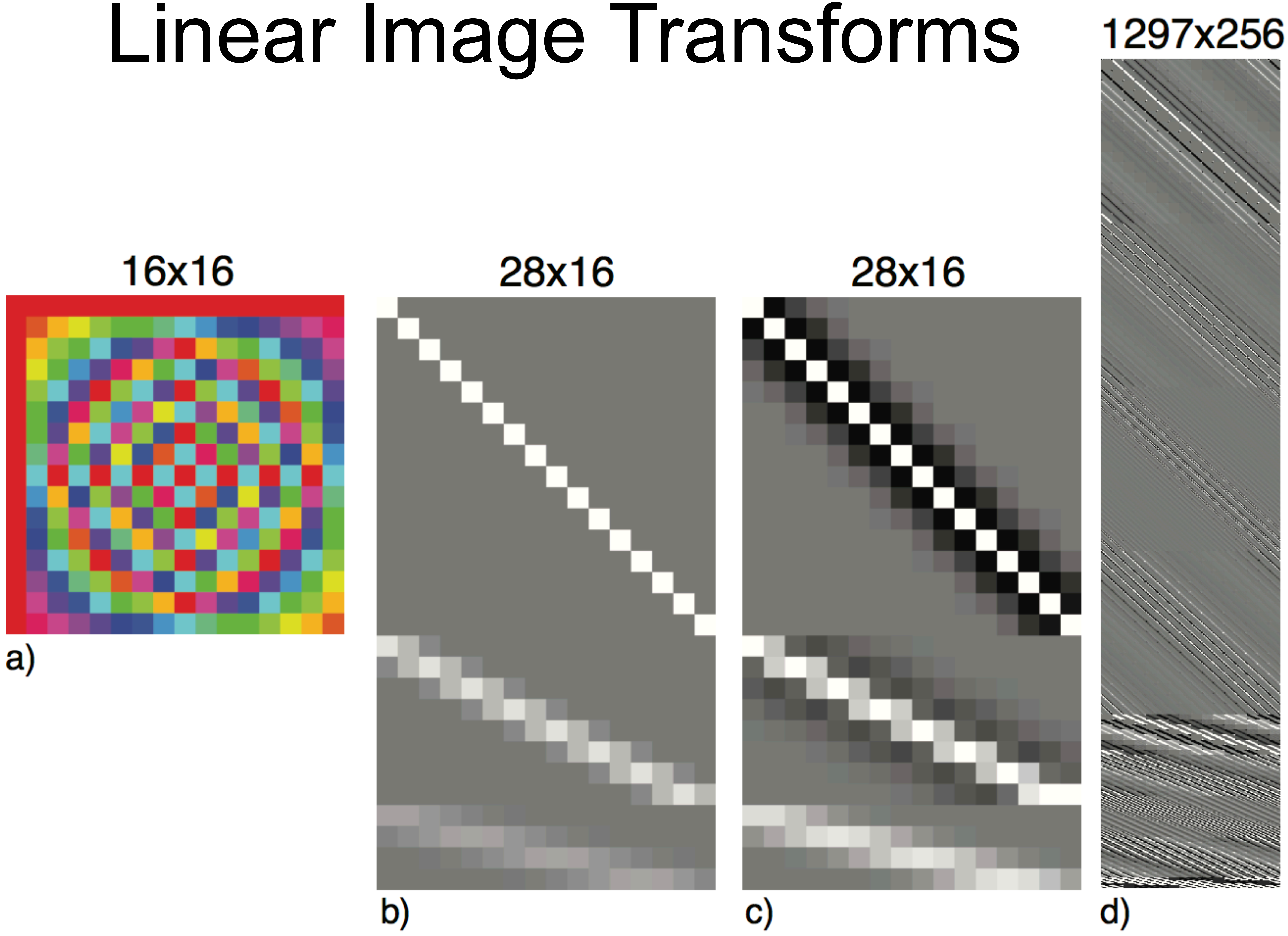
Steerable Pyramid



Steerable pyramid applications

- Texture synthesis
- Noise removal
- Motion analysis
- Motion synthesis, motion magnification

Linear Image Transforms



Making textures

Textures

Stationary

Stochastic


La Bernardine


SPICY BAR-B-Q SAUCE
SWEET & SMOKY BAR-B-Q SAUCE
HOT BAR-B-Q SAUCE
Original BAR-B-Q SAUCE

Nature Vol. 290 12 March 1981

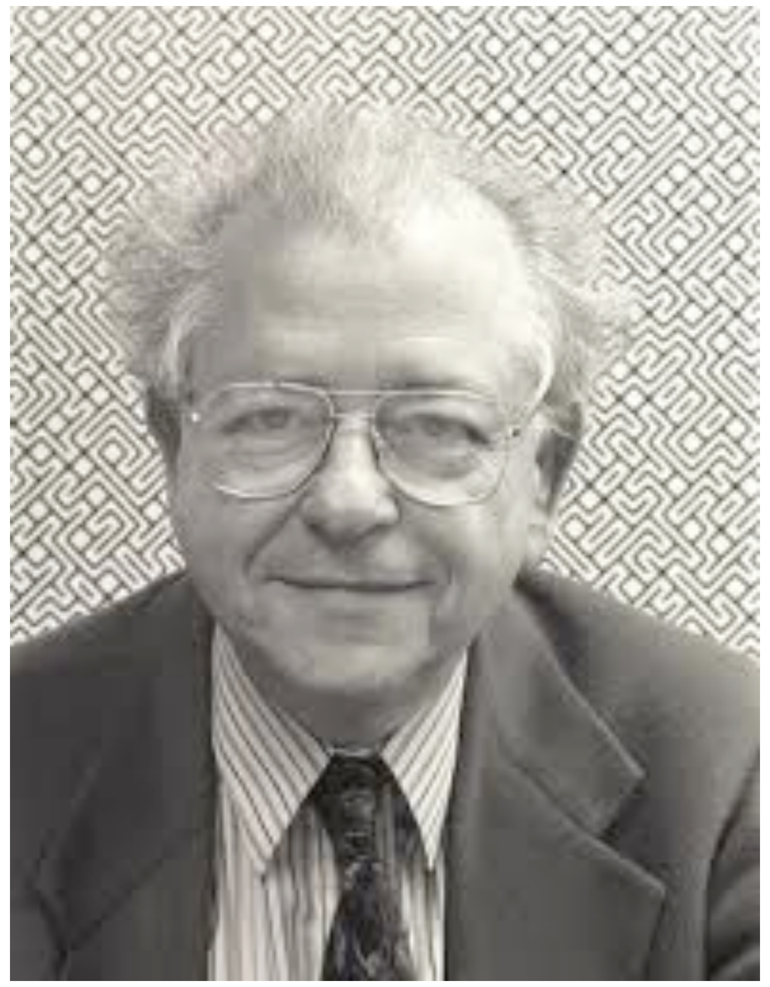
REVIEW ARTICLES

Textons, the elements of texture perception, and their interactions

Bela Julesz

Bell Laboratories, Murray Hill, New Jersey 07974, USA

Research with texture pairs having identical second-order statistics has revealed that the pre-attentive texture discrimination system cannot globally process third- and higher-order statistics, and that discrimination is the result of a few local conspicuous features, called textons. It seems that only the first-order statistics of these textons have perceptual significance, and the relative phase between textons cannot be perceived without detailed scrutiny by focal attention.



Bela Julesz, "Textons, the Elements of Texture Perception, and their Interactions". Nature 290: 91-97. March, 1981.

Pre-attentive texture discrimination

Bela Julesz, "Textons, the Elements of Texture Perception, and their Interactions". Nature 290: 91-97. March, 1981.

Pre-attentive texture discrimination

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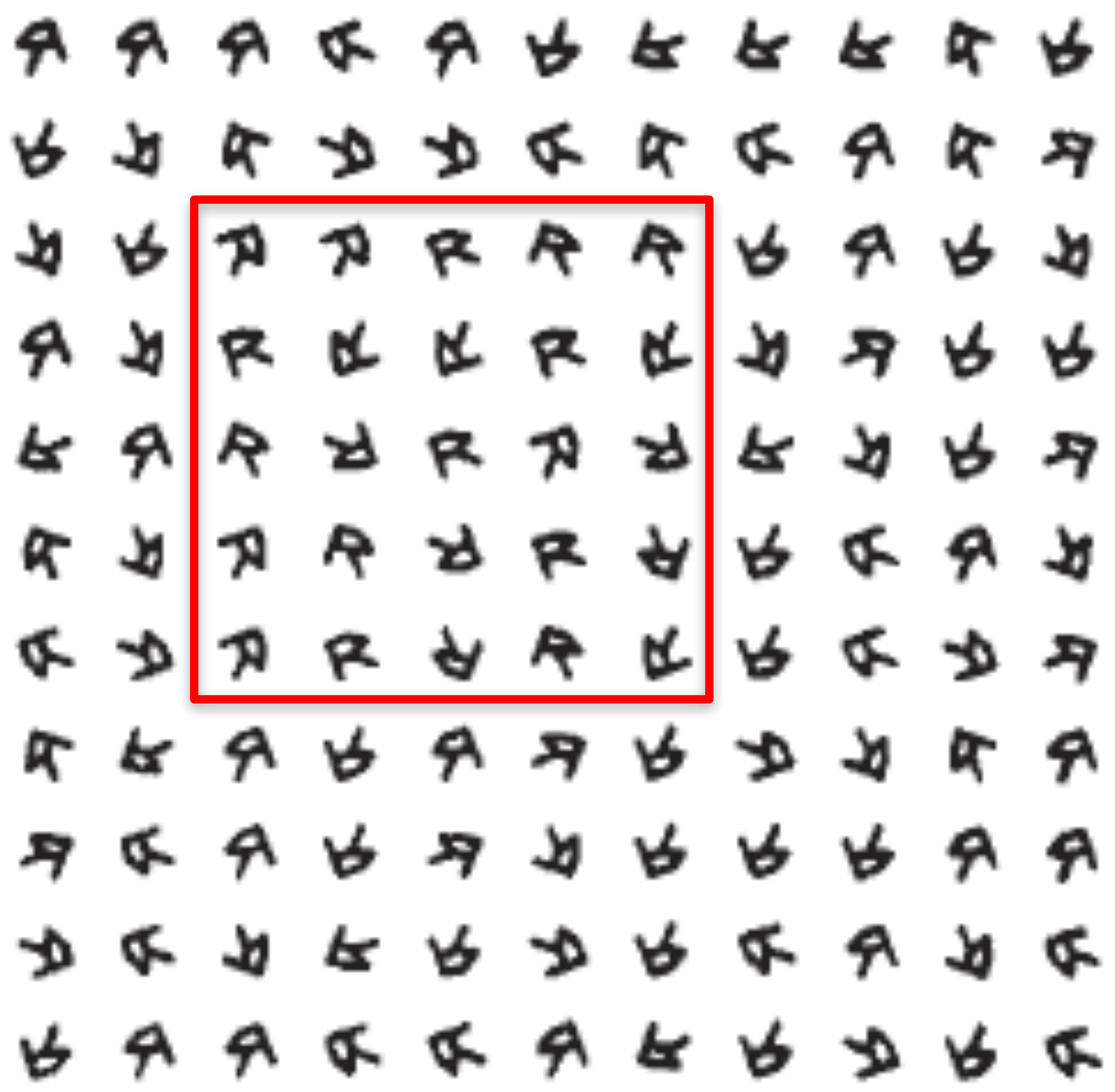
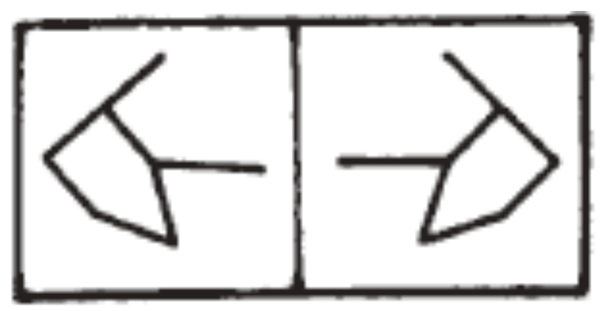
Pre-attentive texture discrimination





This texture pair is pre-attentively indistinguishable. Why?

Bela Julesz, "Textons, the Elements of Texture Perception, and their Interactions". Nature 290: 91-97. March, 1981.

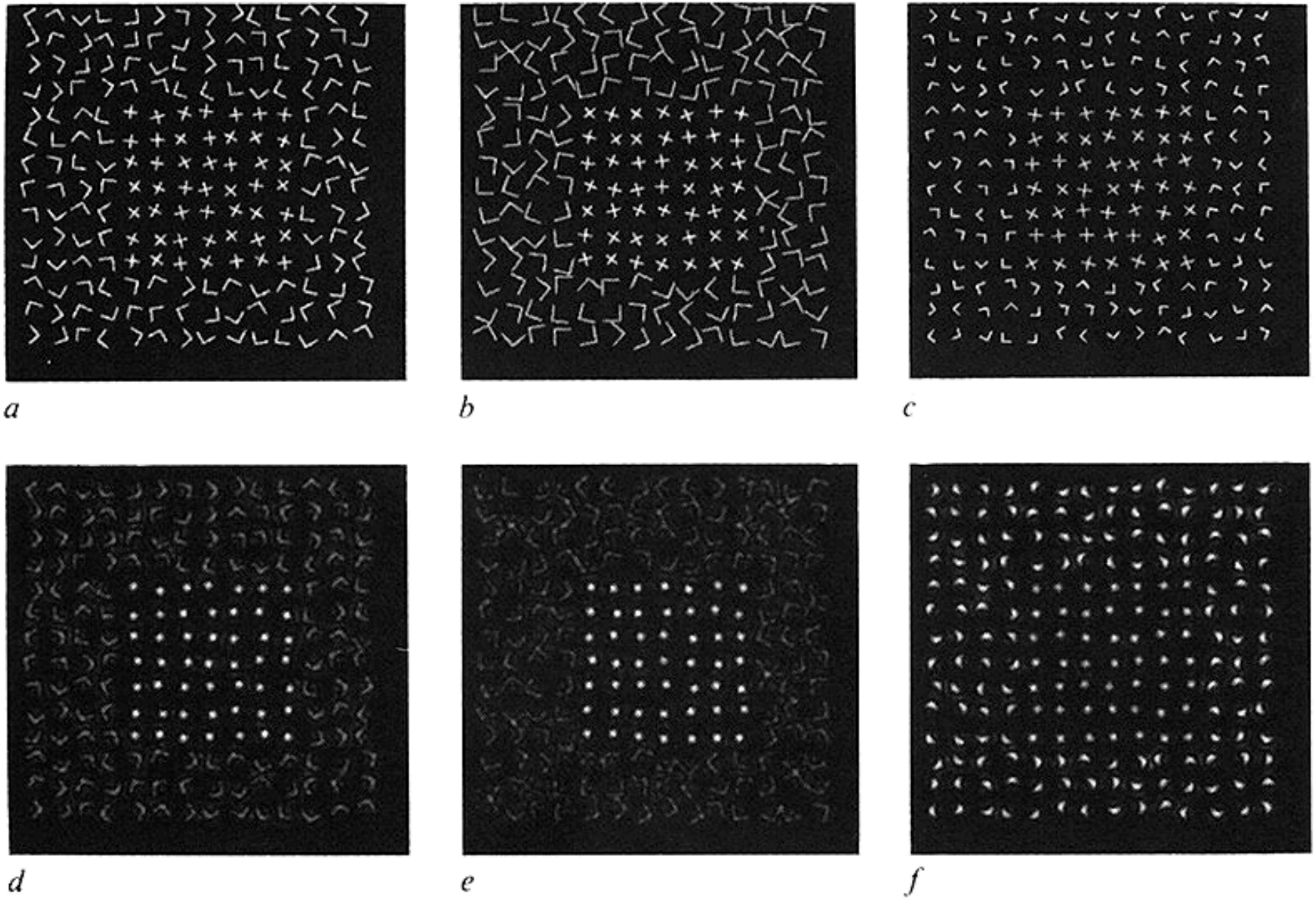


Fig. 1 *Top row*, Textures consisting of Xs within a texture composed of Ls. The micropatterns are placed at random orientations on a randomly perturbed lattice. *a*, The bars of the Xs have the same length as the bars of the Ls. *b*, The bars of the Ls have been lengthened by 25%, and the intensity adjusted for the same mean luminance. Discriminabitity is enhanced. *c*, The bars of the Ls have been shortened by 25%, and the intensity adjusted for the same mean luminance. Discriminabitity is impaired. *Bottom row*: the responses of a size-tuned mechanism *d*, response to image *a*; *e*, response to image *b*; *f*; response to image *c*.

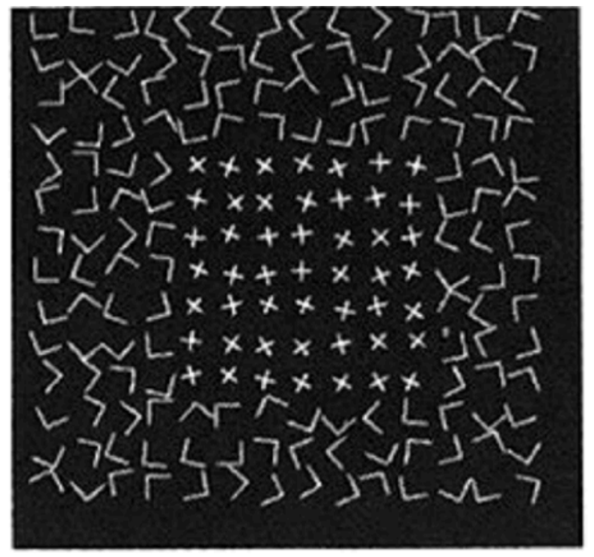
Early vision and texture perception

James R. Bergen* & Edward H. Adelson**

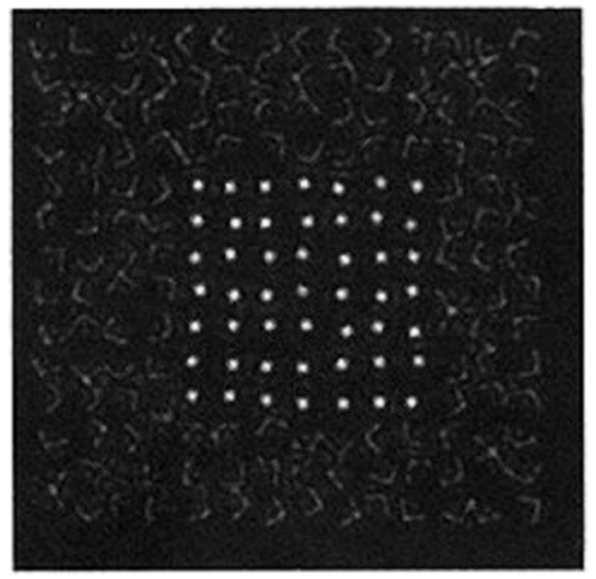
* SRI David Sarnoff Research Center, Princeton, New Jersey 08540, USA
** Media Lab and Department of Brain and Cognitive Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

Reprinted from Nature, Vol. 333. No. 6171. pp. 363-364, 26 May 1988

Jim Bergen's conjecture



b



e

"If matching the mean amplitude of a bandpass spatial filter's response goes a little way towards mimicking human texture perception, then maybe matching the histogram of the responses of filters (the marginal statistics) will do an even better job of capturing human texture perception."

Dave Heeger's response

"You mean if I took a steerable pyramid of some random noise, and forced the histograms of each subband level to match those of some target texture, that the modified noise image would then look like that texture??? No way! I'll prove it to you; here, let me try it out… hmm, gee, that worked pretty well…"

Pyramid-Based Texture Analysis/Synthesis

David J. Heeger*
Stanford University

James R. Bergen†
SRI David Sarnoff Research Center

Abstract

This paper describes a method for synthesizing images that match the texture appearance of a given digitized sample. This synthesis is completely automatic and requires only the "target" texture as input. It allows generation of as much texture as desired so that any object can be covered. It can be used to produce solid textures for creating textured 3-d objects without the distortions inherent in texture mapping. It can also be used to synthesize texture mixtures, images that look a bit like each of several digitized samples. The approach is based on a model of human texture perception, and has potential to be a practically useful tool for graphics applications.

1 Introduction

Computer renderings of objects with surface texture are more interesting and realistic than those without texture. Texture mapping [15] is a technique for adding the appearance of surface detail by wrapping or projecting a digitized texture image onto a surface. Digitized textures can be obtained from a variety of sources, e.g., cropped from a photoCD image, but the resulting texture chip may not have the desired size or shape. To cover a large object you may need to repeat the texture; this can lead to unacceptable artifacts either in the form of visible seams, visible repetition, or both.

Texture mapping suffers from an additional fundamental problem: often there is no natural map from the (planar) texture image to the geometry/topology of the surface, so the texture may be distorted unnaturally when mapped. There are some partial solutions to this distortion problem [15] but there is no universal solution for mapping an image onto an arbitrarily shaped surface.

An alternative to texture mapping is to create (paint) textures by hand directly onto the 3-d surface model [14], but this process is both very labor intensive and requires considerable artistic skill.

Another alternative is to use computer-synthesized textures so that as much texture can be generated as needed. Furthermore, some of the synthesis techniques produce textures that tile seamlessly.

Using synthetic textures, the distortion problem has been solved in two different ways. First, some techniques work by synthesizing texture directly on the object surface (e.g., [31]). The second solution is to use *solid textures* [19, 23, 24]. A solid texture is a 3-d array of color values. A point on the surface of an object is colored by the value of the solid texture at the corresponding 3-d point. Solid texturing can be a very natural solution to the distortion problem: there is no distortion because there is no mapping. However, existing techniques for synthesizing solid textures can be quite cumbersome. One must learn how to tweak the parameters or procedures of the texture synthesizer to get a desired effect.

This paper presents a technique for synthesizing an image (or solid texture) that matches the appearance of a given texture sample. The key advantage of this technique is that it works entirely from the example texture, requiring no additional information or adjustment. The technique starts with a digitized image and analyzes it to compute a number of texture parameter values. Those parameter values are then used to synthesize a new image (of any size) that looks (in its color and texture properties) like the original. The analysis phase is inherently two-dimensional since the input digitized images are 2-d. The synthesis phase, however, may be either two- or three-dimensional. For the 3-d case, the output is a solid texture such that planar slices through the solid look like the original scanned image. In either case, the (2-d or 3-d) texture is synthesized so that it tiles seamlessly.

2 Texture Models

Textures have often been classified into two categories, deterministic textures and stochastic textures. A deterministic texture is characterized by a set of primitives and a placement rule (e.g., a tile floor). A stochastic texture, on the other hand, does not have easily identifiable primitives (e.g., granite, bark, sand). Many real-world textures have some mixture of these two characteristics (e.g. woven fabric, woodgrain, plowed fields).

Much of the previous work on texture analysis and synthesis can be classified according to what type of texture model was used. Some of the successful texture models include reaction-diffusion [31, 34], frequency domain [17], fractal [9, 18], and statistical/random field [1, 6, 8, 10, 12, 13, 21, 26] models. Some (e.g., [10]) have used hybrid models that include a deterministic (or periodic) component and a stochastic component. In spite of all this work, scanned images and hand-drawn textures are still the principle source of texture maps in computer graphics.

This paper focuses on the synthesis of stochastic textures. Our approach is motivated by research on human texture perception. Current theories of texture discrimination are based on the fact that two textures are often difficult to discriminate when they produce a similar distribution of responses in a bank of (orientation and spatial-frequency selective) linear filters [2, 3, 7, 16, 20, 32]. The method described here, therefore, synthesizes textures by matching distributions (or histograms) of filter outputs. This approach depends on the principle (not entirely correct as we shall see) that all of the spatial information characterizing a texture image can be captured in the first order statistics of an appropriately chosen set of linear filter outputs. Nevertheless, this model (though incomplete) captures an interesting set of texture properties.

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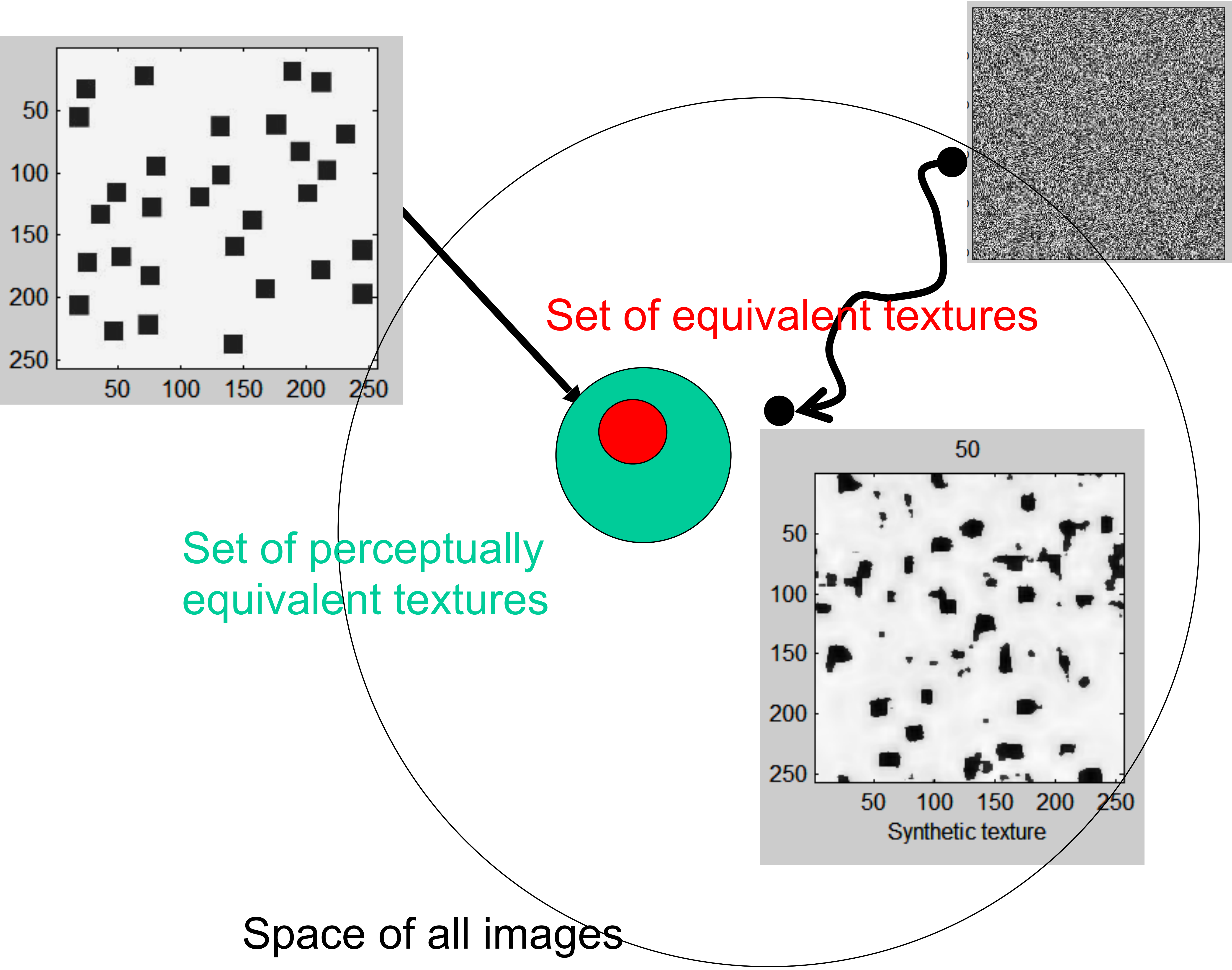
Figure 5: (Top Row) Original digitized sample textures: red granite, berry bush, figured maple, yellow coral. (Bottom Rows) Synthetic solid textured teapots.

https://www.cns.nyu.edu/heegerlab/content/publications/Heeger-siggraph95.pdf

SIGGRAPH 1995

The main idea: it works by ‘kind of’ projecting a random image into the set of equivalent textures



Overview of the algorithm

```
Match-texture(noise,texture)
 Match-Histogram (noise,texture)
 analysis-pyr = Make-Pyramid (texture)
 Loop for several iterations do
    synthesis-pyr = Make-Pyramid (noise)
    Loop for a-band in subbands of analysis-pyr
         for s-band in subbands of synthesis-pyr
         do
         Match-Histogram (s-band,a-band)
    noise = Collapse-Pyramid (synthesis-pyr)
    Match-Histogram (noise,texture)
```

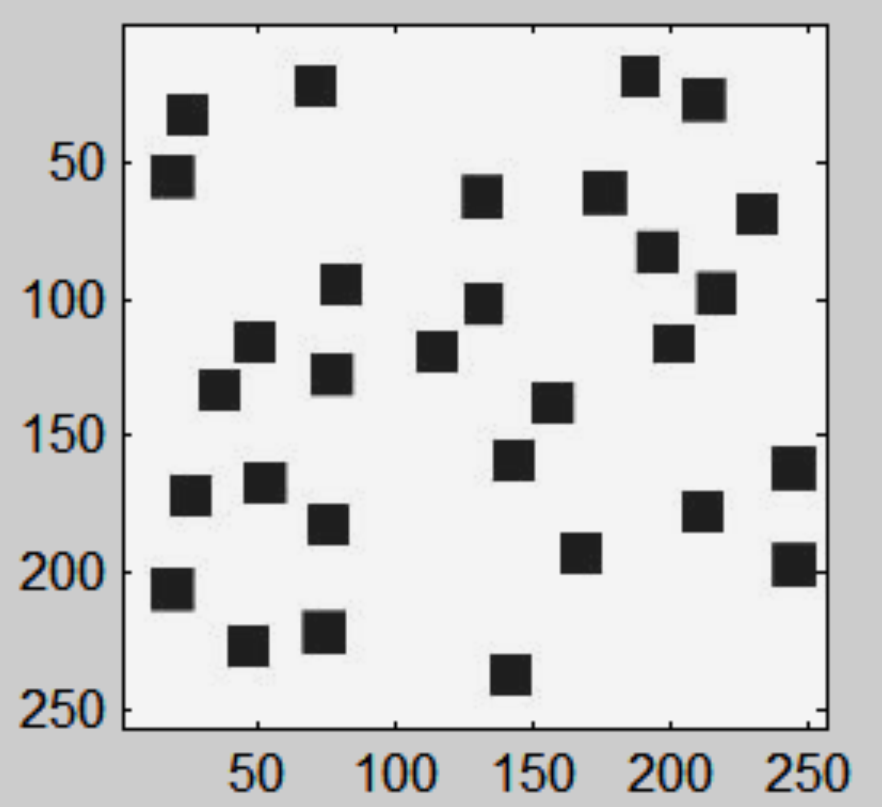
Two main tools:

1- steerable pyramid

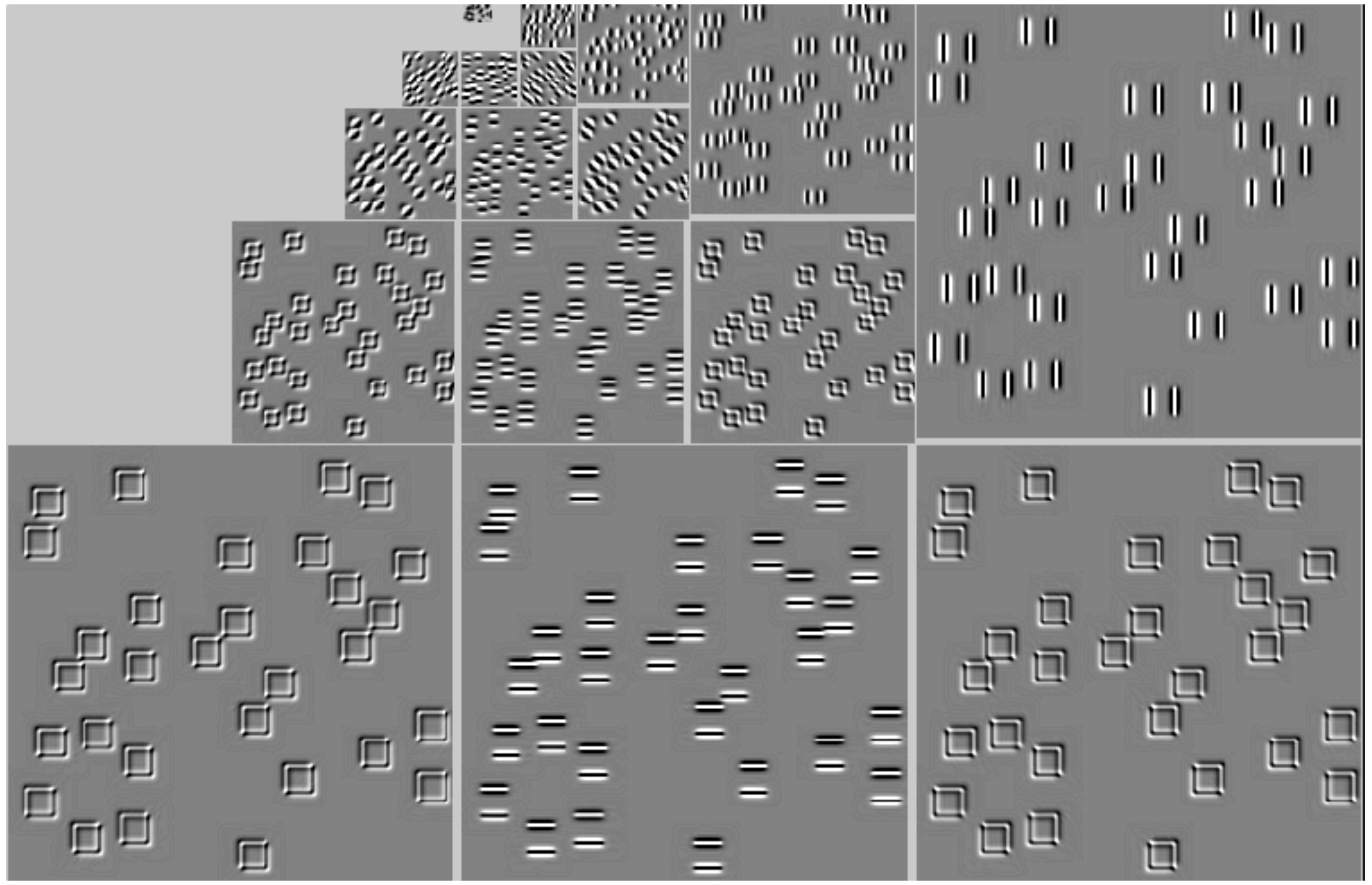
2- matching histograms

1-The steerable pyramid

Input texture



Steerable pyr



Overview of the algorithm

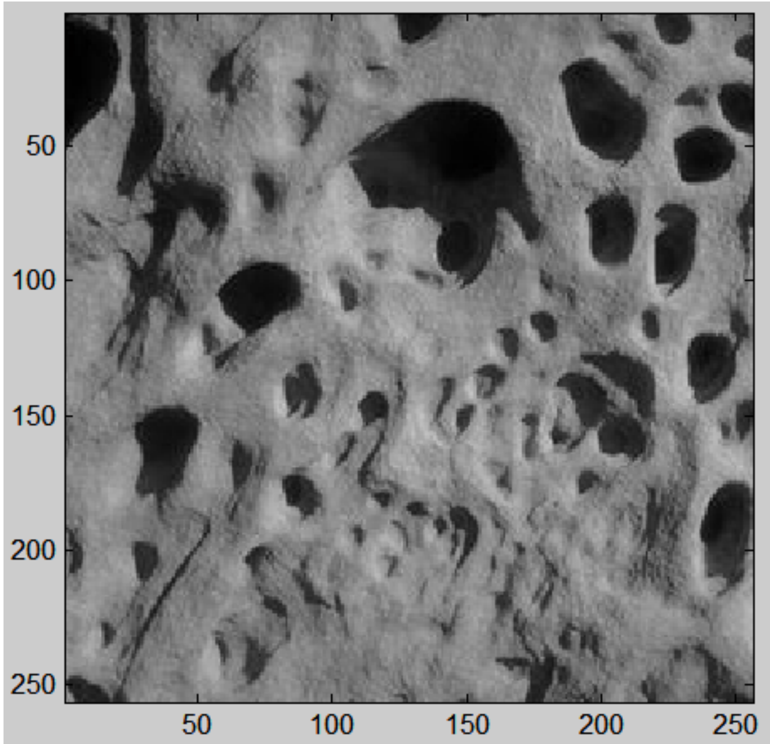
```
Match-texture(noise,texture)
  Match-Histogram (noise,texture)
  analysis-pyr = Make-Pyramid (texture)
  Loop for several iterations do
     synthesis-pyr = Make-Pyramid (noise)
     Loop for a-band in subbands of analysis-pyr
          for s-band in subbands of synthesis-pyr
          do
          Match-Histogram (s-band,a-band)
     noise = Collapse-Pyramid (synthesis-pyr)
     Match-Histogram (noise,texture)
```

Two main tools:

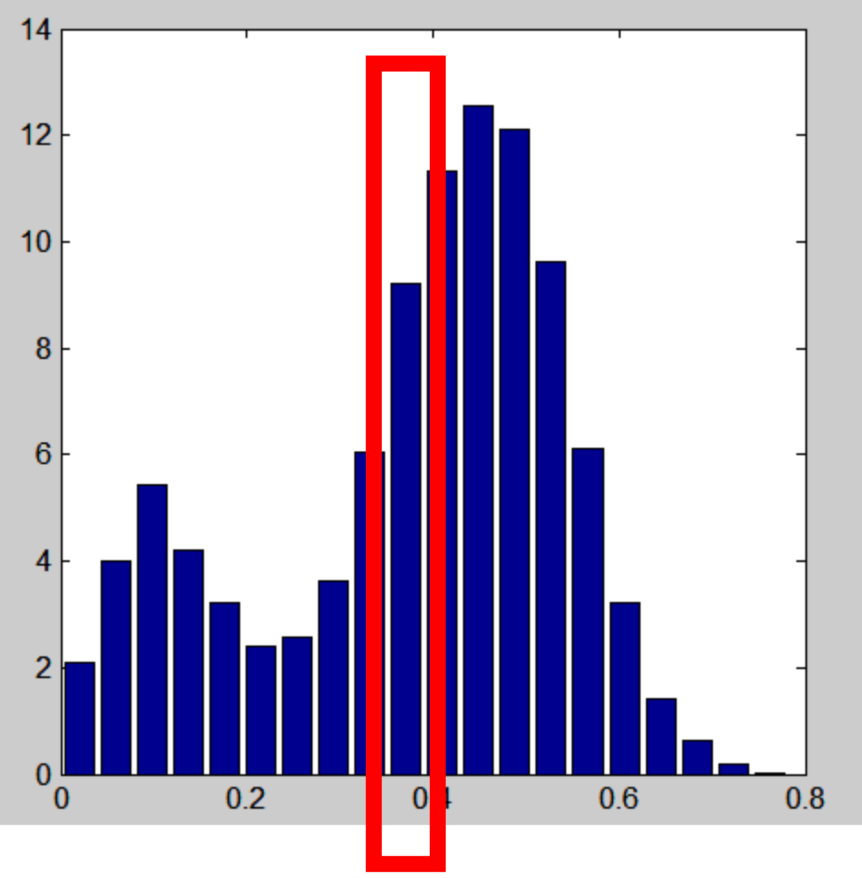
1- steerable pyramid

2- matching histograms

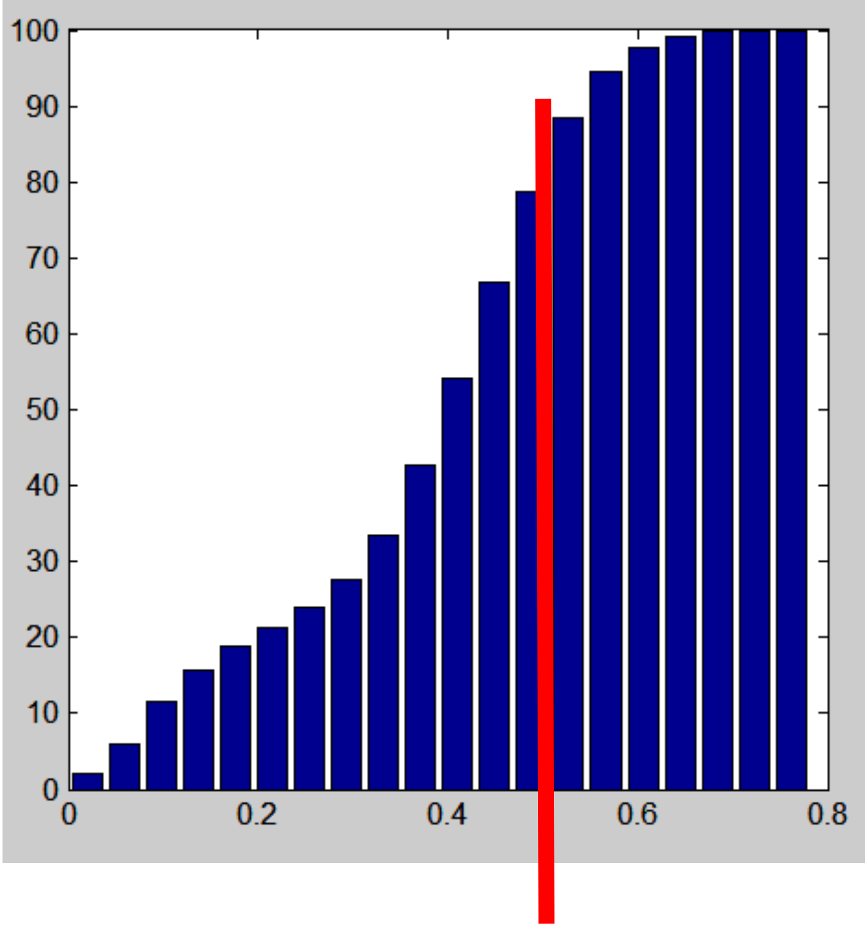
2-Matching histograms



Histograms

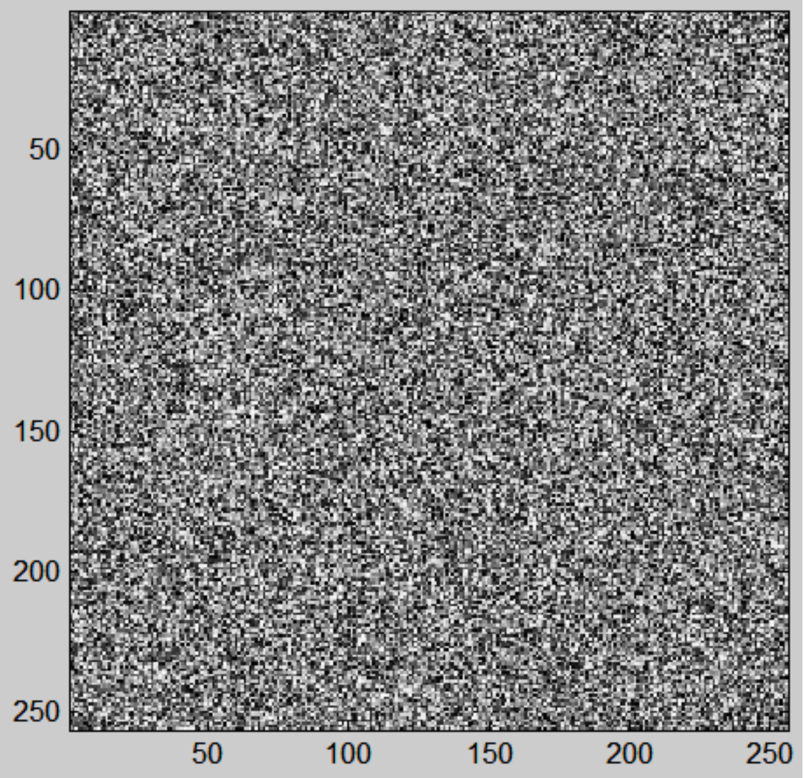


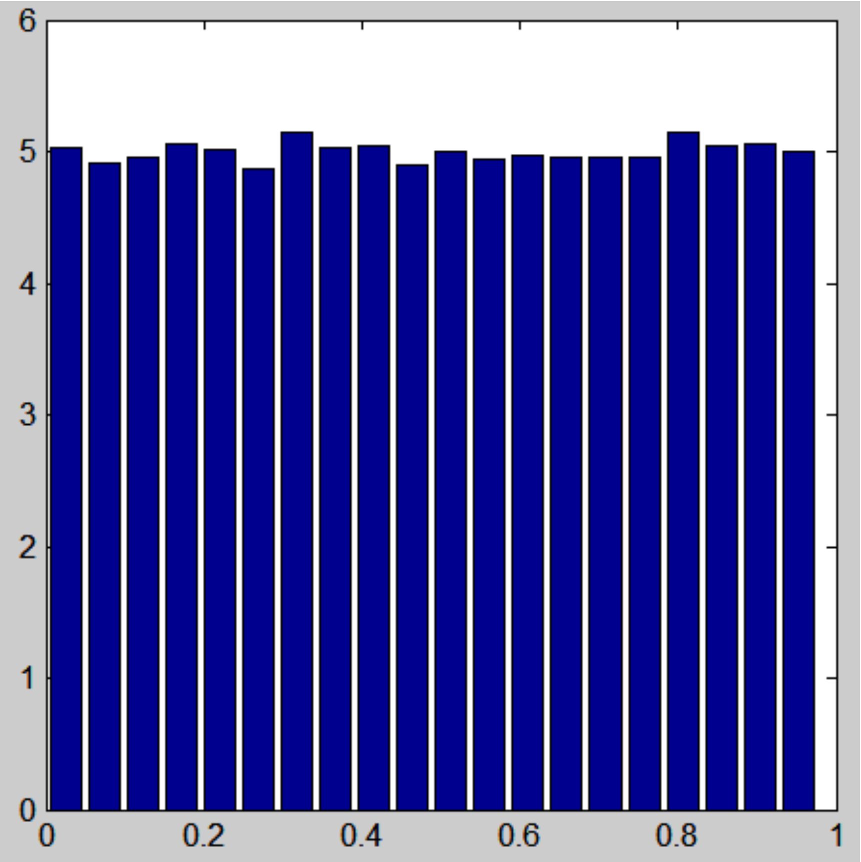
Cumulative Histograms

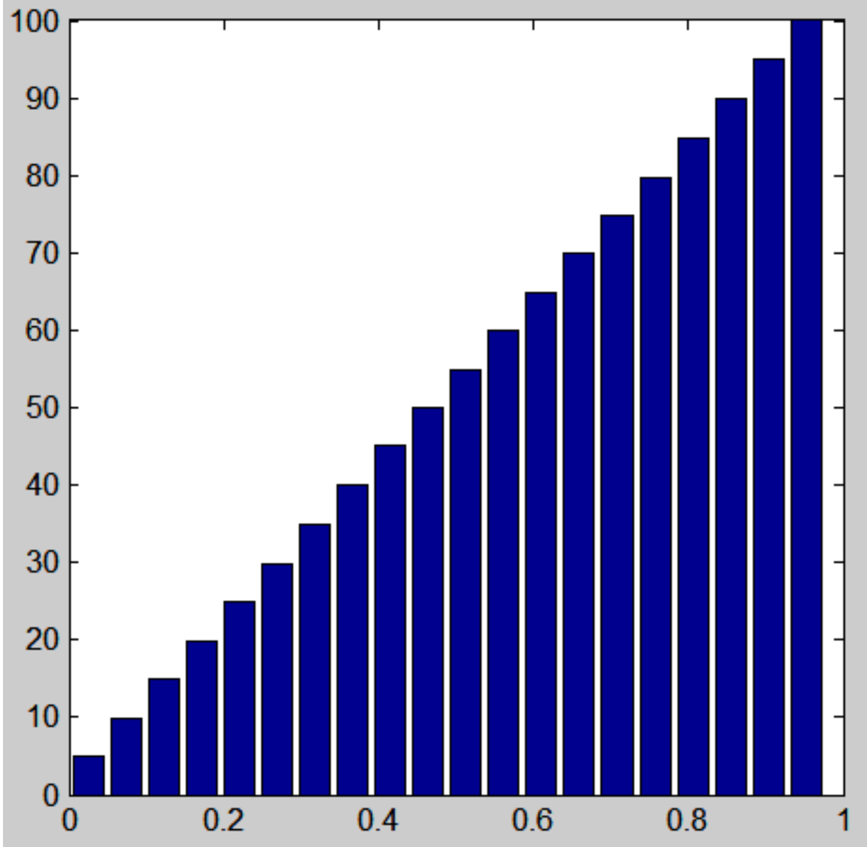


9% of pixels have an intensity value within the range[0.37, 0.41]

75% of pixels have an intensity value smaller than 0.5



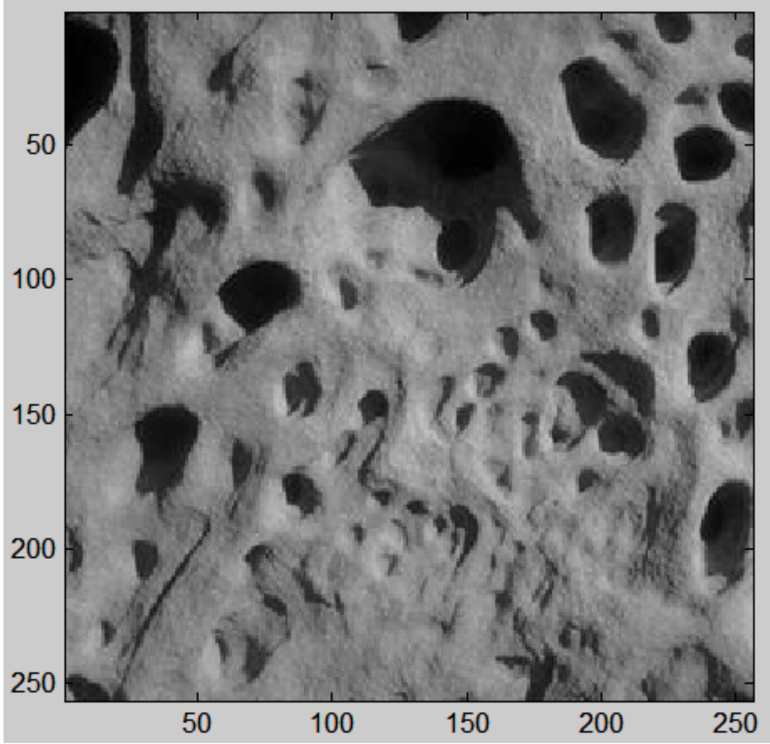


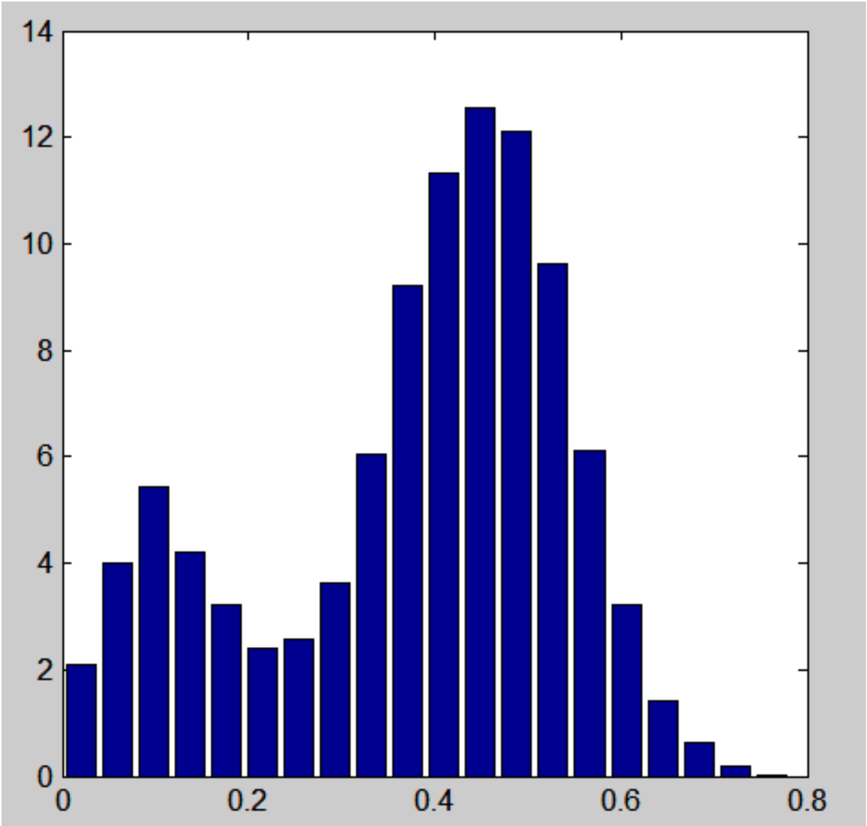


5% of pixels have an intensity value within the range[0.37, 0.41]

2-Matching histograms

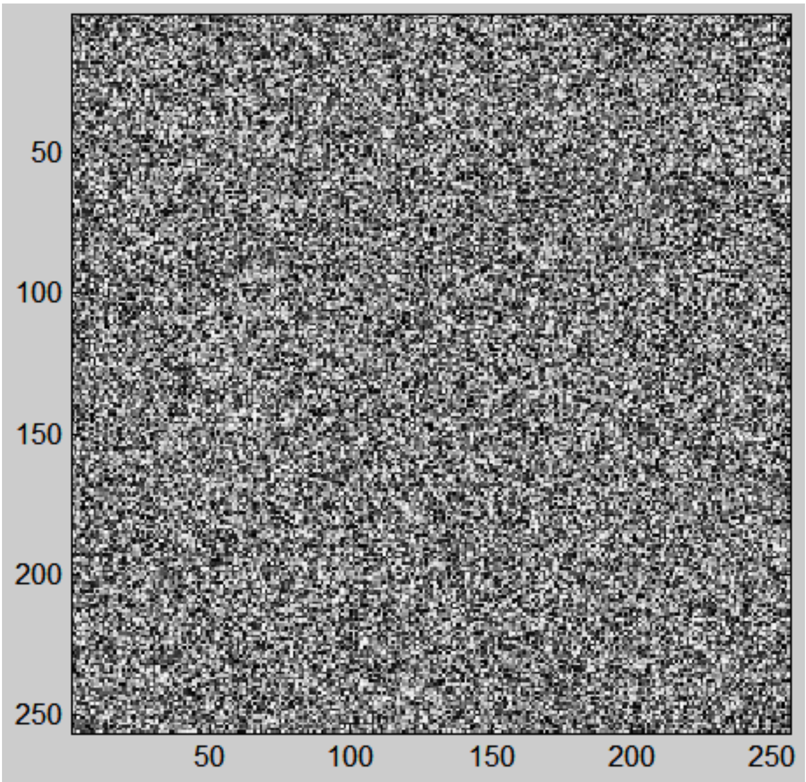
Z(x,y)

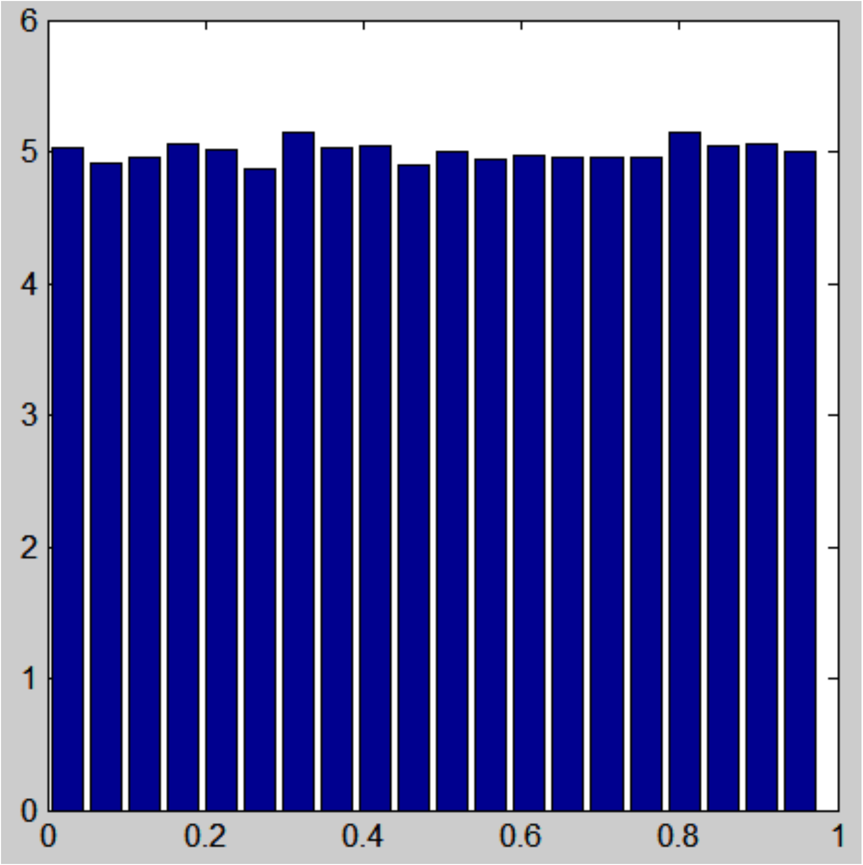




?

Y(x,y)





We look for a transformation of the image Y

Y' = f (Y)

Such that
Hist(Y) = Hist(f(Z))

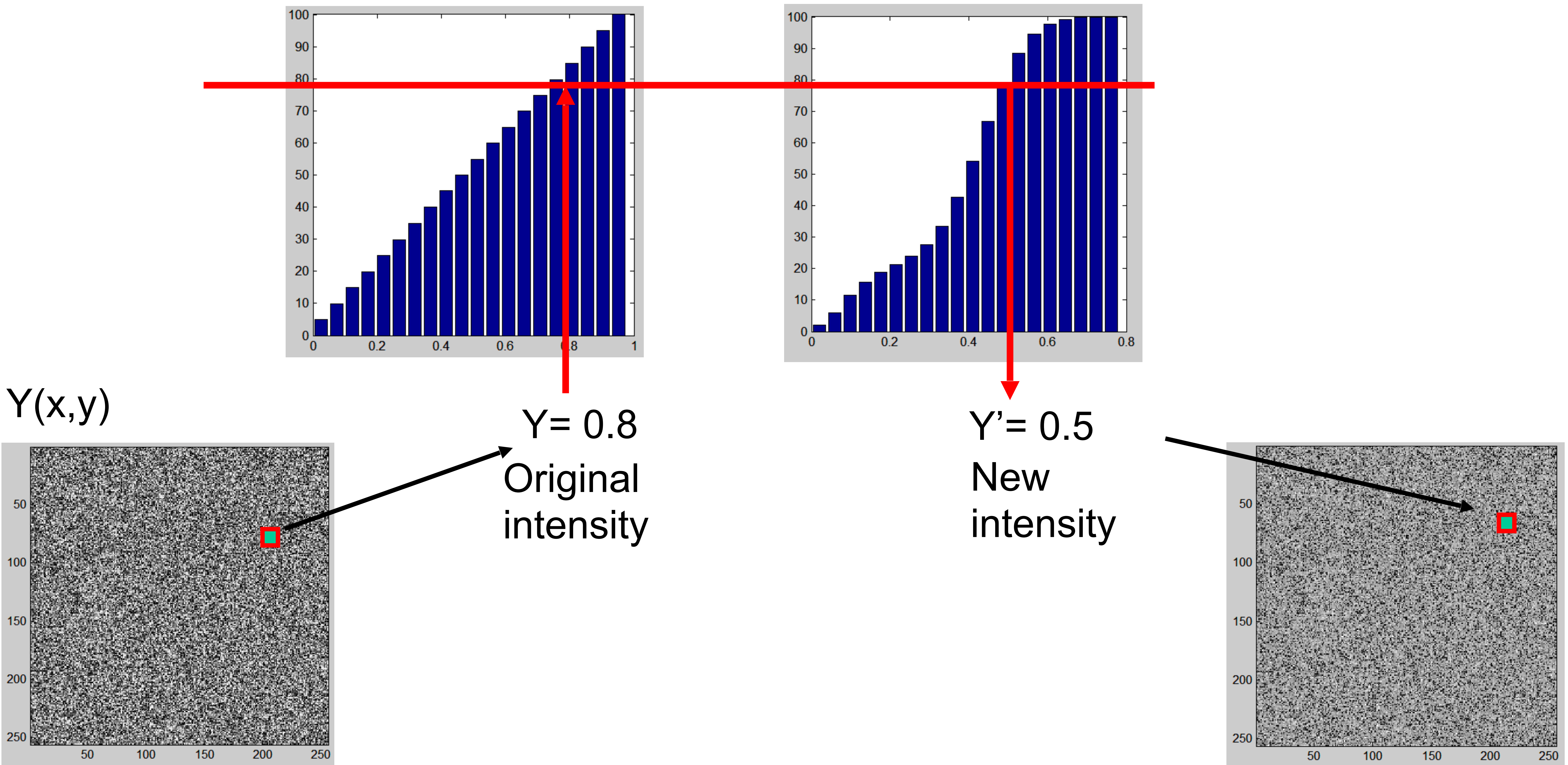
Problem: there are infinitely many functions that can do this transformation.

A natural choice is to use *f* being:
- pointwise non linearity
- stationary
- monotonic (most of the time invertible)

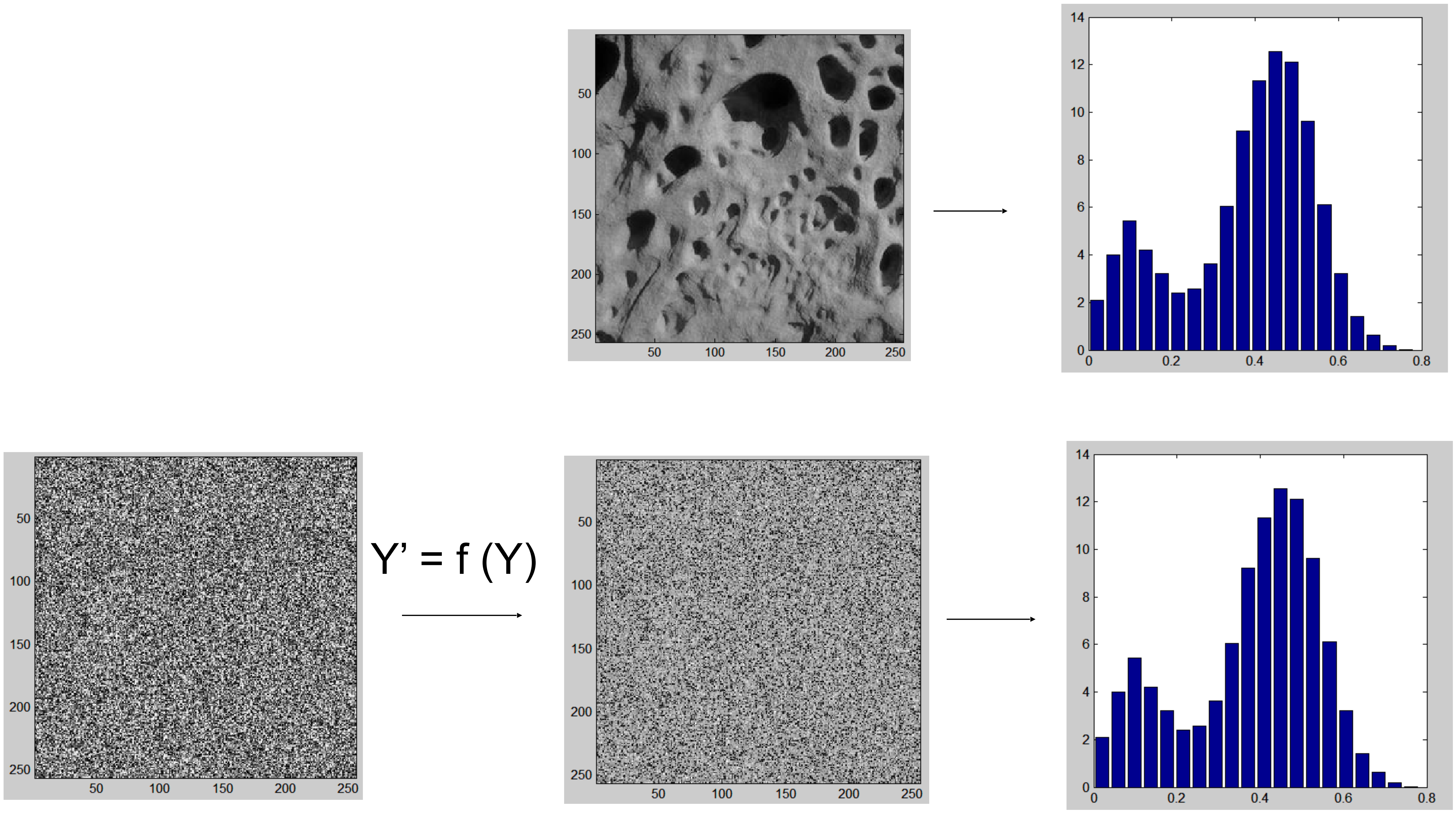
2-Matching histograms

The function f is just a look up table: it says, change all the pixels of value Y into a value f(Y).

$$Y' = f\,(Y)$$



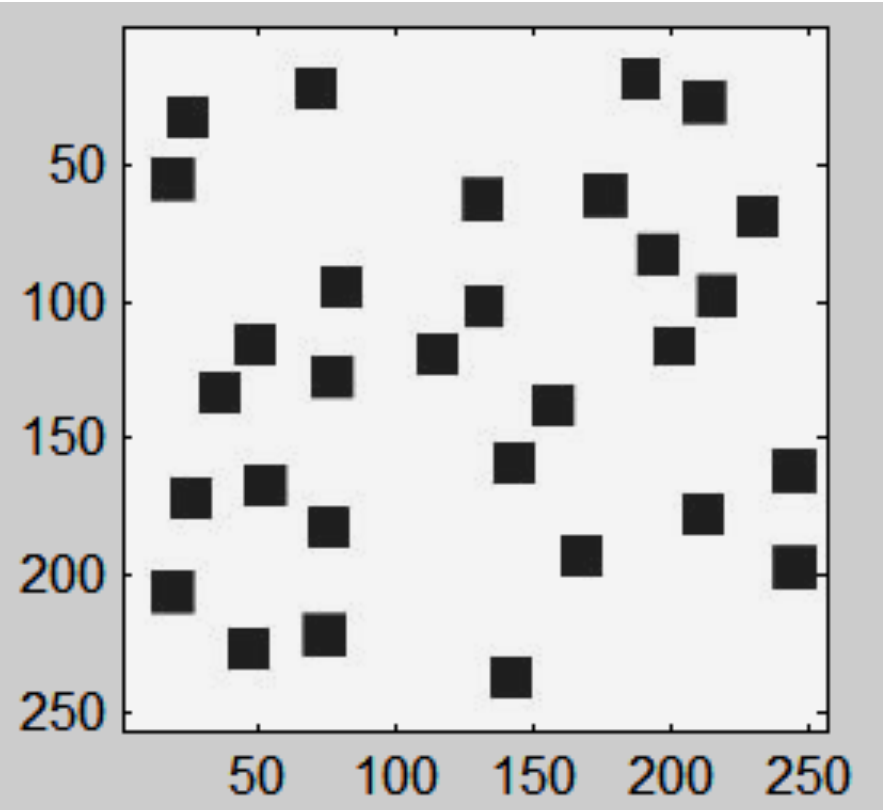
2-Matching histograms

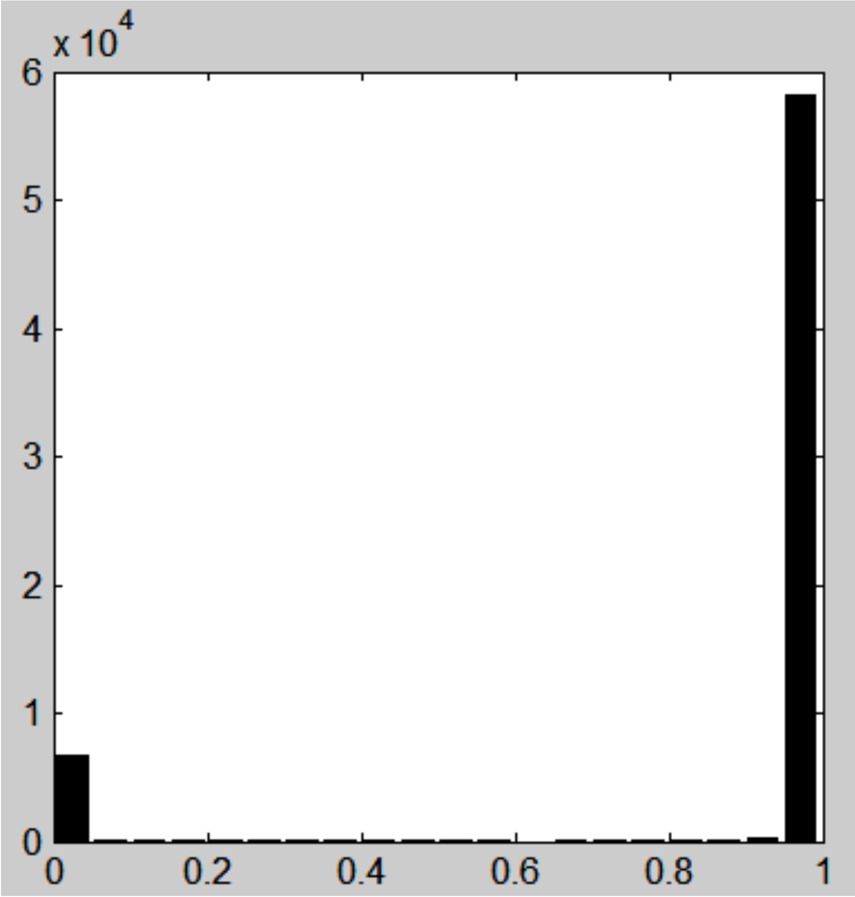


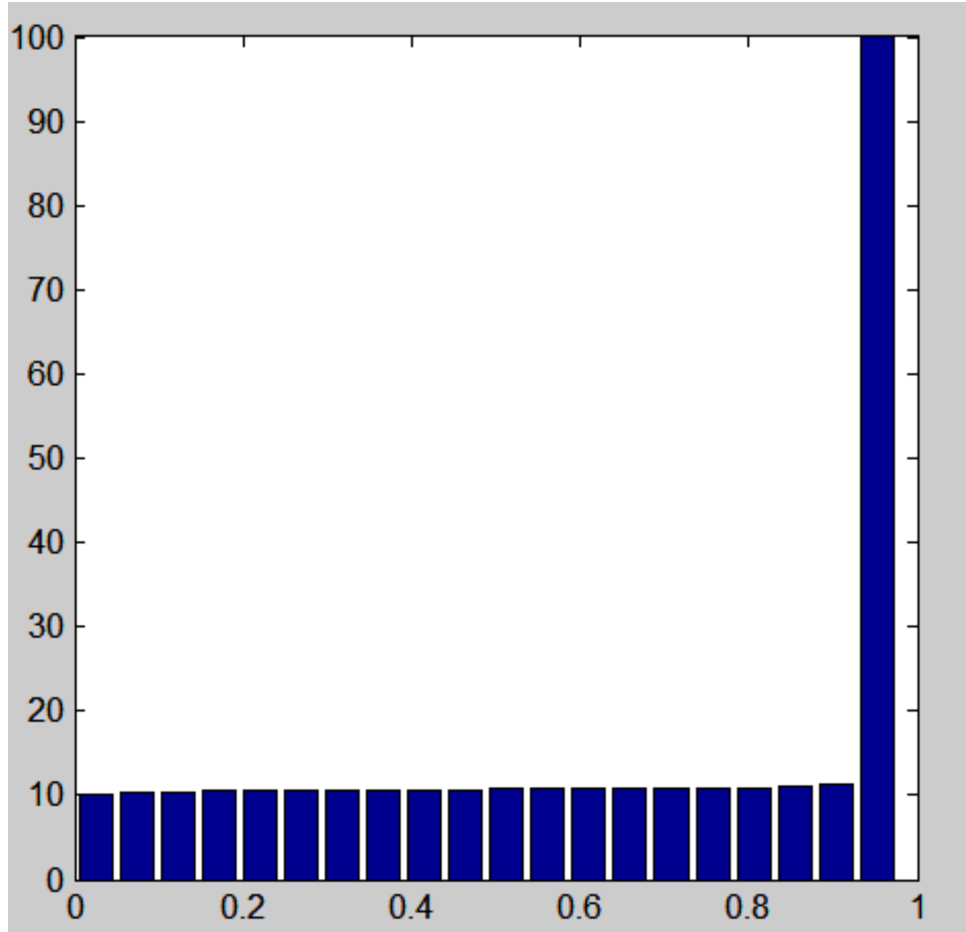
Matching histograms

Histograms

Cumulative Histograms



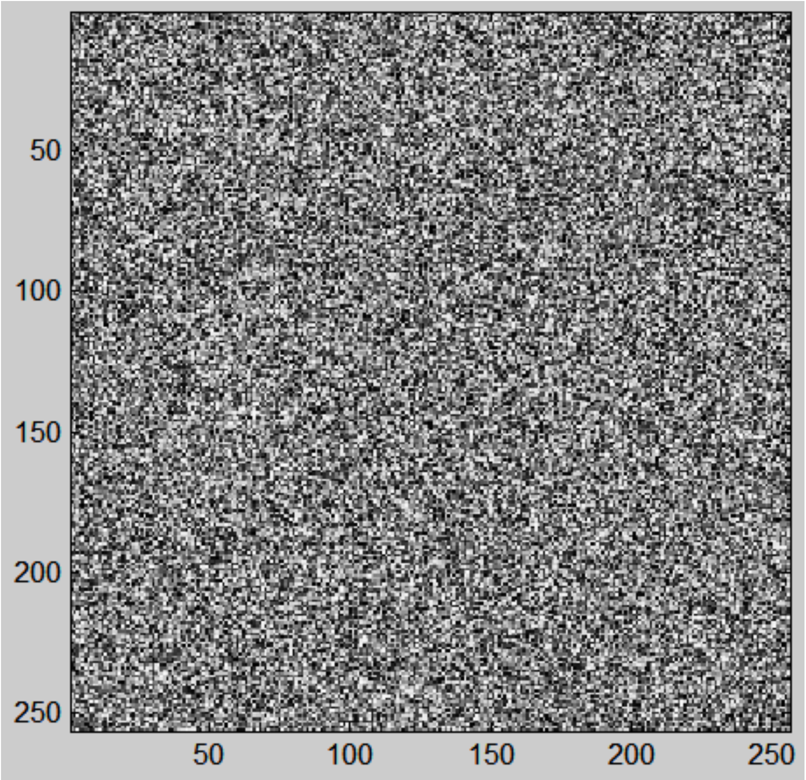


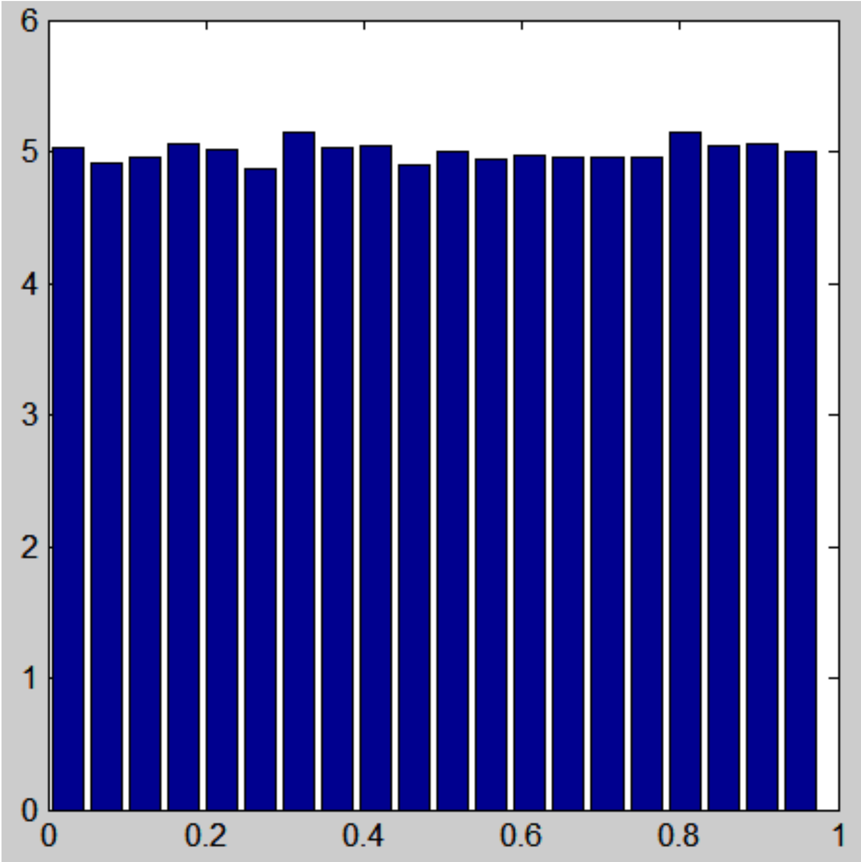


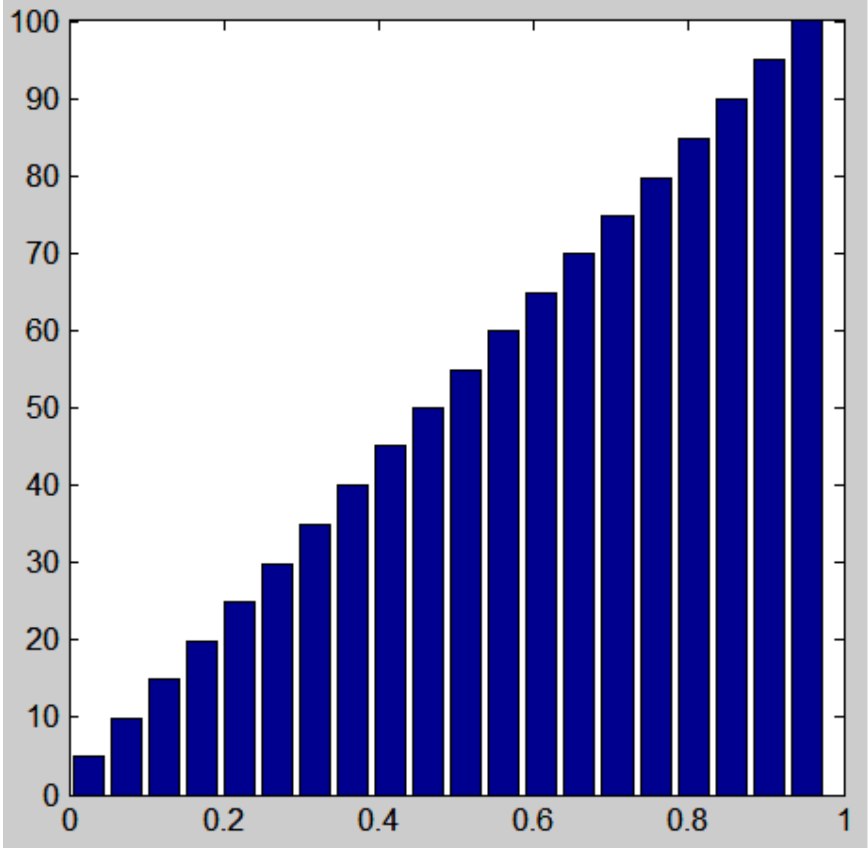
?

?

10% of pixels are black
and 90% are white



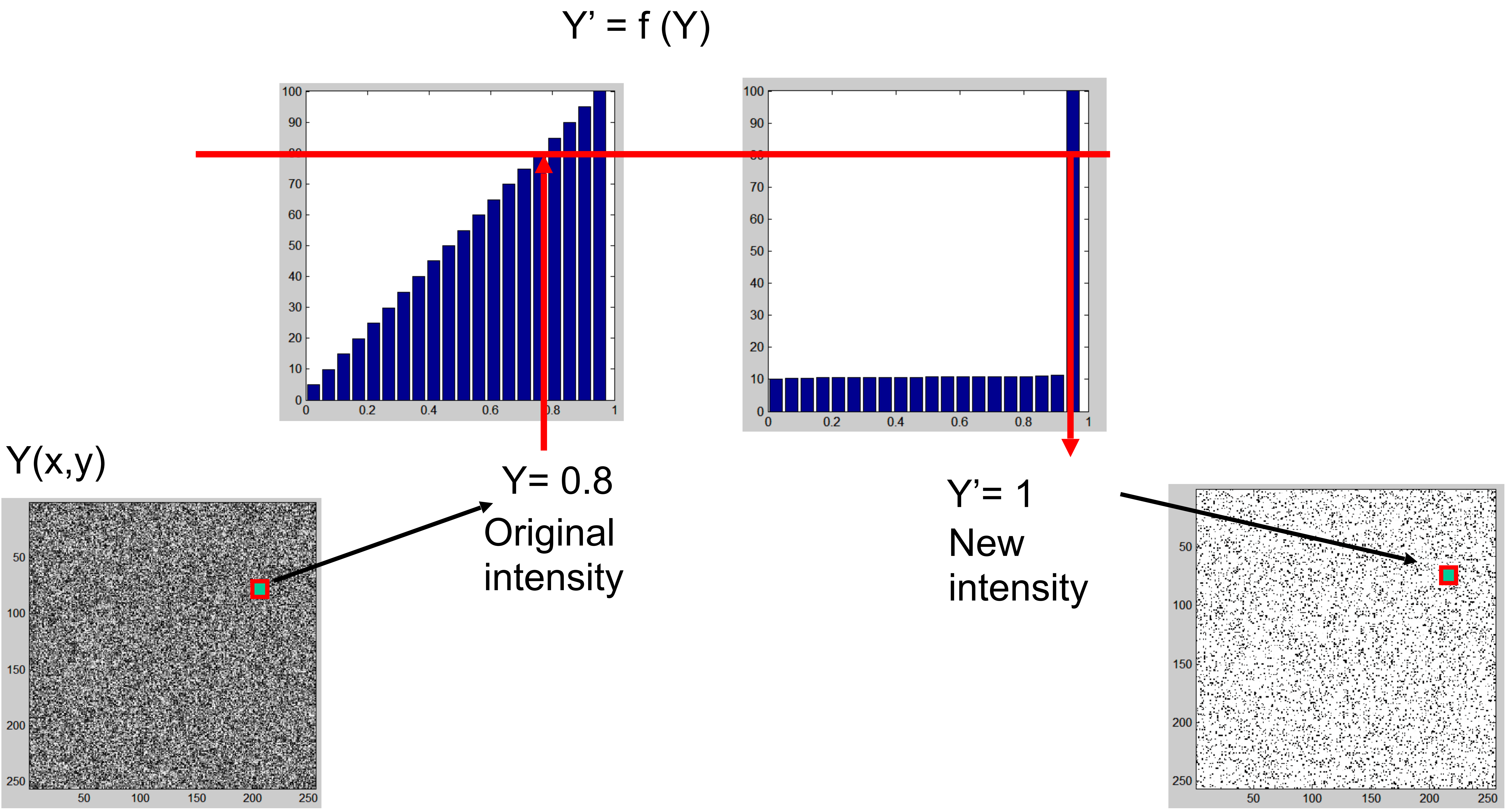




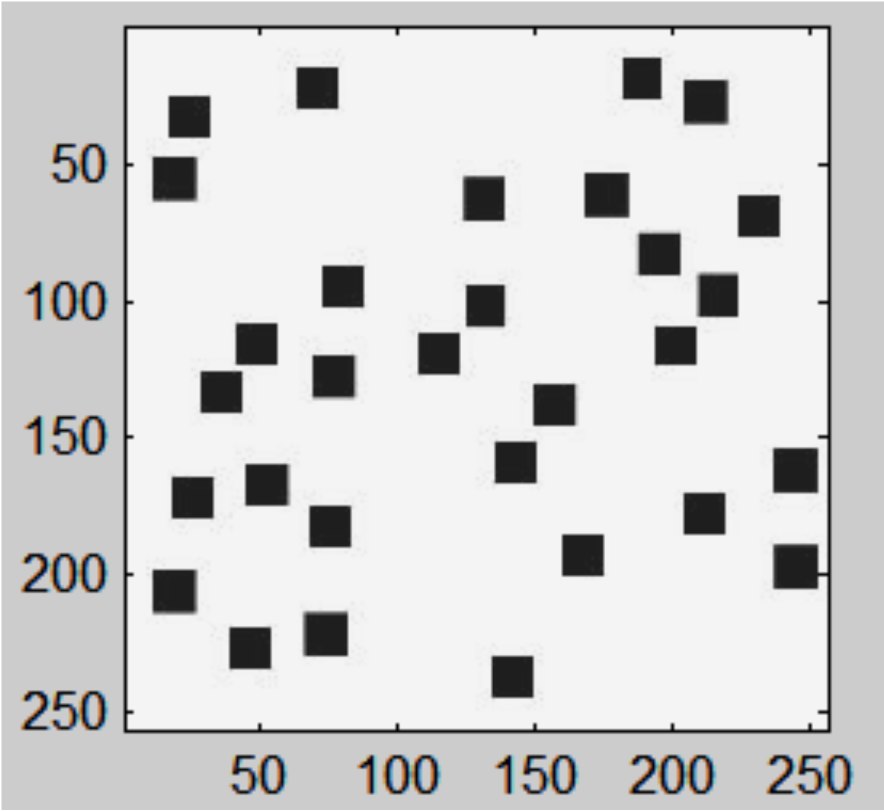
5% of pixels have an intensity value
within the range[0.37, 0.41]

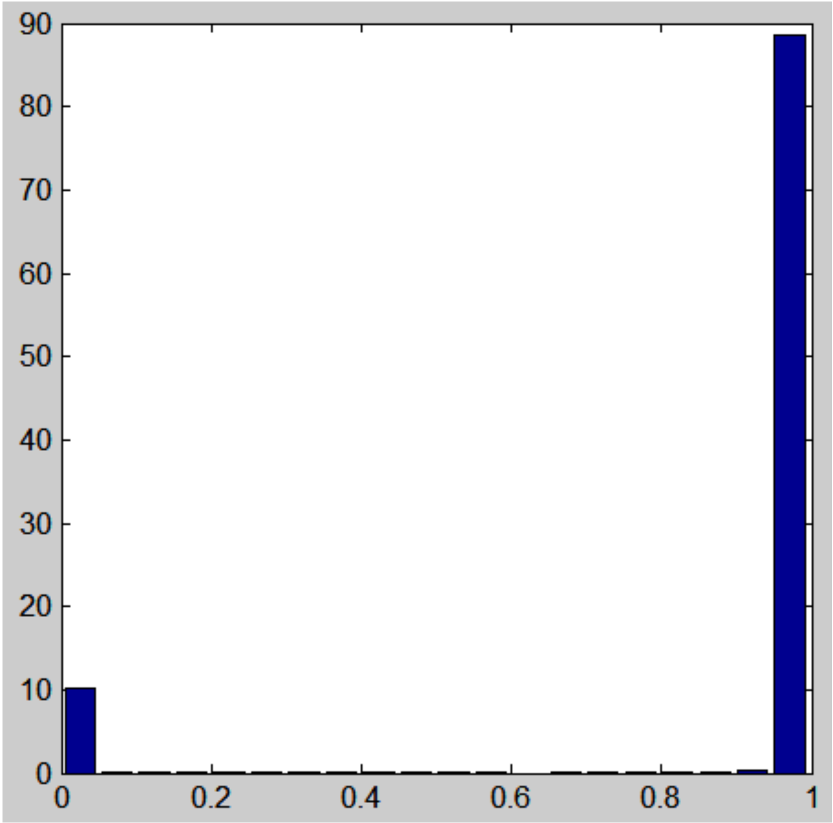
Matching histograms

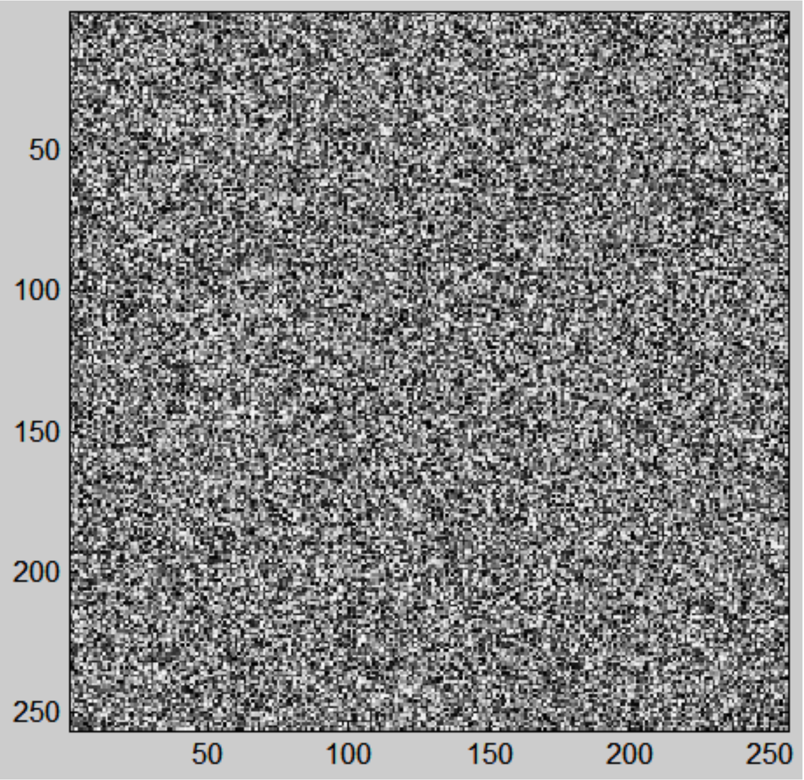
The function f is just a look up table: it says, change all the pixels of value Y into a value f(Y).



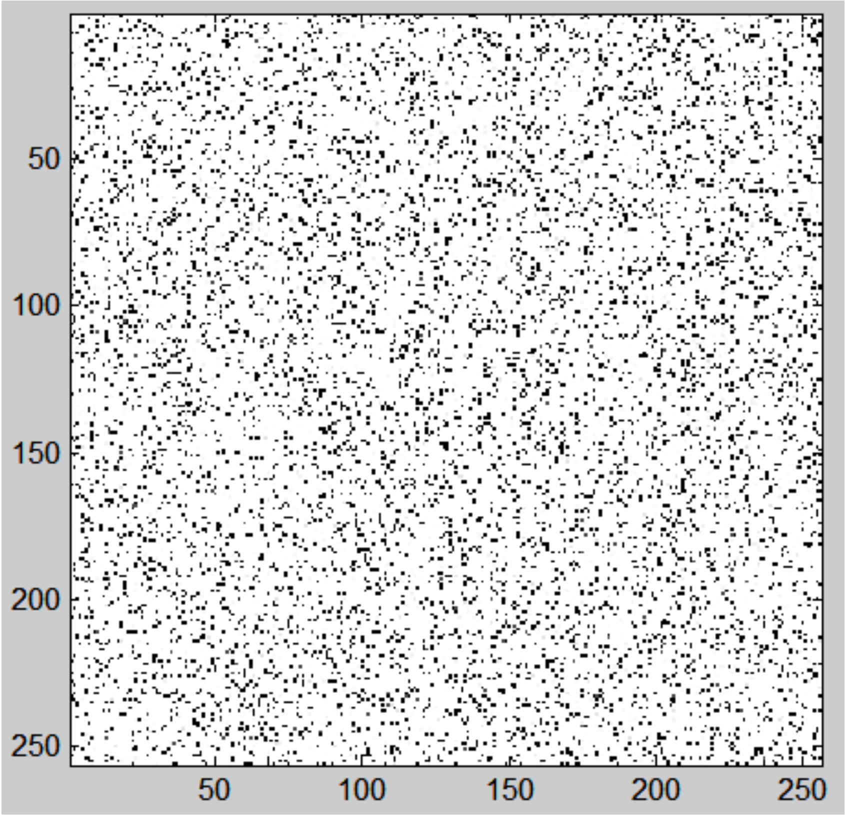
Matching histograms

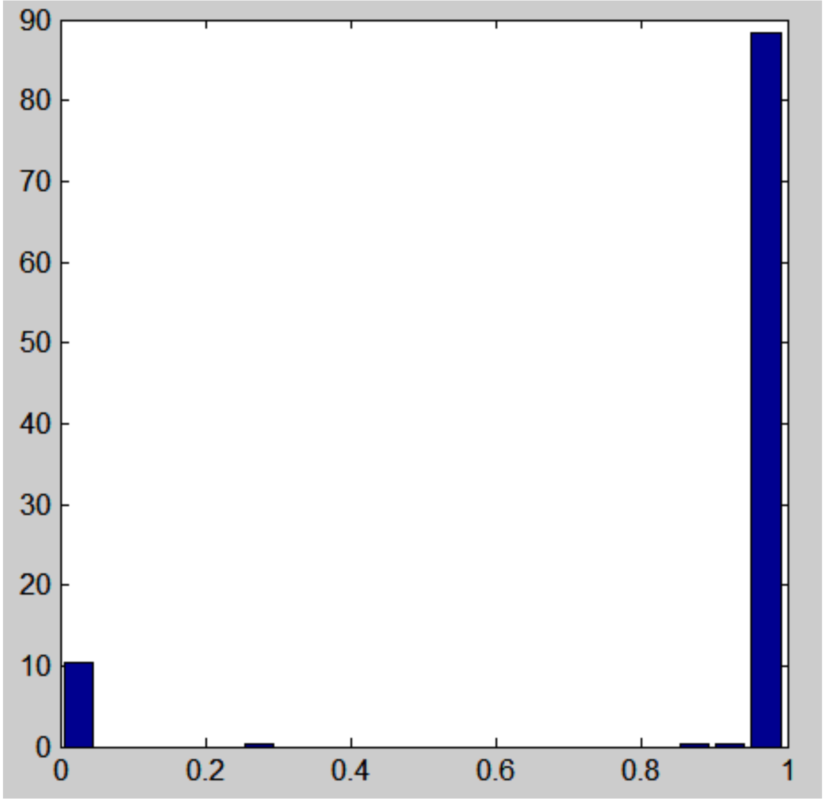






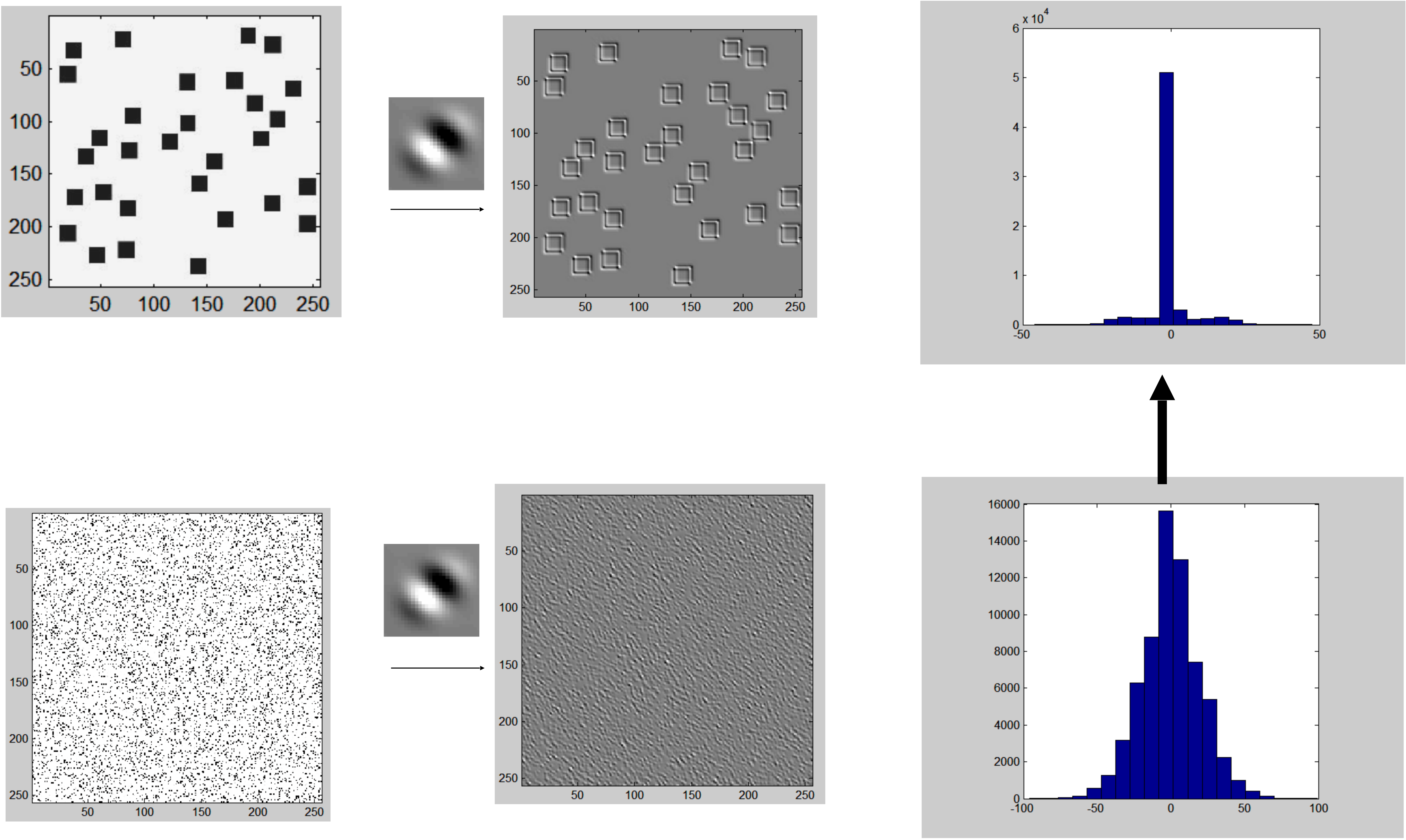
Y’ = f (Y)



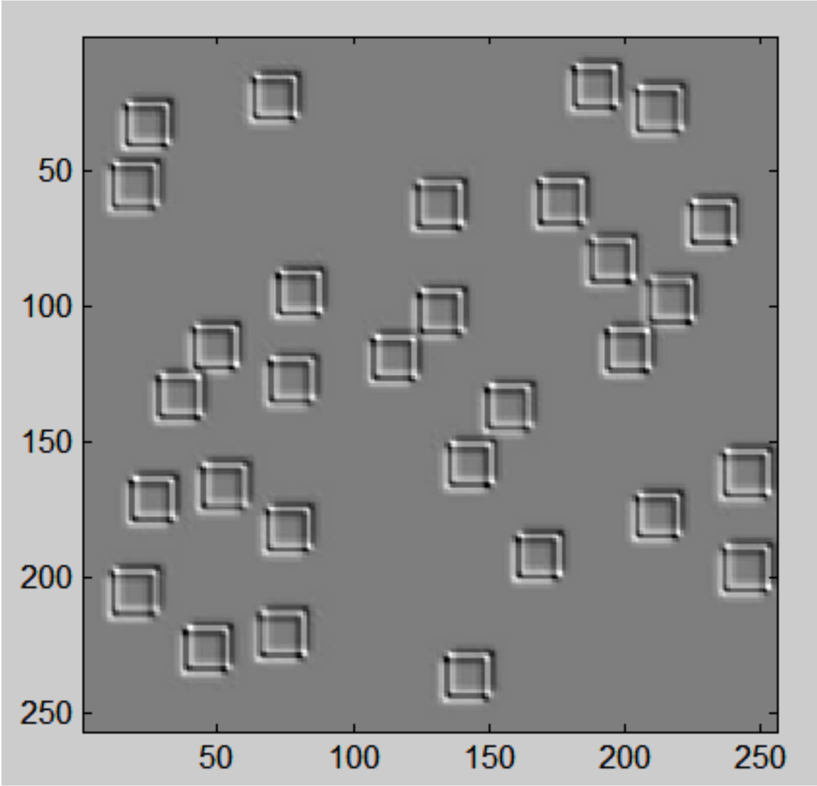


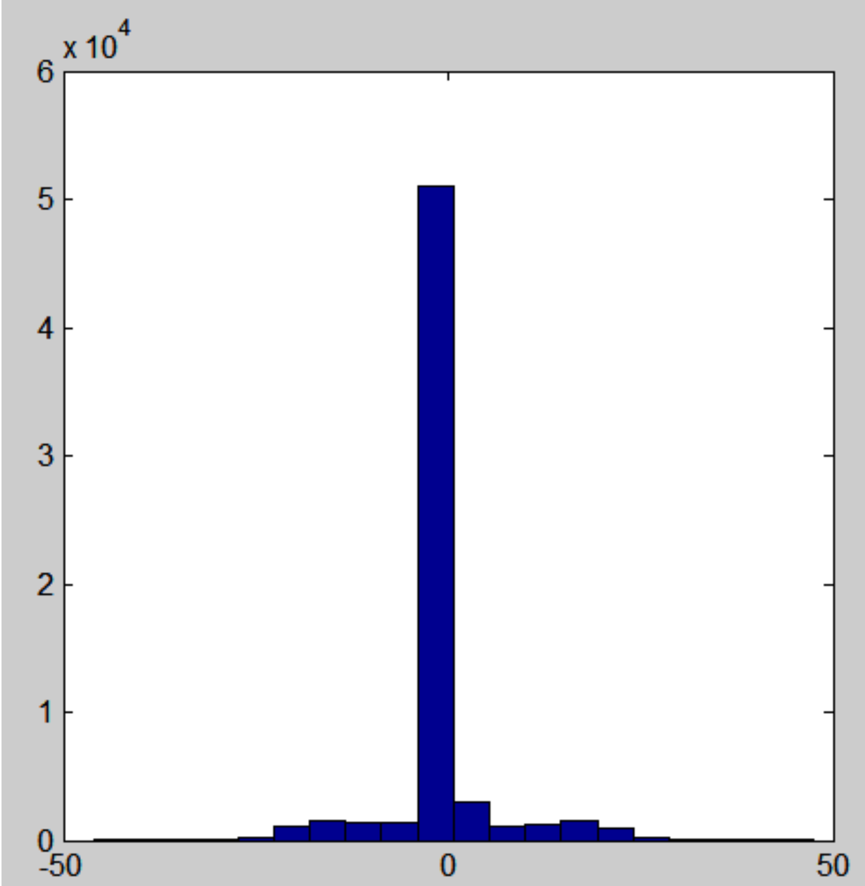
In this example, f is a step function.

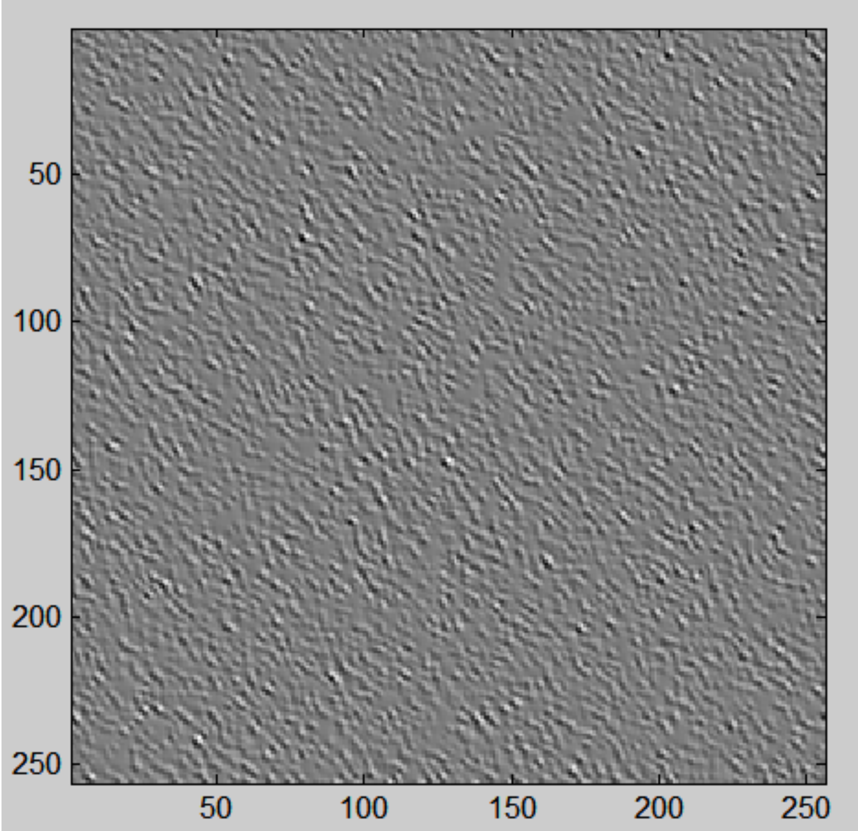
Matching histograms of a subband



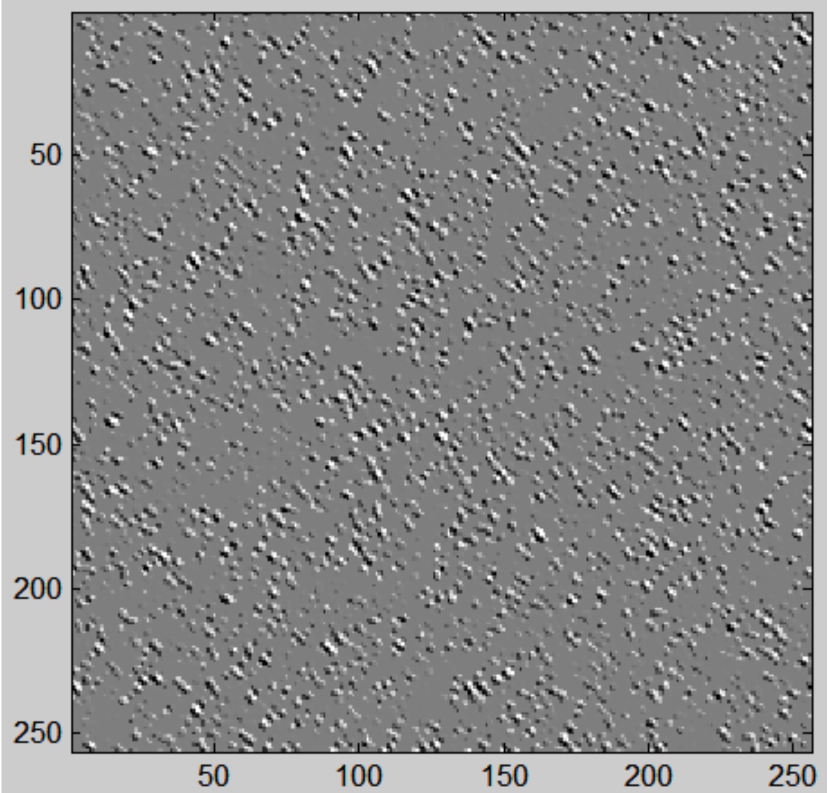
Matching histograms of a subband

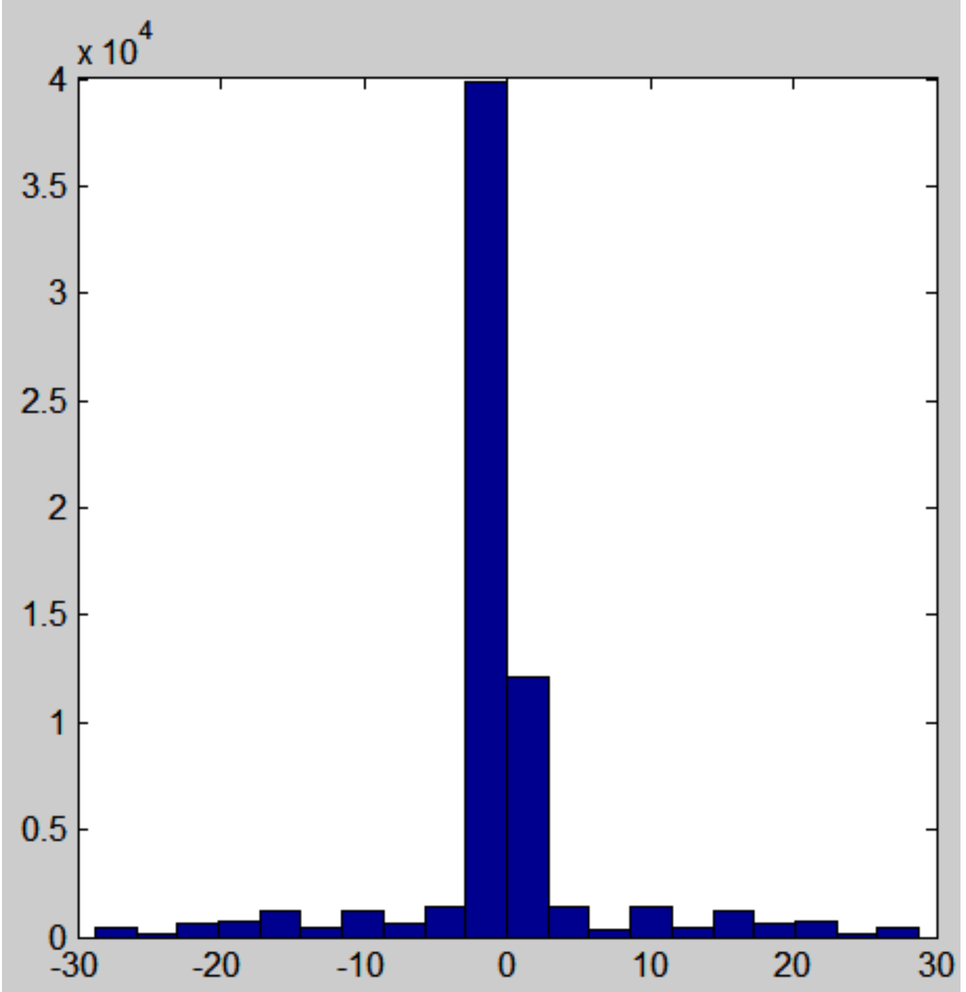






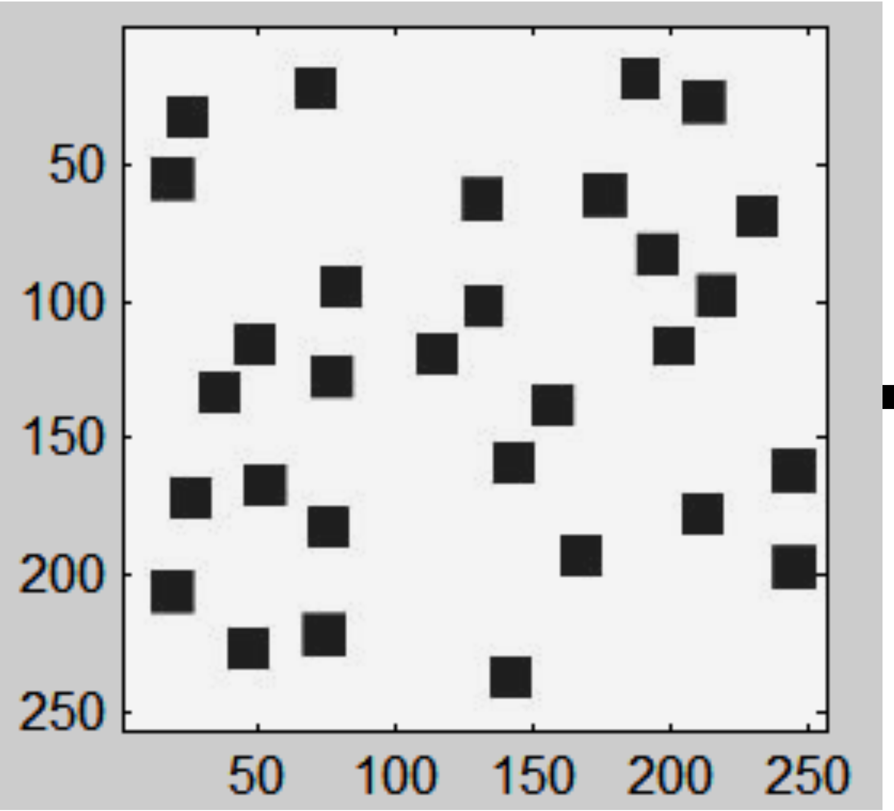
Y' = f (Y)



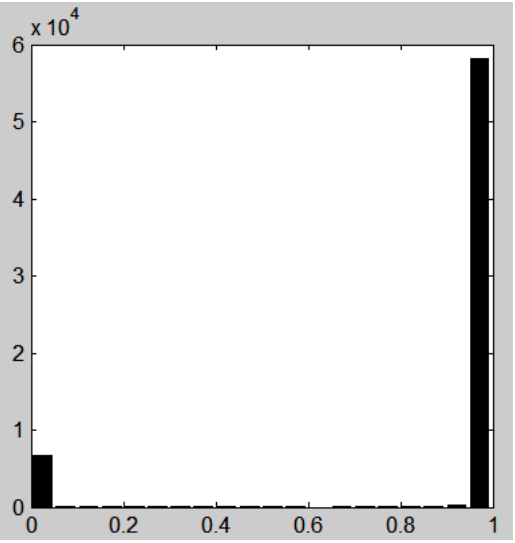


Texture analysis

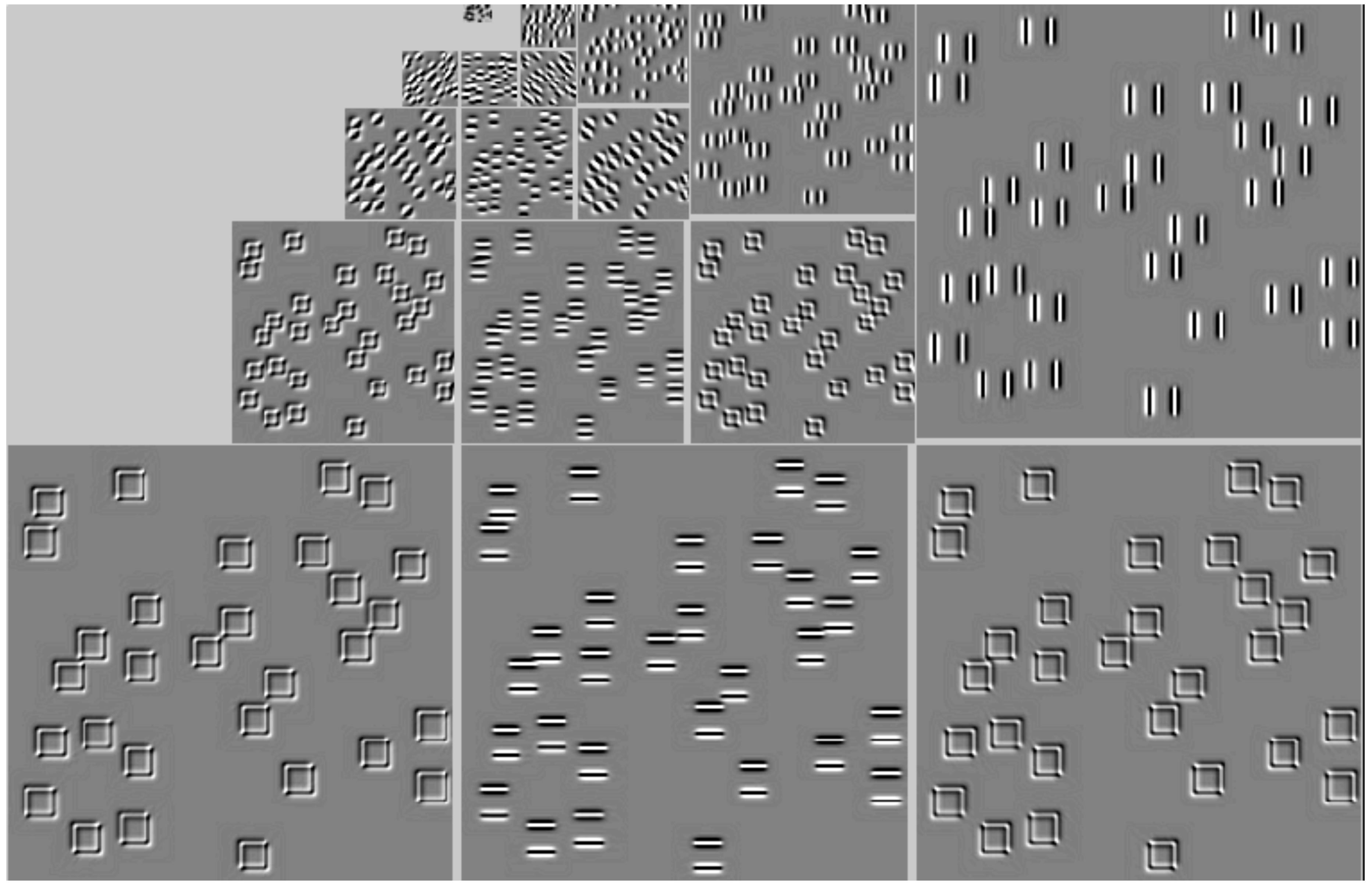
Input texture



(histogram)

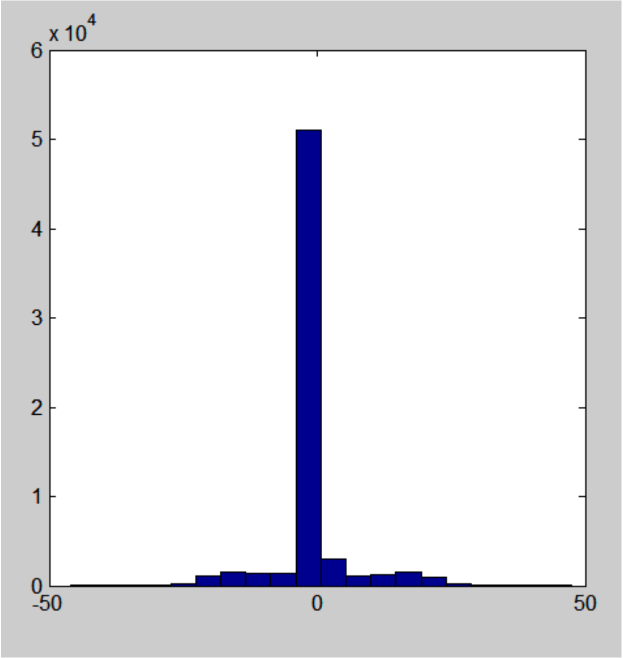


Wavelet decomposition (steerable pyr)



(Steerable pyr; Simoncelli & Freeman, ’95)

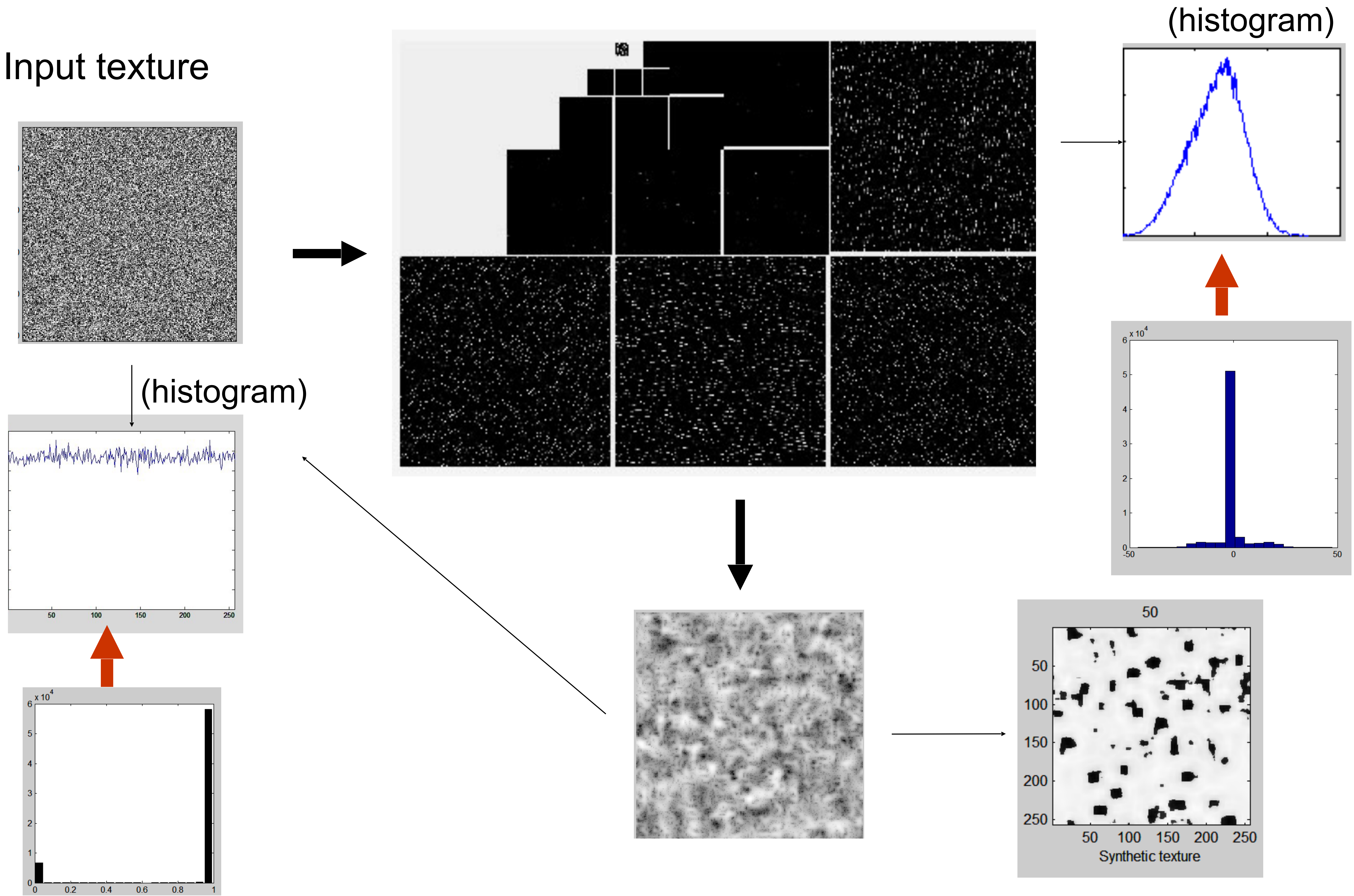
(histogram)



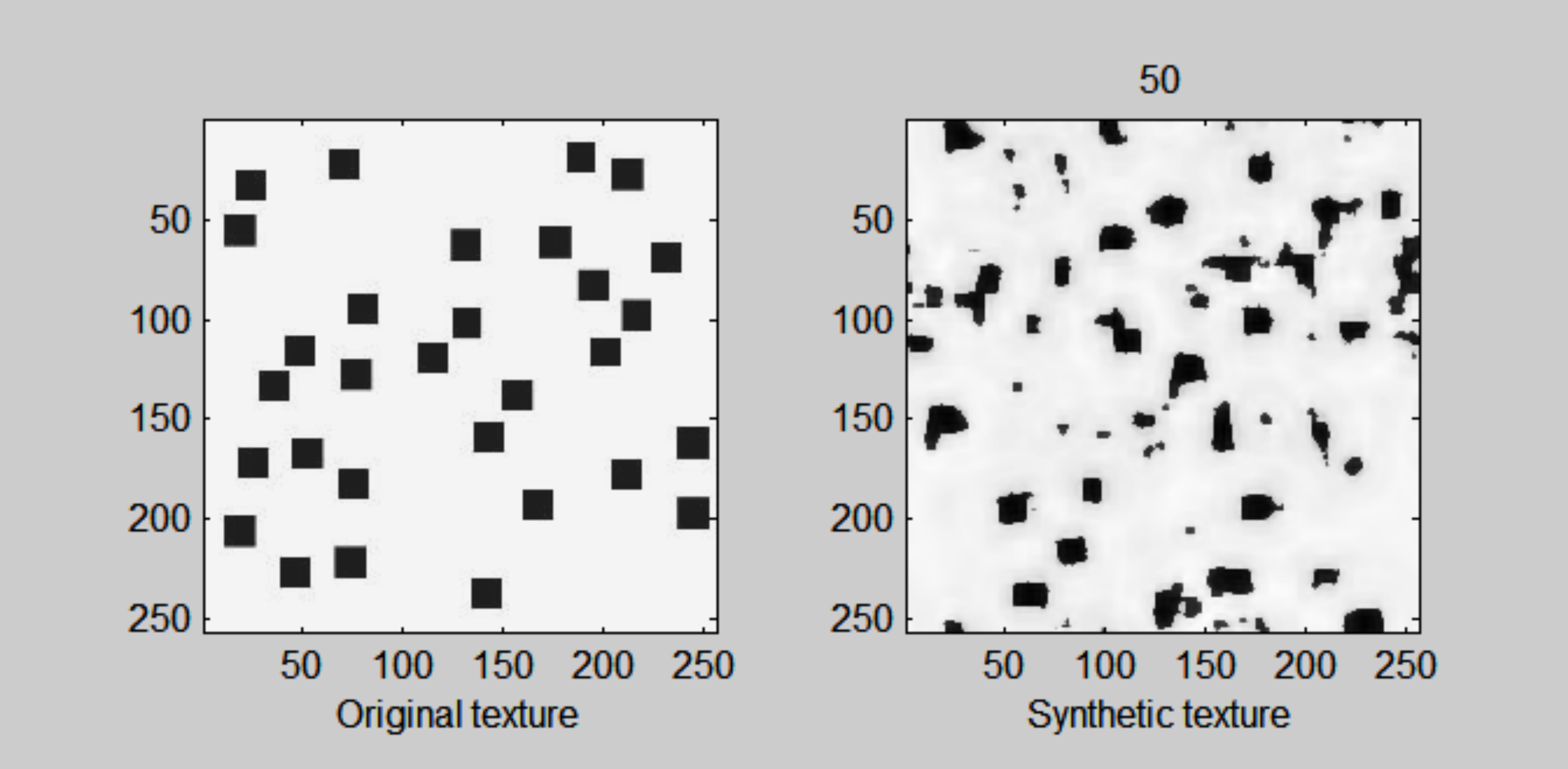
The texture is represented as a collection of marginal histograms.

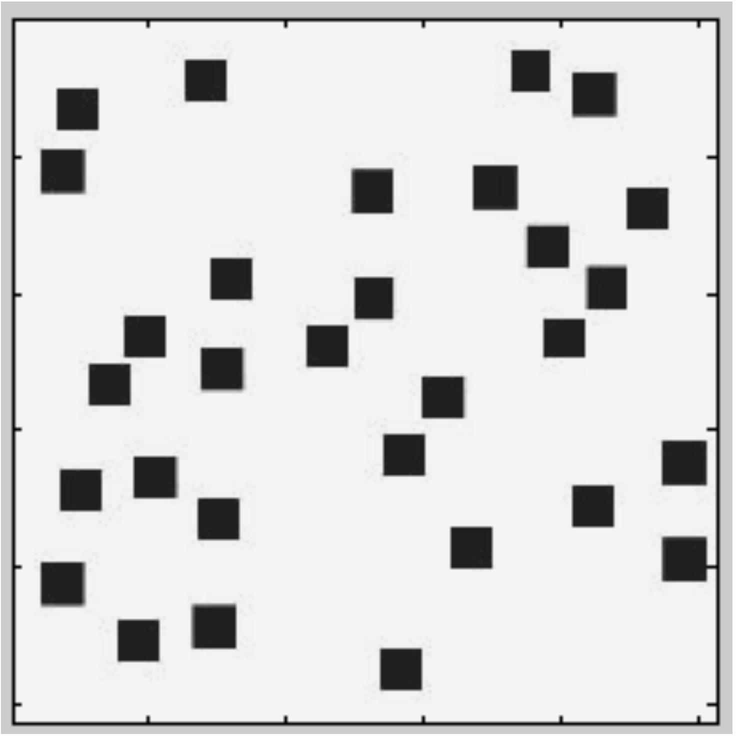
Texture synthesis

Heeger and Bergen, 1995



Why does it work? (sort of)



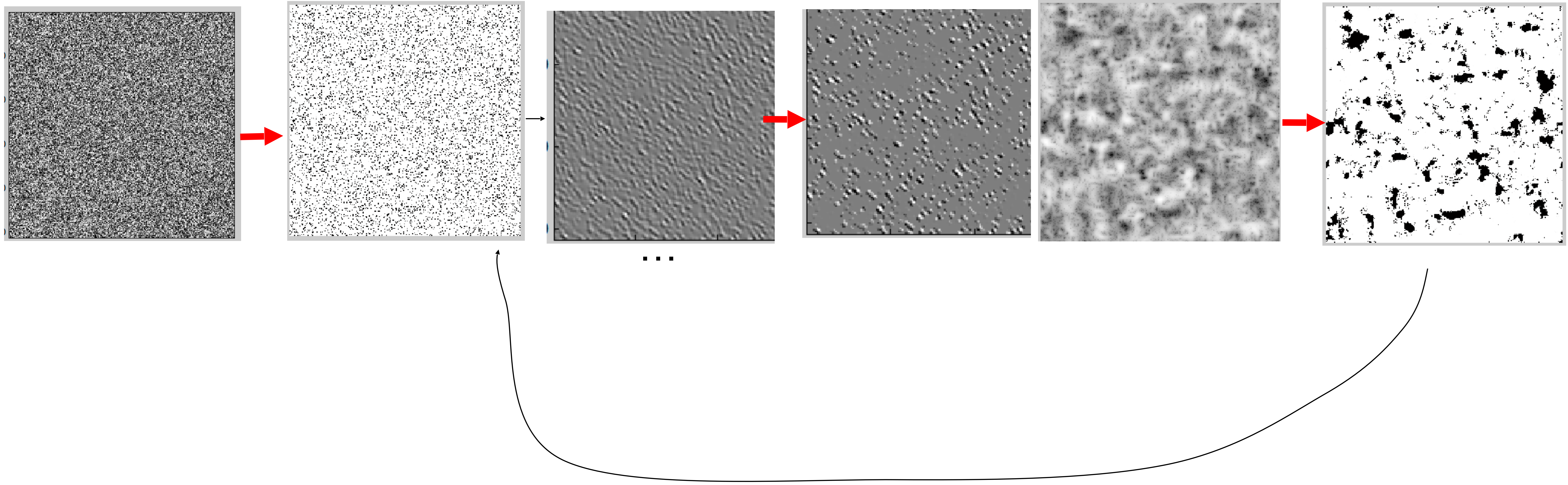


Why does it work? (sort of)

The black and white blocks appear by thresholding (f) a blobby image

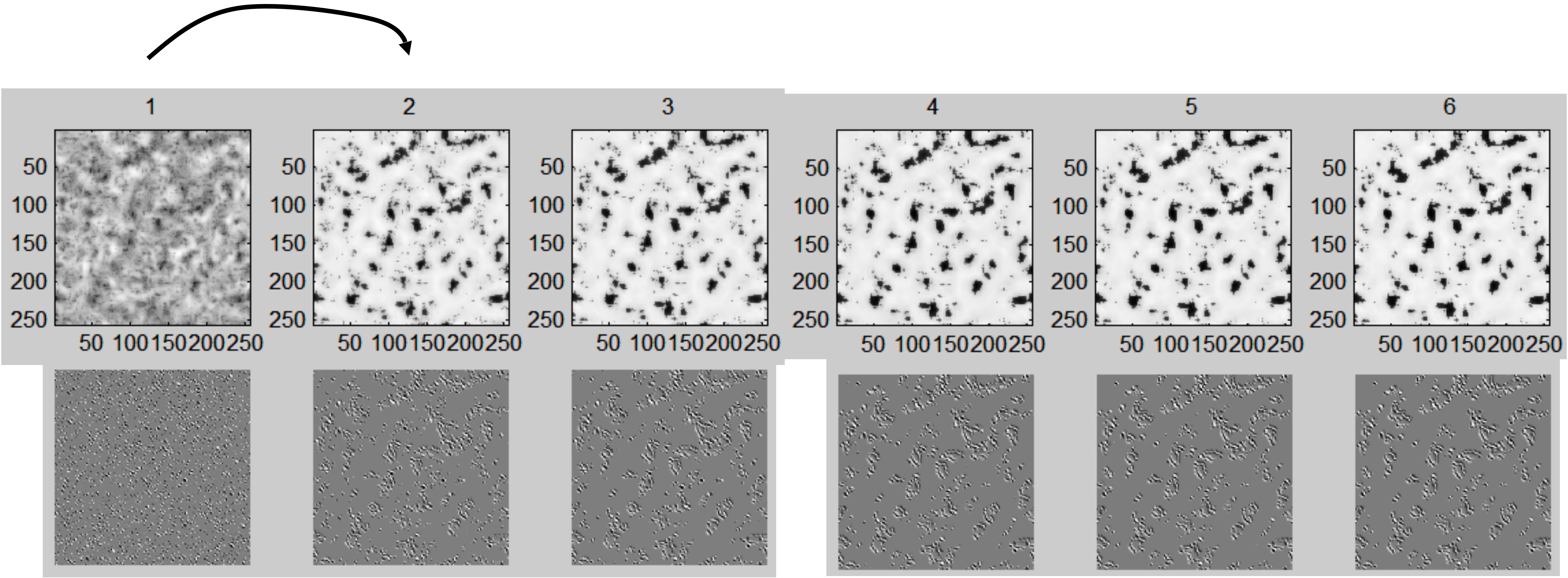
Iteration 0

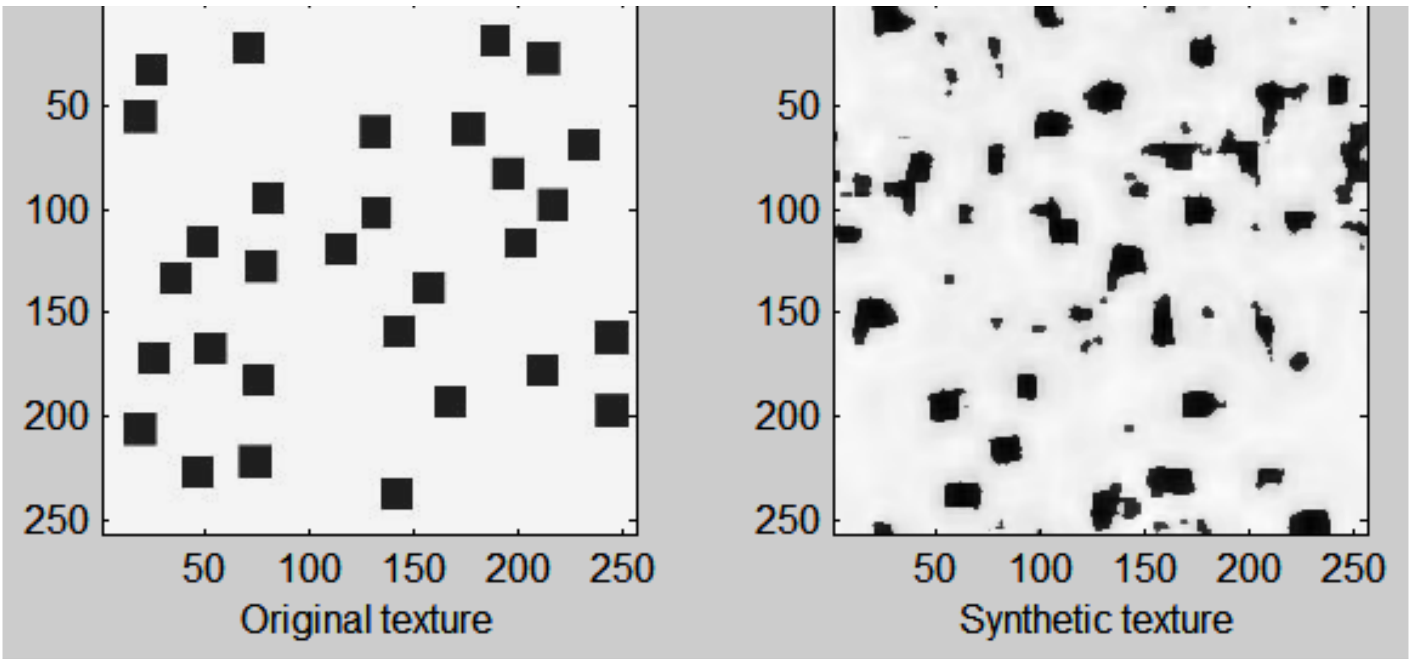
Filter bank



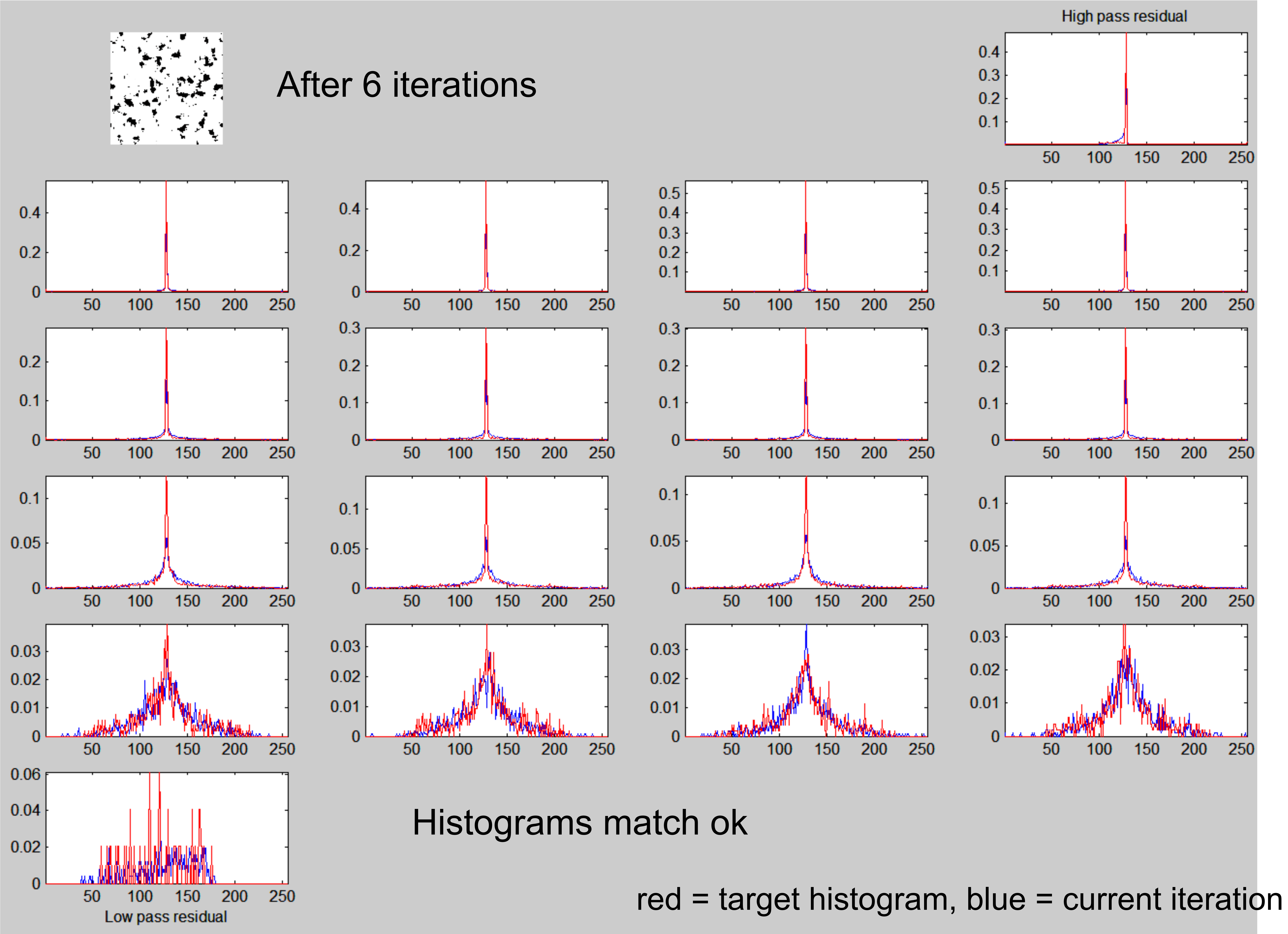
Why does it work? (sort of)

The black and white blocks appear by thresholding (f) a blobby image

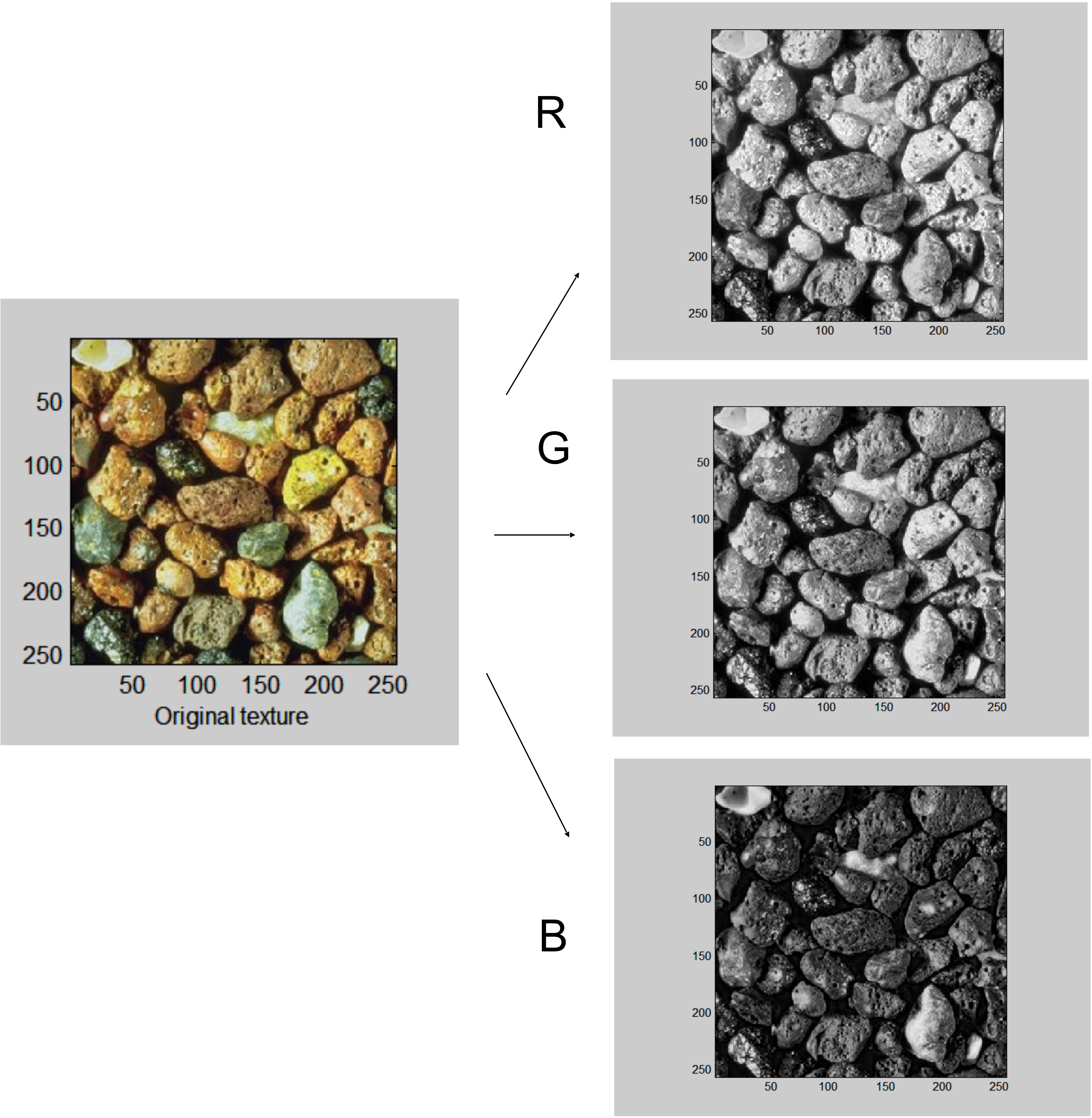




Why does it work? (sort of)

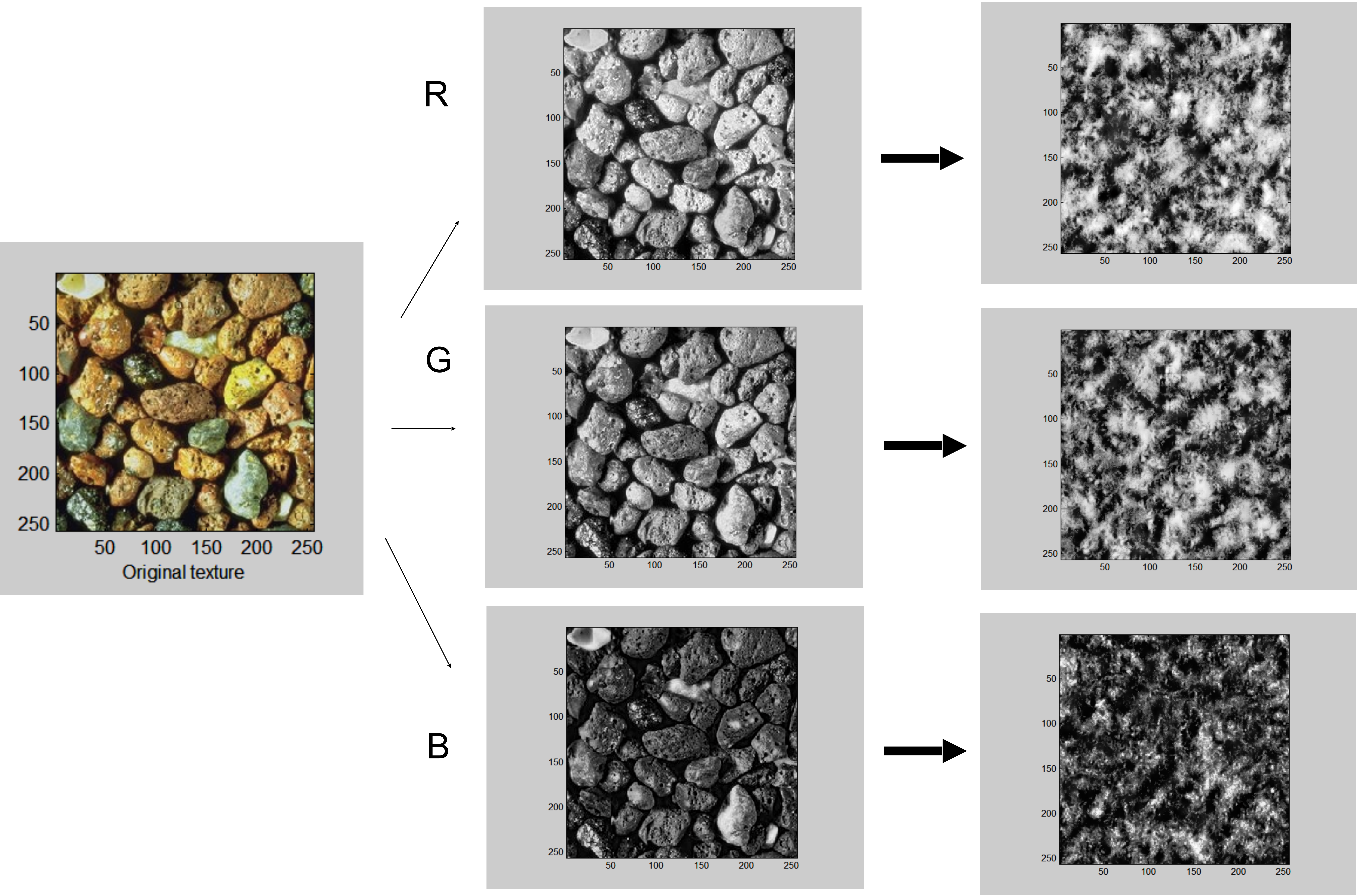


Color textures



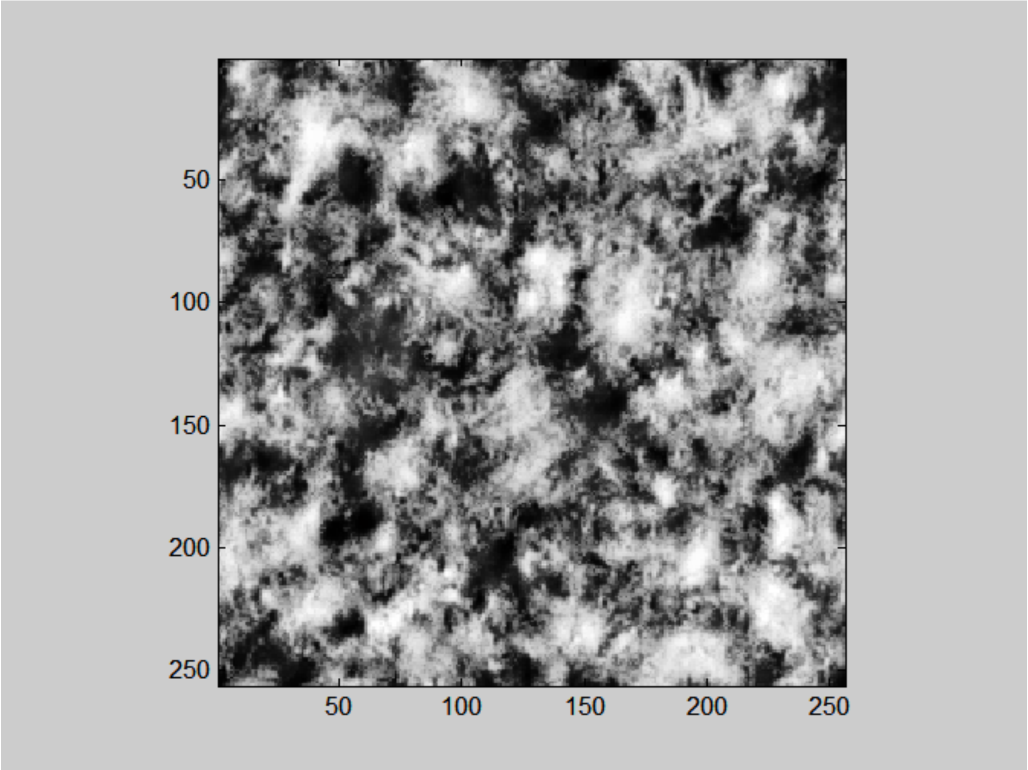
Three textures

Color textures

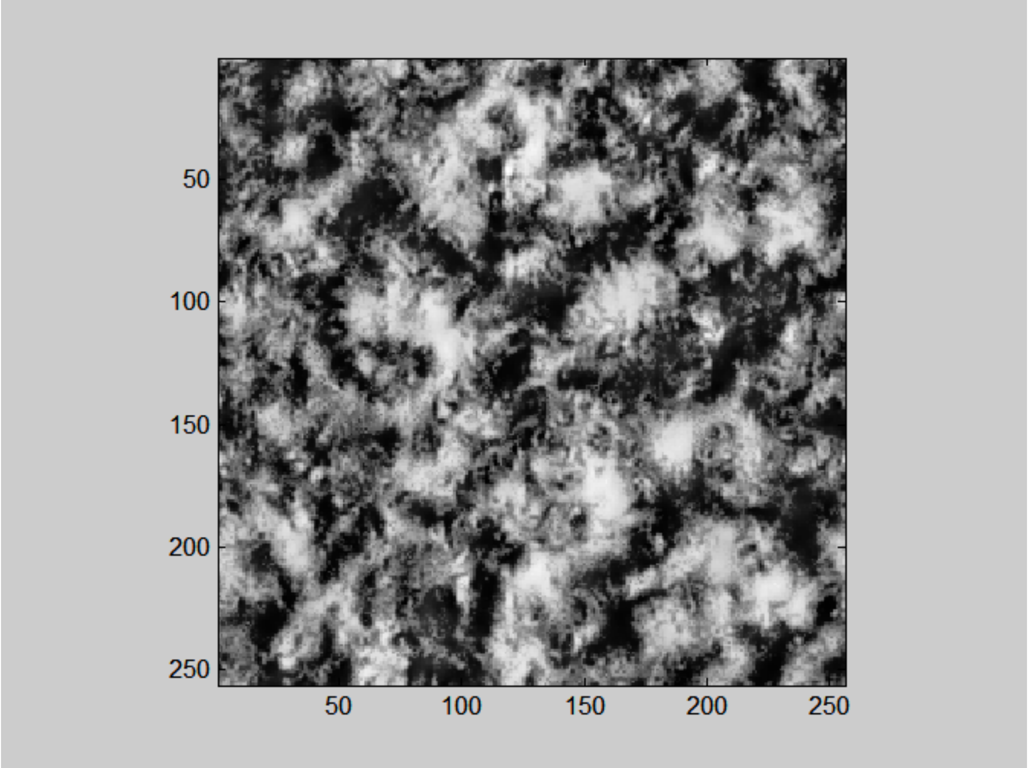


Color textures

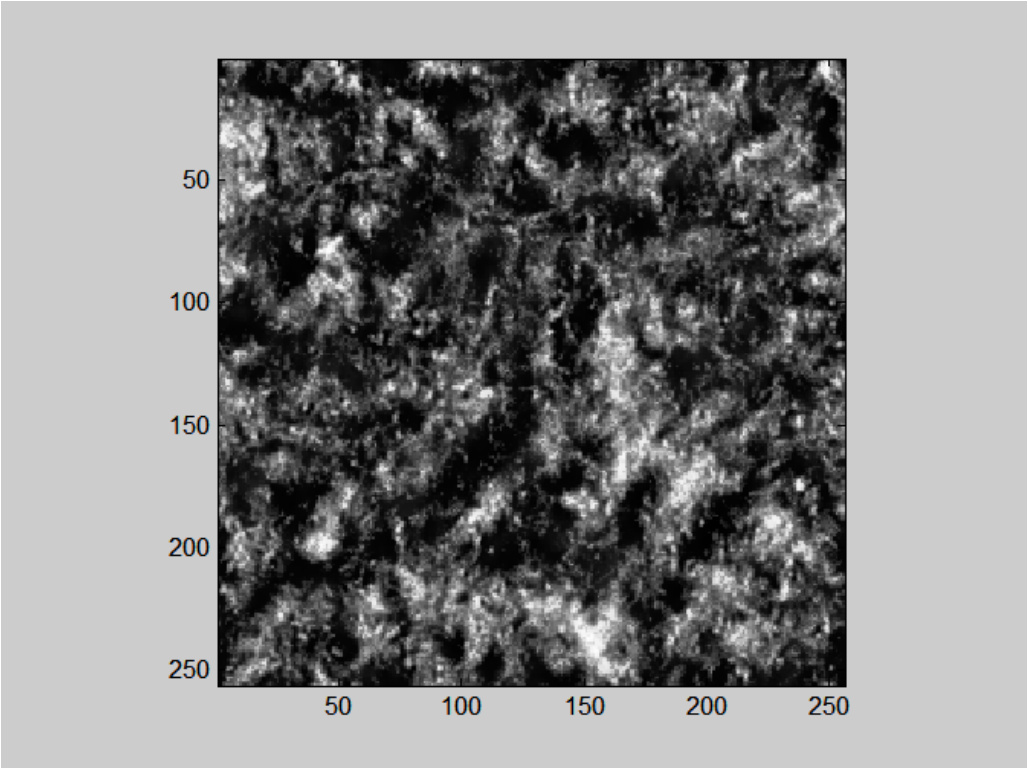
R



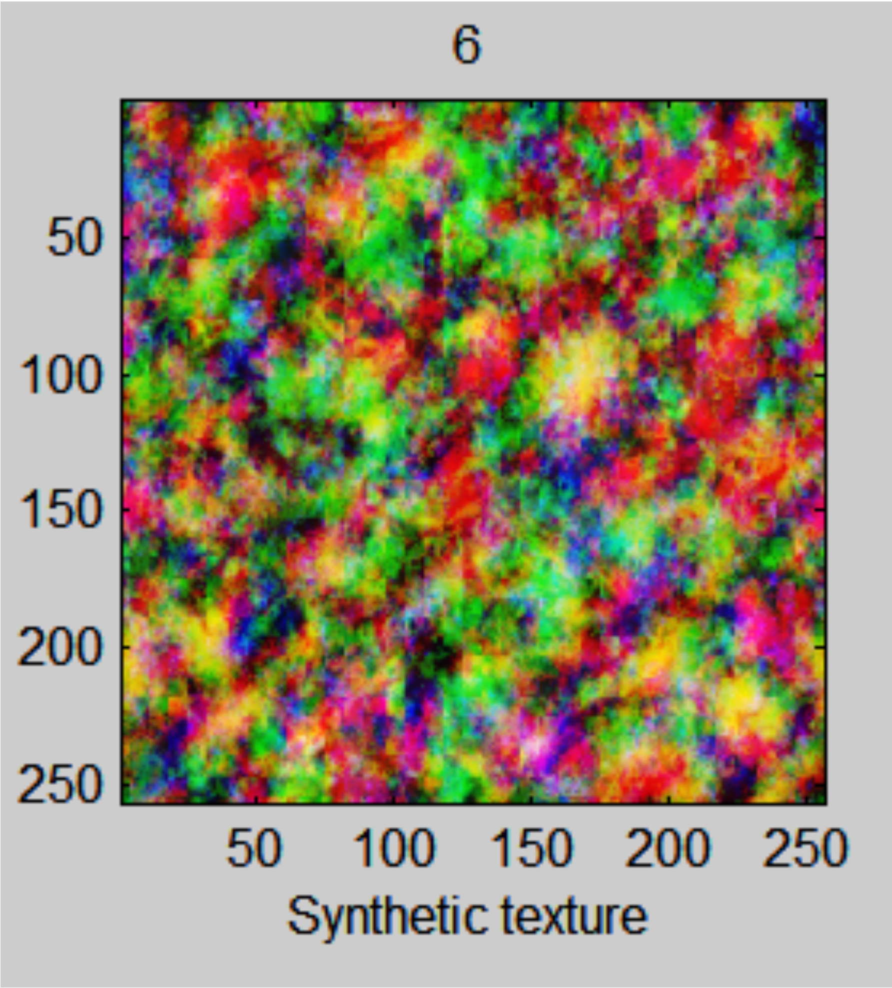
G

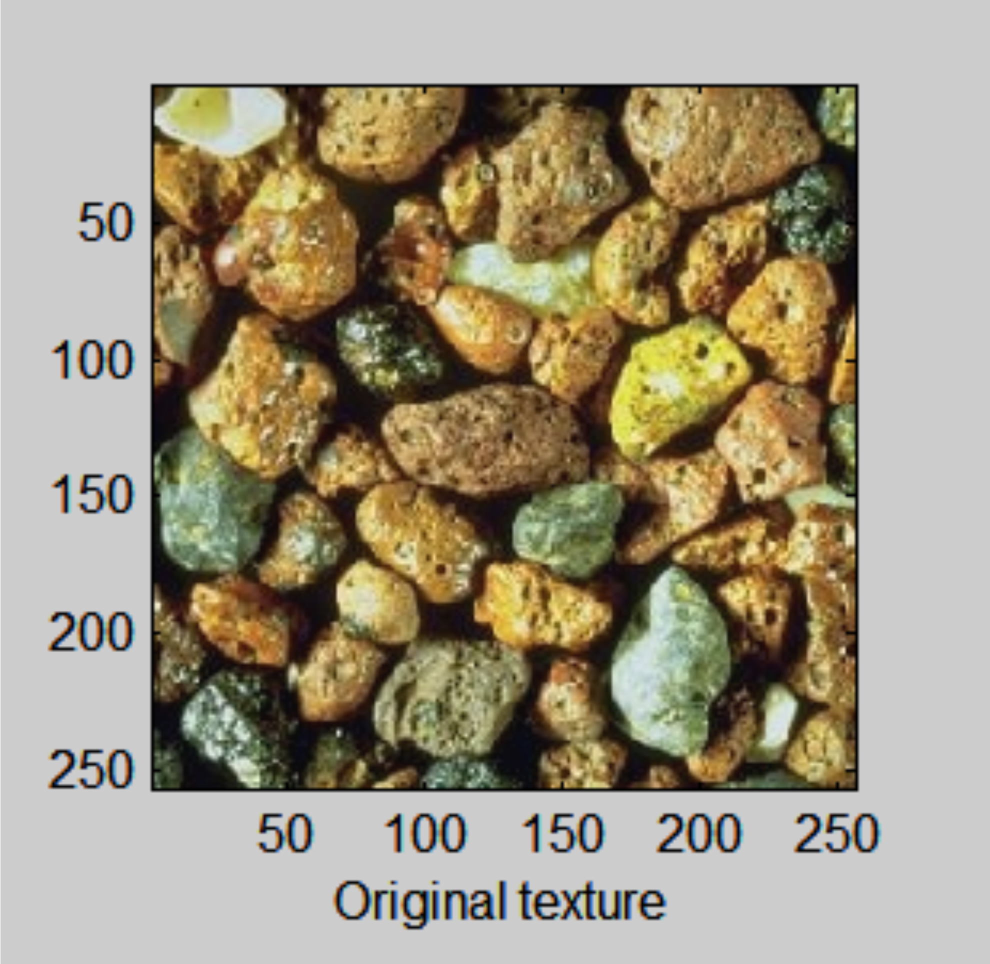


B



This does not work

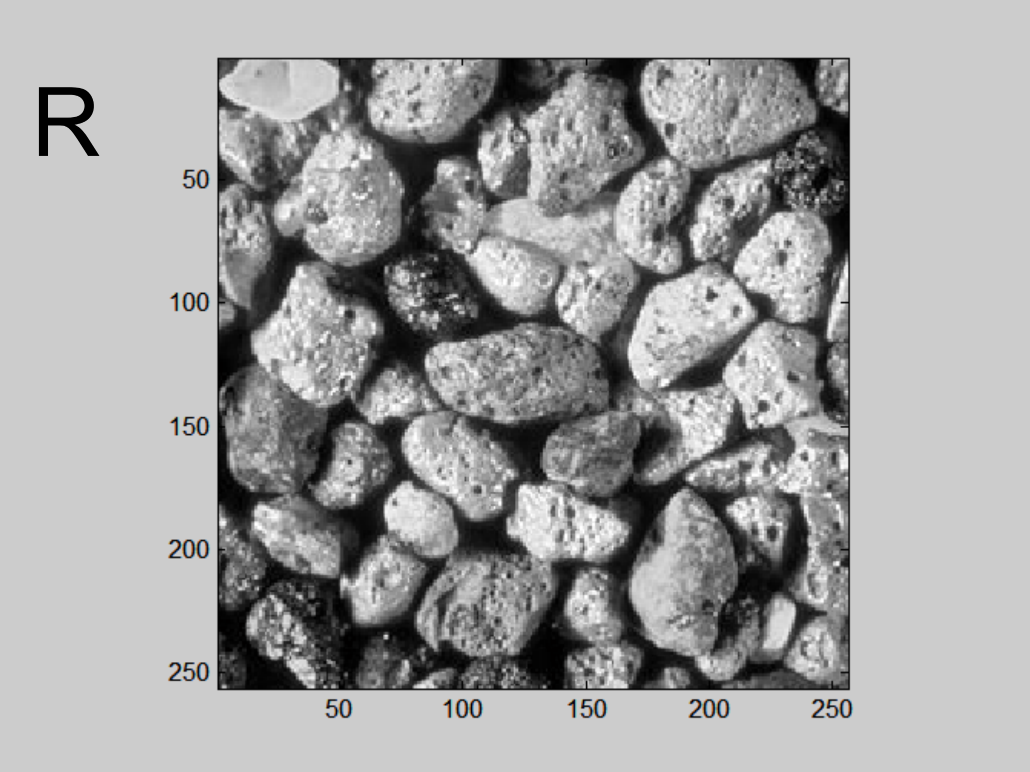


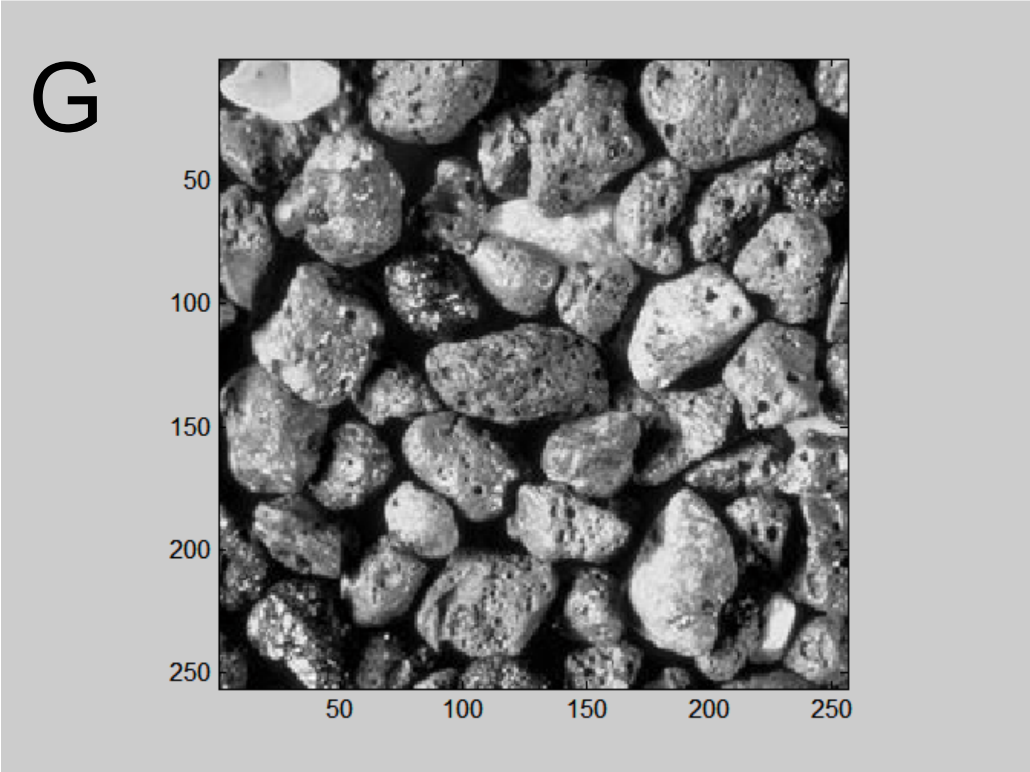


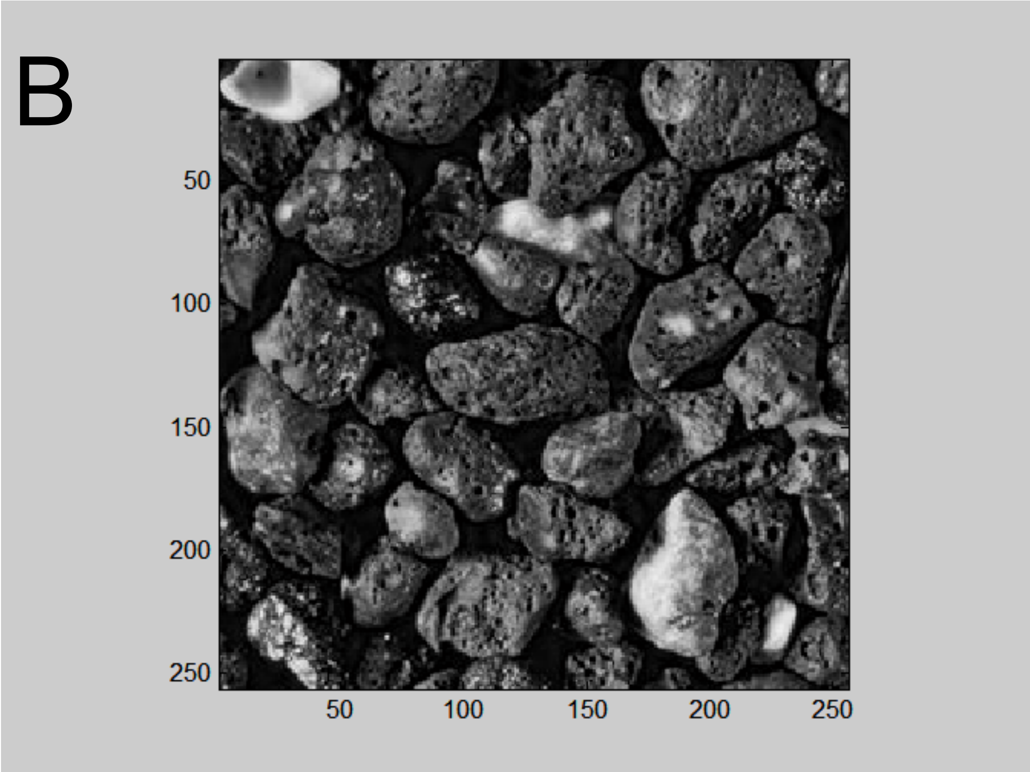
Color textures

Problem: we create new colors not present in the original image.

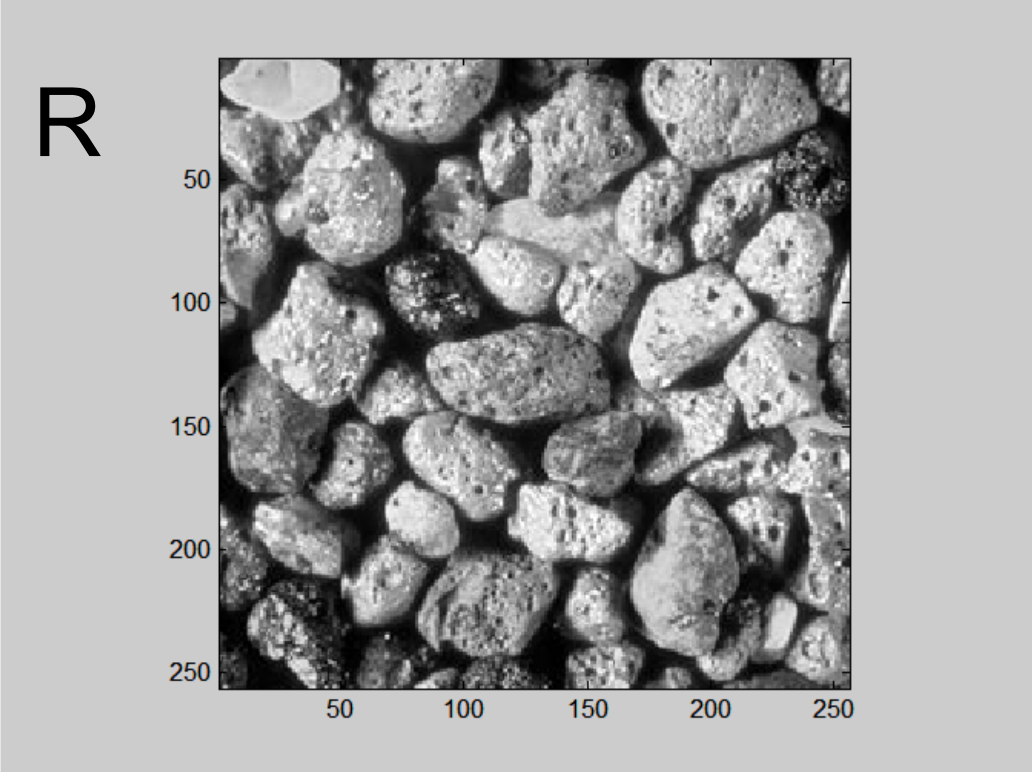
Why? Color channels are not independent.

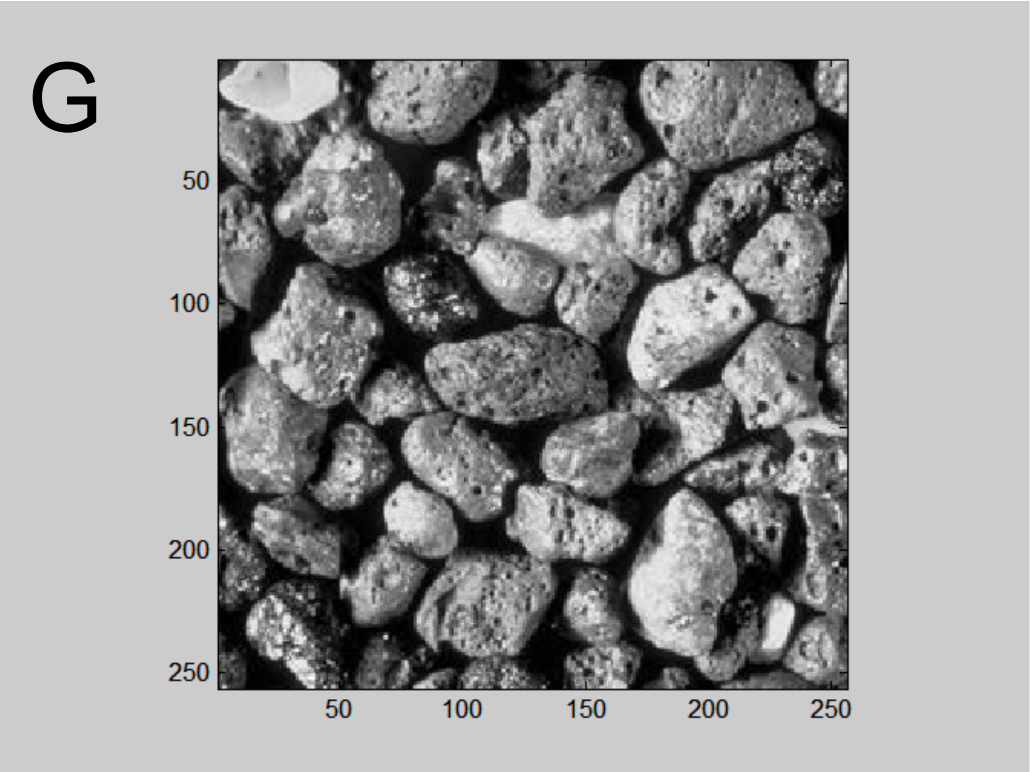


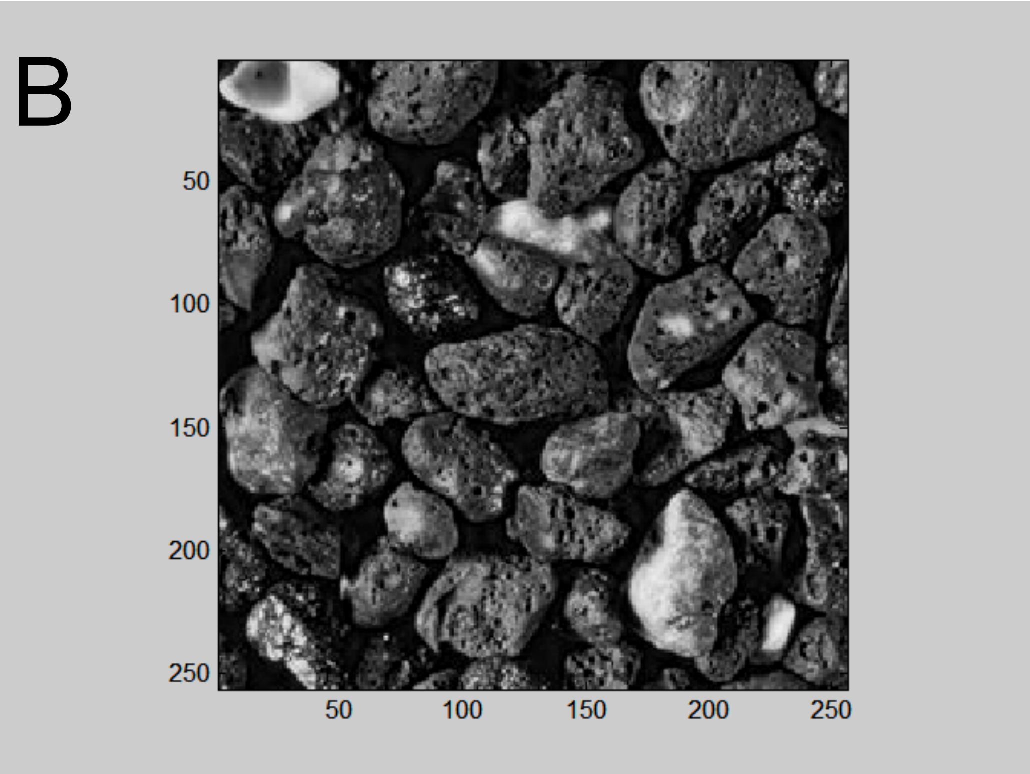




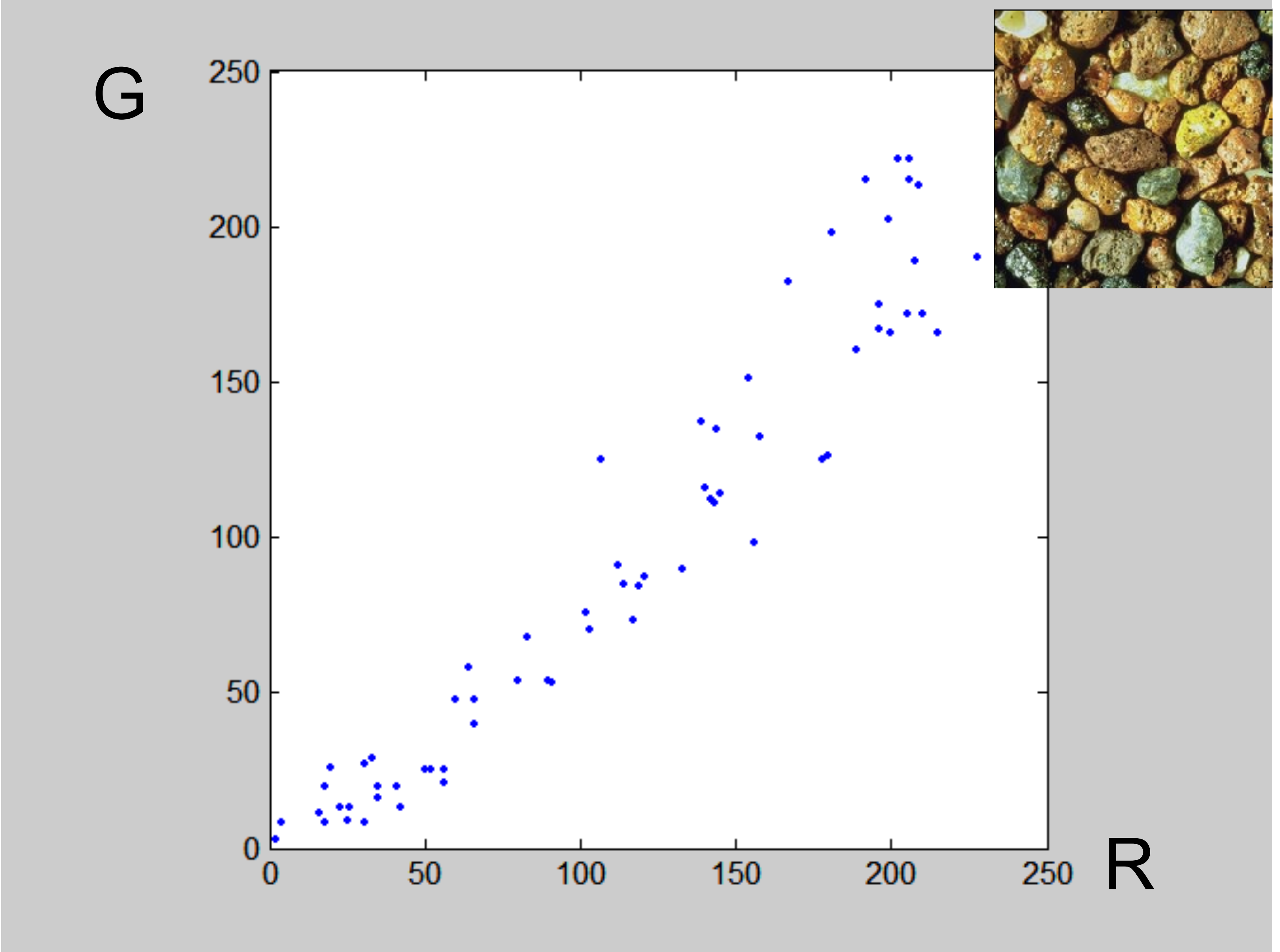
PCA and decorrelation

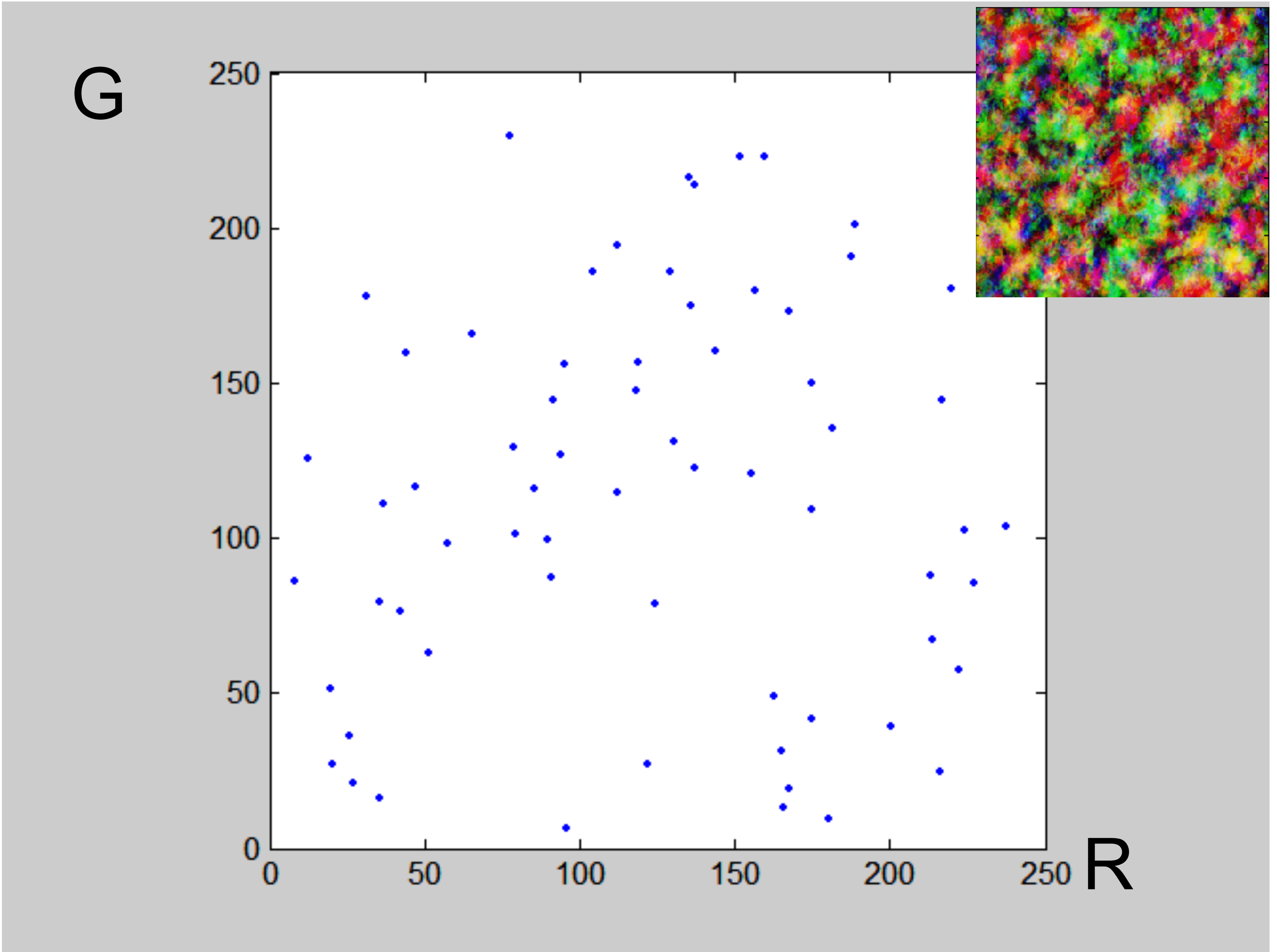






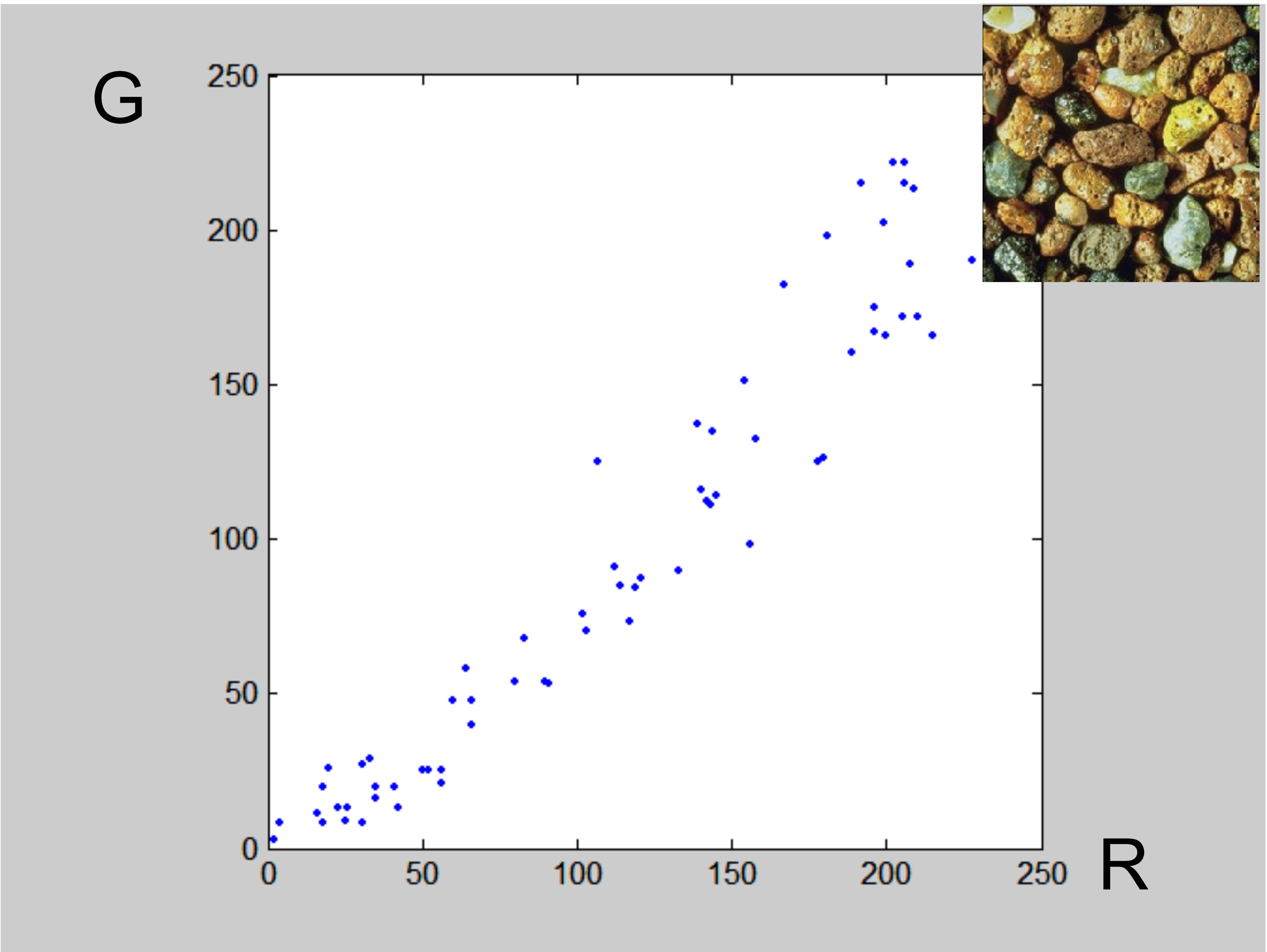
In the original image, R and G are correlated, but, after synthesis,…



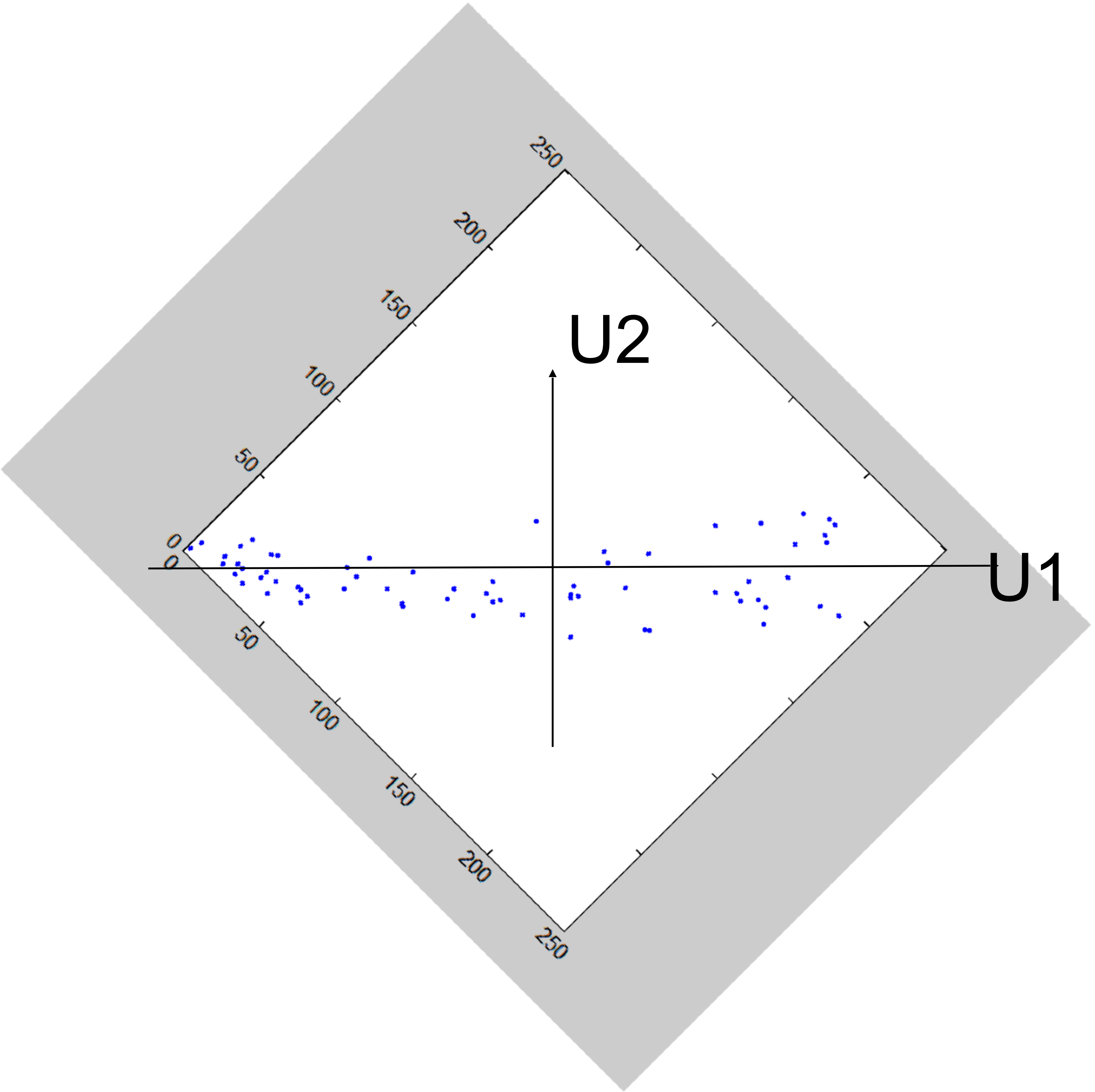


PCA and decorrelation

The texture synthesis algorithm assumes that the channels are independent.
What we want to do is some rotation

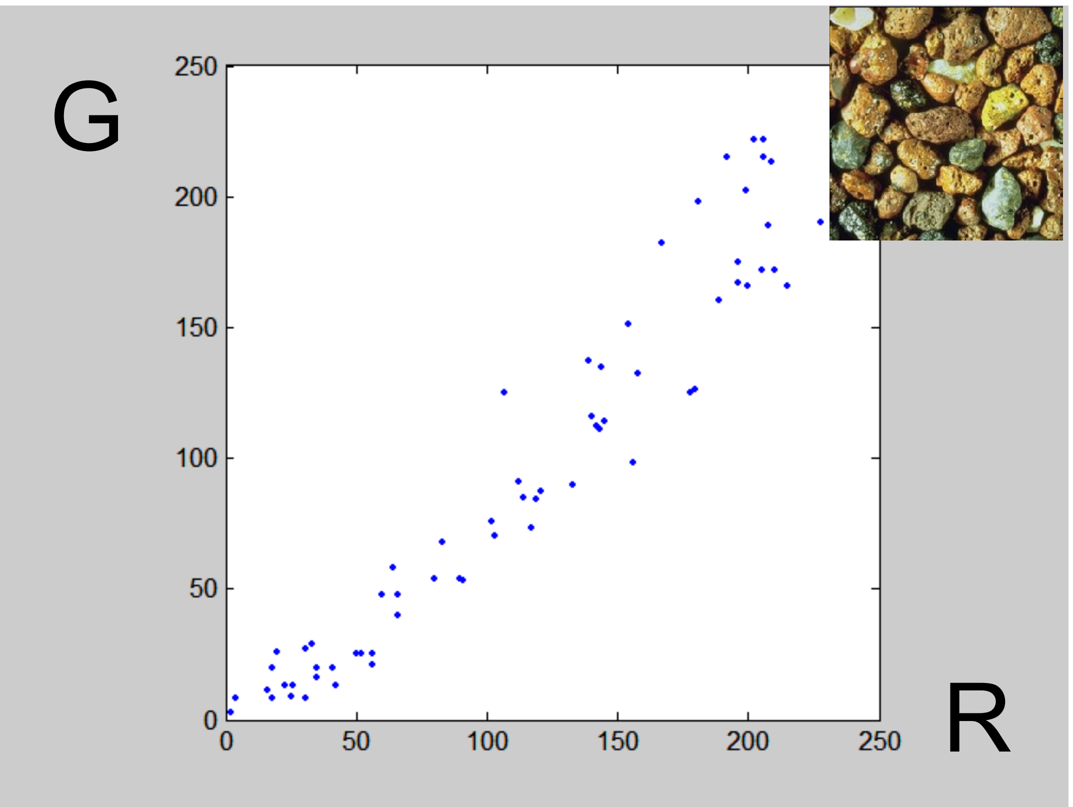


Rotation



See that in this rotated space, if I specify one coordinate the other remains unconstrained.

PCA and decorrelation



correlation(R,G)

$$C = \begin{bmatrix} 1.0000 & 0.9303 & 0.6034 \\ 0.9303 & 0.9438 & 0.6620 \\ 0.6034 & 0.6620 & 0.5569 \end{bmatrix}$$

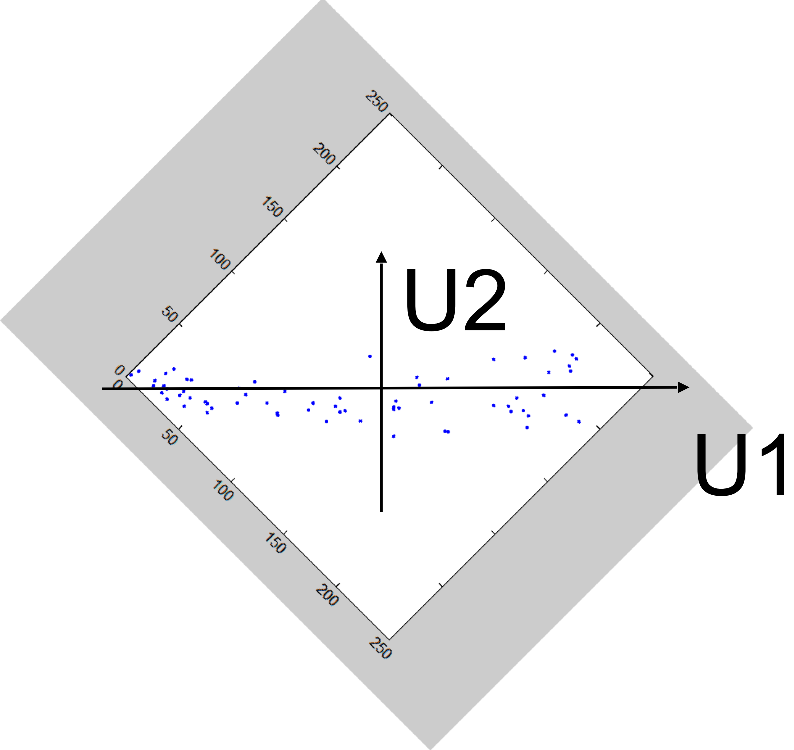
PCA finds the principal directions of variation of the data.
It gives a decomposition of the covariance matrix as:

$$C = D\,D'$$

$$D = \begin{bmatrix} 0.6347 & 0.6072 & 0.4779 \\ 0.6306 & -0.0496 & -0.7745 \\ 0.4466 & -0.7930 & 0.4144 \end{bmatrix}$$

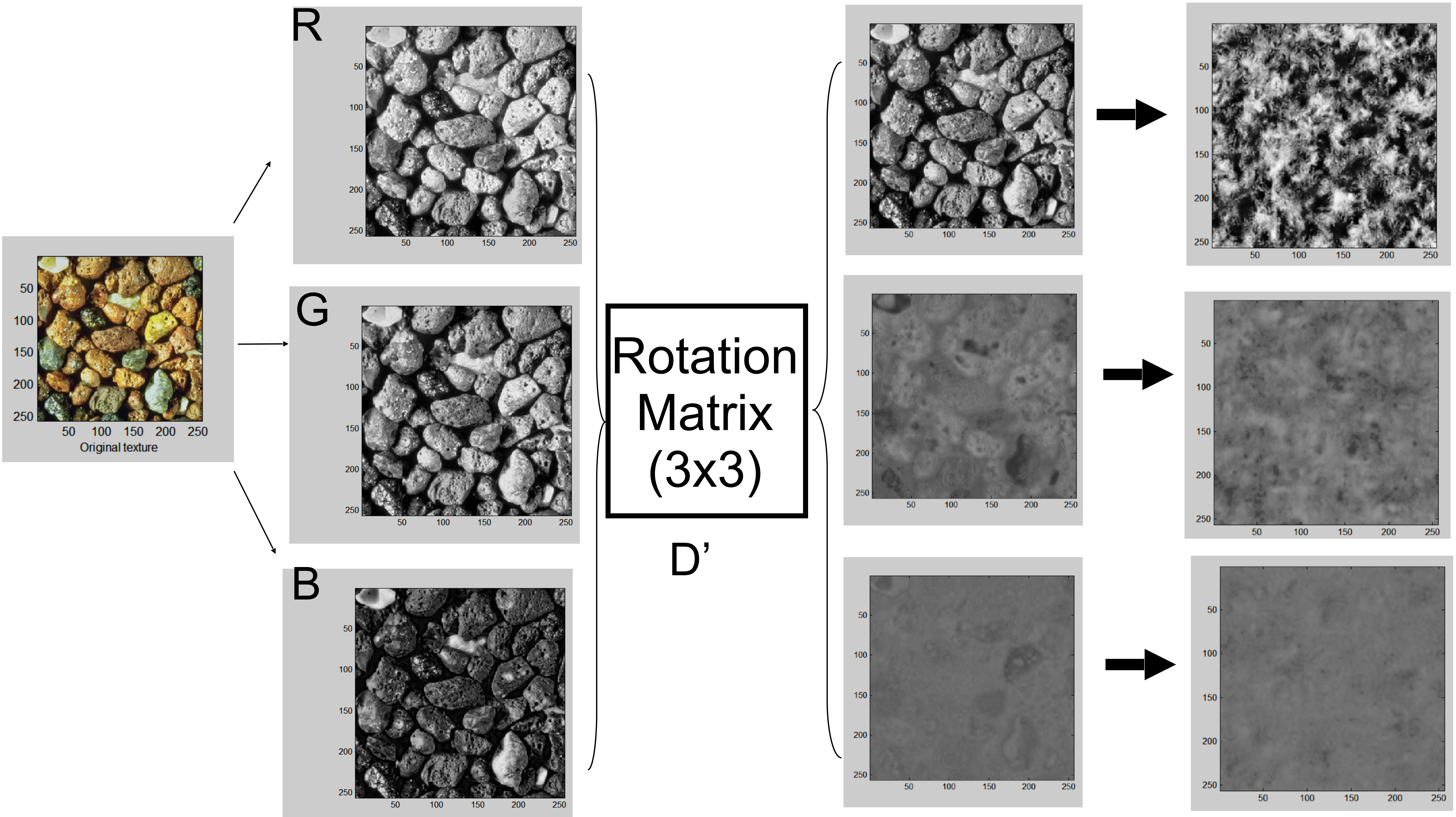
By transforming the original data (RGB) using D we get:

$$\underbrace{\begin{bmatrix} U1 \\ U2 \\ U3 \end{bmatrix}}_{3 \times Npixels} = \underbrace{\begin{bmatrix} D' \end{bmatrix}}_{3 \times 3} \underbrace{\begin{bmatrix} R \\ G \\ B \end{bmatrix}}_{3 \times Npixels}$$



The new components (U1,U2,U3) are decorrelated.

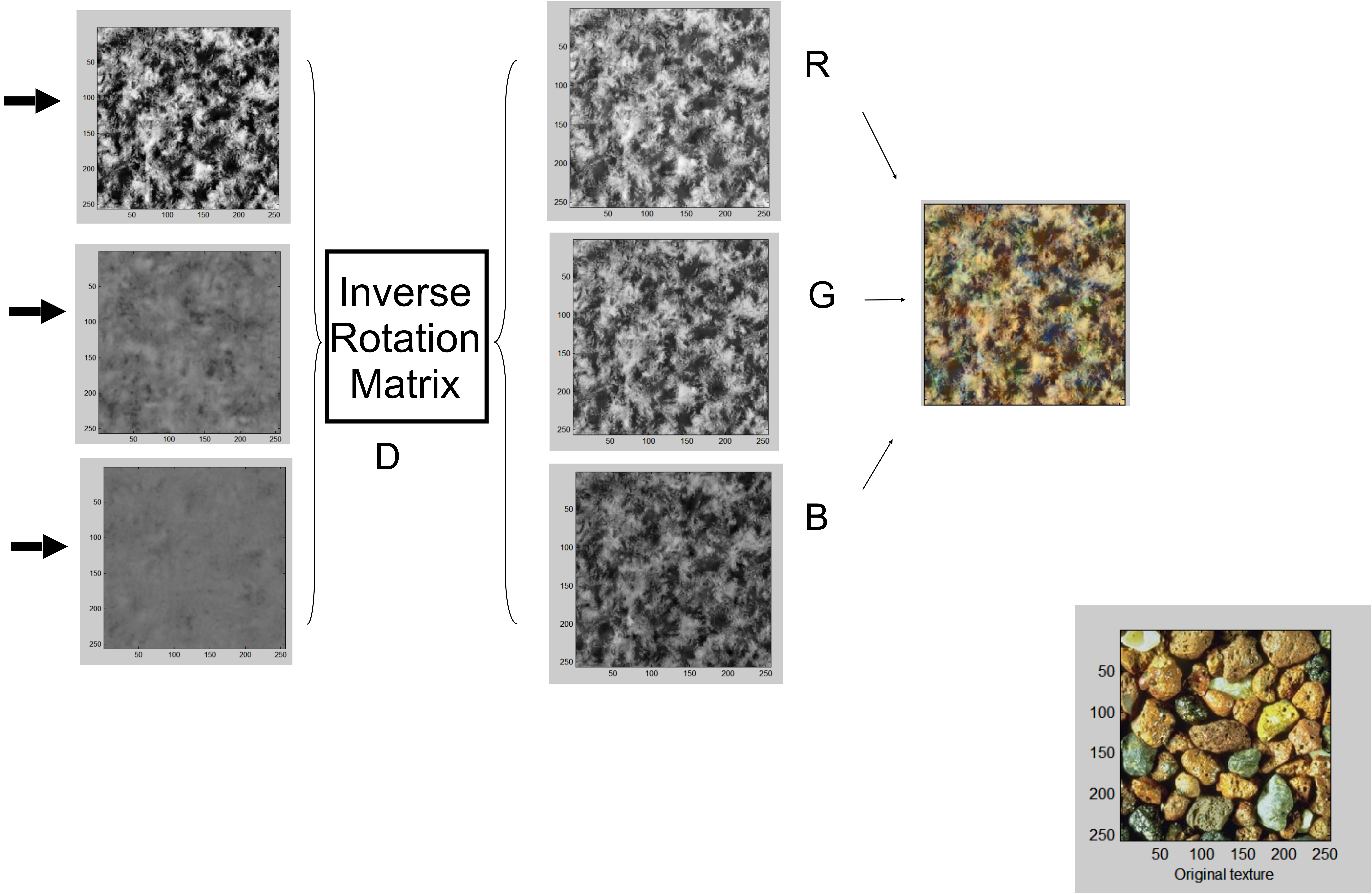
Color textures



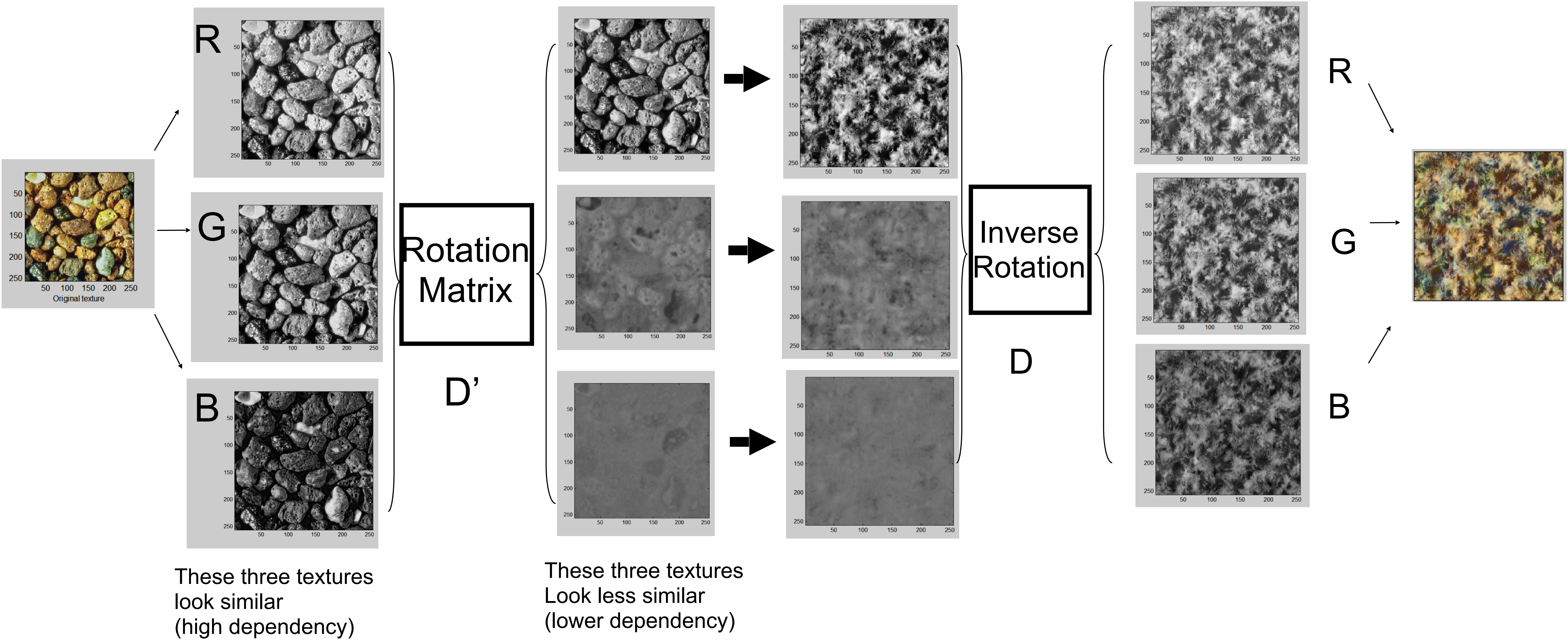
These three textures look similar (high dependency)

These three textures Look less similar (lower dependency)

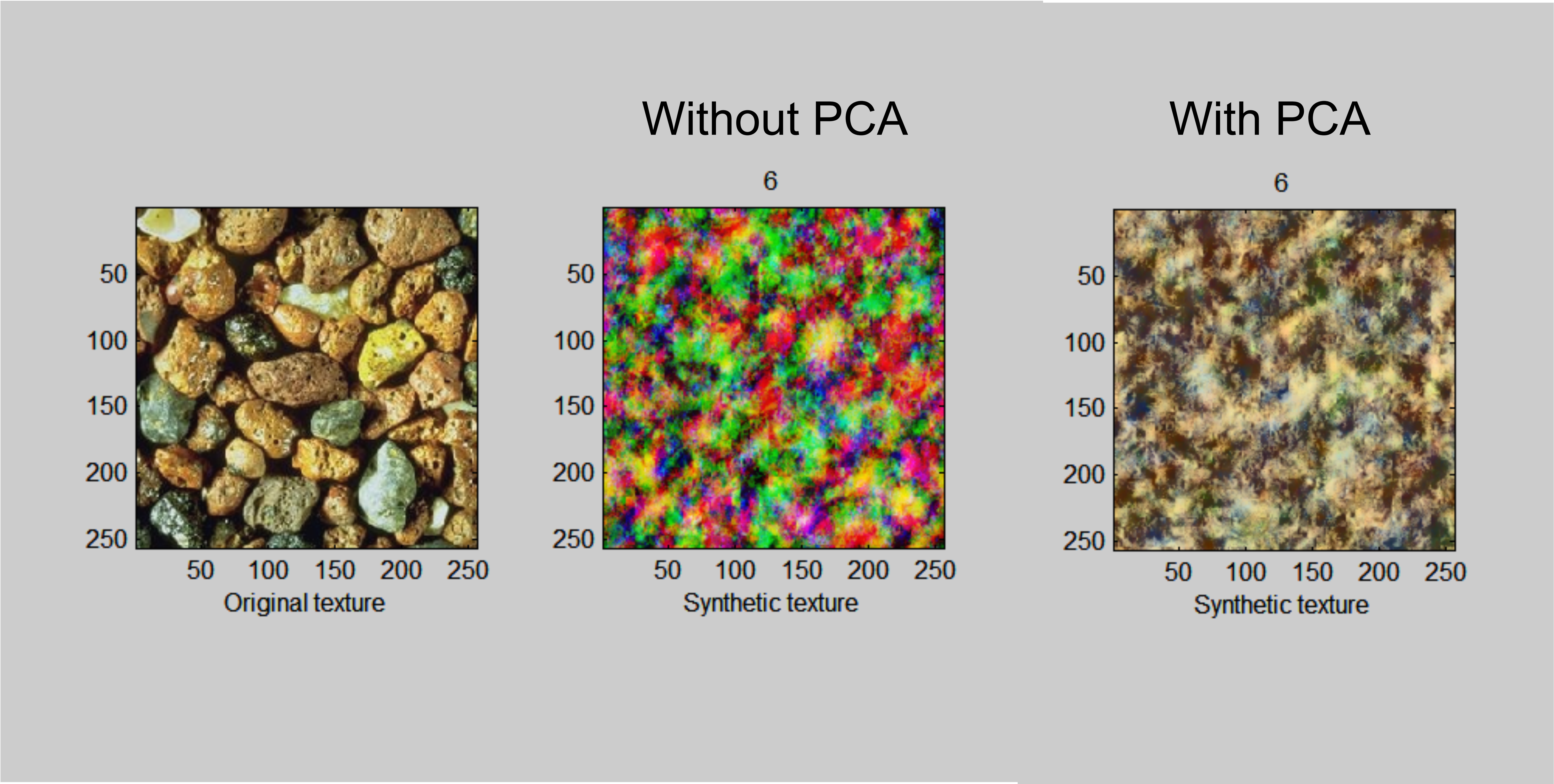
Color textures



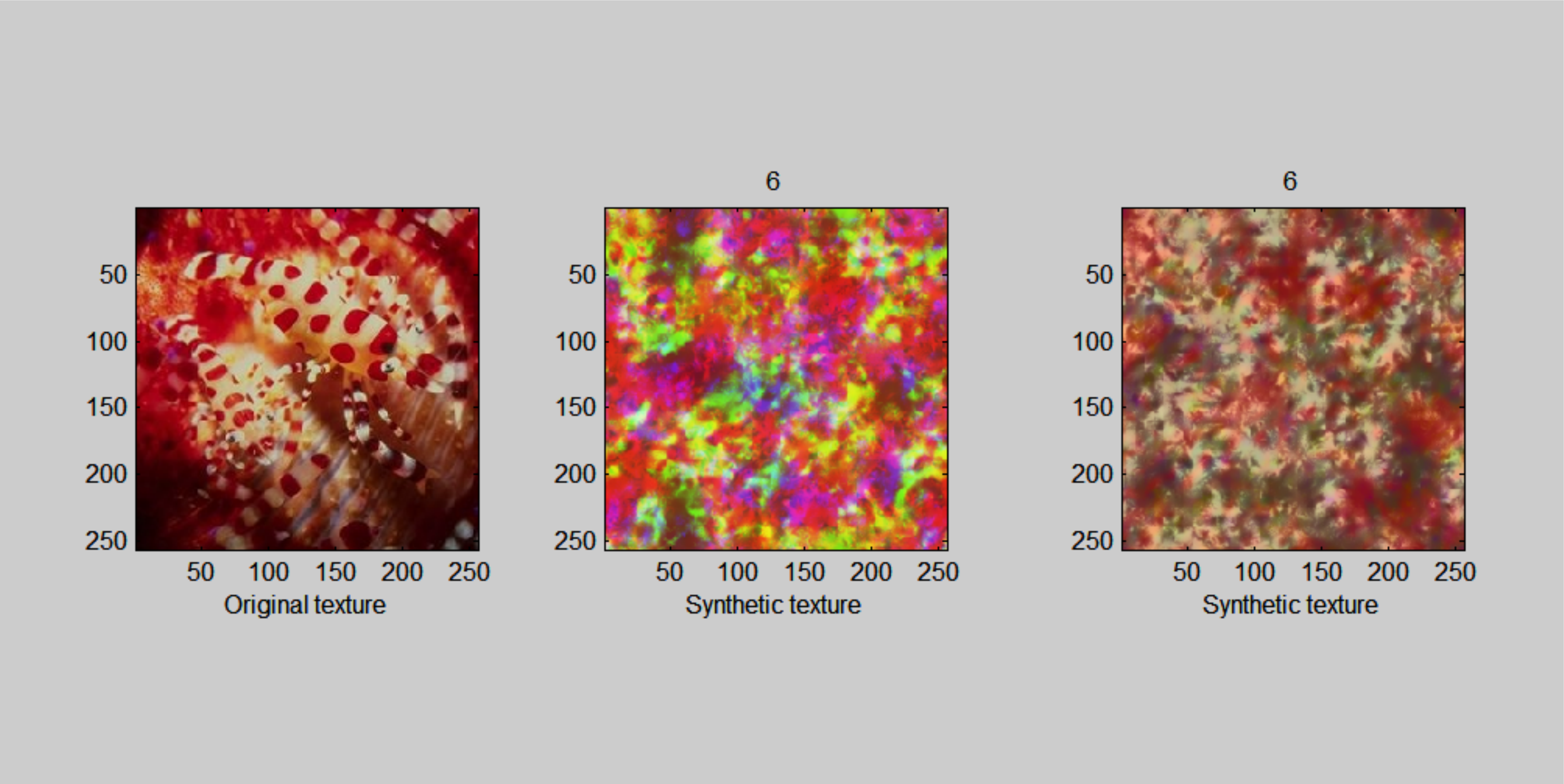
Color textures



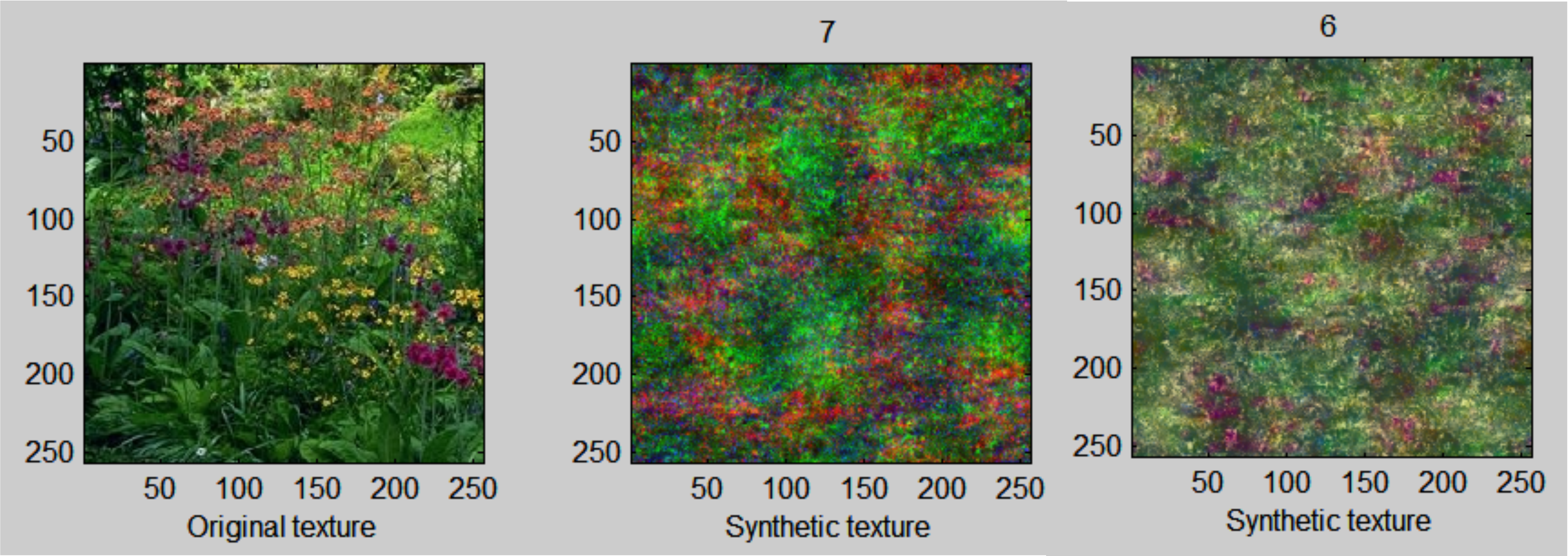
Color channels



Color channels



Color channels



Examples from the paper

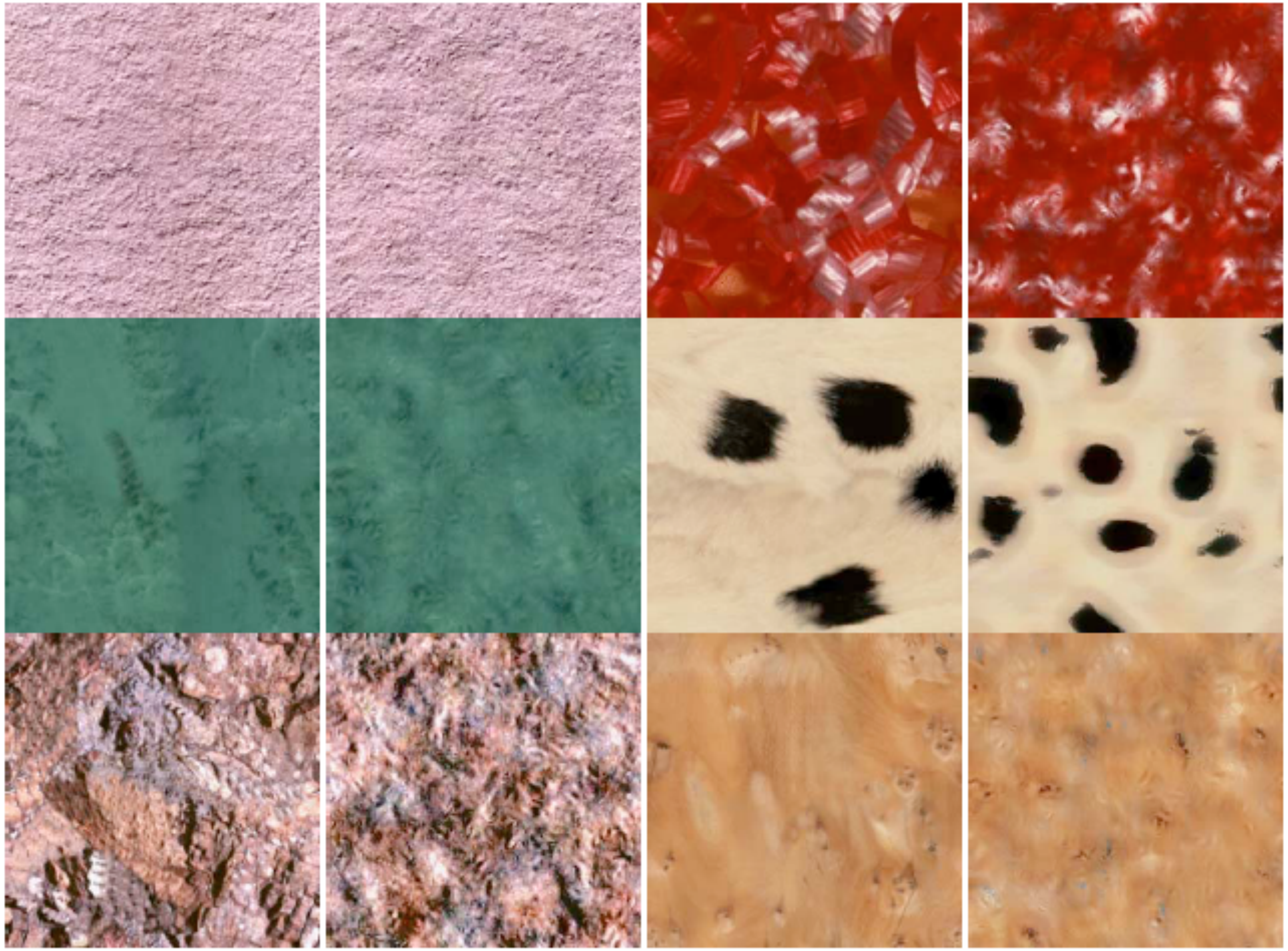


Figure 3: In each pair left image is original and right image is synthetic: stucco, iridescent ribbon, green marble, panda fur, slag stone, figured yew wood.

Heeger and Bergen, 1995

Examples from the paper

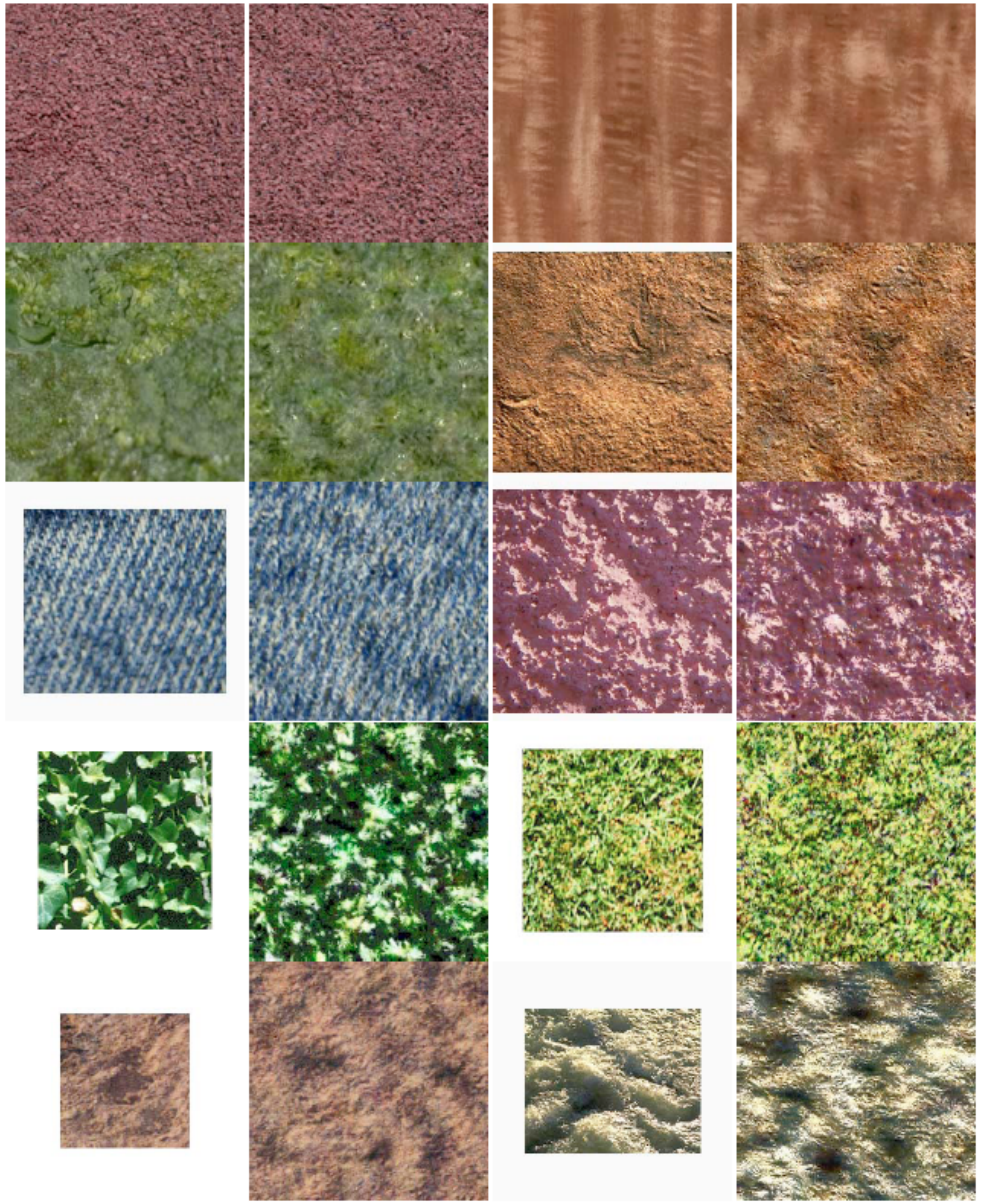
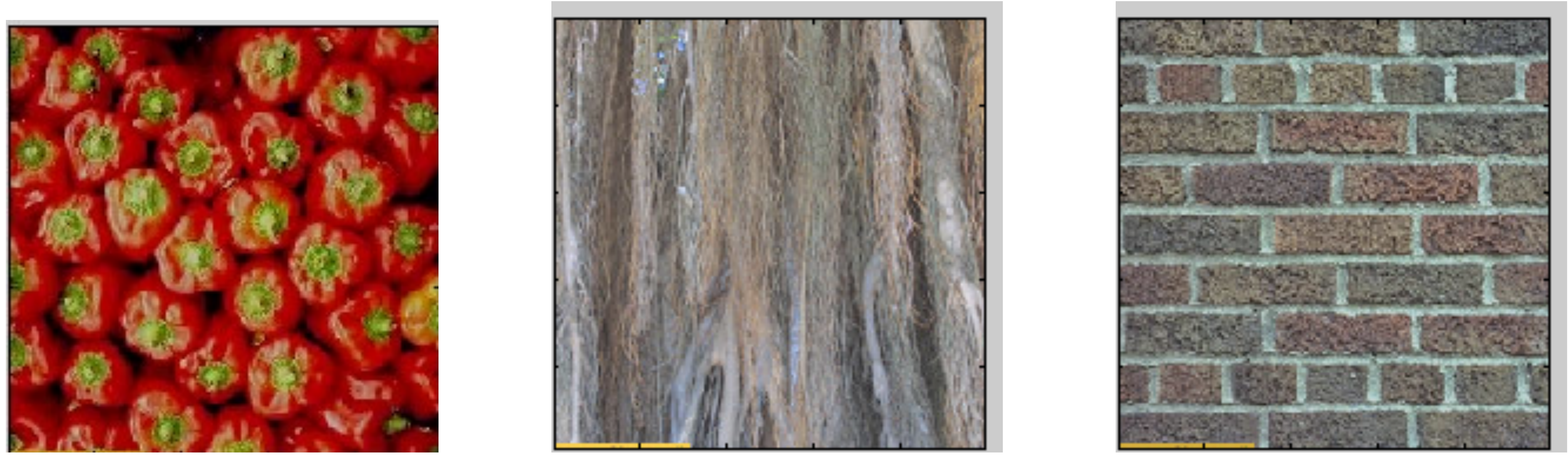


Figure 4: In each pair left image is original and right image is synthetic: red gravel, figured sepele wood, brocolli, bark paper, denim, pink wall, ivy, grass, sand, surf.

Examples not from the paper

Input
texture



Synthetic
texture

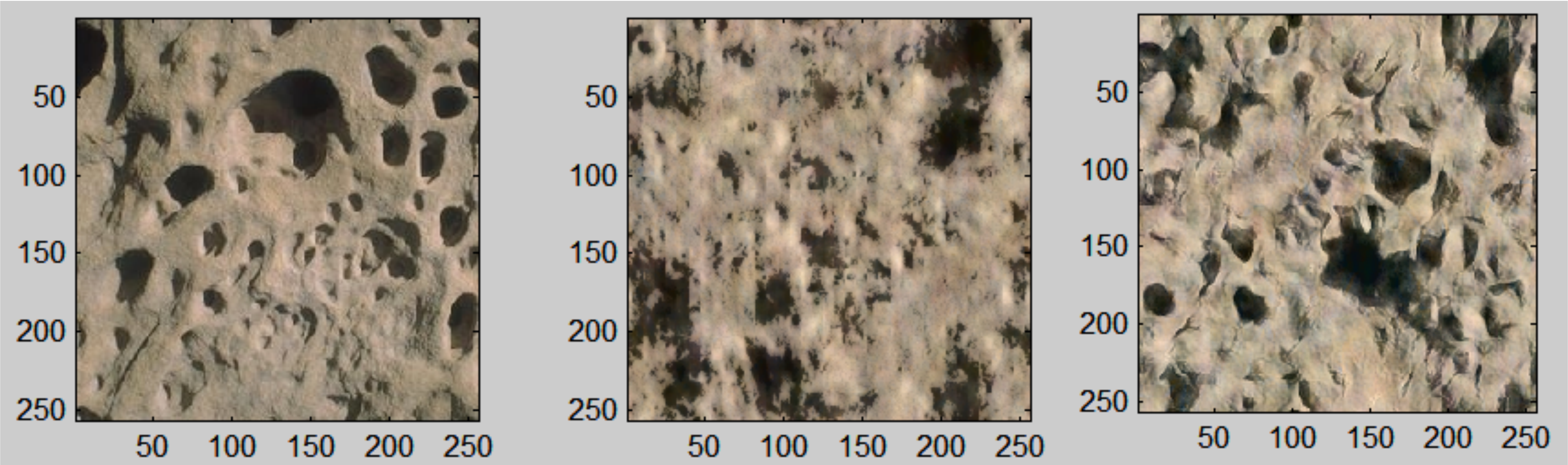
But, does it really work even when it seems to work?

Portilla and Simoncelli

- Parametric representation, based on Gaussian scale mixture prior model for images.
- About 1000 numbers to describe a texture.
- Ok results; maybe as good as DeBonet.

Portilla and Simoncelli

Portilla & Simoncelli



Heeger & Bergen

Portilla & Simoncelli