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Chapter 4

Imaging

4.1 Light interacting with surfaces

Visible light is electromagnetic radiation, exhibiting wave effects like diffraction. For many imaging models it is helpful to introduce the abstraction of a light ray, describing the light radiation heading in a particular direction from a particular location in space. A light ray is specified by its position, direction, intensity as a function of wavelength, and polarization.

Light sources, like the sun or artificial lights, flood our world with light rays. These reflect off surfaces, generating a field of light rays heading in all directions through space. As the light rays reflect from the surfaces, they generally change in some attributes—their brightness or their color. It is those changes upon surface reflection that let us interpret what we see. In this chapter, we describe how light interacts with surfaces and how those interactions are recorded with a camera.

Let an incident light ray be of direction $\hat{p}$ and of power, $I_{in}(\lambda)$, as a function of spectral wavelength $\lambda$. The power of the outgoing light, reflected in the direction, $\hat{q}$, is determined by what is called the bidirectional reflection distribution function (BRDF), $F$, of the surface. If the surface normal is $\hat{n}$, then outgoing power is some function, $F$, of the surface normal, the in- and out-going ray directions, the wavelength, and the incoming light power:

$$I_{out} = F(I_{in}, \hat{n}, \lambda, \hat{p}, \hat{q})$$ (4.1)

In general, BRDF’s can be quite complicated (see [Matusik et al.]), describing both diffuse and specular components of reflection. But surfaces with completely diffuse reflections, called Lambertian surfaces, have a particularly simple BRDF, which we denote $F_L$. The outgoing ray intensity is only a function of the surface orientation relative to the incoming ray direction, and the wavelength and the incoming light power.

$$I_{out} = F_L(I_{in}, \hat{n}, \hat{p}) = a I_{in}(\lambda) \cos(\hat{n} \cdot \hat{p}),$$ (4.2)

where $a$ is the surface reflectance, or albedo, $\hat{n}$ is the surface normal vector, and $\hat{p}$ is the direction of the incident light. Note that the brightness of the outgoing light ray depends on the orientation of the surface relative to the incident ray, as well as the reflectance, $a$, of the surface. For a Lambertian surface, the intensity of the reflected light is a function of the direction of the incoming light ray, but not a function of the outgoing direction of the ray, $\hat{q}$. In general, surface reflection behaves linearly: the reflection from the sum of two rays is the sum of the reflections from the two individual rays.

To analyze the surfaces that reflect light, we need to know which light rays came from which direction in space. That requires that we form an image, which we discuss next.
4.2 The Pinhole Camera and Image formation

Naively, one might wonder, when we look at a blank wall, why we don’t see an image of the scene facing that wall? The light reflected from the wall integrates light from every reflecting surface in the room, so the reflected intensities are an average of light intensities from many different directions and many different sources. Mathematically, integrating the equation for Lambertian reflections, Eq. (4.2), over all possible incoming light directions $\hat{p}$, we have for the intensity reflecting off a Lambertian surface, $I_L$:

$$I_L = \int_{\hat{p}} aI(\hat{p}) \cos(\hat{n} \cdot \hat{p}) \tag{4.3}$$

The intensity, $I_L$, reflecting off a diffuse wall, tells us very little about the light intensity $I(\hat{p})$ from any given direction $\hat{p}$. To learn about $I(\hat{p})$, we need to form an image. Forming an image involves identifying which rays came from which directions. The role of a camera is to organize those rays, to convert from the cacophony of light rays going everywhere to a set of measurements of intensities coming from different surfaces.

Perhaps the simplest camera is a pinhole camera. A pinhole camera requires a light-tight enclosure, a small hole that lets light pass, and a projection surface where one senses or views the illumination intensity as a function of position. Figure 4.1 (a) shows the the geometry of a scene, the pinhole, and a projection surface. For any given point on the projection surface, the light that falls there comes from only from one direction, along the straightline joining the surface position and the pinhole. This creates an image of what’s in the world on the projection plane, Figure 4.1 (b).

Figure 4.2 shows how to make a pinhole camera using a paper bag with a hole in it. One sticks their head inside the bag, which has been padded to be opaque. We encourage readers to make their own pinhole camera designs. The needed elements are an aperture to let light through, mechanisms to block stray light, projection screen, and some method to view or record the image on the projection screen.
4.2. THE PINHOLE CAMERA AND IMAGE FORMATION

Figure 4.1: (a) Pinhole camera geometry, showing some light rays passing through the pinhole aperture to the projection plane. (b) Image on the projection plane resulting from lighting projecting through a pinhole from the subject, (c). Fig. 4.6 (a) shows the configuration of the object, pinhole, and projection plane.

Figure 4.2: A pinhole camera made from paper bags. (a) We can also use a paper bag to make a light-tight pinhole camera, with the viewer inside. Newspapers can be added between two layers of paper bags to make a light-tight enclosure. (b) Use of the paper bag pinhole camera.
4.2.1 Image formation by perspective projection

A pinhole camera projects 3d coordinates in the world to 2d positions on the projection plane of the camera through the straightline path of each light ray through the pinhole. The simple geometry of the camera lets us identify the projection by inspection.

Let the origin of a Cartesian coordinate system be the camera’s pinhole. The coordinates of 3d position of a surface in the world will be \( X, Y, Z \), where the \( Z \) axis is perpendicular to the camera’s sensing plane. Let the coordinates in the camera projection plane be \( x, y \), parallel to the coordinate axes \( X \) and \( Y \), respectively. If the distance from the sensing plane to the pinhole is \( d \) (see Fig. (4.3)) then similar triangles gives us the following relations,

\[
x = -\frac{dX}{Z} \tag{4.4}
\]
\[
y = -\frac{dY}{Z} \tag{4.5}
\]

Eqs. (4.5) are called the perspective projection equations. Under perspective projection, distant objects become smaller, through the inverse scaling by \( Z \). As we will see, the perspective projection equations apply not just to pinhole cameras but to most lens-based cameras, and human vision as well.

4.2.2 Image formation by orthographic projection

Perspective projection is not the only feasible projection from 3d coordinates of a scene down to the 2d coordinates of the sensor plane. Different camera geometries can lead to other projections. One alternative to perspective projection is orthographic projection, where the size of objects is independent of the distance to the camera, scaled by a constant factor, \( k \):

\[
x = kX \tag{4.6}
\]
\[
y = kY \tag{4.7}
\]

This is a good model for telephoto lenses, where the size of objects is roughly independent of their distance away. It’s also a model for the “soda straw camera”, shown in Fig. 4.4. A set of parallel straws allow parallel light rays to pass from the scene to the projection plane, but extinguish rays passing in all other directions. That camera doesn’t invert the image.

4.3 Cameras with lenses

While pinhole cameras can form good images, they suffer from a serious drawback: the images are very dim, because not much light passes through the small pinhole to the sensing
4.3. CAMERAS WITH LENSES

Figure 4.4: Straw camera example (a) The subject: a hand in sunlight. (b) Showing the straw camera and the resulting image. For a straw camera, the image projection is orthographic, with a unity scale factor—the object sizes on the projection plane are the same as those of the objects in the world.

plane of the pinhole camera. As shown in Figs. 4.5 and 4.6, one can try to let in more light by making the pinhole aperture bigger. But that allows light from many different positions to land on the sensor plane, resulting in a bright, but blurry, image. Putting a lens in the larger aperture can give the best of both worlds, capturing more light, while providing a focussed image on the sensor plane.

4.3.1 Lensmaker’s formula

In general, light changes both its wavelength and (in proportion) its speed as it passes from one material to another. Those changes at the material interface will cause the light to bend, an effect called refraction. The amount of light bending depends on the change of speed of light within each material, and the orientation of the light ray with respect to the interface surface, according to Snell’s Law. Both the wavelength and the speed of light in a medium are inversely proportional to the index of refraction of that medium, denoted $n$. $n$ for a vacuum is 1. At a material boundary, for the geometry illustrated in Fig. 4.7, we have

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \quad (4.8)$$

where $\theta_1$ and $\theta_2$ are the angles with respect to the surface normal of the incident and outgoing light rays, and $n_1$ and $n_2$ are the indices of refraction of the materials in region 1 and region 2. Snell’s law can be derived by equating the the wavelength of light along the interface boundary at each side of the boundary.

A lens is a specially shaped piece of transparent material, positioned to focus light from a surface point onto a sensor. In idealization, it has the property that every light ray from the surface point which passes through the lens is refracted onto a common position at the sensor, no matter what part of the lens the ray from the surface hits. This dramatically increases the light-gathering ability of the camera system, overcoming the poor light-gathering properties of a pinhole camera system.

To achieve that property, we must find a surface shape that allows for this focusing. Modern lens surfaces are designed by numerical optimization methods, trading off engineering constraints to achieve the best design, often involving several optical elements and different
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(a) Image from small pinhole

(b) Image from large pinhole

(c) Image from lens within the large pinhole

Figure 4.5: Brightness/sharpness tradeoffs in image formation. (a) A small pinhole will create a sharp image, but lets in little light, so the image may appear dark, for a given exposure time. (b) A larger pinhole lets in more light, generating a bright image. But each sensor element records light from many different image positions, creating a blurry image. (c) A lens can collect light reflected over many different angles from a single point, allowing a bright, sharp image.
Figure 4.6: Physical demonstration of the tradeoffs illustrated in Fig. fig:pinholes. (a) Right to left: Gumby subject, illumination light, barrier with the three apertures and white projection screen. (b) Images formed by light through the three apertures. (c) the three apertures (small pinhole, large pinhole, and lens). (d) The subject.
CHAPTER 4. IMAGING

Figure 4.7: Snell’s law describes the bending of light at interfaces of differing indices of refraction, \( n_1 \) and \( n_2 \) in terms of the angles, \( \theta_1 \) and \( \theta_2 \), relative to the interface, or surface, normal.

materials. But to gain insight into the properties of lenses, we can analytically design a lens surface shape provided we simplify the optical system.

For small angles \( \theta \) denoted in radians, \( \sin(\theta) \approx \theta \). If we also assume the index of refraction of air is 1 (it is 1.0003) and denote the index of refraction of lens glass as \( n \), then Snell’s Law, as shown in Fig. 4.7, becomes

\[
\theta_1 = n\theta_2, \tag{4.9}
\]

for small bending angles \( \theta_1 \) and \( \theta_2 \).

Consider a lens, and two points along its optical axis at a distance \( a \) and \( b \) from the lens, as shown in Fig. 4.8 a. We seek to find a surface shape for a lens which creates the light paths shown in Fig. 4.8 (a): light leaving from any direction at point \( a \) will be focused to arrive at point \( b \). Figure 4.8 (b) shows a view of (a), with angles and distances distorted for clarity of labeling.

We make use of several approximations which commonly hold for imaging systems: The deviations in angle from the optical axes are very small, and the lens is modeled to have negligible thickness. Under those approximations, we can write simple expressions for the bending angles shown in Fig. 4.8 (b) as a function of \( \theta_S \). \( \theta_S \) is the lens surface orientation at height \( c \).

<table>
<thead>
<tr>
<th>Angle</th>
<th>Description</th>
<th>Relation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 )</td>
<td>initial angle from optical axis</td>
<td>( \theta_1 = \frac{c}{a} )</td>
<td>small angle approx.</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>wrt front surface normal</td>
<td>( n\theta_2 = \theta_1 + \theta_S )</td>
<td>Snell’s law, small angle approx.</td>
</tr>
<tr>
<td>( \theta_3 )</td>
<td>angle of refracted ray</td>
<td>( 2\theta_S = \theta_2 + \theta_3 )</td>
<td>symmetry of lens, thin lens approx.</td>
</tr>
<tr>
<td>( \theta_S + \theta_4 )</td>
<td>angle of ray exiting lens</td>
<td>( n\theta_3 = \theta_S + \theta_4 )</td>
<td>Snell’s law, small angle approx.</td>
</tr>
<tr>
<td>( \theta_4 )</td>
<td>final angle from optical axis</td>
<td>( \theta_4 = \frac{c}{b} )</td>
<td>small angle approx.</td>
</tr>
</tbody>
</table>

If we start from the relation in Table 4.3.1 for \( \theta_4 \), and substitute for each angle using the
4.3. CAMERAS WITH LENSES

Figure 4.8: (a) The geometry of a thin lens. (b) Showing the labels of angles referenced in Table 4.3.1, used to describe the conditions for the lens shape, \( \theta_3 \) to give the desired focussing. (c) Showing relation between angles \( \theta_2 \) and \( \theta_3 \).
Figure 4.9: A spherical lens surface satisfies the property, Eq. (4.10), required for focusing a thin lens: that the surface slope increase linearly with distance \( c \) from the optical axis. Note that parallel rays (from infinity) focus at a distance \( f \) behind the lens.

relation in each line above, up through \( \theta_1 = \frac{c}{a} \), we can algebraically eliminate the angles \( \theta_1 \) through \( \theta_4 \) to find the condition on \( \theta_S \) which allows for the desired focusing to occur:

\[
\theta_S = \frac{c}{2(n-1)} \left( \frac{1}{a} + \frac{1}{b} \right)
\]  

(4.10)

This specifies the lens surface angle, \( \theta_S \), as a function of distance from the optical axis, \( c \), which creates the effect that every ray emanating at a small angle from point \( a \) will be focused to point \( b \). Eq. 4.10 shows that the lens surface angle, \( \theta_S \), must deviate from flat in linear proportional to the distance \( c \) from the center of the lens.

For thin lenses, both parabolic and spherical shapes satisfy that constraint. For a spherical lens surface, curving according to a radius \( R \), we have \( \sin(\theta_S) = \frac{c}{R} \). For small angles \( \theta_S \), this reduces to

\[
\theta_S = \frac{c}{R}
\]  

(4.11)

where \( R \) is the radius of the sphere, which has the desired property that \( \theta_S \propto c \). Substituting Eq. (4.11) into the focusing condition, Eq. (4.10) yields the **Lensmaker’s Formula**,

\[
\frac{1}{a} + \frac{1}{b} = \frac{1}{f}
\]  

(4.12)

where the lens *focal length*, \( f \) is

\[
f = \frac{R}{2(n-1)}
\]  

(4.13)

It is straightforward to show, by rotating the lens in Fig. 4.8 (a) by an angle \( \theta_R \) in the above derivation, that the lensmaker’s equation also holds for light originating off the optical axis. One can also generalize the equation for the case of a lens with different radii of curvature, \( R_1 \) and \( R_2 \) on the front and back faces of the lens. Then, following the arguments above leads to

\[
\frac{1}{f} = (n-1)\left( \frac{1}{R_1} + \frac{1}{R_2} \right)
\]  

(4.14)

Fig. 4.11 shows a demonstration of the focussing property for a thin lens of a 20cm focal length. The hand, (a), is flicking a laser pointer back and forth, sending light rays in many directions from a central point. All the the light rays which strike the lens, (b), are focussed to the same green spot, (c), while the light rays passing outside the lens at (b) reveal their straight line trajectories on the wall at (c).
4.3. CAMERAS WITH LENSES

Figure 4.10: Considering light from off-axis sources is equivalent to rotating the lens surface by $\theta_R$, for small angles $\theta_R$. The lens focusses rays from off-axis points like $P_1$, as well as from the on-axis point, $P_0$.

Figure 4.11: Demonstration showing that a lens focusses a fan of rays, such as those reflecting from a diffuse surface, to a point. (a) Here the right hand wiggles a laser pointer back and forth, generating a fan of light rays, approximating rays reflecting from one point on a diffuse surface. At (b), the fan of rays sweeps across a lens, which focusses each ray passing through the lens to the same spot at the wall, (c), regardless of where each ray had entered the lens. The rays which pass outside the lens form the two line segments also seen at (c).
4.4 Cameras as linear systems

Lens-based and pinhole cameras are special-case devices where the light falling on the sensors or the photographic film form an image. For other imaging systems, such as medical or astronomical systems, the intensities recorded by the camera may look nothing like an interpretable image. Because the optical elements of imaging systems are often linear in their response to light, it is convenient and powerful to describe cameras using linear algebra, which then provides a powerful machinery to recover an interpretable image from the recorded data.

Let the light intensities in the world be represented by a vector, \( \vec{x} \). The value of the \( j \)th component of \( \vec{x} \) gives the brightness of the light at position \( j \), heading in the direction of the camera. If the camera sensors respond linearly to the light intensity at each sensor, their measurements, \( \vec{y} \), will be some linear combination of the light brightnesses, given by the matrix, \( A \):

\[
\vec{y} = A \vec{x}
\]  

For the case of conventional lens and pinhole cameras, where the observed intensities, \( \vec{y} \) are an image of the reflected intensities in the scene, \( \vec{x} \), then \( A \) is approximately an identity matrix.

For more general cameras, \( A \) may be very different from an identity matrix, and we will need to estimate \( \vec{x} \) from \( \vec{y} \). In the presence of noise, there may not be a solution \( \vec{x} \) that exactly satisfies Eq. (4.15), so we often seek to satisfy it in a least squares sense. In most cases, \( A \) is either not invertable, or is poorly conditioned. It is often useful to introduce a regularizer, an additional term in the objective function to be minimized. If the regularization term favors small \( \vec{x} \), then the objective term to minimize, \( E \), could be

\[
E = |\vec{y} - A\vec{x}|^2 + \lambda |\vec{x}|^2
\]  

Setting the derivative of Eq. (4.16) with respect to the elements of the vector \( \vec{x} \) equal to zero, we have

\[
0 = \nabla_x |\vec{y} - A\vec{x}|^2 + \nabla_x \lambda |\vec{x}|^2
\]  

or

\[
\vec{x} = (A^T A + \lambda I)^{-1} A^T \vec{y}
\]

For the simplicity of visualization, let’s consider 1-dimensional imaging systems: As shown in Fig. 4.5 the sensor is 1-dimensional and the scene lives in flatland. Let the sensor measurements be \( \vec{y} \) and the unknown scene be \( \vec{x} \). We represent the camera by the matrix, \( A \). For the case of a pinhole camera, and assuming 13 pixel sensor observations, the camera matrix is just an 13x13 identity matrix, depicted in Fig. 4.5.

Next, consider the case of a wide aperture pinhole camera, shown in Fig. (4.13). If a single pixel in the sensor plane covers exactly two positions of the scene intensities, then the geometry is as shown in Fig. (4.13) (a). The imaging matrix, \( A \), and its inverse, \( A^{-1} \), and shown in (b). (b) also shows the regularized inverse of the imaging matrix, which will usually give image reconstructions with fewer artifacts.
4.4. CAMERAS AS LINEAR SYSTEMS

Figure 4.12: (a) Schematic drawing of a small-hole 1-d pinhole camera. (b) Visualization of imaging matrices: The imaging matrix relating scene intensities to sensor readings; the inverse of that matrix; the regularized inverse. For the small-pinhole imager, all three matrices are approximately identity matrices.

Figure 4.13: (a) Large-hole 1-d pinhole camera. (b) Visualization of imaging matrices: The imaging matrix relating scene intensities to sensor readings; the inverse of that matrix; the regularized inverse. For the small-pinhole imager, all three matrices are approximately identity matrices.
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Figure 4.14: (a) An edge camera (b) Visualization of idealized imaging matrices: The imaging matrix relating scene intensities to sensor readings; the inverse of that matrix; the regularized inverse. (c) A blurred edge imaging matrix, and its inverse and regularized inverse

4.5 More general imagers

Many different optical systems can form cameras. Even a simple edge will do. Consider the example of Fig. 4.14. For the edge camera of (a), we have

\[
A = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]  

Figure 4.14 illustrates the imaging matrix, \( A \), and reconstruction matrices for the imager of Eq. 4.21, as well as for an edge imager where the responses are blurred across several sensor elements.

4.5.1 Corner camera

To show a real-world example of a more general imager, let us consider the “corner camera” [Bouman et al.2018]. This is related to the edge camera of Eq. 4.21 and Fig. 4.14, but with slightly more complicated geometry. As shown in Fig. 4.15 (a), a vertical edge partially blocks a scene from view, creating intensity variations on the ground, observable by viewing those intensity variations from around the corner.
In practice, we will subtract a mean image from our observations of the ground plane, so in the rendering equation below, we will only consider components of the scene that may change over time, under the assumption that only a person behind the corner is moving. We will call these intensities $S(\phi, \theta)$ ("S" for the subject), where $\phi$ measures vertical inclination and $\theta$ measures azimuthal angle, relative to position where the vertical edge intersects the ground plane. Integrating the light intensities falling on the ground plane, the observed intensities on the ground will be $y(r, \theta)$, where the polar coordinates $r$ and $\theta$ are measured with respect to the corner. Assuming Lambertian diffuse reflection from the ground plane, we have

$$y(r, \theta) = \int_{\phi=0}^{\phi=\pi} \int_{\xi=0}^{\xi=\theta} \cos(\phi) S(\phi, \xi) d\phi d\xi,$$

(4.22)

where the $\cos(\phi)$ term in Eq. (4.22) follows from Eq. 4.2.

The dependence of the observation, $y$, on vertical variations in $S(\phi, \theta)$ is very weak, just through the $\cos(\phi)$ term. We can integrate over $\phi$ first, to form the 1-d signal, $x(\xi)$:

$$x(\xi) = \int_{\phi=0}^{\phi=\pi} \cos(\phi) S(\phi, \xi) d\phi$$

(4.23)

Then Eq. (4.22) has the form,

$$y(r, \theta) = \int_{\xi=0}^{\xi=\theta} x(\xi) d\xi,$$

(4.24)

where $x(\xi)$ is a 1-dimensional image of the scene around the corner from the vertical edge.

If we sample Eq. (4.24) in its continuous variables, we can write it in the form $y = Ax$. Solving Eq. (4.20) for the multiplier to apply to $y$ to estimate $x$ yields the form show in Fig. 4.15 (b). We see that the way to "read-out" the 1-d signal from the ground plane is to take a derivative with respect to angle. This makes intuitive sense, as the light intensities on the ground integrate all the light up to the angle of the vertical edge. To find the 1-d signal at the angle of the edge, we ask, "what does one pie-shaped ray from the wall see that the pie-shaped ray next to it doesn’t see?"

It can be shown [Bouman et al.2018] that the image intensities from around-the-corner scenes introduce a perturbation of about $\frac{1}{1000}$ to the light reflected from the ground from all sources. By averaging image intensities over the appropriate pie-shaped regions on the ground at the corner Fig. 4.15 (b), one can extract a 1-dimensional image as a function of time from the scene around the corner. Figure 4.16 shows two 1-d videos reconstructed from one (a) and two (b) people walking around the corner.

Figure 4.15: Corner camera geometry.
Figure 4.16: (a) Showing camera recording ground plane intensities. (b) Two people walking around the corner, hidden from direct view of the camera. (c) corner camera trace with one person moving (d) with two people moving. Angle from the corner is plotted horizontally, and time vertically.
Bibliography
